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FEM-based Nonlinear Controller for a Soft Trunk Robot

Ke Wu¹ and Gang Zheng²

Abstract—Soft robots have been attracting much research attention in the past decades owing to their unique properties that are inherently different from conventional rigid robots. The intrinsic characteristics of soft robots, such as compliance and flexibility, have made them a promising supplement for many existing robotic applications. For instance, soft robots provide safer human-machine interaction and better adaptability to unexplored environments than the conventional rigid ones. However, their “soft” feature would conversely cause complex material deformation due to their high-DoF nature, which has made their kinematic or dynamic models more difficult to deduce. Logically, the process of designing model-based controllers would therefore suffer from the lack of efficient and accurate modeling methods. In this paper, through adopting Finite Element Method (FEM), we demonstrate a FEM-based nonlinear control method for a soft trunk robot to achieve trajectory tracking objectives. Basically, the proposed controller is developed by utilizing the essential information from the dynamic FE model of the studied robot. The details of deriving this nonlinear control method are demonstrated, then followed by 4 practical experiments on the soft truck robot to verify the feasibility of the control method. The corresponding experimental results are presented accordingly.

I. INTRODUCTION

Soft robots, distinguished from conventional rigid robots, have nowadays displayed many promising properties over conventional rigid robots, gradually bringing about thriving development in different industries such as manufacturing and specialized robotics [1]. They are inherently flexible, compliant and lighter than conventional rigid robots, empowering themselves to change their deformation to transform into a more suitable configuration for a given task or an unfamiliar work environment [2]. Due to the mentioned advantageous features, these soft robots can achieve safe interaction with operators and environments [3], [4], and they can also be powered by remote actuation with simple structure [5]. Obviously, these advantageous attributes have revealed the potential of soft robots to be integrated into many more industrial applications.

However, the beneficial properties due to their compliant nature can also lead to difficulties in modeling. The soft robot’s complex material deformation results in nearly infinite degrees of freedom (DoF) and is essentially a high-dimensional nonlinear system which is problematic to handle. The large number of designs including geometry and actuation have made every single soft robot unique, making it difficult to find a universal method to model them. From

the perspective of controller design, the nonlinear hysteresis of these robots owing to compliance and flexibility also limits high-frequency control [7].

To realize fast, reliable, accurate and energy-efficient control, many representative studies have been completed which can be mainly categorized into two types: model-based controllers relying on analytical models and model-free controllers based on machine learning techniques or empirical methods.

One of the most notable studies is a modified Denavit Hartenberg method called Constant Curvature (CC) assumption proposed in [8] for mapping its configuration space to task space, which has been adopted in many static model-based controllers. In terms of multi-section continuum robots, the Piece-wise Constant Curvature (PCC) model logically becomes a recommended model partially owing to the success of CC model, which has as well been validated in many cases [9]. Apart from the above mentioned, some more complex modeling approaches adopting beam theory [10] and Cosserat rod theory [11] were investigated for soft/continuum robots as well. Logically, dynamic controllers cannot be developed purely based on the mentioned static models as they are not able to capture the dynamics of soft robots. In the current literature, the studies about developing dynamic model-based controllers for soft robots are rather limited. The authors of [12] have completed one of the first remarkable contribution regarding dynamic control of a planar multi-section continuum robot where a special exponential trajectory tracking has been achieved. Under the framework of PCC assumption [9], the contributors of [13] achieved dynamically control of a pneumatic robot in a closed-loop manner where they used an approximated dynamic model to design the dynamic controller. It is also worth noting that some representative studies on developing the kinematics and dynamics of pneumatically actuated continuum robots have been completed via model shape functions (MSF) [14], [15]. Machine learning or empirical methods have been playing a gradually important role in rigid robots [16]. Likewise, these data-dependent methods have also been adopted for soft robotics. Representatively, the authors of [17] took the lead in developing a static model-free controller, which can correctly predict the desired cable tensions to reach a target in simulations. Different from model-based methods, model-free approaches can bypass the need of obtaining information from the robot itself, but depends on a large number of training data, performing well in systems that are highly nonlinear and complicated, easily influenced by gravity [18], or operate nicely in unexplored environments where modeling is barely possible [19]. However, due to

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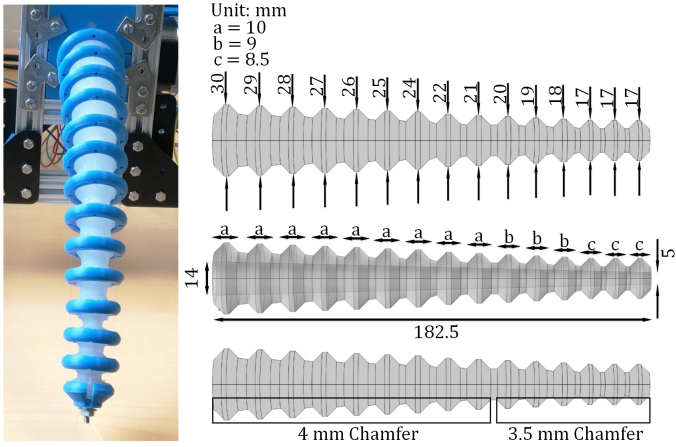


Fig. 1: The studied silicone-made trunk robot [24]

the uncertain nature of model-free approaches, stability and convergence analysis is challenging.

In this paper, Finite Element Method (FEM) [20] is chosen to model soft robots. The mentioned popular models are quite restricted by the geometry of soft robots and the models' quasi-static assumption. Notably, FEM has gradually turned out to be one of the desired approaches of modeling and developing controllers for soft robots [21] [22] [23] [24] as FEM does not require soft robots to have certain types of geometries, structures and quasi-static assumptions, serving as quite a universal strategy for modeling soft robots.

In our previous work [24], we proposed a gain-scheduling control method in local manner where the gains of the linearized controller are scheduled from one sub-workspace to another to achieve operation within the whole workspace. Besides, only path following is concerned without stressing the dynamics or dynamic control that much in the process. However in this paper, as an extension of the former work, we propose to conduct FEM-based nonlinear dynamic control of a soft trunk robot in a global manner within its workspace to track trajectories as well as achieving more globally spatial and dynamic aims. The proposed method has been proved to be valid for enabling the robot to track the trajectory even under various disturbances, showing great robustness. The theoretical derivation and the experimental testing on a soft trunk-like robot are presented accordingly.

II. PROBLEM STATEMENT

A. Investigated soft manipulator

This paper aims to study the control of a silicone-made trunk robot with its exact geometry shown in Fig.1. Particularly, to place a position sensor at the robot's end effector, we made a long and uniform hole along the robot's body from its base to its tip and mounted 4 inextensible cables for tendon-driven actuation. 15 3D-printed rigid rings are mounted along its body (see Fig. 1) to reduce the friction between the cables and the soft silicone. The material we used for shaping the robot is isotropic silicone rubber with

the following material properties:

Young's modulus: $E = 6.5 \times 10^6$ Pa

Poisson ratio: $\nu = 0.45$

Mass: $m = 0.045$ kg

B. Control objective

The main work of this paper aims to control the end-effector of the trunk-like soft robot (see Fig. 1) to relatively quickly and stably track a given desired trajectory with (or without) the interfere of various bounded disturbances.

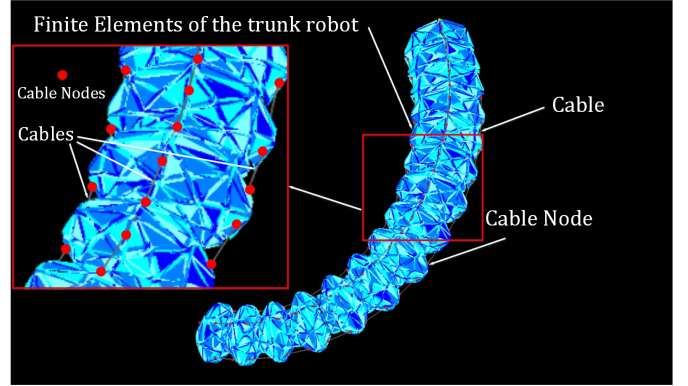


Fig. 2: The mesh elements of the trunk robot [24]

III. MODELING

As is known to all, it is always difficult to dynamically or statically model soft robots since they are able to present unexpected irregular deformation due to the strong flexibility and compliance [25]. Here, FEM is utilized to derive the dynamics of the studied soft trunk-like robot. Via synthesizing its model information from a FEM simulator (SOFA Framework), we can design a FEM-based dynamic controller for this robot.

In terms of deriving the FE dynamical model of the robot, readers are referred to [20] and [26]. Essentially speaking, the procedure is similar to the classic FEM. First, the number and type of mesh elements need to be chosen to discretize the geometry of the modeled deformable robot (demonstrated in Fig. 2). By doing this, the displacement of any arbitrary point within mesh elements can be determined through shape functions, as well as the corresponding strain and stress. Finally, Euler-Lagrange equations are utilized to derive the FE dynamical model (Eq. 1) of the studied soft robot.

$$M(q)\ddot{q} + D(\dot{q}, q)\dot{q} + K(q)q = H(q)^T u \quad (1)$$

where $M(q) \in \mathbb{R}^{3n \times 3n}$ stands for mass matrix; $D(\dot{q}, q) \in \mathbb{R}^{3n \times 3n}$ stands for damping matrix; $K(q) \in \mathbb{R}^{3n \times 3n}$ stands for stiffness matrix; $H(q) \in \mathbb{R}^{4 \times 3n}$ stands for the input control matrix; $q \in \mathbb{R}^{3n \times 1}$, $\dot{q} \in \mathbb{R}^{3n \times 1}$ and $\ddot{q} \in \mathbb{R}^{3n \times 1}$ stand for the displacement, velocity and acceleration of all the n mesh nodes respectively in the robot's FE model; $u \in \mathbb{R}^{4 \times 1}$ is an input vector that contains the four tension forces exerted at the four cables respectively.

Here, the function of $H(q)$ is to only distribute u to all the elements containing the cable nodes (the red nodes in Fig. 2) since the friction between the cables and trunk robot is neglected in the derived FE model. $M(q)$ is the corresponding mass matrix at q and can be reduced as a constant diagonal matrix due to its inherent seldom varying characteristics so that it will be replaced by M in the following sections for simplicity.

Remark 1: Using the above model (1), the hysteresis and other nonlinear effects can be also modeled by $K(q)$, the dynamic friction could be modeled by $D(\dot{q}, q)$ and the static friction could be considered as a matched external disturbance f added to the channel u (i.e., replace u in (1) by $u + f$). Logically, the accuracy of the developed model plays a very important role in simulation and open-loop control. However, we would like to emphasize that it does not need a rather accurate model to achieve the control objective as long as the controller has access to essential model information, since those nonlinear effects might be eliminated or attenuated via robust controllers. We will discuss this issue in Section IV.

In the robot's FE model, tetrahedron mesh elements are chosen and the robot's body is spatially discretized into $n = 1484$ nodes. Logically speaking, the model will have a larger number of mesh elements if finer mesh elements are chosen to discretize the robot, which leads to more accurate results but more computational time. The FE model of the robot is demonstrated in Fig. 2. Note that the dynamic position control of the robot's end effector (noted as $q_e \in \mathbb{R}^{3 \times 1}$) is our main focus. Therefore, under the framework of the derived FE model, this problem can be transformed into an output regulation problem of the following nonlinear dynamic system:

$$\begin{aligned} M(q)\ddot{q} + D(\dot{q}, q)\dot{q} + K(q)q &= H(q)^T u \\ q_e &= Cq \end{aligned} \quad (2)$$

where $C \in \mathbb{R}^{3 \times 3n}$ is used to pick out the end-effector node from all $n = 1484$ nodes defined in the FE model. Logically, we can easily define the workspace of the robot's end effector as follow:

$$W_e = \{q_e \in \mathbb{R}^{3 \times 1} \mid \dot{q} = \ddot{q} = 0, q_e = Cq\} \quad (3)$$

constraint by (2). Identically, it can also be formulated as:

$$W_e = \{q_e \in \mathbb{R}^{3 \times 1} \mid q_e = Cq, K(q)q = H(q)^T u, \forall u \in U\} \quad (4)$$

where U denotes the set of all admissible inputs.

Given two points inside W_e , noted as the initial position $q_e^{init} \in W_e$ and the final position $q_e^{fin} \in W_e$, there exist different techniques in the literature, such as differential flatness [27], interpolation via polynomials [28], to generate smooth trajectories. Therefore, this paper will not present the details on how to plan a feasible end-effector trajectory $q_e^r(t)$ in W_e , but just assumes that there exists such a smooth feasible end-effector trajectory $q_e^r(t)$ over the pre-defined time interval $[0, T_f]$ such that $q_e^r(0) = q_e^{init}$ and $q_e^r(T_f) = q_e^{fin}$. The objective of this paper is then to design

a robust controller u such that $q_e(t)$ converges to $q_e^r(t)$ in the presence of external disturbances.

IV. NONLINEAR CONTROLLER DESIGN

As is stated above, the control problem of the robot's end-effector is actually a problem of output regulation, as described in (2). Therefore, we can re-formulated it into the following projected space:

$$\ddot{q}_e + \alpha(q, \dot{q}) = CM^{-1}H(q)^T u \quad (5)$$

with

$$\alpha(q, \dot{q}) = CM^{-1}D(\dot{q}, q)\dot{q} + CM^{-1}K(q)q$$

With the planned smooth feasible end-effector trajectory $q_e^r(t)$ over $[0, T]$, let us define the following tracking error:

$$e(t) = q_e^r(t) - q_e(t) \quad (6)$$

where $q_e^r(t)$ stands for the given desired end-effector trajectory and $q_e(t)$ represents the measurable end-effector position.

With the introduced notations, (6) can be rewritten as:

$$\ddot{e} = \alpha - CM^{-1}H(q)^T u + \ddot{q}_e^r \quad (7)$$

In order to achieve the convergence of e to 0, the following nonlinear controller is proposed:

$$u = \Delta u + u_o \quad (8)$$

where u_o is the initial control input for the soft robot at the rest configuration and

$$\begin{aligned} \Delta u &= (CM^{-1}H(q)^T)^{-1} \left(\ddot{q}_e^r(t) + \int_0^t (K_1 \ddot{e}(s) \right. \\ &\quad \left. + K_2 \dot{e}(s) + K_3 e(s) + K_4 \int_0^s e(\tau) d\tau) ds \right) \end{aligned} \quad (9)$$

where K_1 , K_2 , K_3 and K_4 are the controller gains to be designed.

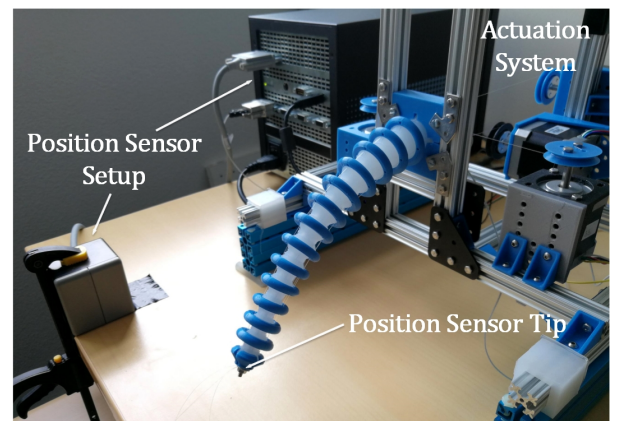


Fig. 3: Overall experimental setup

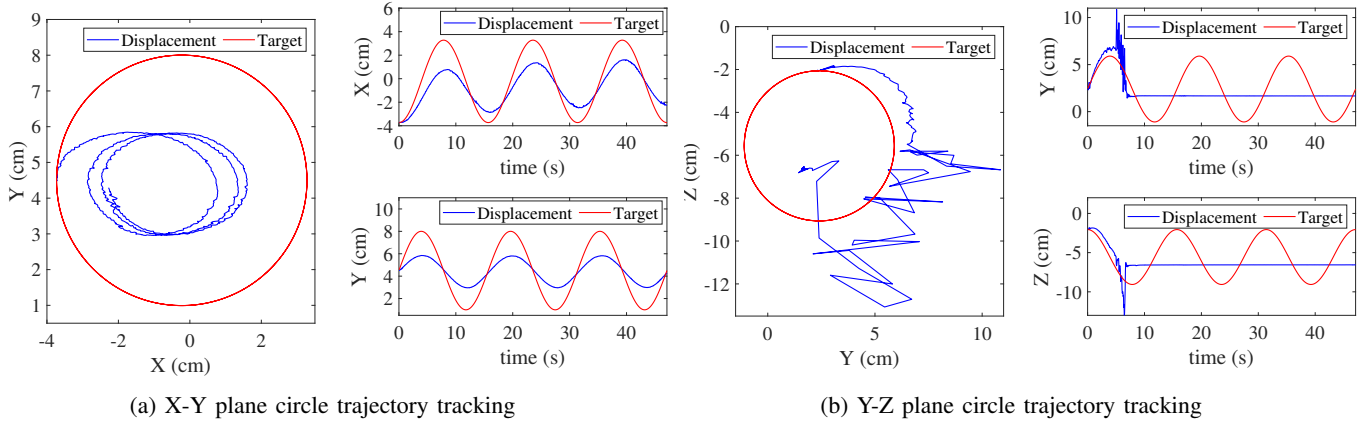


Fig. 4: Trajectory tracking using a common PID controller

Inserting the above nonlinear controller into (7), we can finally obtain:

$$\ddot{e}(t) = \alpha - \int_0^t (K_1 \ddot{e}(s) + K_2 \dot{e}(s) + K_3 e(s) + K_4 \int_0^s e(\tau) d\tau) ds - CM^{-1} H^T(q) u_o$$

Now, let us note

$$\begin{aligned} \alpha^*(t) &= \alpha - CM^{-1} H^T u_o \\ &= CM^{-1}(q) [D(\dot{q}, q) \dot{q} + K(q) q - H^T(q) u_o] \end{aligned} \quad (10)$$

and regard $\alpha^*(t)$ as a bounded disturbance. Then, the above equation becomes

$$\ddot{e} + \int_0^t \left(K_1 \ddot{e} + K_2 \dot{e} + K_3 e + K_4 \int_0^s e(\tau) d\tau \right) ds = \alpha^*(t) \quad (11)$$

which is equivalent to:

$$e^{(4)} + K_1 e^{(3)} + K_2 e^{(2)} + K_3 e^{(1)} + K_4 e = \ddot{\alpha}^*(t) \quad (12)$$

where $e^{(i)}$ means the i -th derivative of e with respect to time. It is worth noting that (12) is a classical 4-order linear system with a bounded disturbance $\ddot{\alpha}^*(t)$, therefore we can always find matrices K_1 , K_2 , K_3 and K_4 to guarantee that the tracking error $e(t)$ will converge to a neighborhood of 0 whose size depends on the magnitude of $\ddot{\alpha}^*(t)$ [29].

The above analysis shows that the performance of the proposed controller (8) depends on the characteristics of $\alpha^*(t)$. It is clear that if $\alpha^*(t)$ contains high frequency, then the magnitude of $\ddot{\alpha}^*(t)$ will become larger than that of $\alpha^*(t)$. Due to the same reason, the magnitude of $\ddot{\alpha}^*(t)$ might be smaller if the highest frequency of $\alpha^*(t)$ is less than $\frac{1}{2\pi}$. Moreover, the smaller $\ddot{\alpha}^*(t)$ is, the closer e converges to 0, i.e., $q_e(t)$ will converge to a smaller neighborhood of $q_e^r(t)$. Consequently, in order to get a satisfactory performance for the proposed controller (8), the planned trajectory should not vary too quickly. In practice, such a constraint can be easily satisfied by choosing a relatively larger value of T_f , i.e., the time interval of the planned trajectory.

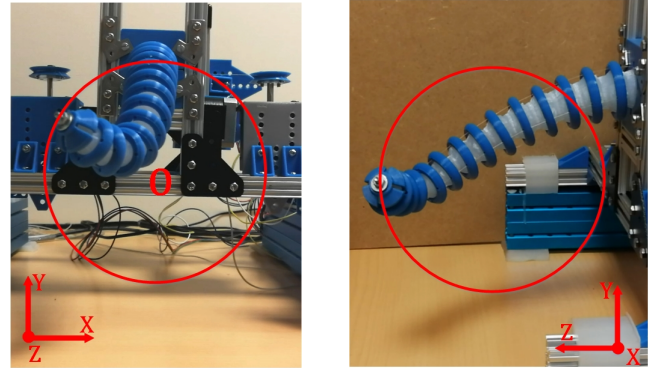
Remark 2: When considering nonlinear effects and frictions discussed in Remark 1, according to (10), it is clear that

those issues are included into $\alpha^*(t)$. Therefore, the controller proposed in this paper is robust even if we have no precise model of those nonlinearities. The same assertion is true when the measurement is corrupted with noise. To show this, let us model the output of sensor as $q_e = Cq + \epsilon(t)$ where $\epsilon(t)$ represents the noise in measurement channel. Following the similar procedure, we will end up with:

$$e^{(4)} + K_1 e^{(3)} + K_2 e^{(2)} + K_3 e^{(1)} + K_4 e = \ddot{\alpha}^*(t) - \epsilon(t)$$

Again, by regarding $\bar{\alpha} = \alpha^*(t) - \epsilon(t)$, we can draw the same robustness conclusion, i.e., if $\bar{\alpha}$ is bounded, then $e(t)$ will converge to the neighborhood of 0.

The above discusses how to design the nonlinear controller for the trunk robot, the next section presents several experimental tests regarding trajectory tracking.



(a) X-Y plane circle trajectory (b) Y-Z plane circle trajectory

Fig. 5: Circle trajectory tracking

V. EXPERIMENTAL TESTING

The overall experimental setup of the testing platform is shown in Fig. 3, mainly including the mechanical setup, the Polhemus magnetic position sensor (which is placed at the end effector through the long internal hole of the studied robot as described in Fig. 1) and the actuation system.

In our work, a front circle trajectory (Fig. 5a) and a side circle trajectory (Fig. 5b) are selected for validation

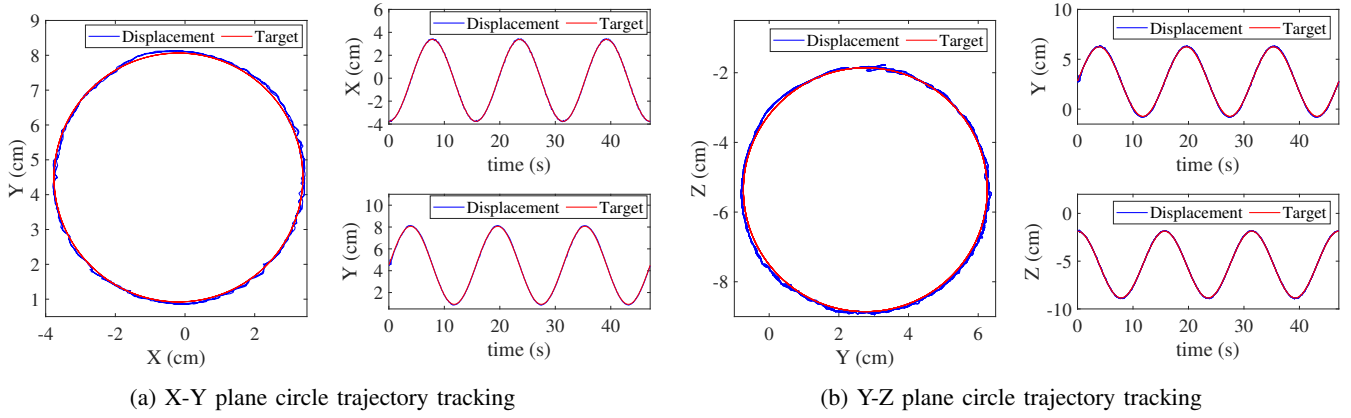


Fig. 6: Trajectory tracking using the proposed nonlinear controller

where a common PID controller and our proposed nonlinear controller are compared. Besides, the robustness (against temporary and permanent disturbances) of the proposed controller has also been experimentally proved. In the current literature, the experimental validation for most trunk-like robots is quite limited to front trajectories whereas we have added the side circle trajectory (Fig. 5b) in a highly nonlinear zone to further prove the validity of our proposed controller.

A. Trajectory tracking tests

1) *Trajectory tracking using a common PID controller:* In this test, the trunk robot was expected to track two different circle trajectories in X-Y plane and Y-Z plane respectively (see Fig. 5a and Fig. 5b) using a common PID controller.

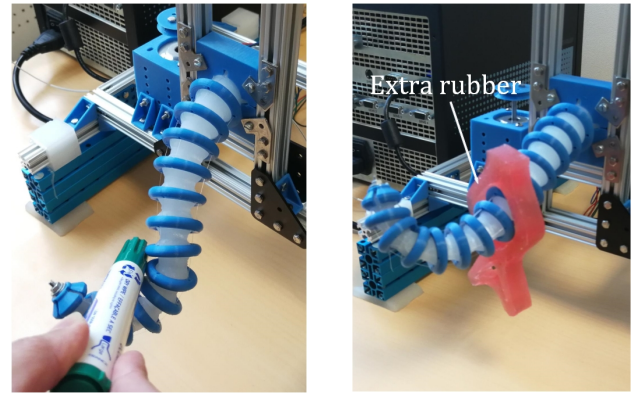
It is well-known that tuning criteria for PID exists for SISO (single-input-single-output) systems. However, it is generally difficult, and there does not exist universal methods to tune those paramters when dealing with MIMO (multi-input-multi-output) systems. In this paper, we adopted our former work [24] to tune PID controller.

The results are demonstrated in Fig. 4. As shown in Fig. 4a, the end-effector tended to track the X-Y plane circle trajectory but was not able to do so. This is because the common PID cannot effectively deal the hysteresis when the tracked trajectory is time-varying (in this case, the average speed of the trajectory is around 1.5 cm/s). Fig. 4b indicates that the robot was out of control when tracking the Y-Z plane circle trajectory, and the common PID is invalid in this area of the workspace.

2) *Trajectory tracking using the proposed nonlinear controller:* In this test, we used the proposed nonlinear controller to control the robot, and the trunk end-effector was expected to track the two mentioned circle trajectories (Fig. 5). The corresponding results are displayed in Fig. 6a and Fig. 6b. It can be easily noticed that the proposed nonlinear controller enabled the robot's end-effector to track both X-Y plane circle trajectory and Y-Z plane circle trajectory with almost no hysteresis. This means the proposed nonlinear controller can more rapidly react to real-time trajectory error, presenting dominant advantages over the common PID controller.

B. Robustness Analysis

To test the robustness of the proposed controller, two more trajectory tracking tests were conducted under two different scenarios (temporary disturbance and permanent disturbance) shown in Fig. 7.



(a) Temporary disturbance by touching the robot with a pen

(b) Permanent disturbance by adding an extra piece of rubber on the robot.

Fig. 7: External disturbance tests

1) *Temporary disturbance:* In this test, the trunk robot was expected to track the same X-Y plane circle trajectory (Fig. 5a) under temporary disturbances exerted by a pen (Fig. 7a). During the testing, the robot was interfered by 7 temporary disturbances when tracking the trajectory. According to the results in Fig. 8, the end-effector quickly reacted to the disturbances and caught up with the trajectory, which presents great robustness of the proposed controller against temporary disturbances.

2) *Permanent disturbance:* In this test, the trunk robot was expected to track the X-Y circle trajectory while carrying an extra piece of 0.033 kg rubber (almost the same weight as the trunk robot, see Fig. 7b). According to the results demonstrated in Fig. 9, the robot quickly caught up with the trajectory at the beginning of the trajectory, and then stably tracked the circle trajectory, which therefore proved the robustness of the controller against permanent disturbances.

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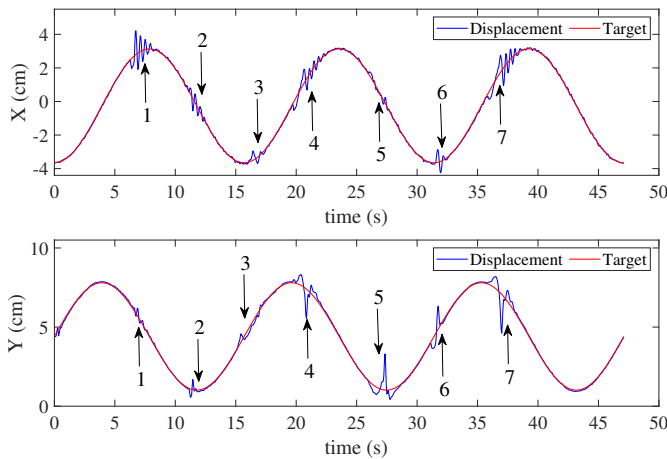


Fig. 8: Temporary disturbance results

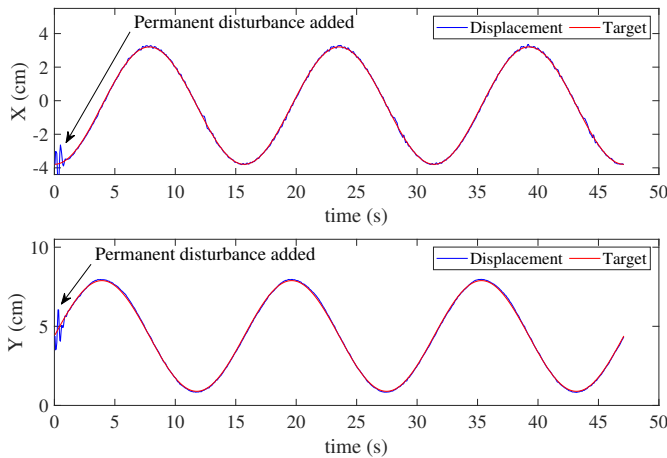


Fig. 9: Permanent disturbance results

VI. CONCLUSIONS

Soft robotics has attracted much research attention, presenting great potential as well in a wide variety of practical applications due to its unique characteristics compared to conventional rigid robots. Nevertheless, we still need to face many tough challenges regarding dynamically controlling them nowadays. In this paper, we propose a nonlinear controller for a soft trunk robot to track user-defined global trajectories. The feasibility has been proved via 4 different tests, and the robustness of the proposed method has been illustrated by rejecting the temporary and permanent external disturbances. However, as stated previously, the proposed controller also has some limitations, for example, the planned trajectory should not vary too rapidly. Therefore, in our future work, we will propose to employ a new control strategy to deal with control objectives regarding faster trajectory tracking.

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