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An imperative programming language characterizing FBQP
Emmanuel Hainry, Romain Péchoux, Mário Silva
Université de Lorraine, CNRS, Inria, LORIA, F-54000 Nancy, France

Introduction

Quantum programming languages are a useful tool, not only for writing complex algorithms, but also for abstracting and reasoning about their properties and to learn more about what can be done efficiently by quantum computers [1].

However, merely being able to describe a quantum program does not inform us on its complexity, meaning that it may not ultimately be efficiently run on a quantum computer. Consequently, there is a need for static analysis tools and techniques for reasoning about and certifying the complexity of these quantum algorithms.

We introduce an imperative programming language called QPT (Quantum Poly-Time) with recursion rules that captures the bounded-error class FBQP (Functions Bounded-error Quantum Polytime), the class of functions computable in polynomial time by a quantum Turing machine with at most 1/3 probability of error, commonly accepted as the class of feasible problems for quantum computers. Our result takes advantage of a function algebra proposed by Yamakami [2] characterizing FBQP.

Rule for termination (R1)

Let proc1 ∼ proc2 mean that proc1 and proc2 are mutually recursive procedures. The following condition ensures that a program with procedure declarations D will always terminate:

∀ Proc proc1, proc2 ∈ D, 
∀ call proc1(σ) ∈ Sn, 
proc1 ∼ proc2 ⇒ σ is a proper subset of Php

i.e., any call to a mutually recursive function must strictly decrease the amount of qubits available.

Rule for poly-time (R2)

We prevent an exponential number of procedure calls by limiting the number of recursive calls in each branch of a Case. Let RCalls(·) represent the maximum number of recursive calls for any branch, then

∀ Proc proc1, proc2 ∈ D, RCalls(proc1) ≤ 1,

i.e., multiple recursive calls can be done only on different branches of the Case structure.

Example: QPT

The quantum Fourier transform (QFT) is in FBQP, appearing as a subroutine of Shor’s algorithm. It contains two recursive patterns, following rules R1 and R2, highlighted in the circuit of Figure 2.

QPT ∼ FBQP

Soundness: For any program P in QPT following rules R1 and R2, there exists a poly-sized uniform family of circuits (Cn)n∈N for each input size n that simulates P.

Completeness: For any function f ∈ FBQP with size-bounding polynomial p, and any constant ε ∈ [0, 1/2), there exists a program P in QPT following rules R1 and R2 such that, running P on polynomially extended input state |ψ⟩, for x ∈ {0, 1}n, a measurement of the first |f(x)| of the output qubits will result in f(x) with probability at least 1 − ε.

Building poly-sized circuits

Dealing with procedures that include recursive branching, a straightforward approach to building the circuit will readily require an exponential number of gates, e.g. in the following procedure:

Proc f(ψ) { 
if |ψ| > 1 : 
Case C(|ψ|) = 0 then 
call f(remove(ψ, 1)) 
else Case C(|ψ|) = 1 then 
call f(remove(ψ, 2)) 
else ψ|σ⟩ = U : 
call f(σ) 
}

As a proof of Soundness, we provide an algorithm to build all programs following rule R2 with a polynomially large set of gates and wires, such as in the example of Figure 1 for the above function applied on an input q of size n = 5.

References

[1] Peter Selinger.
Towards a quantum programming language.

A schematic definition of quantum polynomial time computability.