



**HAL**  
open science

# Activation Sites Estimation using a Fast Algorithm Based on an Eikonal Equation

Jérôme Fehrenbach, Lisl Weynans

► **To cite this version:**

Jérôme Fehrenbach, Lisl Weynans. Activation Sites Estimation using a Fast Algorithm Based on an Eikonal Equation. Computing in Cardiology 2022, Sep 2022, Tampere, Finland. hal-03913733

**HAL Id: hal-03913733**

**<https://inria.hal.science/hal-03913733>**

Submitted on 27 Dec 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Activation Sites Estimation using a Fast Algorithm Based on an Eikonal Equation

Jérôme Fehrenbach, Lisl Weynans -  
- Univ. Toulouse, IMT and Univ. Bordeaux, Inria, IHU Liryc, France



## Context and objectives

- ▶ Classical algorithms for ECGI solve at each time instant an static and extremely ill-posed inverse problem, requiring an adequate regularization strategy (or instance Tikhonov)
- ▶ An alternative way of regularizing the inverse problem is to **consider information from other time steps, taking into account that electric potentials in the heart are obtained by the propagation of an electric wave.**
- ▶ Our objectives: design a fast algorithm based on a simple activation front model to retrieve significant features of the ECGI inverse problem

## Propagation model

- ▶ We consider in the present work only the surface of the heart
- ▶ **Anisotropic eikonal equation:** simplified model of electrical activation front in asymptotic regime:

$$\begin{cases} \|\nabla T\|_{D(x)}^2 = 1, \\ T(x_i^0) = \tau_i, i = 1 \dots s, \end{cases}$$

with

- ▶  $T$  depolarization time of heart surface,
- ▶  $D(x)$  tensor quantity describing the anisotropic conduction
- ▶  $\tau_i$  activation times at sources  $x_i^0$  (earliest activation sites)  
⇒ parameters relevant in the context of ECGI
- ▶  $T(x)$  is also the length of the shortest path joining the sources to  $x$

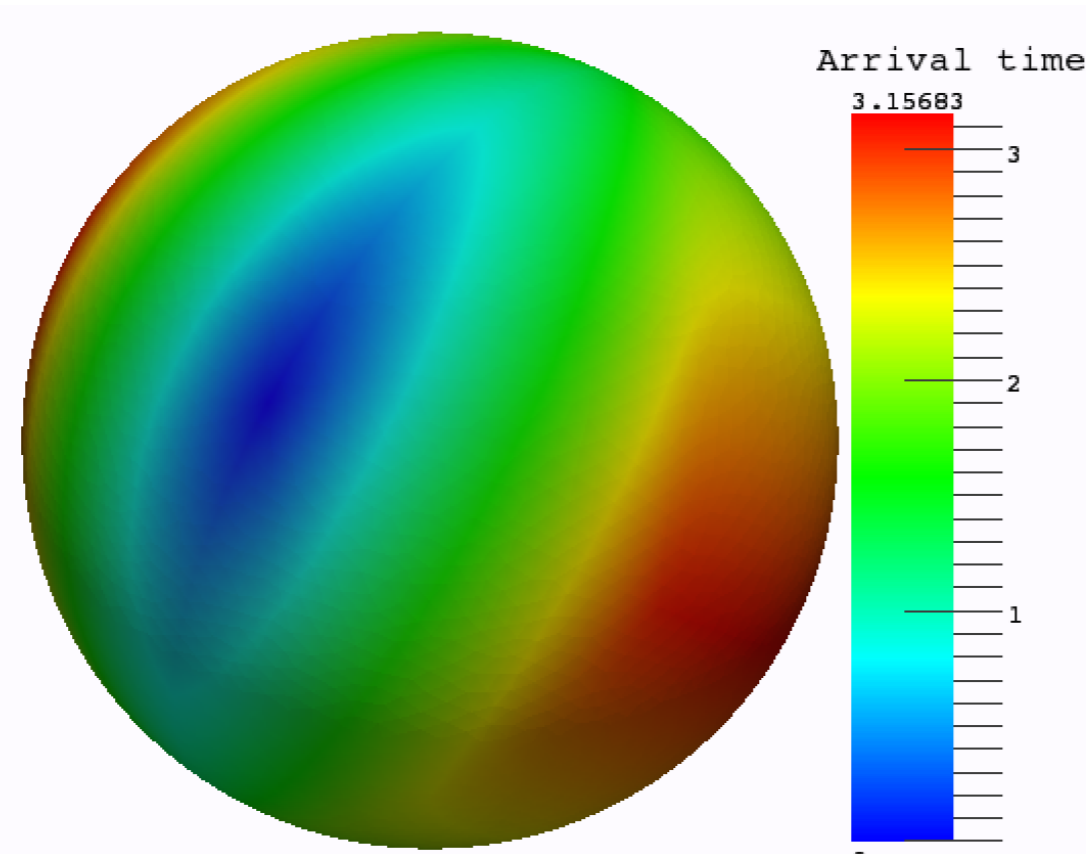


Figure: Example of solution of eikonal equation with anisotropic conduction

## Methods

- ▶ Iterative algorithm to minimize misfit between observations and predictions of the model

### Cost function

$$J(x, \tau) = \frac{1}{2} \|G(\tau + \phi_x) - g^{\text{OBS}}\|_Y^2.$$

with  $G$  observation operator (on heart or torso surface),  $x$  sources location,  $g^{\text{OBS}}$  observations with noise

- ▶ Least square minimization with Gauss Newton algorithm
- ▶ Sensitivity of cost function with respect to source locations and activation times depends on **Logarithmic map**:

if  $\gamma$  is a unique shortest geodesic path from point  $x$  to point  $y$  then

$$\text{Log}_x(y) = \gamma'(0) = \text{initial direction of } \gamma$$

- ▶ **Vector Heat Method:** efficient computational method to compute Logarithmic map, avoiding to compute all geodesics

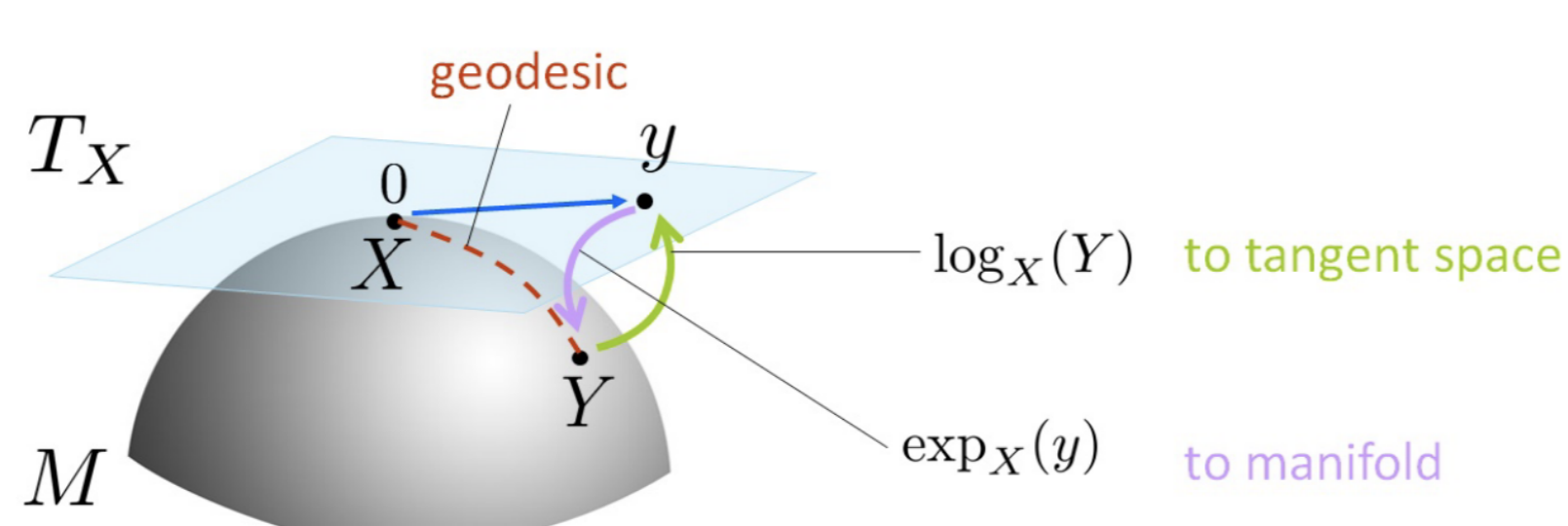


Figure: Illustration of Log map from et al

## Test case: Localization of 2 sources and metric estimation

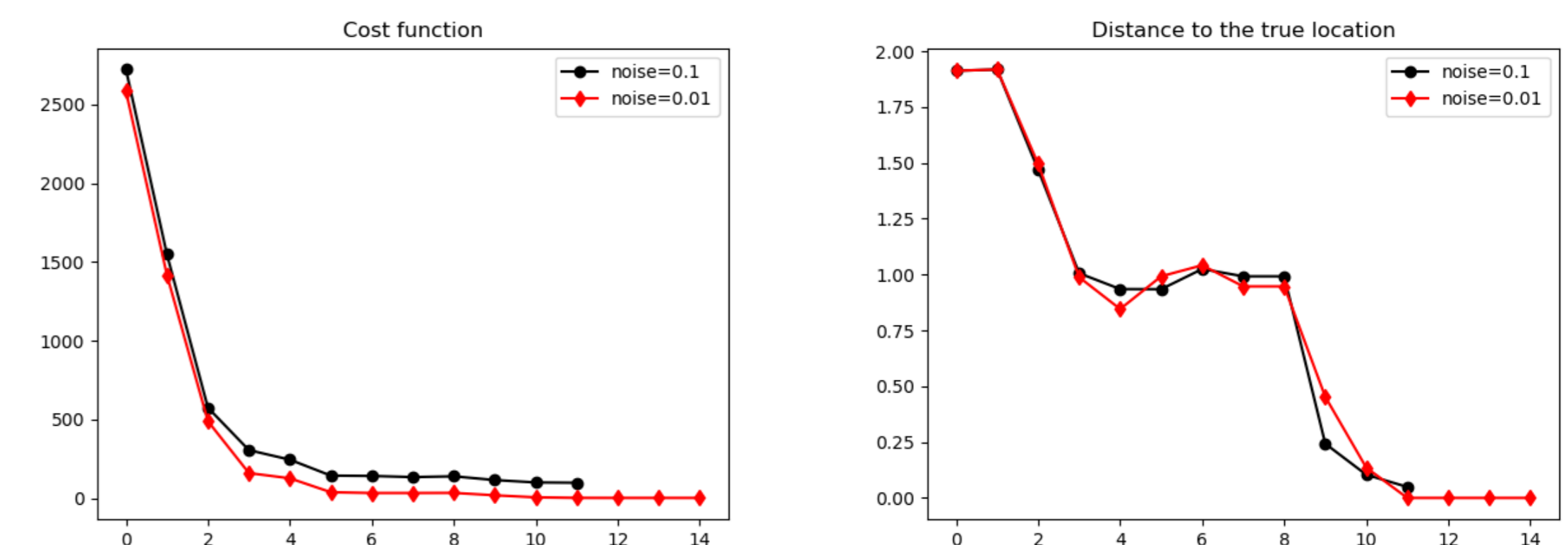


Figure: Left: evolution of the cost function  $J$ . Right: evolution of the distance to the solution  $\|x^k - x^*\|$ .

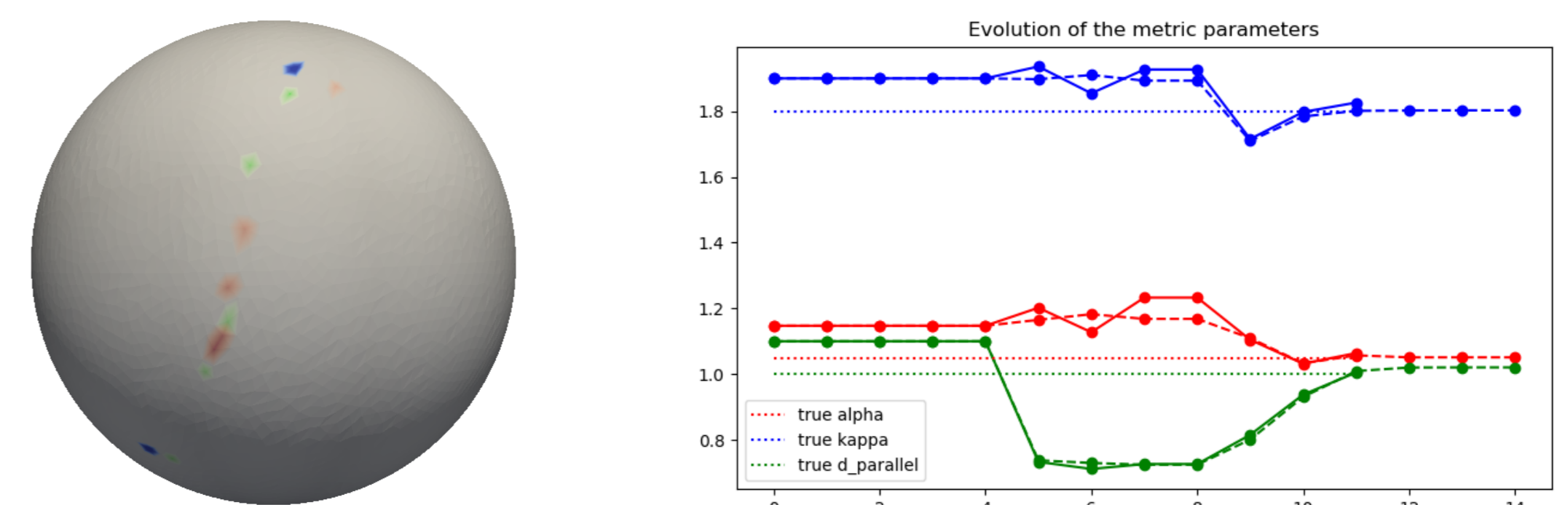


Figure: Left: location of the successive iterates  $x^k$  (red) and the true source point  $x^*$  (blue). Right: evolution of the parameters of the metric.

## Test case: mimicking an ECG inversion

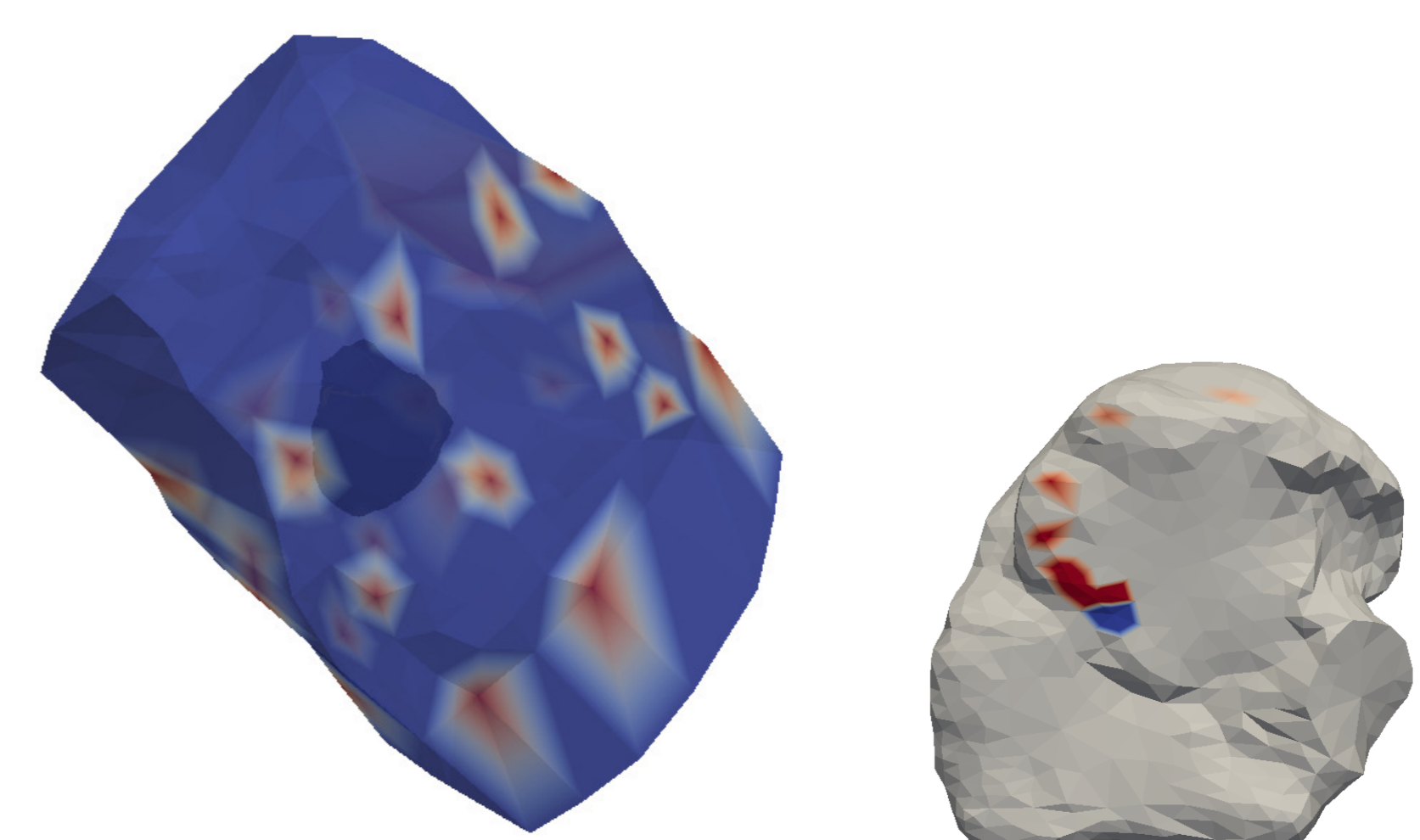


Figure: Left: configuration of the torso (electrodes in red) and the heart surface (visible by transparency). Right: location of the successive iterates  $x^k$  (red) and the true source point  $x^*$  (blue)

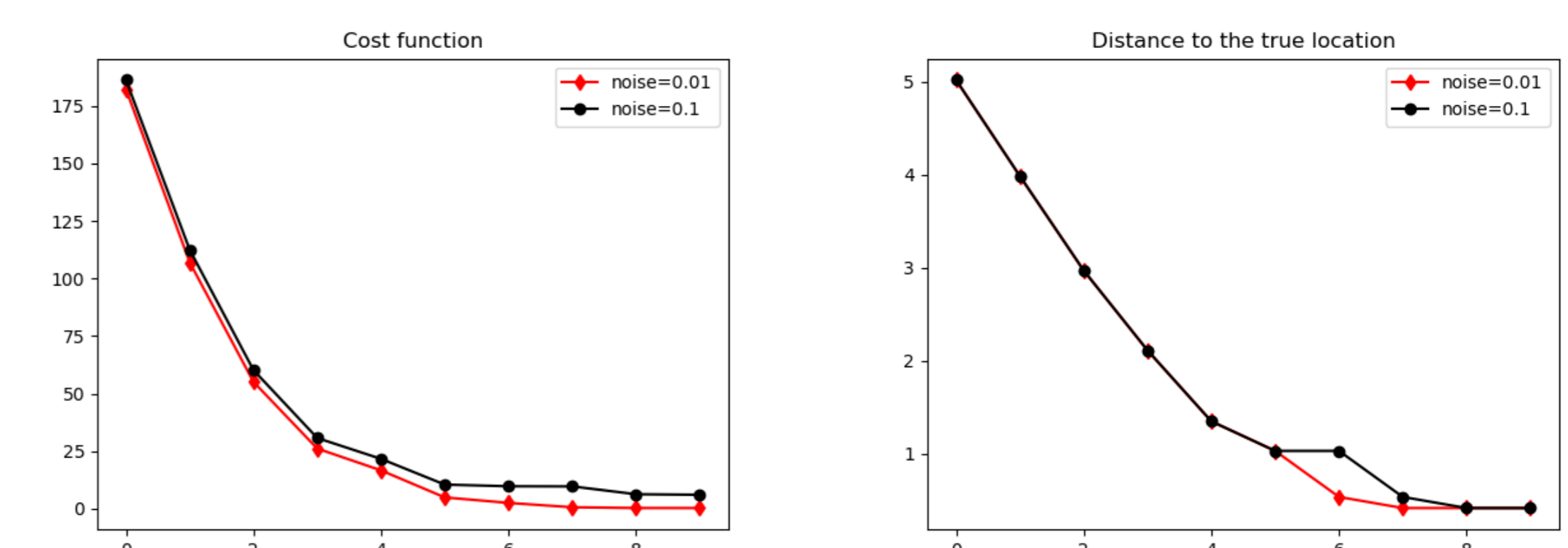


Figure: Left: evolution of the cost function  $J$ . Right: evolution of the distance to the solution  $\|x^k - x^*\|$ .

## Conclusion

- ▶ Efficient new method to estimate sources and activation times in the eikonal equation on a surface ⇒ applicable to ECGI.
- ▶ In case of complex or pathological propagation patterns, need to extend the method to a cardiac volume.

## References

- [1] J. Fehrenbach, L. Weynans, Source and metric estimation in the eikonal equation using optimization on a manifold, under revision
- [2] Grandits et al, GEASI: Geodesic-based Earliest Activation Sites Identification in cardiac models, *Int. J. Numer. Meth. Biomed Engng*, 2021
- [3] N. Sharp et al, The vector heat method. *ACM Transactions on Graphics*, 2019.