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Abstract. Supply Chains have been more and more suffering from unexpected industrial, natural events, or epidemics that might disrupt the normal flow of materials, information, and money. The recent pandemic triggered by the outbreak of the new COVID-19 has pointed out the increasing vulnerability of supply chain networks, prompting companies (and governments) to implement specific policies and actions to control and reduce the spread of the disease across the network, and to cope with exogenous shocks. In this paper, we present a stochastic Susceptible-Infected-Susceptible (SIS) framework to model the spread of new epidemics across different distribution networks and determine social distancing/treatment policies in the case of local and global networks. We highlight the relevance of adaptability and flexibility of decisions in unstable and unpredictable scenarios.

Keywords: Networks, Stochastic Disruption Shocks, Stochastic Logistics, COVID-19

1 Introduction

Supply Chains (SCs) have been more and more suffering from unexpected industrial, natural events, or epidemics that might disrupt the normal flow of materials, information, and money [2][3][5]. Indeed, in recent years, studies on supply chain disruptions are getting increased attention to both academics and practitioners. Previous scholars (i.e. [6]) distinguished supply chain risks into operational and disruption risks. While the operational risks relate to ordinary issues in the SC operations (i.e. demand fluctuations), the disruption risks concern mainly events which occur with low frequency but high impacts [4] such as epidemic outbreaks. These are special category of risks in terms of duration (from middle to long term), high uncertainty, and ripple effects' propagation [5]. It has been observed that pandemics can threaten SC resilience and robustness. Resilience concerns the ability of SCs to recover their performance after having absorbed change, disturbance, and the disruption effects [4]. Robustness

refers to SCs' ability to maintain its planned performance after a disruption impacts [9]. Both impact on productivity performances. Throughout the history of public health, Cholera Pandemics (1817-1923), Spanish Flu (1918-1919), HIV/AIDS (1981-present), SARS (2002-2003), Ebola (2014-2016), and MERS (2015-present) are some of the most famous and brutal diseases that out-broke across international borders. On March 11, 2020, the World Health Organization officially declared COVID-19 a pandemic, causing 3.881.561 deaths until June 15, 2021 [14]. Scientists, policymakers, and managers all over the world have tried to forecast the pandemic evolution while at the same time keeping it under control by implementing specific policies to manage and reduce the spread of the disease. COVID-19 initially impacted China, which is at the center of many Global Value Chains, thus strong disruptions on supply chains raised. Moreover, the demand side has been affected by lockdown and consumers' physical spending increasing the challenges on the market. The COVID-19 outbreak re-exposes the importance of epidemic researches and the development of mathematical models to describe the behavior of epidemics [11]. Modeling describes the dynamic of epidemics and helps to take informed public health interventions [1]. Although previous research papers have successfully described the mechanism by which epidemics would spread, some control strategies (i.e., vaccination treatment, quarantine, social distancing, etc.) have been often neglected. In this paper, we contribute to the extant literature by adopting a modified Susceptible-Infectious-Susceptible (SIS) framework with a stochastic logistic-type formulation. In this way, we can consider exogenous and external events (i.e. the Indian COVID-19 variant) that might impact on the resilience policy, and thus on the productivity of a supply chain. Our results can help the global supply chain manager to understand the evolution of the epidemics and, therefore, determine the best counteractions to be put in place. The paper is structured as follows. Section 2 presents the Susceptible-Infected-Susceptible (SIS) Model. Section 3 points out the role played by stochastic shocks and Section 4 illustrates the shock propagation. In Section 5, we present the numerical simulation of our model and Section 6 concludes as usual.

2   The Susceptible-Infected-Susceptible Model

The Susceptible-Infected-Susceptible Model is one of the simplest and most widely used framework in mathematical epidemiology. It allows to describe the evolution of a number of infectious diseases which do not confer permanent immunity after recovery as in the case of COVID-19. If we denote by N the total population, by $I(t)$ the number of infected people, and by $S(t) = N - I(t)$ the number of susceptible ones, the model reads as:

$$\begin{cases} \dot{I}(t) = \alpha I(t)S(t) - \delta I(t) \\ \dot{S}(t) = \alpha I(t)S(t) + \delta I(t) \end{cases} \tag{1}$$

where α is the infection rate and δ is the recovery parameter. By doing the substitution $S(t) = N - I(t)$ the model boils down to:

$$\dot{I}(t) = \alpha I(t)(N - I(t)) - \delta I(t) \tag{2}$$

which is a Bernoulli differential equation whose solution is known and provided by the following expression:

$$I(t) = \frac{(1 - \frac{\delta}{\alpha})Ce^{(\alpha - \delta)t}}{1 + Ce^{(\alpha - \delta)t}} \tag{3}$$

where $C = \frac{1 - \frac{\delta}{\alpha} - I(0)}{I(0)}$ ([7]). The SIS model can be used to analyze the spread of common diseases, such as the seasonal flu and the common cold, but also of emerging diseases. This model also applies to the analysis of as COVID-19 since thus far there exists no evidence that people who have recovered from COVID-19 and have antibodies are protected from a second infection ([12,13,14]). In the following we suppose that the total population N is normalized to 1. The following Fig. 1 shows the behavior of COVID infected with the following parameters' values: $\alpha = 0.1328$ and the recovery rate $\delta = 0.0476$ (see [7]). In this scenario the amount of infected converges to a plateau representing the long run endemic equilibrium. Fig. 2, instead, shows the behavior of COVID-19 infected people with the following parameters' values: $\alpha = 0.1328$ and the recovery rate $\delta = 0.476$. This scenario corresponds to the case in which the adoption of treatment o vaccination campaigns produces an increment of the recovery parameter. As a result we can observe that the number of infected people gets reduced in the long run; we also notice that disease eradication is not possible in finite time.
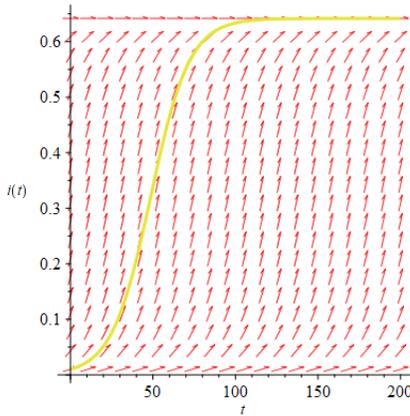

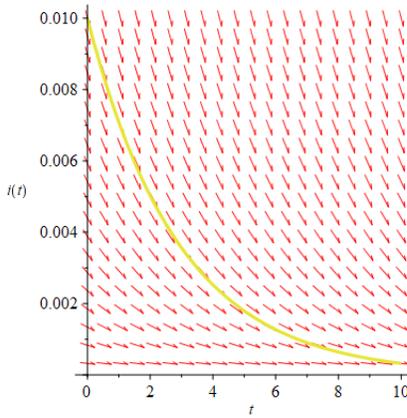**Fig. 1.** Deterministic evolution of the number of infected $I(t)$

Fig. 2. Deterministic evolution of the number of infected $I(t)$

3   SIS with Stochastic Shocks

The previous section presented a fully deterministic SIS model. In the following paragraph we make an effort to model the effects of exogenous shocks on the epidemic evolution in order to present a more realistic scenario. Therefore we suppose that the number of infected people is subject to exogenous shocks driven by a Wiener process $W(t)$ as follows:

$$dI(t) = [\alpha - \delta - \alpha I(t)] \, I(t)dt + \sigma I(t)dW(t), \quad I(0) = I^0 \tag{4}$$

Let us recall that a Wiener process is characterized by the following properties:

1. $W(0)$ is deterministic and given,
2. $W(t)$ has independent increments,
3. $W(s) - W(t)$ is normally distributed with zero mean and variance equal to $t - s$

Other stochastic processes could be considered as well. For instance Levy-type or jump processes could be used to model other possible non-continuous shocks. From the perspective of the extant literature, this model can be identified as the geometric stochastic Verhulst diffusion [10]. Verhulst work was built on a previous paper by Malthus [8] who was among the first to notice the existence of two different regimes in the growth of world population. Verhulst model has been at the heart of an interdisciplinary work by researcher coming from many different field. In this context the notion of deterministic equilibrium has to be replaced by the notion of steady state or stationary density. If we denote by $g[I(s), s; I(t), t]$ the probability density of $I(s)$ at time s, conditional upon its

value $I(t)$ at time t, then it is well known that g satisfies the Fokker-Planck equation, which reads as:

$$\frac{\partial g(I,t)}{\partial t} = -\frac{\partial \left(g(I,t)I(\alpha - \delta - \alpha I)\right)}{\partial I} + \frac{1}{2}\sigma^2 \frac{\partial^2 \left(g(I,t)I^2\right)}{\partial I^2}. \tag{5}$$

The steady state density $g(I(\infty), \infty, s; I(t), t)$ can be found by solving the stationary equation $\frac{\partial g(I,t)}{\partial t} = 0$. This yields to a second order ordinary differential equation for g whose solution is provided by:

$$g[I(\infty), \infty, s; I(t), t] = \frac{I^{d-1}e^{-cI}(c)^d}{\Gamma(d)} \tag{6}$$

which is the Gamma distribution. Mean and variance of this distribution are known and provided by $\frac{v-1}{c} = (\theta - \frac{\sigma^2}{2\alpha})$ and $\frac{\theta\sigma^2}{2\alpha} - \frac{\sigma^4}{4\alpha^2}$, respectively. Under the condition that $d = \frac{2(\alpha-\delta)}{\sigma^2} - 1 > 0$ the previous quantities are strictly positive.

Fig. 3 shows the stochastic behavior of COVID-19 infected people with the following parameters' values: $\alpha = 0.1328m$, $\delta = 0.0476$, and $\sigma^2 = 0.01$. This corresponds to the scenario in which the number of infected people fluctuates around an endemic equilibrium. Fig. 4 shows the behaviour of $I(t)$ with the following parameters' values: $\alpha = 0.1328m$, $\delta = 0.0476$, and $\sigma^2 = 0.05$. A greater value of the variance causes more amplified oscillations around the endemic equilibrium and thus more challenges for SC managers.
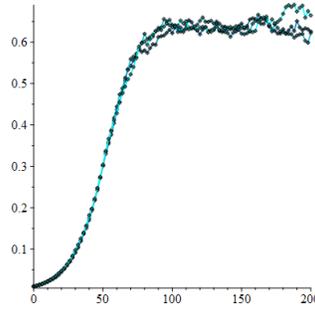
Fig. 3. Stochastic evolution of the number of infected $I(t)$

4 Shock Propagation on a Network

As we are interested in analyzing the epidemic propagation over a supply chain we refer to a network that is modeled by a graph G, composed by N different nodes x_i, $i = 1...N$. Each pair of nodes (i,j) can or cannot be connected through an edge γ_{ij}. γ_{ij} will be zero if the nodes are disconnected and a positive number
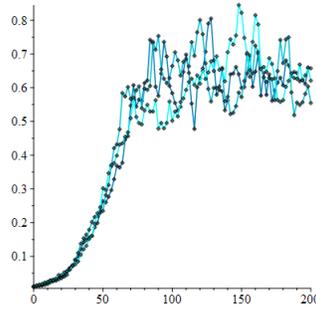
Fig. 4. Stochastic evolution of the number of infected $I(t)$

when the nodes are connected, with the number itself providing the linking intensity. At each node $i \in G$, the total number of infected people is described by:

$$d\dot{I}_i(t) = \left(\alpha_i - \delta_i - \alpha_i I_i(t) + \sum_{j \neq i} \gamma_{ij} I_j(t) \right) I_i(t)dt + \sigma_i I_i(t)dW_i(t), \quad (7)$$

with initial conditions $I_i(t_0) = I_i^0$. The above system of N stochastic differential equations describes the spread of the epidemic across the network. The amount of infected people at the node i grows as consequence of two effects:

1. the local spread of the epidemics,
2. the immigration of infected people moving from the other nodes j, $j \neq i$, to the node i

The amount of infected is also subject to exogenous shocks, all of them driven by a Geometric Wiener Process W_i where σ_i is the volatility term and the co-variance is given by:

$$\mathrm{E}(dW_i(t)dW_j(t)) = \rho_{i,j} \quad (8)$$

where $\rho_{i,i} = 1$. The spread of the epidemic causes a loss of productivity. If we define by θ_i, $i = 1...N$ the per-capita productivity at the node i, the total loss of productivity $L(t)$ is given by:

$$L(t) = -\sum_{i=1} \theta_i I_i(t) \quad (9)$$

subject to

$$d\dot{I}_i(t) = \left(\alpha_i - \delta_i - \alpha_i I_i(t) + \sum_{j \neq i} \gamma_{ij} I_j(t)\right) I_i(t)dt + \sigma_i I_i(t)dW_i(t), \quad I_i(t_0) = I_i^0$$

(10)

L is a stochastic process that describes the loss of productivity over time.

When a strict lockdown policy is put in place each node of the network is isolated and, therefore, we can assume that $\gamma_{ij} = 0$. We can also suppose that the Wiener processes W_i are independent as the nodes are totally disconnected. In this scenario the above system boils down to:

$$d\dot{I}_i(t) = (\alpha_i - \delta_i - \alpha_i I_i(t)) I_i(t)dt + \sigma_i I_i(t)dW_i(t), \quad I_i(t_0) = I_i^0 \qquad (11)$$

5   Numerical Simulations

As the number of infected people can affect the productivity level of a supply chain, our model enables decision makers to better understand the impact of lockdown measures. Through a numerical simulation we provide a visual representation of different scenarios. Indeed, the numerical simulations compare the behavior of the number of infected people over medium and large size networks. We consider two scenarios, which correspond either to the presence or to the absence of lockdown restrictions. We also report the average behavior and thus the impact on the supply chain networks. In particular, Figs. 5 and 6 show the behavior over a medium size network with 11 nodes. One can immediately observe that the absence of lockdown restrictions allows internal flows among the different nodes thus it increases, on average, the number of infected people even in presence of exogenous shocks (negative or positive) and localized treatment policies. The same conclusion is supported by Figs. 7 and 8 that show the behavior over a large network with 50 nodes.

This numerical simulation shows the effects of network connectivity on the spread of the disease at the global level. As the spread of exogenous shocks across the network might become relevant and not controllable in the case of connected networks. Thus it is crucial to intervene combing flow barriers between different nodes and local intervention policies. In other words, connectivity might compromise the benefits of implementing local vaccination campaigns.

6   Conclusion

As the virus spread and most governments imposed lockdown orders, supply chain disruptions increased. Indeed, COVID-19 illustrated that many companies are not fully aware of the vulnerability of their supply chain relationships to global shocks. SC managers need to balance and combine actions to serve

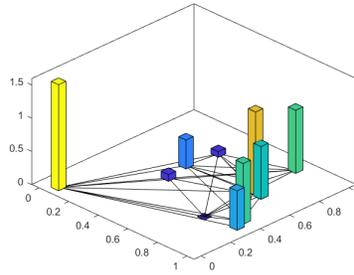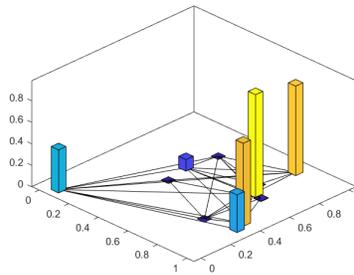**Fig. 5.** Evolution of infected people over a connected network with 11 nodes.

Fig. 6. Evolution of infected people over a disconnected network with 11 nodes.

Fig. 7. Evolution of infected people over a connected network with 50 nodes.

Fig. 8. Evolution of infected people over a connected network with 50 nodes.
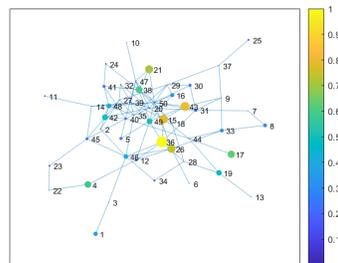
their customers, as well as protect and support their workers. In this paper, we aim at analyzing the stochastic effects of the epidemic spread on a supply chain network. We present a stochastic SIS model which assumes the form of a stochastic logistic differential equation. Exogenous shocks are modeled by means of a stochastic Wiener process. We present a numerical simulation and we draw insights to support local supply chain managers to decide about the social distancing policy: he/she can take into account costs, governmental policies, and infection parameters. We also discuss the flow of infected people from one node to another and we provide some results to control the spread of the epidemics across the network. These results can help the global supply chain manager to understand the evolution of the epidemic and, therefore, determine the best counteractions to put in place: it is evident that lockdown policies and treatment measures have to coexist. Employees around the global supply chain need to receive the vaccine on the same timescale to ensure the best results for everyone. Further research involves the design of a stochastic optimal control model able to identify the best compromise between economic costs of lockdown restrictions and implementation costs of vaccination campaigns.

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