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# On the use of the "Augmented" autocovariance matrix for the classification of BCI-EEG

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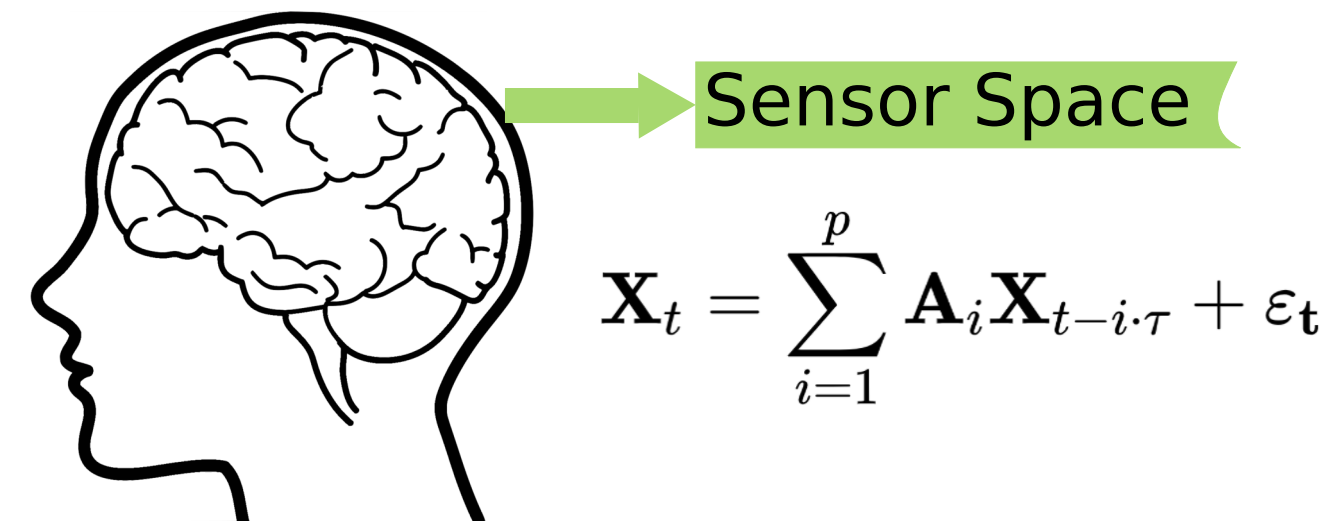


## ABSTRACT

Electroencephalogram (EEG) signal is recorded as a multidimensional dataset, which can be interpreted using autoregressive (AR) models [1]. In this way, it is possible to extract relevant information about the signal, which can be used for Brain Computer Interface (BCI) classification. Yule-Walker equations can be derived from the AR equation, which yields the autocovariance matrix that is a symmetric positive definite matrix (SPD) which is classified using Riemann geometry approach [3]. We can also think of formulating the problem from another point of view: since the autocovariance matrix is a matrix of delayed covariances, we can obtain the same result by creating an embedding of the original system in a high dimensional space. Thus, it seems natural to connect our approach with the delay embedding theorem proposed by Takens [4] in the context of dynamical systems. Such an embedding method depends on two parameters: the delay parameter  $\tau$  and the embedding dimension  $D$ , respectively the lag and the order in the context of AR model. We use grid search to find various parameters of the compared approaches. We test our approach on several datasets and compare it to the state of the art.

## METHODS

### 1 Autoregressive Model



Derive the Yule Walker equations

$$\Gamma(0) = \sum_{k=1}^p \mathbf{A}_k \Gamma(-k \cdot \tau) + \mathbf{U} \text{ for } i = 0$$
$$\Gamma(i) = \sum_{k=1}^p \mathbf{A}_k \Gamma(i - k \cdot \tau) \text{ for } i \neq 0$$

### 2 Takens Theorem

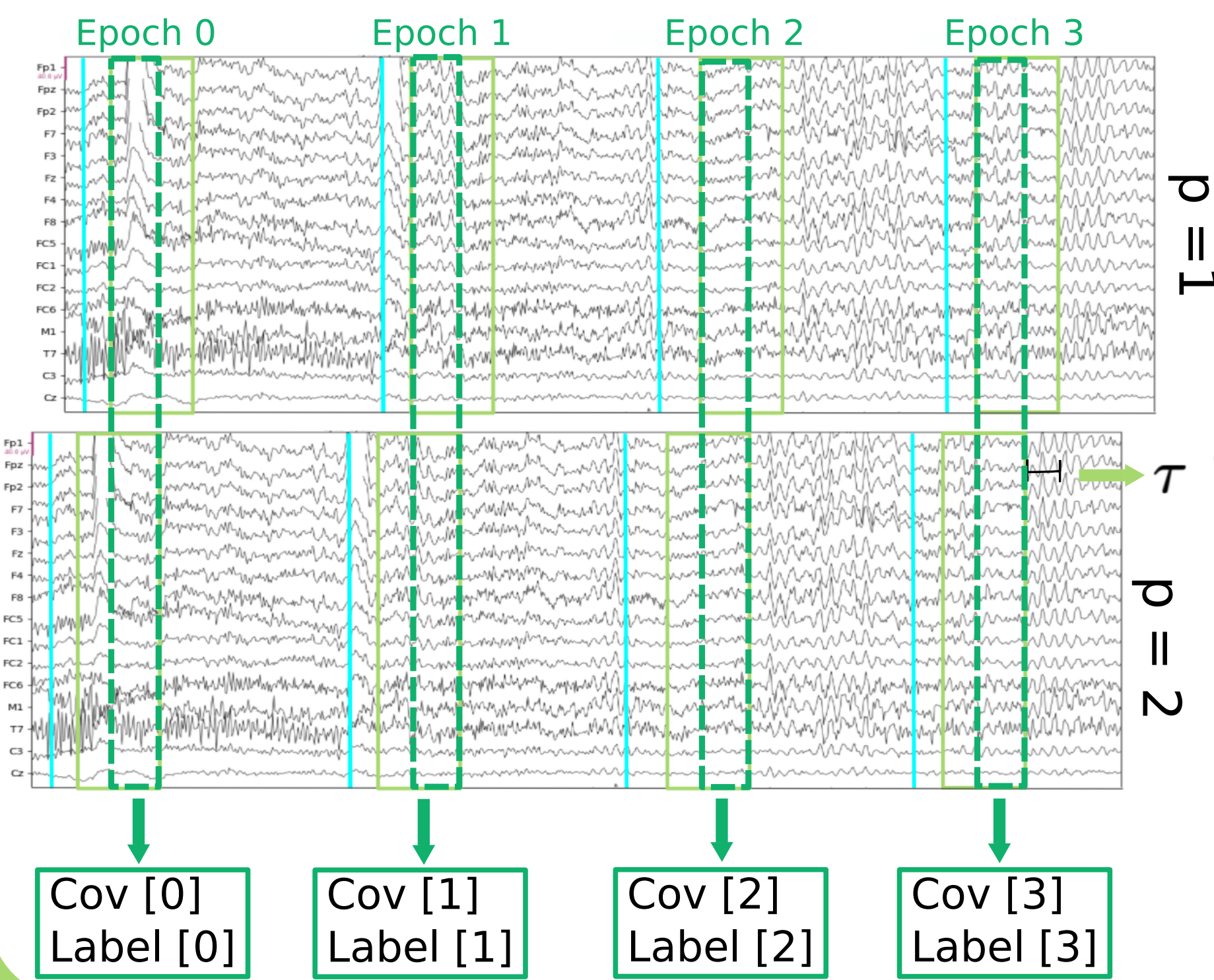
Let  $M$  to be a compact manifold of dimension  $m$ . For pairs  $(\Phi, Y)$ ,  $\Phi: M \rightarrow M$  a smooth diffeomorphism and  $Y: M \rightarrow \mathbb{R}$  a smooth function, it is a generic property that the map  $\varphi_{(\Phi, Y)}: M \rightarrow \mathbb{R}^{2m+1}$ , defined by

$$\varphi_{(\Phi, Y)}(x) = (Y(x), Y(\Phi(x)), \dots, Y(\Phi^{2m}(x)))$$

is an embedding; by "smooth" we mean at least  $\mathbb{C}^2$ . We refer to the function  $Y$  as measurement function.[4]

$$\begin{bmatrix} \Gamma_0 & \Gamma_{-1} & \Gamma_{-2} & \cdots \\ \Gamma_1 & \Gamma_0 & \Gamma_{-1} & \cdots \\ \Gamma_2 & \Gamma_1 & \Gamma_0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ \Gamma_{p-1} & \Gamma_{p-2} & \Gamma_{p-3} & \cdots \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_p \end{bmatrix} = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \vdots \\ \Gamma_p \end{bmatrix}$$

"Augmented" Covariance  
This matrix is a SPD matrix

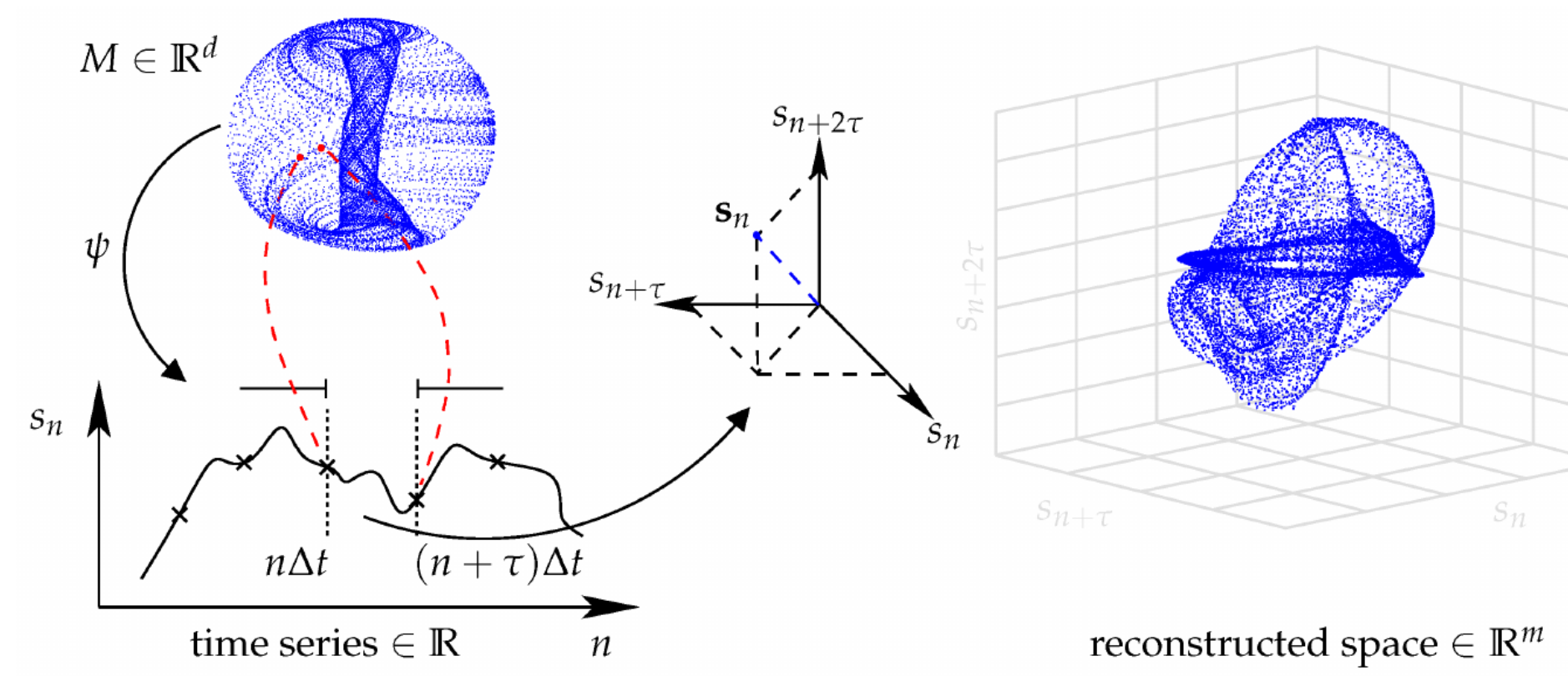


Classification using  
Riemann Distance

To compute the  $\Gamma_{aug}$  matrix  
create an "Augmented" dataset  
by embedding the standard dataset  
in a dimension equal to the order of  
the AR mode, with a  $\tau$  lag.  
So now we compute the **Standard  
Covariance directly on the  
"Augmented" Dataset.**

### List of Symbols

$X \in \mathbb{R}^{ch \times T}$  = Multivariate EEG dataset  
 $A_i \in \mathbb{R}^{ch \times ch}$  = Autoregressive Coefficient  
 $\Gamma(i)$  = Autocovariance matrix of lag  $i$   
 $\Gamma_{aug}$  = "Augmented" Covariance matrix  
 $D$  or  $p$  = Embedding dimension or Autoregressive Order  
 $\tau$  = Embedding Delay  
 $\epsilon$  = White Noise  
 $f_s$  = Sampling Frequency



The geometric structure of the  
**multivariable system can be  
unfolded** from the observable  
 $s_n$  in a **D-dimensional space**  
constructed from the new vector

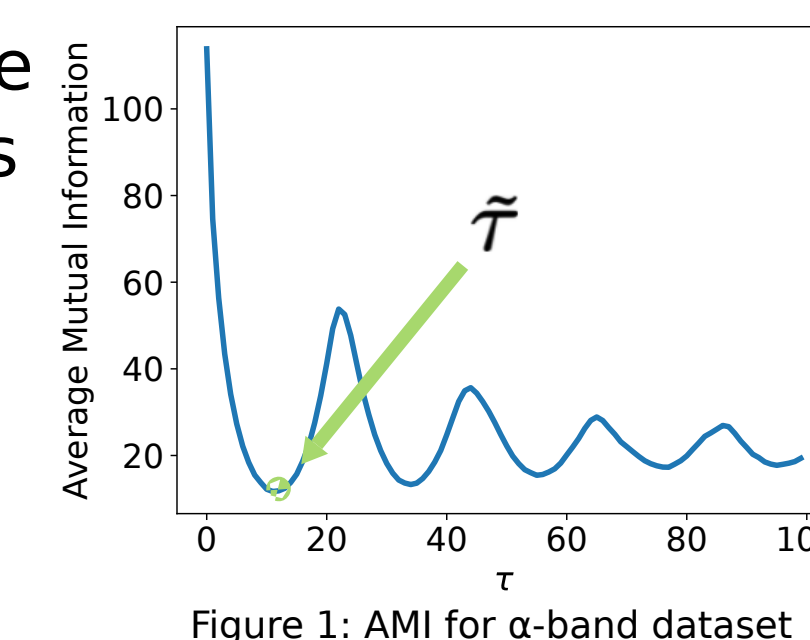
$$s_R(n) = [s(n), s(n-\tau), \dots, s(n-(D-1)\tau)]^T$$

### Average Mutual Information (AMI)

The Average Mutual Information quantifies  
the dependence between the original time  
series  $x(t)$  and the shifted one  $x(t+\tau)$

$$I(x(t), x(t+\tau)) = \sum_{i,j} Pr_{ij}(\tau) \log \left( \frac{Pr_{ij}(\tau)}{Pr_i Pr_j} \right)$$

Optimal  $\tilde{\tau}$  using the  
method of Vlachos  
et al [6].  
Final  $\tilde{\tau}$  is  
defined as the  
average value  
across epoch

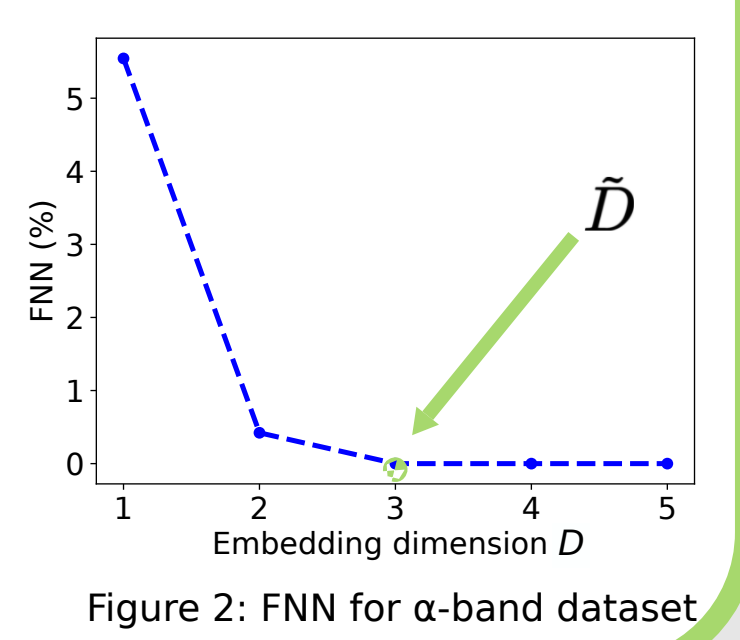


### False Nearest Neighbors (FNN)

Two points are considered as real  
neighbors if the squared Euclidean  
distance  $R^2$  between them remains  
constant as the  $D$  increases.

$$\sqrt{\frac{R_{(D+1)-d}^2(t, r) - R_{D-d}^2(t, r)}{R_{D-d}^2(t, r)}} > R$$

Optimal  $\tilde{D}$  using  
the method of  
Wallot et al [5].  
Final  $\tilde{D}$  is defined  
as the average  
value across  
epoch



## RESULTS: alpha-band

**Preprocessing:** Bandpass Filter = [8 ; 12]Hz,  $f_s = 512$ Hz

**Task:** Class 1 eyes closed, presence of alpha-band  
Class 2 eyes open, absence of alpha-band

	Parameter Used	Accuracy Train Dataset	Accuracy Test Dataset	Computational Time(s)	Memory Usage(Mb)
Standard Covariance		0.96	0.82	3.82 s	0.73 Mb
Standard Covariance XDawn	Xdown = 10	0.96	0.84	2.30 s	0.77 Mb
Augmented Covariance Takens	Order = 2 Lag = 13	1.00	0.88	28.92 s	1.73 Mb
Augmented Covariance GridSearch	Order = 2 Lag = 1	1.00	0.88	25.60 s	1.57 Mb

Table 1: Result alpha classification

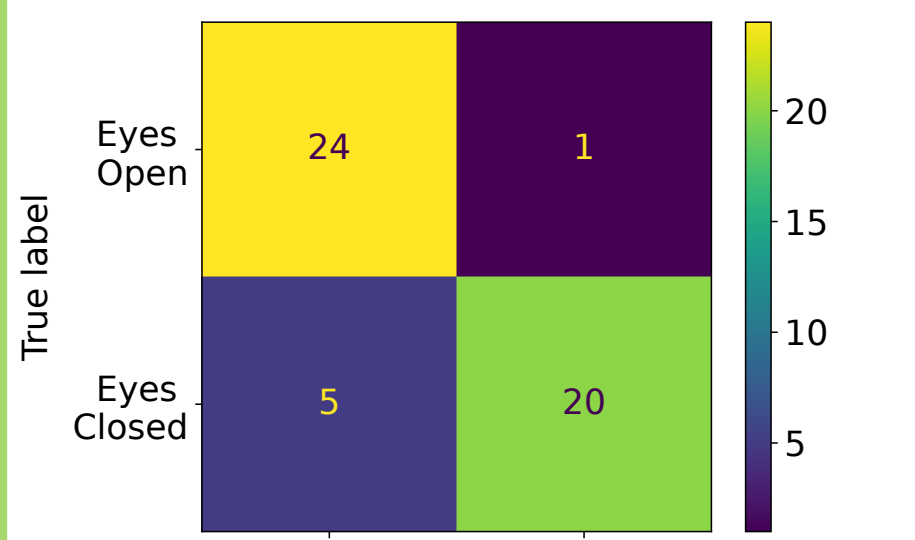


Figure 3: Confusion Matrix alpha-band Augmented Covariance Takens

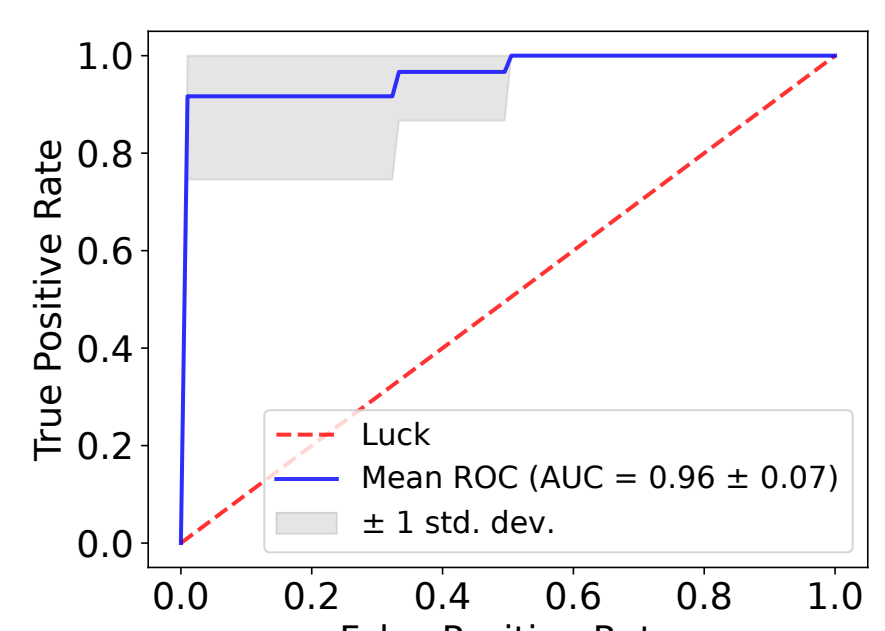


Figure 4: ROC curve alpha-band Augmented Covariance Takens

## RESULTS: calculus

**Preprocessing:** Bandpass Filter = [4 ; 8]Hz,  $f_s = 512$ Hz

**Task:** Class 1 eyes closed, absence of calculus operation  
Class 2 eyes open, presence of calculus operation

	Parameter Used	Accuracy Train Dataset	Accuracy Test Dataset	Computational Time(s)	Memory Usage(Mb)
Standard Covariance		1.00	0.84	8.01 s	1.79 Mb
Standard Covariance XDawn	Xdown = 16	1.00	0.84	9.25 s	1.79 Mb
Augmented Covariance Takens	Order = 1 Lag = 20	1.00	0.84	7.88 s	18.73 Mb
Augmented Covariance GridSearch	Order = 3 Lag = 13	1.00	0.92	55.87 s	18.56 Mb

Table 2: Result Calculus classification

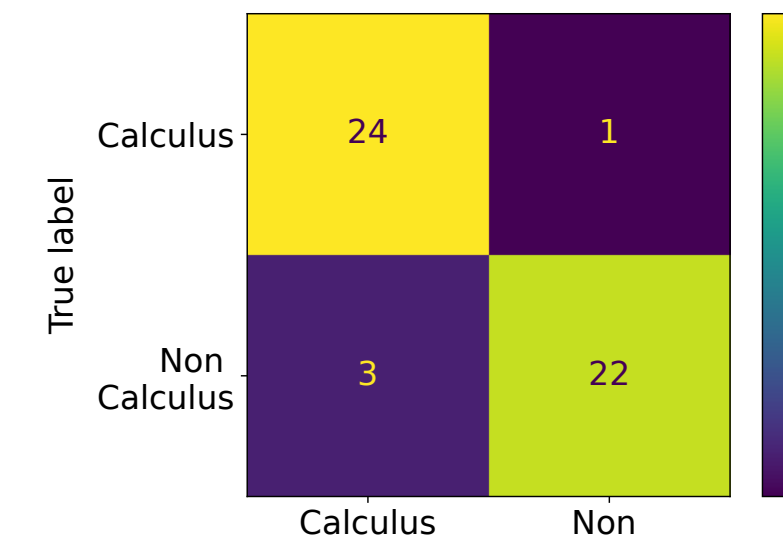


Figure 5: Confusion Matrix calculus Augmented Covariance GridSearch

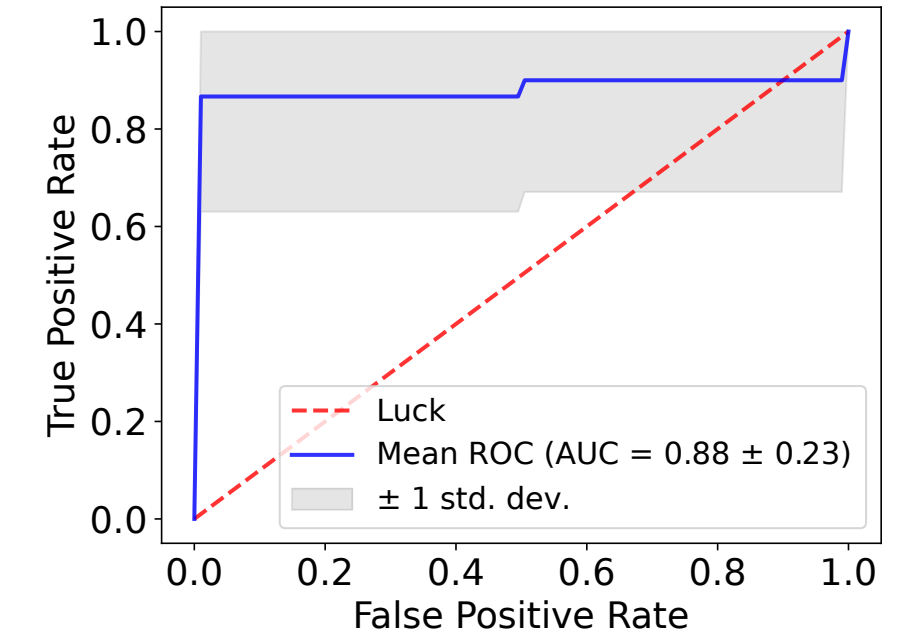


Figure 6: ROC curve calculus Augmented Covariance Grid Search

## RESULTS: Motor Imagery

**Preprocessing:** Bandpass Filter = [0 ; 40]Hz,  $f_s = 150$ Hz

**Task:** Class 1 auditory stimulus (tone) to the left ear  
Class 2 auditory stimulus (tone) to the right ear  
Class 3 visual stimulus (checkerboard) to the left visual field  
Class 4 visual stimulus (checkerboard) to the right visual field

	Parameter Used	Accuracy Train Dataset	Accuracy Test Dataset	Computational Time(s)	Memory Usage(Mb)
Standard Covariance XDawn	Xdown = 1	0.59	0.53	2.56 s	4.42 Mb
Augmented Covariance Takens	Xdown = 1 Order = 3 Lag = 5	0.70	0.64	3.35 s	0.91 Mb
Augmented Covariance GridSearch	Xdown = 1 Order = 5 Lag = 9	0.93	0.87	7.36 s	4.94 Mb

Table 3: Result Motor Imagery Standard Covariance

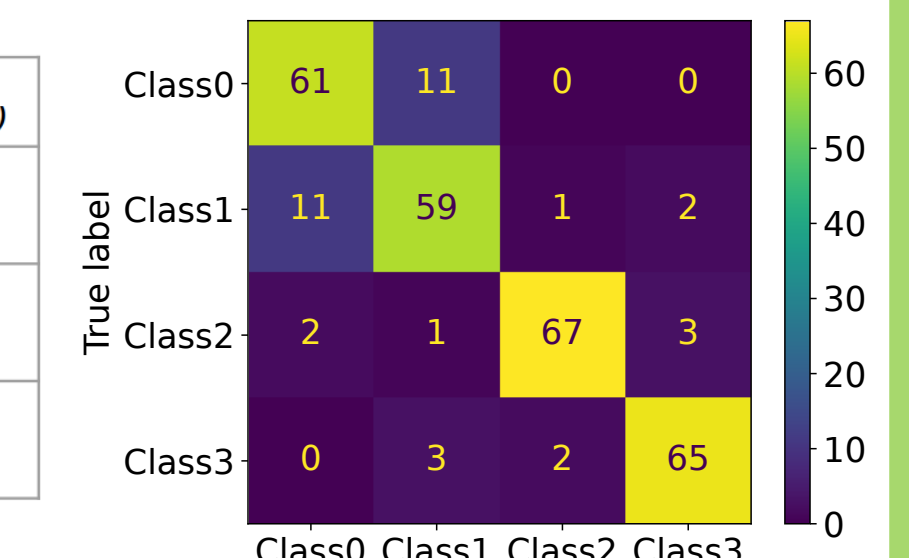


Figure 7: Confusion Matrix Motor Imagery Augmented Covariance GridSearch

	Parameter Used	Accuracy Train Dataset	Accuracy Test Dataset	Computational Time(s)	Memory Usage(Mb)
XDawn Covariance XDawn	Xdown = 3	0.93	0.90	3.09 s	3.61 Mb
Augmented XDawn Covariance Takens	Xdown = 1 Order = 3 Lag = 5	0.98	0.95	3.92 s	0.87 Mb
Augmented XDawn Covariance GridSearch	Xdown = 1 Order = 4 Lag = 3	0.98	0.96	5.60 s	5.43 Mb

Table 4: Result Motor Imagery XDawnCovariance

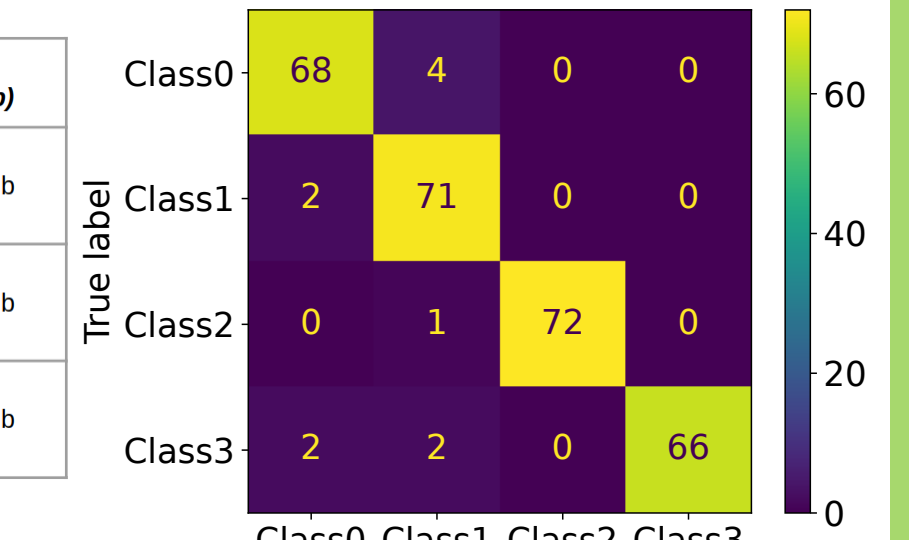


Figure 8: Confusion Matrix Motor Imagery Augmented XDawnCovariance GridSearch

## CONCLUSION

In this work, we propose a new method to classify the task in EEG signal using Autoregressive model and the "Augmented" covariances, based on Riemann framework for classification. We test our approach on three different EEG datasets and the new method show a slight improvement in classification performance in every dataset considered. The approach using the Takens method selects parameters that are not the optimal ones (considering the result of the grid search) but still show a slight improvement in classification performance in every dataset considered. These preliminary results show that "Augmented" covariances can induce some performance gains. More work is needed to fully qualify the gains in accuracy w.r.t. the speed of the classification procedure.

## ACKNOWLEDGEMENTS

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