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On the use of the "Augmented" autocovariance matrix for the classification of BCI-EEG

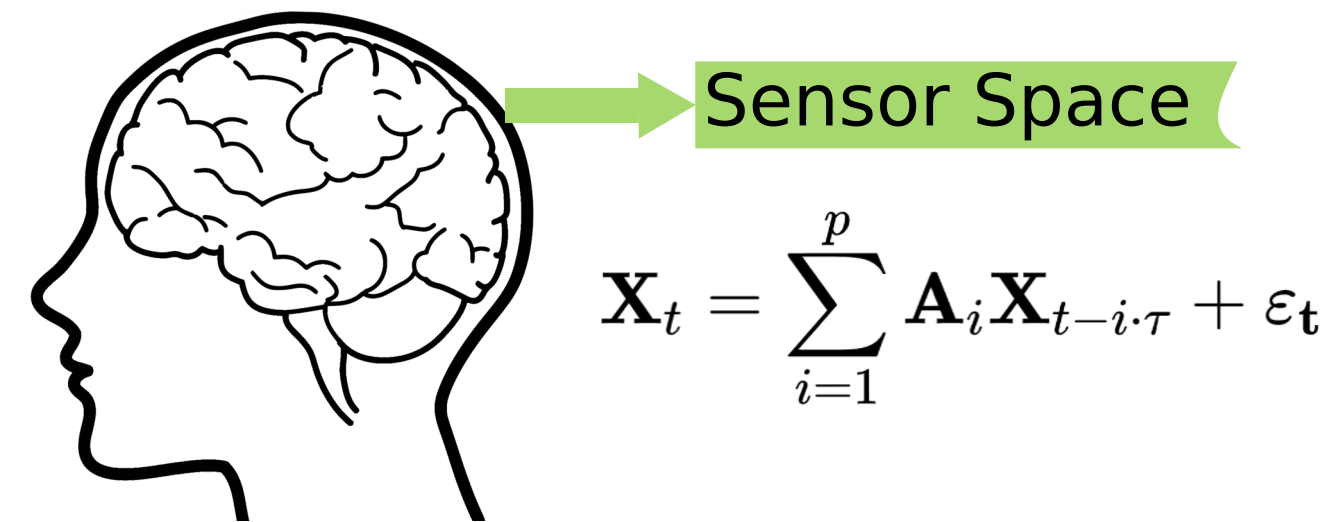
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ABSTRACT

Electroencephalogram (EEG) signal is recorded as a multidimensional dataset, which can be interpreted using autoregressive (AR) models [1]. In this way, it is possible to extract relevant information about the signal, which can be used for Brain Computer Interface (BCI) classification. Yule-Walker equations can be derived from the AR equation, which yields the autocovariance matrix that is a symmetric positive definite matrix (SPD) which is classified using Riemann geometry approach [3]. We can also think of formulating the problem from another point of view: since the autocovariance matrix is a matrix of delayed covariances, we can obtain the same result by creating an embedding of the original system in a high dimensional space. Thus, it seems natural to connect our approach with the delay embedding theorem proposed by Takens [4] in the context of dynamical systems. Such an embedding method depends on two parameters: the delay parameter τ and the embedding dimension D , respectively the lag and the order in the context of AR model. We use grid search to find various parameters of the compared approaches. We test our approach on several datasets and compare it to the state of the art.

METHODS

1 Autoregressive Model



Derive the Yule Walker equations

$$\Gamma(0) = \sum_{k=1}^p \mathbf{A}_k \Gamma(-k \cdot \tau) + \mathbf{U} \text{ for } i = 0$$

$$\Gamma(i) = \sum_{k=1}^p \mathbf{A}_k \Gamma(i - k \cdot \tau) \text{ for } i \neq 0$$

2 Takens Theorem

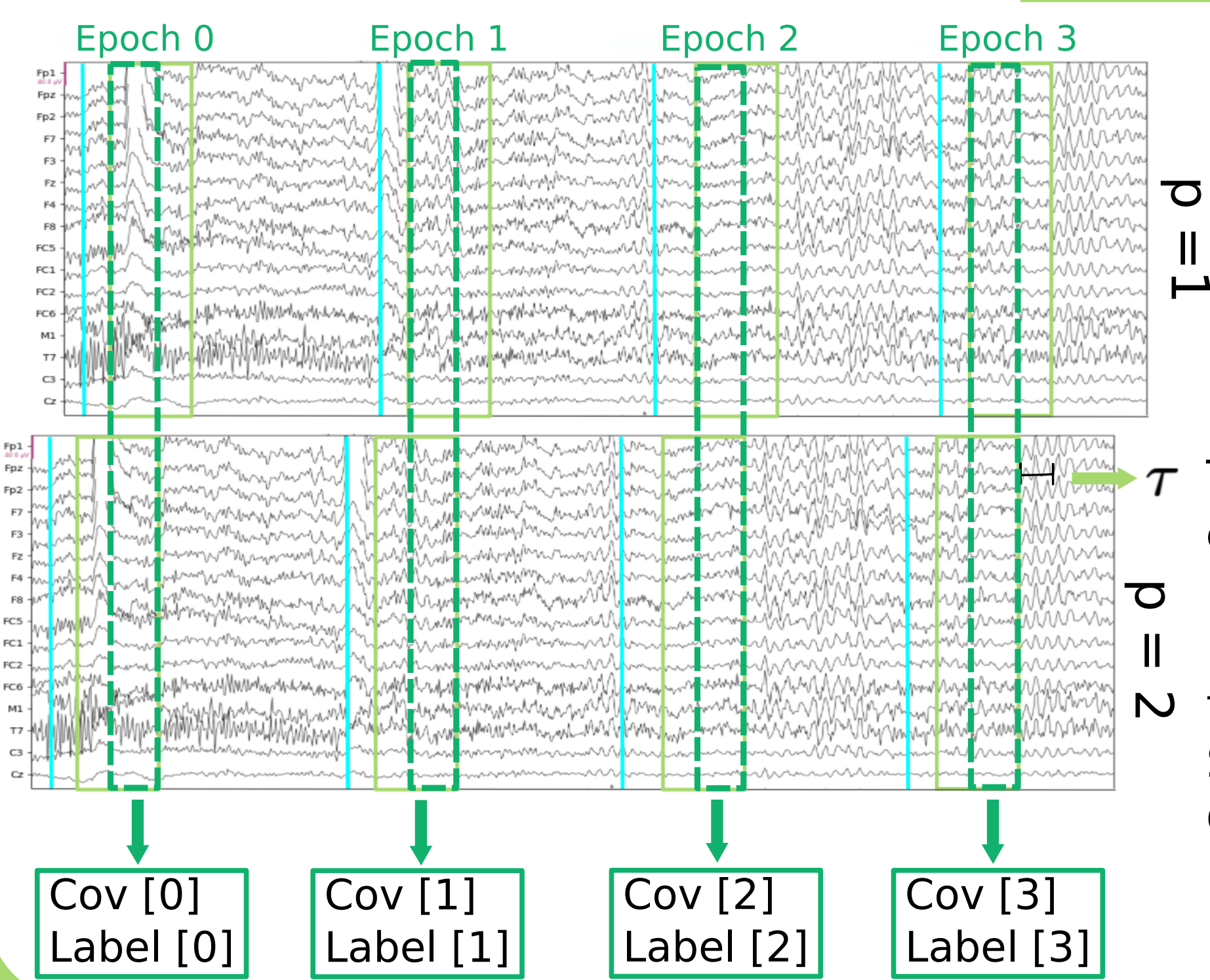
Let M to be a compact manifold of dimension m . For pairs (Φ, Y) , $\Phi: M \rightarrow M$ a smooth diffeomorphism and $Y: M \rightarrow \mathbb{R}$ a smooth function, it is a generic property that the map $\varphi_{(\Phi, Y)}: M \rightarrow \mathbb{R}^{2m+1}$, defined by

$$\varphi_{(\Phi, Y)}(x) = (Y(x), Y(\Phi(x)), \dots, Y(\Phi^{2m}(x)))$$

is an embedding; by "smooth" we mean at least C^2 . We refer to the function Y as measurement function.[4]

$$\begin{bmatrix} \Gamma_0 & \Gamma_{-1} & \Gamma_{-2} & \dots \\ \Gamma_1 & \Gamma_0 & \Gamma_{-1} & \dots \\ \Gamma_2 & \Gamma_1 & \Gamma_0 & \dots \\ \vdots & \vdots & \vdots & \ddots \\ \Gamma_{p-1} & \Gamma_{p-2} & \Gamma_{p-3} & \dots \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_p \end{bmatrix} = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \vdots \\ \Gamma_p \end{bmatrix}$$

"Augmented" Covariance
This matrix is a SPD matrix



Classification using Riemann Distance

To compute the Γ_{aug} matrix create an "Augmented" dataset by embedding the standard dataset in a dimension equal to the order of the AR mode, with a τ lag. So now we compute the Standard Covariance directly on the "Augmented" Dataset.

List of Symbols

$X \in \mathbb{R}^{ch \times T}$ = Multivariate EEG dataset
 $A_i \in \mathbb{R}^{ch \times ch}$ = Autoregressive Coefficient
 $\Gamma(i)$ = Autocovariance matrix of lag i
 Γ_{aug} = "Augmented" Covariance matrix
 D or p = Embedding dimension or Autoregressive Order
 τ = Embedding Delay
 ε = White Noise
 f_s = Sampling Frequency

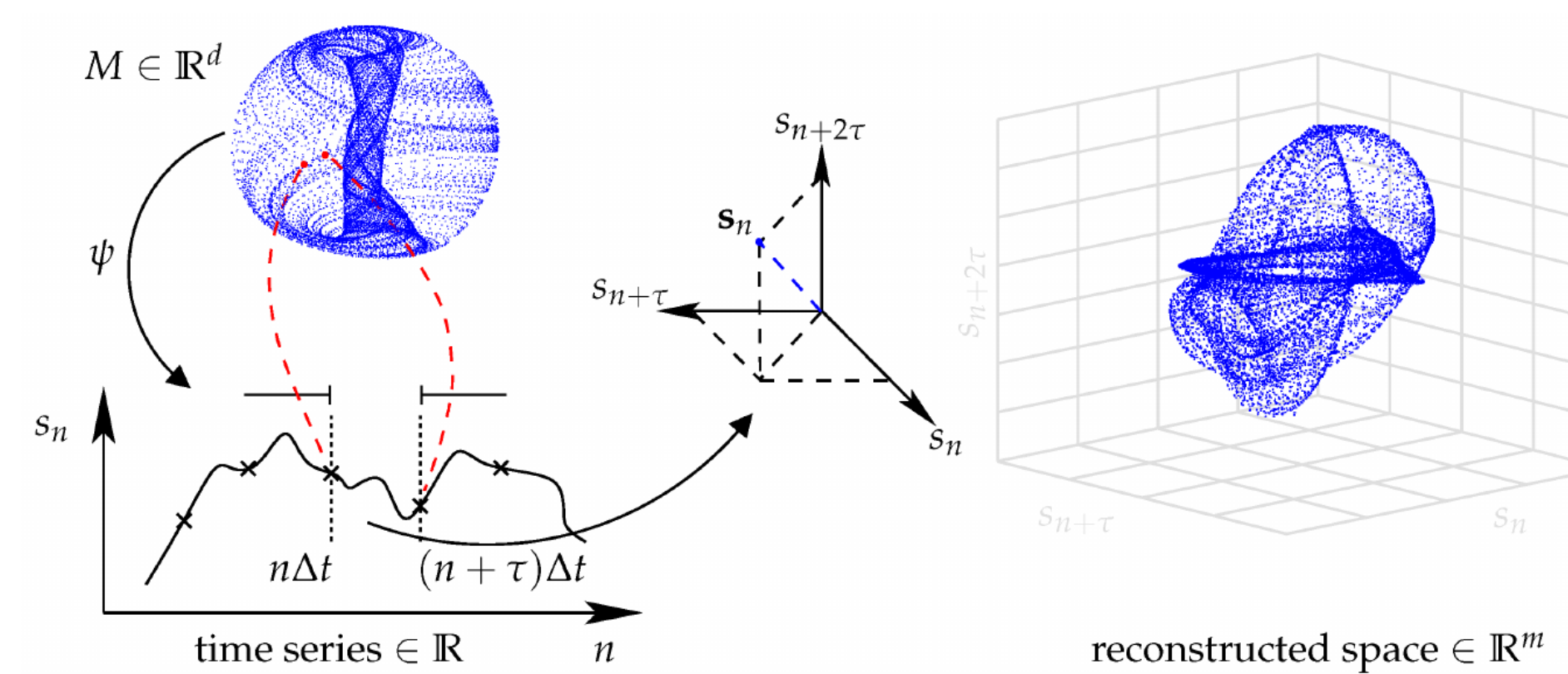


Figure: Embedding Procedure extracted from [7]

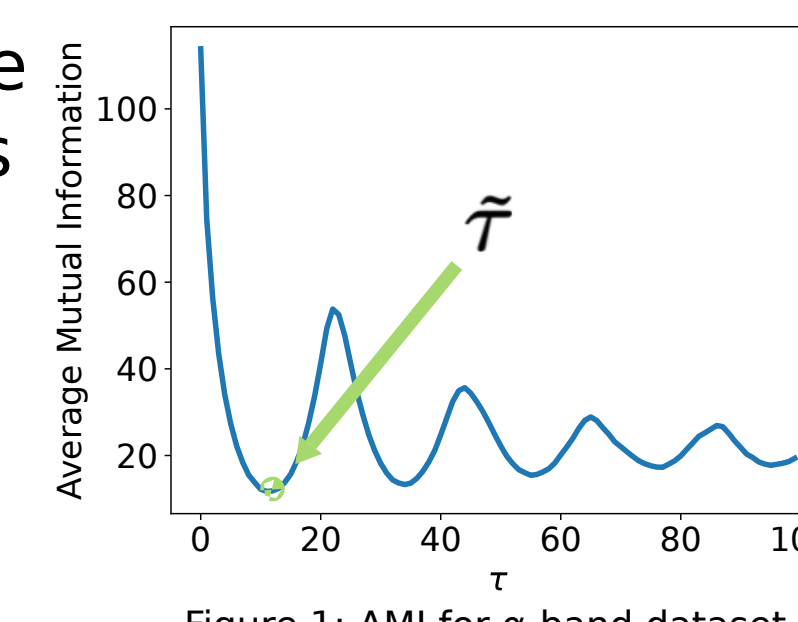
The geometric structure of the multivariable system can be unfolded from the observable s_n in a D -dimensional space constructed from the new vector

Average Mutual Information (AMI)

The Average Mutual Information quantifies the dependence between the original time series $x(t)$ and the shifted one $x(t+\tau)$

$$I(x(t), x(t+\tau)) = \sum_{i,j} Pr_{ij}(\tau) \log \left(\frac{Pr_{ij}(\tau)}{Pr_i Pr_j} \right)$$

Optimal $\tilde{\tau}$ using the method of Vlachos et al [6]. Final $\tilde{\tau}$ is defined as the average value across epoch

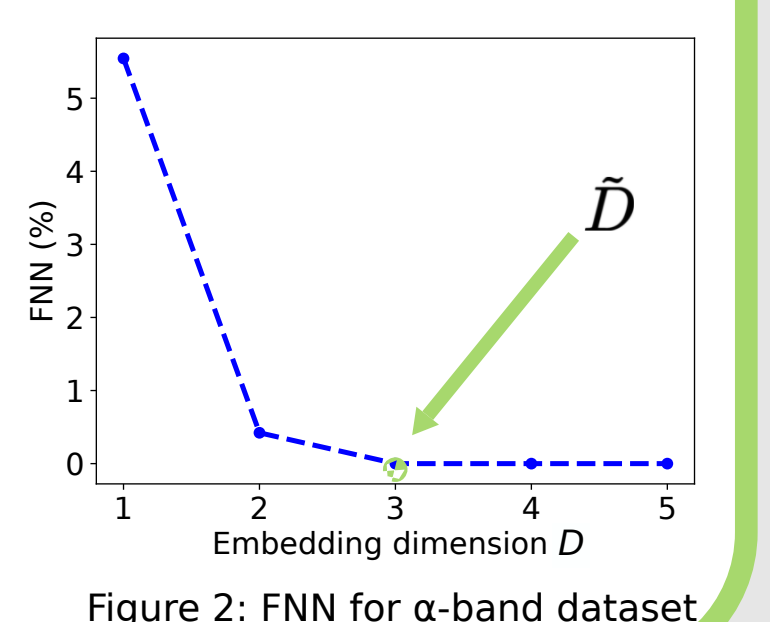


False Nearest Neighbors (FNN)

Two points are considered as real neighbors if the squared Euclidean distance R^2 between them remains constant as the D increases.

$$\sqrt{\frac{R_{(D+1)-d}^2(t, \tau) - R_{D-d}^2(t, \tau)}{R_{D-d}^2(t, \tau)}} > R$$

Optimal \tilde{D} using the method of Wallot et al [5]. Final \tilde{D} is defined as the average value across epoch



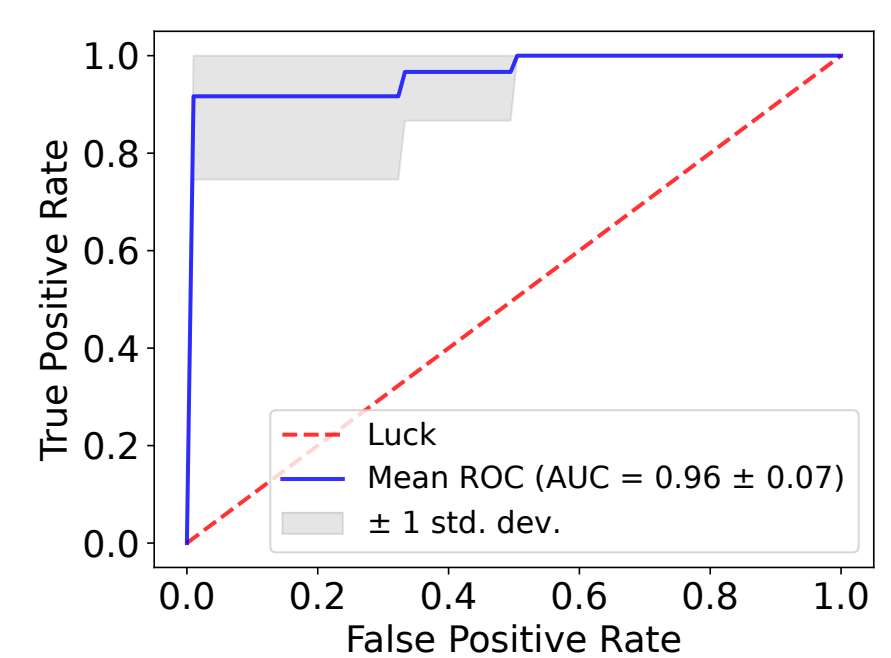
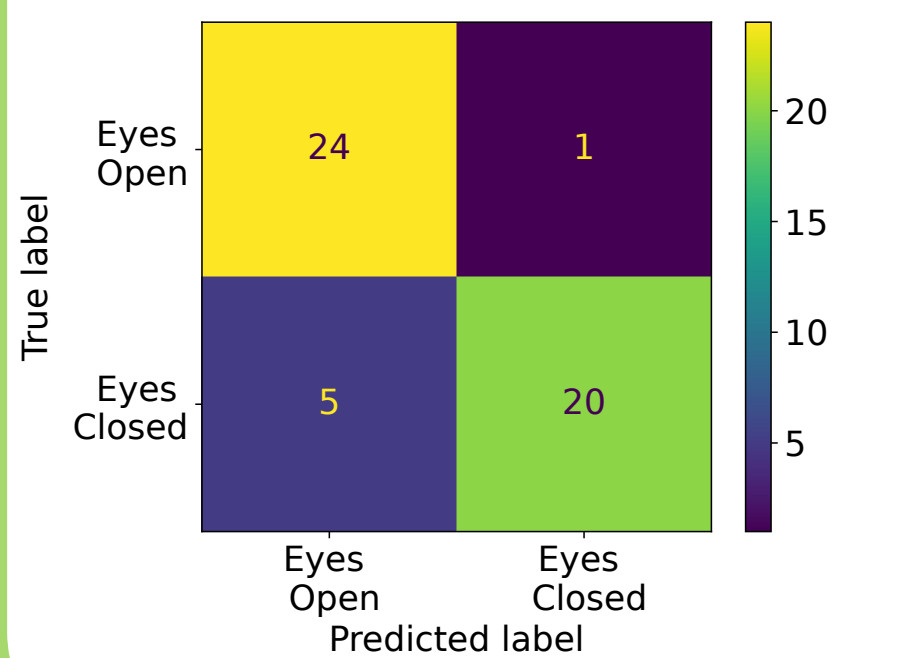
RESULTS: α -band

Preprocessing: Bandpass Filter = [8 ; 12]Hz, $f_s = 512$ Hz

Task: Class 1 eyes closed, presence of α -band
Class 2 eyes open, absence of α -band

| Parameter Used | Accuracy Train Dataset | Accuracy Test Dataset | Computational Time(s) | Memory Usage(Mb) | |
|---------------------------------|------------------------|-----------------------|-----------------------|------------------|---------|
| Standard Covariance | 0.96 | 0.82 | 3.82 s | 0.73 Mb | |
| Standard Covariance XDown | Xdown = 10 | 0.96 | 0.84 | 2.30 s | 0.77 Mb |
| Augmented Covariance Takens | Order = 2 Lag = 13 | 1.00 | 0.88 | 28.92 s | 1.73 Mb |
| Augmented Covariance GridSearch | Order = 2 Lag = 1 | 1.00 | 0.88 | 25.60 s | 1.57 Mb |

Table 1: Result α classification



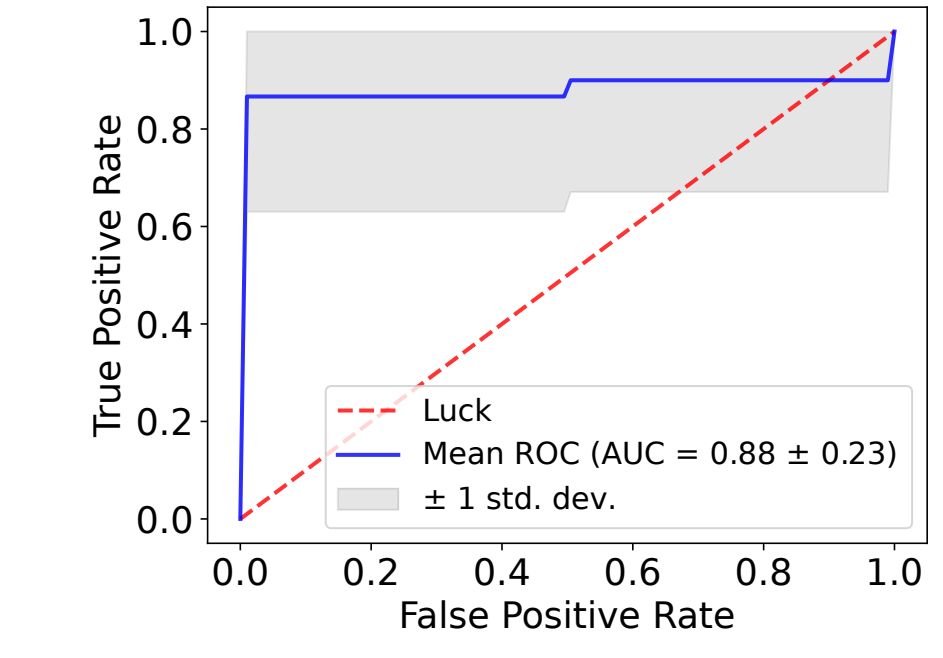
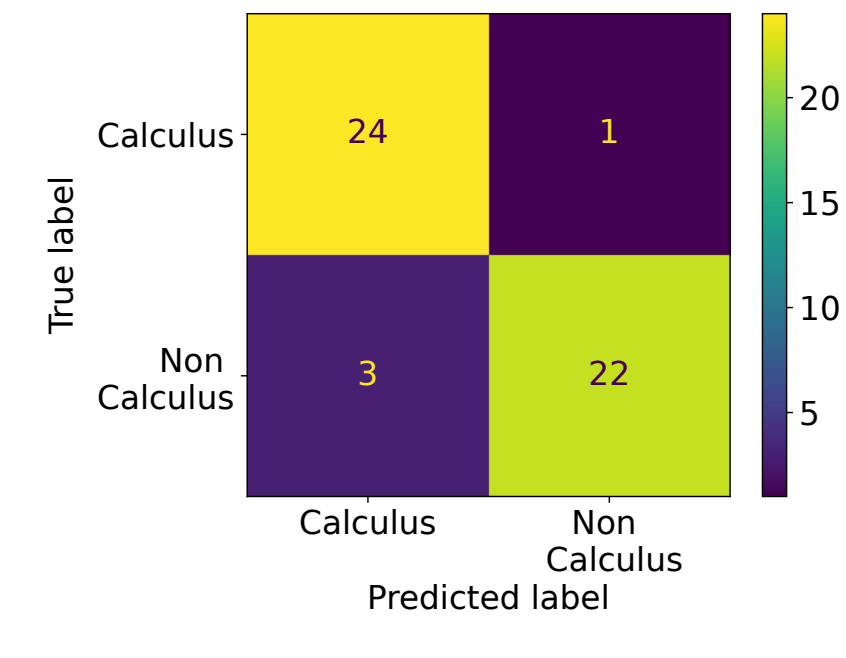
RESULTS: calculus

Preprocessing: Bandpass Filter = [4 ; 8]Hz, $f_s = 512$ Hz

Task: Class 1 eyes closed, absence of calculus operation
Class 2 eyes open, presence of calculus operation

| Parameter Used | Accuracy Train Dataset | Accuracy Test Dataset | Computational Time(s) | Memory Usage(Mb) | |
|---------------------------------|------------------------|-----------------------|-----------------------|------------------|----------|
| Standard Covariance | 1.00 | 0.84 | 8.01 s | 1.79 Mb | |
| Standard Covariance XDown | Xdown = 16 | 1.00 | 0.84 | 9.25 s | 1.79 Mb |
| Augmented Covariance Takens | Order = 1 Lag = 20 | 1.00 | 0.84 | 7.88 s | 18.73 Mb |
| Augmented Covariance GridSearch | Order = 3 Lag = 13 | 1.00 | 0.92 | 55.87 s | 18.56 Mb |

Table 2: Result Calculus classification



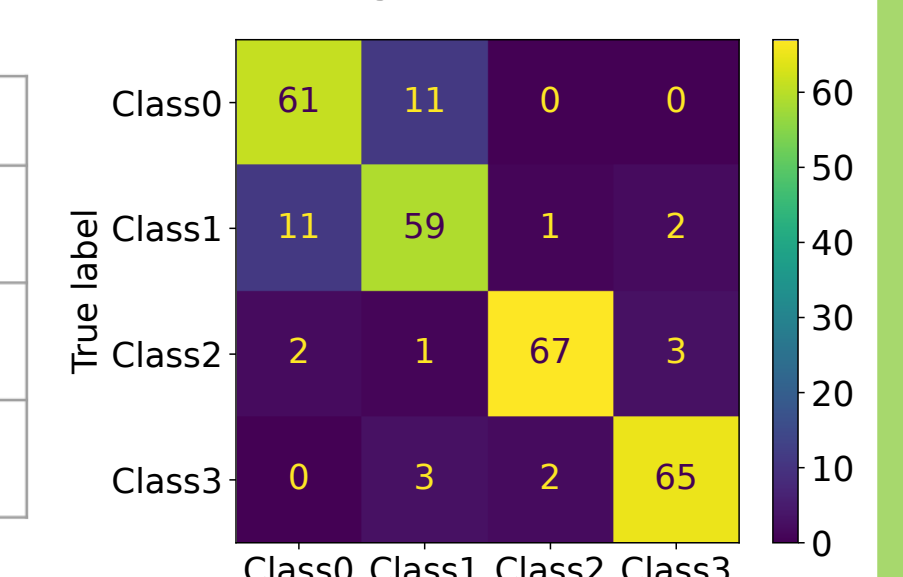
RESULTS: Motor Imagery

Preprocessing: Bandpass Filter = [0 ; 40]Hz, $f_s = 150$ Hz

Task: Class 1 auditory stimulus (tone) to the left ear
Class 2 auditory stimulus (tone) to the right ear
Class 3 visual stimulus (checkerboard) to the left visual field
Class 4 visual stimulus (checkerboard) to the right visual field

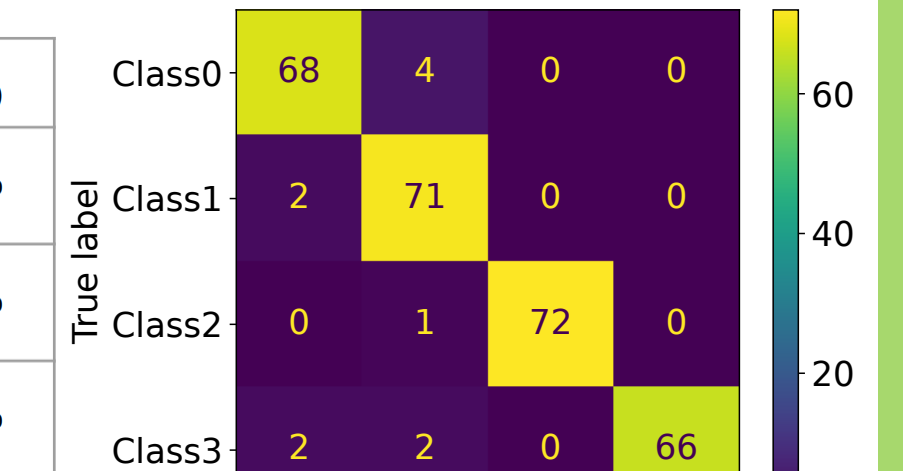
| Parameter Used | Accuracy Train Dataset | Accuracy Test Dataset | Computational Time(s) | Memory Usage(Mb) | |
|---------------------------------|-----------------------------------|-----------------------|-----------------------|------------------|---------|
| Standard Covariance XDown | Xdown = 1 | 0.59 | 0.53 | 2.56 s | 4.42 Mb |
| Augmented Covariance Takens | Xdown = 1 Order = 3 Lag = 5 | 0.70 | 0.64 | 3.35 s | 0.91 Mb |
| Augmented Covariance GridSearch | Xdown = 1 Order = 4 Lag = 9 | 0.93 | 0.87 | 7.36 s | 4.94 Mb |

Table 3: Result Motor Imagery Standard Covariance



| Parameter Used | Accuracy Train Dataset | Accuracy Test Dataset | Computational Time(s) | Memory Usage(Mb) | |
|---------------------------------------|-----------------------------------|-----------------------|-----------------------|------------------|---------|
| XDown Covariance XDown | Xdown = 3 | 0.93 | 0.90 | 3.09 s | 3.61 Mb |
| Augmented XDown Covariance Takens | Xdown = 1 Order = 3 Lag = 5 | 0.98 | 0.95 | 3.92 s | 0.87 Mb |
| Augmented XDown Covariance GridSearch | Xdown = 1 Order = 4 Lag = 3 | 0.98 | 0.96 | 5.60 s | 5.43 Mb |

Table 4: Result Motor Imagery XDownCovariance



CONCLUSION

In this work, we propose a new method to classify the task in EEG signal using Autoregressive model and the "Augmented" covariances, based on Riemann framework for classification. We test our approach on three different EEG datasets and the new method show a slight improvement in classification performance in every dataset considered. The approach using the Takens method selects parameters that are not the optimal ones (considering the result of the grid search) but still show a slight improvement in classification performance in every dataset considered. These preliminary results show that "Augmented" covariances can induce some performance gains. More work is needed to fully qualify the gains in accuracy w.r.t. the speed of the classification procedure.

ACKNOWLEDGEMENTS

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