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# POD Curves for Natural Frequency Testing

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**Abstract.** Evaluating natural frequencies for damage detection and localization is a standard procedure in non-destructive testing (NDT) and structural health monitoring (SHM). In particular, for cables and other prestressed tendons, natural frequencies exhibit high sensitivities to structural changes, which is often demonstrated by comparing the frequencies before and after damage occurred, e.g., using probability of detection (POD) or probability of localization (POL) curves. In this paper, an approach is developed to predict POD/POL curves without any data from the damaged state. The method is based on data from the undamaged state and a numerical model, which is utilized to obtain sensitivity vectors that linearize the relation between frequency changes and changes in structural parameters. At the same time, statistical uncertainties in the frequency estimation process are considered. Based on this method, a cable monitoring strategy is suggested, where sensitivity vectors are first used to calibrate decisive cable properties using model updating, followed by a performance assessment through POD/POL curves, and a continuous damage detection and localization based on statistical tests. The synergic effects of the presented methods are demonstrated by means of a numerical case study on a prestressed cable, where data from a single uni-axial vibration sensor is evaluated. The results indicate that changes in stiffness and axial forces can be distinguished using localization tests, but additional sensors and features are necessary for damage localization along the cable.

## Introduction

Structural health monitoring (SHM) describes the regular observation and the automated damage detection on civil and mechanical engineering structures [1]. Basic steps include the acquisition of measurement data through permanently installed sensors, the extraction of damage-sensitive features (such as natural frequencies), and the automated damage diagnosis, which is often split into four hierarchical steps of increasing diagnostic complexity, i.e., damage detection, localization, quantification or type assessment, and lifetime prognosis [2]. Considering the unstoppable trend toward the digitalization of engineering systems through virtual representations (digital twins, building information models, etc.), SHM takes up a special role, as it is suitable to provide real-time feedback from the structure to designers and operators, which enables an optimized maintenance schedule and an extension of the design lifetime.

Global vibration-based methods are particularly suitable for SHM, as they can be applied under normal operating conditions and without human interaction. They are based on the premise that structural damage leads to changes in the global stiffness, mass, or damping characteristics of a structure, which in turn, lead to changes in the ambient vibration behavior



that can be measured through a small number of sensors at selected measurement locations, sometimes at considerable distances to the damage location. In particular, for cable monitoring (stay cables or other pretensions tendons and rods), vibration-based features such as resonance frequencies, mode shapes, and damping ratios have shown to be suitable damage-sensitive features of sufficient sensitivity [3]. However, an aspect that is often neglected is the proper performance assessment of monitoring systems, meaning the system operator needs to know which damages can be detected and localized or how severe the damage has to be for a reliable diagnosis. On this background, the research question of this paper is specified as follows: is it possible to place a single uni-axial sensor on a cable and to develop a method that (i) autonomously identifies all relevant cable properties (stiffness, mass, and prestress), (ii) assesses the performance of the monitoring system before damage occurs, and (iii) detects and localizes damage in quasi-real-time, without human interaction. The paper will show that sensitivity-based methods are appropriate to reach this goal, and a decisive element is added to the diagnostic chain, so-called probability of detection (POD) curves and probability of localization curves (POL), which enable a predictive performance assessment.

The paper is organized as follows: Section 1 gives some background on sensitivity vectors and matrices, and explains how they can be employed to analyze the ill-conditioning of the damage localization problem through cluster trees. Section 2 recaps sensitivity-based model updating and sensitivity-based statistical tests. Moreover, an approach is developed to construct POD and POL curves based on data from undamaged specimens or structures. In Section 3, a numerical case study is presented, where all presented methods are combined to reach the objective of this paper, followed by some conclusions in Section 4.

## 1. Sensitivity Analysis

This section explains how damage-sensitive features can be linked to structural parameters using sensitivity vectors, and describes a clustering approach to assess the ill-conditioning of the damage localization problem.

### 1.1 Damage-sensitive Features

Damage diagnosis is based on measurement data  $\mathbf{y} \in \mathbb{R}^{N \times r}$  measured through  $r$  measurement channels and  $N$  data points. Typically, measurements are not evaluated directly but several preprocessing steps are applied to extract damage-sensitive features  $\mathbf{f} \in \mathbb{R}^{N_f}$  (e.g., natural frequencies). The evaluation of features instead of measurements reduces the amount of data to be transmitted and stored, and increases the robustness toward environmental effects. The estimated feature vector  $\hat{\mathbf{f}}$  can be evaluated directly, or a reference vector  $\mathbf{f}^0$  is defined and subtracted to form a damage-sensitive residual  $\hat{\mathbf{r}} = \hat{\mathbf{f}} - \mathbf{f}^0$ . By multiplying the residual with the square root of the sample size  $\sqrt{N}$ , an asymptotically Gaussian residual  $\hat{\boldsymbol{\zeta}}$  can be obtained

$$\hat{\boldsymbol{\zeta}} = \sqrt{N}(\hat{\mathbf{f}} - \mathbf{f}^0). \quad (1)$$

Every quantity estimated from data is subject to uncertainties, which is indicated through the hat symbol in Eq. (1). The reference value  $\mathbf{f}^0$  lacks the hat symbol, because it can be evaluated based on data or deterministic models. Assuming that the feature vector can be approximated through a Gaussian distribution, the data-driven uncertainties can be quantified by computing the covariance matrix based on  $n_b$  data blocks,

$$\boldsymbol{\Sigma} = \frac{1}{n_b - 1} \sum_{k=1}^{n_b} \hat{\boldsymbol{\zeta}}_k \hat{\boldsymbol{\zeta}}_k^T. \quad (2)$$

For uncorrelated features, the covariance matrix simplifies to a diagonal matrix. For correlated features, such as natural frequencies, the covariance matrix contains off-diagonal values that describe the correlation between different features. Knowing the features' distribution is key to determining the significance of residual deviation from zero.

### 1.2 Sensitivity Clustering

Sensitivity analysis is an efficient approach to determine the most appropriate features and to assess the ill-conditioning of the damage localization problem. The core idea is to build a numerical model of the examined structure (e.g., a finite element model), to define damage as a change in structural parameters  $\boldsymbol{\theta} \in \mathbb{R}^{N_p}$  (material properties, cross-sectional value, etc.) and to analyze the rate of change in data-driven features for small perturbations in the structural parameters. Mathematically speaking, the data-driven residual is a function of structural parameters, possibly superimposed with measurement noise  $\mathbf{e}$

$$\mathbf{r} = F(\boldsymbol{\theta}) + \mathbf{e}, \quad (3)$$

and sensitivity analysis is equivalent to calculating the slope or tangent of the function  $F(\boldsymbol{\theta})$  at the origin, i.e., for  $\boldsymbol{\theta} = \boldsymbol{\theta}^0$ . The simplest approach to calculate the sensitivity is the finite difference method, where the feature vector is extracted before and after a perturbation by  $\Delta\theta_h = \theta_h - \theta_h^0$ . In this case, each sensitivity vector  $\mathcal{J}_h \in \mathbb{R}^{N_f}$  can be calculated as

$$\mathcal{J}_h = \left. \frac{\delta E[\mathbf{r}]}{\delta \theta_h} \right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^0} = \frac{\mathbf{f} - \mathbf{f}^0}{\theta_h - \theta_h^0}. \quad (4)$$

For some data-driven features, such as natural frequencies, analytical derivatives exist as well. For example, the modal method [4] describes the frequency derivatives as a function of the mass, stiffness, and damping matrix derivatives, which leads to results that are independent of the step size  $\Delta\theta_h$  if those matrices are linear functions of the examined parameters. The sensitivity vector from Eq. (4) is calculated for each parameter, and finally, all vectors are assembled in one matrix, the sensitivity matrix  $\mathcal{J} \in \mathbb{R}^{N_f \times N_p}$

$$\mathcal{J} = [\mathcal{J}_1 \quad \dots \quad \mathcal{J}_{N_p}]. \quad (5)$$

The sensitivity matrix links changes in data-driven features to structural parameters through

$$\Delta\mathbf{f} \approx \mathcal{J}\Delta\boldsymbol{\theta}, \quad (6)$$

and serves multiple purposes in this paper: it is used for model updating (Section 2.1) and damage detection and localization based on statistical tests (Section 2.2). Furthermore, sensitivity vectors aid in the assessment of the numerical conditioning of the damage localization problem. Damage is localizable if changes in the feature vector can clearly be assigned to one parameter. Since each column vector in the sensitivity matrix describes the features' sensitivity toward one structural parameter, localizability is determined by how similar the sensitivity vectors are regardless of their location within the structure. In this paper, the similarity between two vectors is quantified through the distance

$$d_{ab} = 1 - \frac{|\mathbf{J}_a^T \mathbf{J}_b|}{\|\mathbf{J}_a\| \cdot \|\mathbf{J}_b\|}, \quad (7)$$

where the second term describes the cosine between the two vectors with the subscript  $a$  and  $b$ . The distance  $d_{ab}$  is zero if the two vectors are identical, and it is maximal with  $d_{ab} = 1$  if the vectors are orthogonal. A particularly intuitive way to visualize this distance is a cluster tree, for example, constructed based on hierarchical clustering and complete linkage [5,6]. Clustering is an iterative procedure; in the first step, the two parameters with the shortest distance are combined into one cluster. Secondly, the distance between the newly formed cluster and all other parameters is re-evaluated and the two partitions (vectors or clusters) with the closest distance are merged. The process of evaluating the distance from Eq. (7) and merging the partitions is then repeated until all vectors are combined into one cluster, so the cluster tree can be drawn, see Fig. 1. The closer the first branches meet to the bottom axis the worse the ill-conditioning of the localization problem. This is because the sensitivity vectors are very similar (indicated through a low distance  $d$ ) and it is challenging to identify the parameter that caused the deviation in the features. The higher the intersection of the branches the better the conditioning. One way to improve the conditioning is to remove redundant parameters from the analysis after visually inspecting the cluster tree. Alternatively, the cluster tree can be trimmed and the finite element model can be substructured into damage localization components by merging redundant parameter sensitivities into clusters. This is done by selecting a cut-off distance  $d_{thres}$  in the tree (the horizontal dashed lines in Fig. 1) and replacing the Jacobian matrix with the cluster centers at the cut-off level

$$\mathbf{J}^c = [\mathbf{c}_1 \quad \dots \quad \mathbf{c}_K], \quad (8)$$

where  $\mathbf{c}_k$  are the cluster centers

$$\mathbf{c}_k = \frac{1}{m_k} \sum_{i \in C_k} \mathbf{J}_i, \quad (9)$$

and  $m_k$  is the number of merged parameters in any of the obtained clusters  $C_k$ . In the next section, it will be explained how sensitivity vectors can be used to diagnose damage.

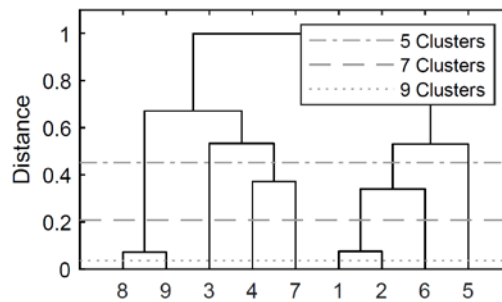


Fig. 1. Representative cluster tree for a monitoring application with  $N_p = 9$  examined parameters.

## 2. Damage Diagnosis

This section explains how sensitivity-based methods can be employed determine the as-built state of structures and continuously monitor the structural health. In this paper, model updating is applied to calibrate the magnitude of the monitoring parameters based on measurement data, and statistical tests are applied for continuous monitoring. The advantage

of sensitivity-based statistical tests is that they allow for a performance assessment of the monitoring system before damage occurs. The core of this paper is the development of predictive POD and POL curves to assess the performance.

### 2.1 Model Updating

Sensitivity-based model updating, also known as gradient-based or Newton-Raphson optimization, is a deterministic updating approach, as it neglects the distribution of structural parameters and treats them as deterministic values. The main idea is to vary the structural parameters  $\boldsymbol{\theta}$  in the model until the numerical features  $\mathbf{f}^0$  are identical to the measured features  $\hat{\mathbf{f}}$ , which represent the true system state. The iterative procedure is terminated once a cost function drops below a user-defined threshold value. The cost function  $t$  is defined as the statistical distance

$$t = \hat{\boldsymbol{\zeta}}^T \boldsymbol{\Sigma}^{-1} \hat{\boldsymbol{\zeta}}, \quad (10)$$

and contains the residual from Eq. (1) as well as the covariance matrix from Eq. (2) as a weighting factor, so changes in features with low uncertainties are emphasized. Moreover, the deviations in all residual entries are reduced to one scalar value  $t$ . If the numerical features deviate from the measured ones, the sensitivity matrix is computed and the parameter vector is updated by rewriting Eq. (6) to

$$\boldsymbol{\theta} = \boldsymbol{\theta}^0 + \Delta\boldsymbol{\theta}, \quad \text{where} \quad \Delta\boldsymbol{\theta} = -\mathbf{J}^{-1} \hat{\mathbf{r}}. \quad (11)$$

Typically, several iterations are necessary until the cost function from Eq. (10) drops below a user-defined tolerance value, as the relation between features and parameters is inherently non-linear. For large models, with time-consuming sensitivity computations, it is sensible to compute the sensitivity once (before the first iteration) and to iterate toward the minimum in the cost function using the modified Newton-Raphson method, without re-computing the sensitivity. Model updating is appropriate to detect, localize, and quantify structural damage in the continuous monitoring phase, but in this paper, it is only used for model calibration.

### 2.2 Statistical Testing

Statistical tests cannot calibrate numerical models, but they can be used to detect and localize damage, and assess the POD and POL before damage occurs. In the simplest form, the statistical test can be written as in Eq. (10) and damage is detected by comparing the test statistic  $t$  against a safety threshold value  $t_{crit}$ , which was determined based on data from the undamaged structure. It is also possible to include the sensitivity matrix in the test leading to [7]

$$t = \hat{\boldsymbol{\zeta}}^T \boldsymbol{\Sigma}^{-1} \mathbf{J} \cdot (\mathbf{J}^T \boldsymbol{\Sigma}^{-1} \mathbf{J})^{-1} \cdot \mathbf{J}^T \boldsymbol{\Sigma}^{-1} \hat{\boldsymbol{\zeta}}, \quad (12)$$

where the bracket term is known as the Fisher information

$$\mathbf{F} = \mathbf{J}^T \boldsymbol{\Sigma}^{-1} \mathbf{J}. \quad (13)$$

The advantage of the formulation in Eq. (12) is that damage can be localized, simply by selecting one single column in the sensitivity matrix  $\mathbf{J}_h$  and applying the test for each parameter  $\theta_h$ . The parameter with the highest test response  $t_h$  beyond the safety threshold is the most likely to be damaged. This approach is known as the sensitivity test and may lead

to false localization alarms when parameters with similar sensitivities are tested, meaning an unchanged parameter can exhibit a strong test response beyond the safety threshold. This is because the test only considers the likelihood of changes in tested parameters but neglects the possibility of changes in untested partitions (parameters or clusters). A more accurate localization result can be achieved through the minmax test, which evaluates the minimum likelihood of changes in the tested parameter against the maximum likelihood of changes in untested partitions. Mathematically, this concept is implemented through a geometrical projection, which is explained in the following. The first step is to reorganize the sensitivity as follows  $\mathcal{J} = [\mathcal{J}_h \ \mathcal{J}_{\bar{h}}]$ , where  $\mathcal{J}_h$  is the vector of the tested parameter and  $\mathcal{J}_{\bar{h}}$  contains all untested partitions, and to project the residuals onto the tested and untested partitions through  $\zeta_h = \mathcal{J}_h^T \Sigma^{-1/2} \hat{\zeta}$  and  $\zeta_{\bar{h}} = \mathcal{J}_{\bar{h}}^T \Sigma^{-1/2} \hat{\zeta}$ , respectively. Then, the minmax test statistic can be written as

$$t_h^* = \zeta_h^{*T} F_h^{*-1} \zeta_h^* \quad (14)$$

where  $r_h^*$  is the projected residual

$$\zeta_h^* = \zeta_h - \mathbf{F}_{h\bar{h}} \mathbf{F}_{\bar{h}\bar{h}}^{-1} \zeta_{\bar{h}} \quad \text{with} \quad \begin{aligned} \mathbf{F}_{h\bar{h}} &= \mathcal{J}_h^T \Sigma^{-1} \mathcal{J}_{\bar{h}} \\ \mathbf{F}_{\bar{h}\bar{h}} &= \mathcal{J}_{\bar{h}}^T \Sigma^{-1} \mathcal{J}_{\bar{h}}, \end{aligned} \quad (15)$$

and  $F_h^*$  is the projected Fisher information

$$F_h^* = F_{hh} - \mathbf{F}_{h\bar{h}} \mathbf{F}_{\bar{h}\bar{h}}^{-1} \mathbf{F}_{\bar{h}h} \quad \text{and} \quad \begin{aligned} F_{hh} &= \mathcal{J}_h^T \Sigma^{-1} \mathcal{J}_h \\ F_{\bar{h}h} &= \mathcal{J}_{\bar{h}}^T \Sigma^{-1} \mathcal{J}_h. \end{aligned} \quad (16)$$

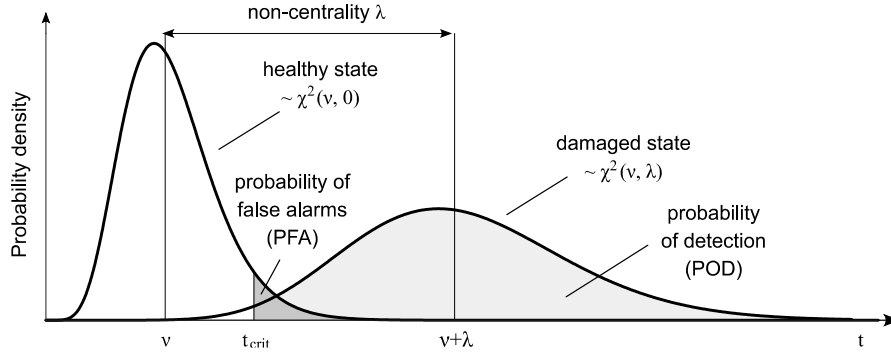
The more severe the damage extent the higher the test statistic  $t_h^*$ , but even undamaged structures lead to a test response unequal to zero. Figure 2 displays the test distribution of the detection test and more details on the distribution properties are given in the next section.

### 2.3 POD/POL Curves

The classical approach to determining the POD is to repeatedly apply the test statistic from Eq. (12), in a Monte Carlo experiment, and to count the relative number of test results beyond the safety threshold. For example, if the test is applied 100 times and 98 points are beyond the safety threshold, the empirical POD is 98%. The same can be applied to the localization test from Eq. (14), in which case the result would be the probability of localization (POL). The approach suggested in this paper is to avoid any Monte Carlo experiments, and to analyze the statistical properties of the tests instead. Then, it will become clear that the mean test response  $\lambda$  can be mathematically linked to hypothetical changes in the parameter vector  $\theta$ . For that purpose, the statistical properties of the tests have to be examined. Both considered tests can be approximated by a  $\chi^2$ -distribution, which is uniquely defined by the number of degrees of freedom  $\nu$  and the non-centrality  $\lambda$ . In the undamaged state, the non-centrality is zero and the number of degrees of freedom is identical to the mean value of the distribution, and the more severe the damage extent the larger  $\lambda$ . If it is assumed that damage is restricted to a single parameter  $\theta_h$ , the mean test response for this parameter can be calculated as [8]

$$\lambda_h = N(\theta_h - \theta_h^0)^2 F_h, \quad (17)$$

where  $F_h$  is the Fisher information from Eq. (13) or (16) depending on whether the detection or localization test is considered, respectively.



**Fig. 2.** Distribution of the test statistic for detection

Note that the mean test response from Eq. (17) can be predicted based on quantities that are available in the undamaged state. On close inspection of Fig. 2, it can be understood that the mean test response can directly be translated into an equivalent POD/POL. This is because the test distribution in the damaged state is uniquely defined through a  $\chi^2$ -distribution, and the POD/POL is equivalent to the complement of the cumulative distribution function evaluated at the safety threshold, i.e., the area under the PDF beyond the safety threshold

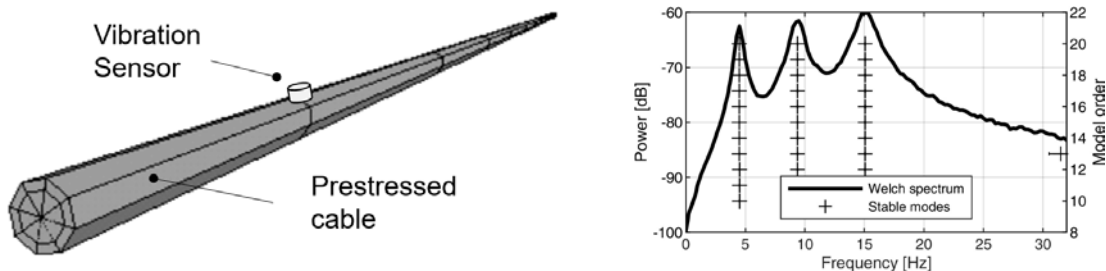
$$\text{POD}(\lambda) = \int_{t_{crit}}^{\infty} f_{\chi^2(v,\lambda)}(t) dt. \quad (18)$$

In summary, the POD from Eq. (18) depends on the mean test response  $\lambda_h$ , and the mean test response depends on hypothetical parameter changes  $\Delta\theta_h$ , see Eq. (17). That means there is a mathematical relation between the POD/POL and hypothetical parameter change. The most intuitive way to plot this relation are so-called POD/POL curves, which plot the parameter change on the x-axis and the corresponding POD/POL on the y-axis, see Fig. 5 and 6.

This section explained an approach to calibrate numerical models using model updating, to automatically assess the performance of the monitoring system, and to continuously monitor structures. One of the main take-away from this section is that sensitivity-based model updating and sensitivity-based statistical test are closely related. They form some sort of synergy effect as the sensitivity and covariance matrices can first be used for model calibration and then re-used for continuous monitoring without additional efforts.

### 3. Numerical Case Study on a Cable

This section contains a numerical proof of concept study for a cable with internal prestress. First, a sensitivity analysis is conducted to evaluate the ill-conditioning of the damage localization problem. Secondly, the numerical model is calibrated based on modal updating. Thirdly, POD curves and POL curves are constructed based on data from the undamaged state, and lastly, damage is applied to the cable to validate the accuracy of the curves.



**Fig. 3.** Prestressed cable in ANSYS with uni-axial vibration sensor (left) and stabilization diagram (right).



The structure under consideration is a 20 m-long cable with clamped supports on both sides, see Fig. 3. The cable has a circular cross-section with a radius of 5 cm and a material that is characterized by the mass density of 7,850 kg/m<sup>3</sup> and an E-modulus of 210,000 MN/m<sup>2</sup>. Since no real data is available, the ANSYS model is used for both the damage parametrization and the generation of vibration data. Cable structures are known to critically depend on the bending stiffness, the mass density, and the prestressing levels [3]; however, a sensitivity analysis (Section 1.2) shows that the sensitivity to changes in the mass density and prestress level is very similar, so masses are excluded from the analysis to better distinguish changes in prestress or stiffness. In this paper, the prestress level is simulated through an initial strain of 0.1% and the stiffness changes are modelled as changes in the cable's radius (e.g., due to strand failure), so the parameter vector is defined as

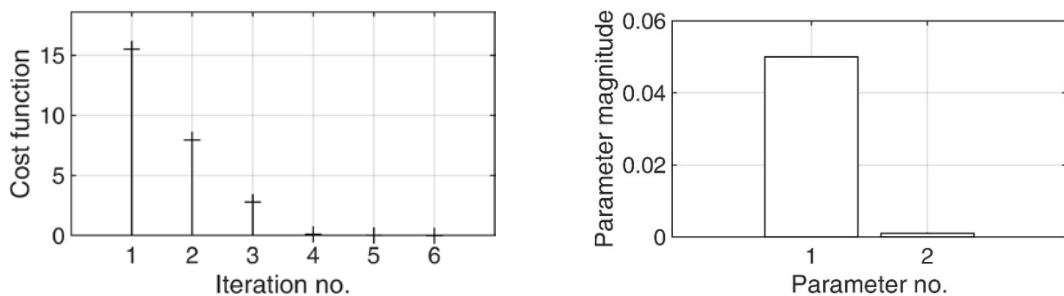
$$\boldsymbol{\theta}^0 = \begin{bmatrix} r \\ \varepsilon \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.001 \end{bmatrix}. \quad (19)$$

The instrumentation consists of a single measurement channel close to one end of the cable, see Fig. 3 (left) that measures vibration velocities. For excitation, normally distributed white noise excitation is applied to all 60 degrees of freedom. The synthetically generated vibration data  $\mathbf{y} \in \mathbb{R}^N$  has a measurement duration of  $T = 10$  min and a sampling frequency of  $f_s = 256$  Hz. To simulate measurement noise, the variance of the generated output signal  $\mathbf{y}$  is computed, and the signal is superimposed with uniformly distributed white noise with a standard deviation that corresponds to 5% of the signal's standard deviation. Operational modal analysis, or more precisely covariance-driven subspace system identification (SSI-Cov) [9], is applied to the synthetically generated measurements to extract natural frequencies, and the first three frequencies are employed as damage-sensitive features  $\hat{\mathbf{f}}$ , see Fig. 3 (right).

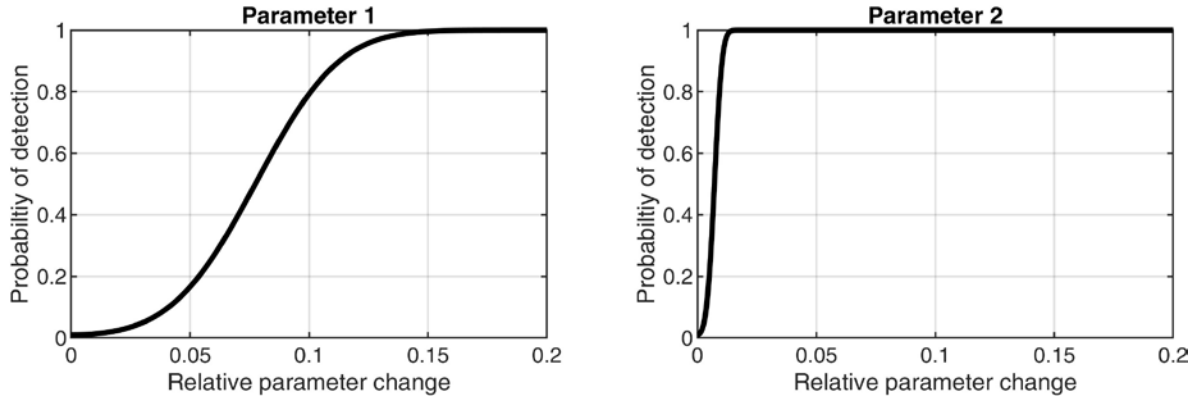
### 3.2 Model Updating

As for real monitoring applications, the first step is to calibrate the numerical model based on real measurement data. In this case, the reference configuration from Section 3.1 is run to extract the true natural frequencies  $\mathbf{f}^0$  of 4.50, 9.37, and 15.08 Hz. In all following considerations, those three frequencies are assumed to be extracted from real measurement data, so they reflect the true system state.

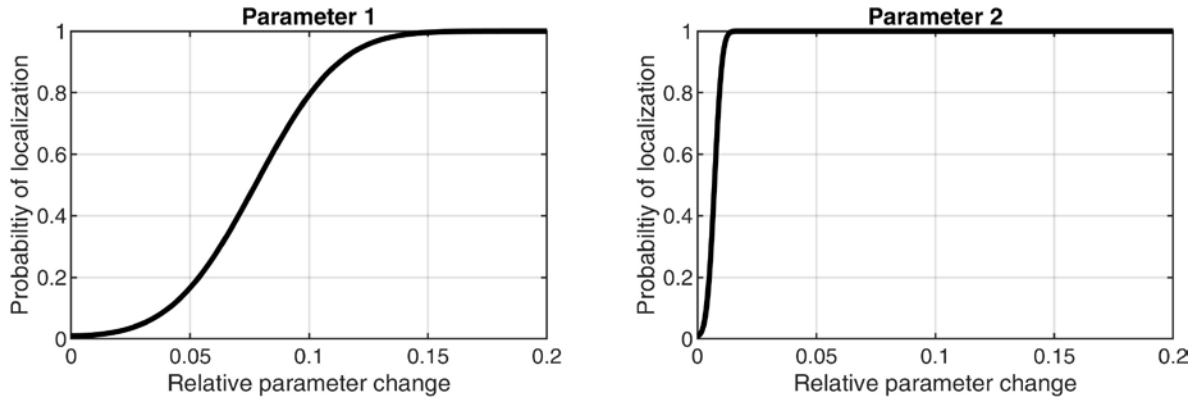
Subsequently, the prestress is changed to almost zero  $\varepsilon = 0.00001$  and the radius is modified from  $r = 5$  cm to  $r = 1$  cm (as if nothing was known about the cable's radius and prestress). The altered parameter vector  $\boldsymbol{\theta} = [0.01 \ 0.00001]$  leads to a different set of frequencies  $\mathbf{f}$ , so a residual vector is formed based on Eq. (1), and the model updating scheme from Section 2.1 is launched to identify the true parameters from the reference configuration. The optimization results are summarized in Fig. 4, which shows the number of iterations until the cost function drops below the user-defined threshold on the left side, and the final parameters on the right side, which are almost identical to the true parameter from Eq. (19).



**Fig. 4.** Cost function during iteration (left) and resulting structural parameters after updating (right).



**Fig. 5.** Probability of detection (POD) curves



**Fig. 6.** Probability of localization (POL) curves

### 3.3 POD/POL Curves

After the model calibration, the cable is continuously monitored based on statistical detection and localization tests. Before damage occurs, the developed approach can be applied to construct the POD curves in Fig. 5, and the POL curves in Fig. 6. It appears that the POD and the POL significantly vary depending on the examined structural parameter. For example, the already discussed 10% change leads to a POL of 79% for changes in the radius but almost 100% for changes in prestress, Fig. 6. A reliable damage diagnosis is given if the POD/POL is close to 100%. Following this logic, a parameter change in the cable's radius or prestress can be detected reliably if the relative change exceeds 15% and 1.5%, respectively. Reliable damage localization is possible for parameter changes of 15% and 1.5%, which is a meaningful result to structural designers and system operators.

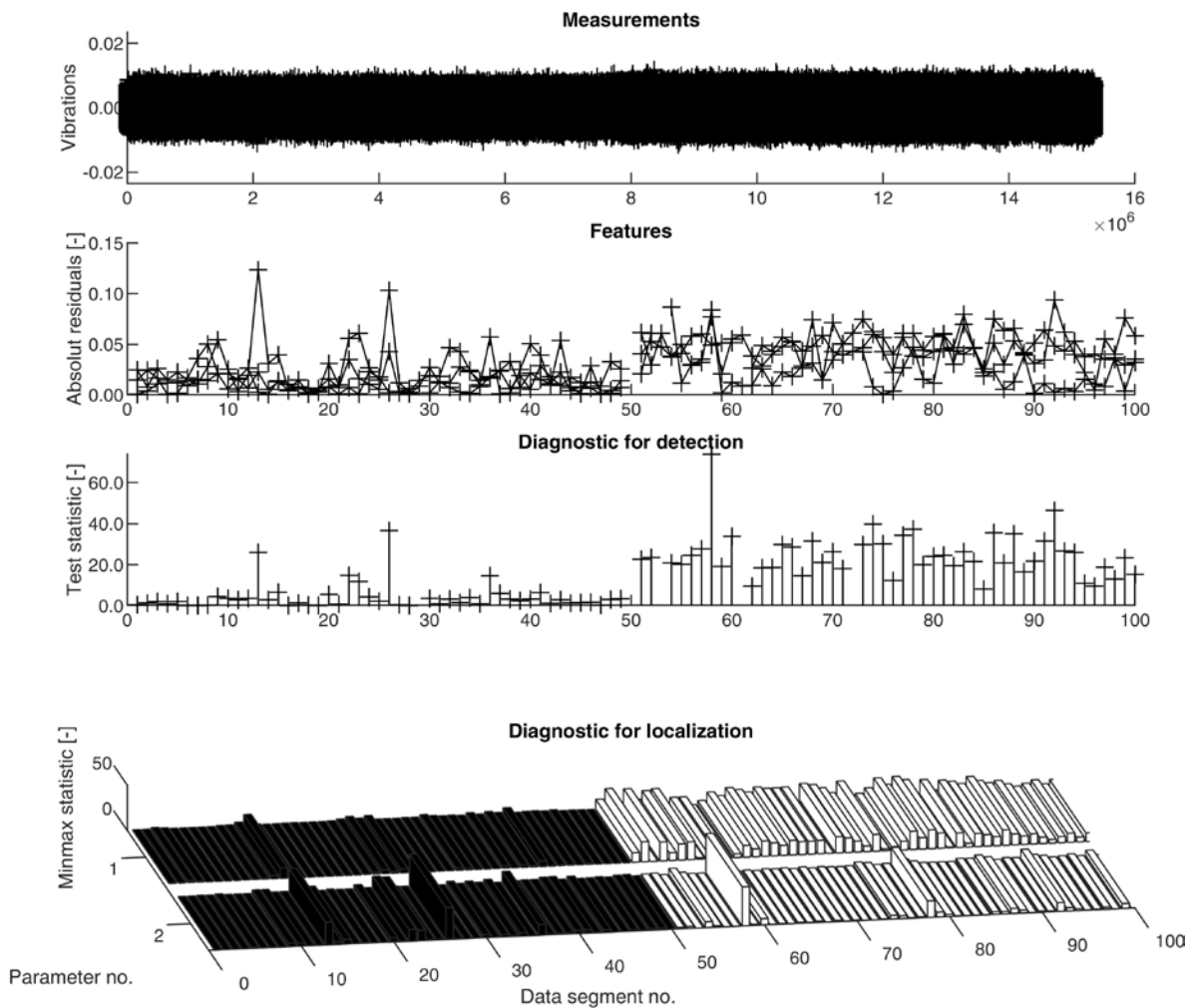
If this result is not satisfactory, several measures could be taken to improve it. Firstly, the sensor location could be optimized [10]. Secondly, the measurement duration could be increased to more than 10 min (which was used in this paper) as it is proportional to the test response  $\lambda_h$  from Eq. (17). Ultimately, sensors with a higher signal-to-noise ratio could be employed (as this affects the covariance matrix), or additional damage-sensitive features could be considered [11]. However, these topics are beyond the scope of this paper and the interested reader is referred to the references.

### 3.4 Statistical Testing

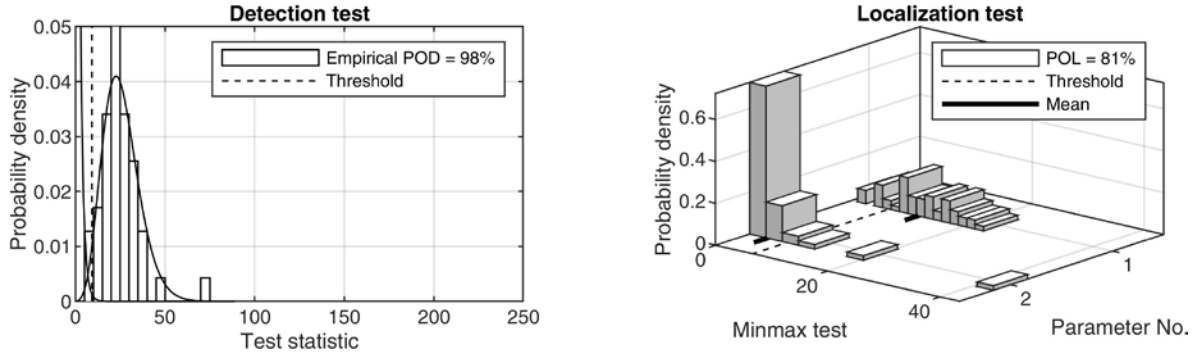
The previous section summarizes the POD/POL curves based on measured changes in the first two natural frequencies. In this section, data from the damaged state is used (for the first time in this study) to validate that the predictive POD/POL curves are accurate. Figure 7

summarizes the entire diagnosis chain where the considered damage scenario was a 10% decrease in the cable's radius. Where the vibration record indicates the damage through a very subtle increase in the vibration amplitudes, the damage-sensitive residuals (evaluated based on natural frequency changes) show a more pronounced change, and the test statistic from Eq. (12) summarizes the information from all residual entries in a single diagnostic value that clearly highlights the significance of the applied structural changes. Some of the plots show missing values, which occur if the natural frequencies could not be estimated and/or tracked. The lowest subplot in Fig. 7 visualizes the damage localization result.

Each data segment leads to a damage diagnosis results that could be interpreted individually, but a more reliable detection and localization result can be obtained by plotting the test distribution in histograms, as shown in Fig. 8. By counting the relative number of localization tests beyond the threshold, the empirical POL of 81% can be determined. Comparing this value to the predictive POL of 79.7% from Fig. 6 (left) demonstrates that the predictive POD is very close to the empirical POD, so the developed approach for POL curves is accurate. Equivalently, the validation could be performed for damage detection.



**Fig. 7.** Control charts for measurements, extracted features, detection results, and localization results for a 10% parameter change in parameter 1 that occurs after 50 data segments



**Fig. 8.** Distribution of the detection test (left) and the localization test (right) for a 10% parameter change in parameter 1

#### 4. Conclusion

The main contribution of this paper is a method to determine probability of detection (POD) and probability of localization (POL) curves for global vibration-based features, such as natural frequencies. The method requires an analytical model of the examined structure (e.g., a finite element model) and a feature vector that can be approximated through a normal distribution, but no data from the damaged state is required.

The method is based on sensitivity vectors, which are used for several purposes in this paper. They are mathematical tools to analyze the effect that changes in structural parameters have on changes in measurable features. By linearizing the relation, sensitivity vectors allow one to predict the test response of statistical damage detection and localization tests to future changes in structural parameters. In this paper, the relation is visualized through POD/POL curves that consider the statistical uncertainty in the feature estimation process. Sensitivity vectors are also helpful means to assess the ill-conditioning of the damage localization problem (and inverse modelling approaches in general). For example, if two sensitivity vectors are similar, i.e., if the cosine between them is equal to one, it is not possible to identify which parameter has changed through changes in measured response variables. In this sense, the notation of localizability is not necessarily defined as the localization of the damage event within the structure but the distinction of different types of parameter change, possibly in one and the same structural component (e.g., changes in prestressing forces and cross-sectional values in a cable). Cluster trees appear to be particularly efficient means to visualize the degree of ill-conditioning, and to decide on the redundant parameters that have to be removed or combined, which corresponds to a substructuring into damage localization units. Besides the assessment of ill-conditioning, sensitivity vectors are appropriate for model updating, damage detection and localization based on statistical tests. In this paper, the synergy between sensitivity-based model updating and sensitivity-based statistical testing is highlighted for the first time and incorporated into a comprehensive framework for cable monitoring.

A numerical case study is included for proof of concept. In this study, a uni-axial vibration sensor is placed on the cable and natural frequencies are evaluated based on ambient excitation and operational modal analysis (SSI-Cov). The developed methods are able to autonomously characterize the decisive dynamic properties of the cable based on model updating, and to monitor them continuously based on statistical tests. Before damage occurs, POD/POL curves can be generated to quantify the efficacy of the monitoring measure, which is critical information for the SHM engineer and the system operator. It is noteworthy that for the cable structure under consideration, it is possible to detect damage and assess the type

of damage (changes in prestress vs. cross-sectional values), but damage localization along the cable is only possible if additional features (e.g., from mode shapes) at multiple measurement locations along the cable is available. Another aspect that will be addressed in future publications is the removal of environmental effects on the extracted natural frequencies, e.g., due to temperature changes.

If there are limitations on the data transmission and storage capabilities, it is possible to preprocess the vibration signal on-site and merely transmit the evaluated frequencies. Ultimately, the small number of sensors and the small amount of data, combined with a high degree of automation and the minimum degree of human interaction makes this technique economic and suitable for the digitalization of cable properties and the embedment in digital representations.

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