



HAL
open science

Geometric Error Modeling and Sensitivity Analysis of a Laser Pipe-Cutting System Based on Lie Group and Sobol Method

Yuze Jiang, Wenyu Yang, Liang Qin, Tong Ding

► **To cite this version:**

Yuze Jiang, Wenyu Yang, Liang Qin, Tong Ding. Geometric Error Modeling and Sensitivity Analysis of a Laser Pipe-Cutting System Based on Lie Group and Sobol Method. IFIP International Conference on Advances in Production Management Systems (APMS), Sep 2021, Nantes, France. pp.465-472, 10.1007/978-3-030-85910-7_49 . hal-03806496

HAL Id: hal-03806496

<https://inria.hal.science/hal-03806496v1>

Submitted on 7 Oct 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License



This document is the original author manuscript of a paper submitted to an IFIP conference proceedings or other IFIP publication by Springer Nature. As such, there may be some differences in the official published version of the paper. Such differences, if any, are usually due to reformatting during preparation for publication or minor corrections made by the author(s) during final proofreading of the publication manuscript.

Geometric error modeling and sensitivity analysis of a laser pipe-cutting system based on Lie group and Sobol method

Yuze Jiang, Wenyu Yang, Liang Qin, Tong Ding

School of Mechanical Science and Engineering, Huazhong University of Science and Technology, 430074 Wuhan, China
mewyang@hust.edu.cn

Abstract. Laser pipe-cutting system, a special machine tool, has been widely used in the precision machining of metal pipe. The geometric errors remarkably affect the machining accuracy of products. Error modeling and sensitivity analysis are key issues to improve the product quality. In this paper, the geometric error model of the laser pipe-cutting system which contains 70 geometric errors is established based on multi-body theory and Lie group. The coupling effects caused by two chucks are considered as the spatial angular deviation in modeling. The sensitivity analysis for the geometric error model is conducted to identify the essential sensitivity errors based on an improved Sobol method with quasi-Monte Carlo algorithm. The results show that not only the linear positioning errors, but also the squareness errors and parallelism errors play crucial roles in the machining accuracy. Based on the result, the essential sensitivity errors are calibrated and the machining accuracy is improved.

Keywords: Lie group, geometric error model, Sobol method, error sensitivity analysis.

1 Introduction

Precision pipes produced by laser cutting are widely used in medical equipment, aerospace and other fields. The feeding accuracy of the pipe feeding system plays an important role in the product quality. Considered as a multi-body system, the geometric error model of the laser pipe-cutting system needs to be established to predict the deviation between the desired and actual position of the working point.

Over the past decades, scholars have employed many approaches in the geometric error modeling, and got many noteworthy achievements. Abbaszadeh-mir [1] used homogeneous transform matrix (HTM) to build the geometric error model of a five-axis machine tool, and identified the sensitive errors based on the sensitivity Jacobian matrix. Zhu [2] established the geometric error model of a five-axis machine tool considered of the PIGEs and PDGEs based on HTM. To express the kinematic chain more clearly, the Lie group [3] is introduced to formulate the geometric error model which only needs to establish the reference frame and end effector frame. Cheng [4] studied

the geometric error model of a five-axis machine tool based on POE model and identified the essential errors with Morris sensitivity analysis. Xiang [5] established a novel method to represent PIGEs and PDGEs using POE model, which was applied to the five-axis machine. Also, the Lie group describes the motion according to its geometric property. Hence, the Lie group theory is suitable for solving kinematic problems of multi-degree-of-freedom complex machinery, and is widely used.

Based on the geometrical error model, sensitivity analysis can estimate the influences of each errors on machining accuracy and identify the critical errors. Common sensitivity analysis methods can be determined as local sensitivity analysis and global sensitivity analysis. As a well-known global sensitivity analysis approach, Sobol method [6] is applied to the sensitivity analysis for complex multi-body system, which can estimate the contribution and coupling effect of errors in the whole workspace. Zou [7] applied the Sobol method to a three-axis diamond turning machine, and identified the crucial errors between 18 geometric errors. Xia [8] established the geometric error model of a gear grinding machine tool and analyzed the error sensitivity based on Sobol method, which reduced the tooth surface errors by 68.75%.

Few researches of geometrical error modeling have been made about the machine tools with two chucks or two spindles, except Liu [9]. In [9], he considered the dual-spindle ultra-precision drum roll lathe as a combination of a five-axis machine tool and a virtual C' axis which was ignored in the error modeling. Therefore, the aim of this paper is (I) to establish the geometrical error model of the laser pipe-cutting system considering the coupling effect between two chucks, (II) to identify the essential sensitivity errors for the calibration.

The rest of the paper is organized as follows. In Section 2, the geometrical error model of the laser pipe-cutting system is established based on the topological structure and Lie group. The global sensitivity analysis of the error model is implemented to identify the essential sensitivity errors using an improved Sobol method in Section 3. Finally, the conclusions are presented in Section 4.

2 Geometric error modeling based on Lie group

The pose of the multi-body system with respect to the base frame can be described by Lie group theory. For an open chain with n-dof, the forward kinematic can be written as:

$$T = e^{\hat{\xi}_1 \theta_1} \dots e^{\hat{\xi}_n \theta_n} \cdot T(0) \quad (1)$$

where $\hat{\xi}_n$ and θ_n represent the twist and joint variable of the n th joint. $T(0)$ is the initial transformation matrix. The equation (1), which is known as POE model, can be used in error modeling of multi-body system. More details can be referenced to Dai [10].

In classical machine tool modeling, the two open-chain kinematic loops, which are workpiece and tool kinematic loop, are closed at the tool-workpiece contact point. The geometric error model is established based on the relative position and orientation of the contact point between two loops. The schematic diagram of the laser pipe-cutting system is illustrated in Fig.1. In the system we introduced, the main chuck and feeding

chuck clamp the pipe simultaneously, which means the workpiece kinematic loop is not a serial chain. According to the characteristics above, the workpiece kinematic model is divided in two levels. The first level is the transformer from the bed to the jaws, named as $0-1-2-3.N$ and $0-4-5.N$, which computes the clamping concentricity of two chucks. Based on the concentricity errors, the pose of pipe is deduced to estimate the actual location of the working point as the second level.

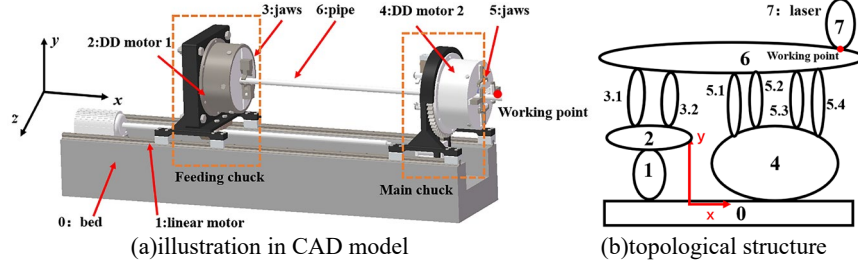


Fig. 1. Structural diagram of the laser pipe-cutting system

As Fig.1 shown, the main chuck with 4 orthogonal jaws is fixed on the bed. The feeding chuck with only 2 jaws is located on a linear motor which provides the motion in X-direction as the feeding process. The jaws move to the central axis until the pipe is clamped by two chucks simultaneously. The twists of each part related to the machine frame are as follow:

$$\begin{aligned}
 \xi_{01} &= [0 \ 0 \ 0 \ 1 \ 0 \ 0]^T \\
 \xi_{12} &= [1 \ 0 \ 0 \ 0 \ 0 \ 0]^T \\
 \xi_{23.1} &= \xi_{23.2} = [0 \ 0 \ 0 \ 0 \ 1 \ 0]^T \\
 \xi_{04} &= [1 \ 0 \ 0 \ 0 \ 0 \ 0]^T \\
 \xi_{45.1} &= \xi_{45.2} = [0 \ 0 \ 0 \ 0 \ 1 \ 0]^T \\
 \xi_{45.3} &= \xi_{45.4} = [0 \ 0 \ 0 \ 0 \ 0 \ 1]^T
 \end{aligned} \tag{2}$$

where $\xi_{23.1}, \xi_{23.2}$ are the twists of jaws on the feeding chuck. $\xi_{45.1}, \xi_{45.2}, \xi_{45.3}, \xi_{45.4}$ are the twists of jaws on the main chuck. Based on the design of opposing-jaw linkage, the relationships between jaw motion variables are $Y_{23.1} = -Y_{23.2}, Y_{45.1} = -Y_{45.2}, Z_{45.3} = -Z_{45.4}$.

According to Dai [10], the ideal POE models of each jaws are obtained as:

$$\begin{aligned}
 P_{03.1} &= e^{\xi_{01}X_{01}} \cdot e^{\xi_{12}\theta_{12}} \cdot e^{\xi_{23.1}Y_{23.1}} \cdot P_{03.1}(0) \\
 P_{03.2} &= e^{\xi_{01}X_{01}} \cdot e^{\xi_{12}\theta_{12}} \cdot e^{\xi_{23.2}Y_{23.2}} \cdot P_{03.2}(0) \\
 P_{05.1} &= e^{\xi_{04}\theta_{04}} \cdot e^{\xi_{45.1}Y_{45.1}} \cdot P_{05.1}(0) \\
 P_{05.2} &= e^{\xi_{04}\theta_{04}} \cdot e^{\xi_{45.2}Y_{45.2}} \cdot P_{05.2}(0) \\
 P_{05.3} &= e^{\xi_{04}\theta_{04}} \cdot e^{\xi_{45.3}Y_{45.3}} \cdot P_{05.3}(0) \\
 P_{05.4} &= e^{\xi_{04}\theta_{04}} \cdot e^{\xi_{45.4}Y_{45.4}} \cdot P_{05.4}(0)
 \end{aligned} \tag{3}$$

According to the definition from both ISO 230-1 and ISO 230-7, 70 error components of PIGEs (position-independent geometric errors) and PDGE (position-dependent geometric errors) are taken into consideration, as shown in Table 1.

Table 1. Error components of pipe feeding system

Adjacent bodies	PDGEs	PIGEs	
		Squareness errors	Parallelism errors
0-1	$\delta_{xX01}, \delta_{xY01}, \delta_{xZ01}$ $\varepsilon_{xX01}, \varepsilon_{xY01}, \varepsilon_{xZ01}$	-	-
1-2	$\delta_{xX12}, \delta_{xY12}, \delta_{xZ12}$ $\varepsilon_{xX12}, \varepsilon_{xY12}, \varepsilon_{xZ12}$	-	$\kappa_{xZ12}, \kappa_{xY12}$
2-3.1	$\delta_{yX23.1}, \delta_{yY23.1}, \delta_{yZ23.1}$ $\varepsilon_{yX23.1}, \varepsilon_{yY23.1}, \varepsilon_{yZ23.1}$	$\beta_{yX23.1}, \beta_{yZ23.1}$	-
2-3.2	$\delta_{yX23.2}, \delta_{yY23.2}, \delta_{yZ23.2}$ $\varepsilon_{yX23.2}, \varepsilon_{yY23.2}, \varepsilon_{yZ23.2}$	$\beta_{yX23.2}, \beta_{yZ23.2}$	-
0-4	$\delta_{xX04}, \delta_{xY04}, \delta_{xZ04}$ $\varepsilon_{xX04}, \varepsilon_{xY04}, \varepsilon_{xZ04}$	-	$\kappa_{xZ04}, \kappa_{xY04}$
4-5.1	$\delta_{yX45.1}, \delta_{yY45.1}, \delta_{yZ45.1}$ $\varepsilon_{yX45.1}, \varepsilon_{yY45.1}, \varepsilon_{yZ45.1}$	$\beta_{yX45.1}, \beta_{yZ45.1}$	-
4-5.2	$\delta_{yX45.2}, \delta_{yY45.2}, \delta_{yZ45.2}$ $\varepsilon_{yX45.2}, \varepsilon_{yY45.2}, \varepsilon_{yZ45.2}$	$\beta_{yX45.2}, \beta_{yZ45.2}$	-
4-5.3	$\delta_{zX45.3}, \delta_{zY45.3}, \delta_{zZ45.3}$ $\varepsilon_{zX45.3}, \varepsilon_{zY45.3}, \varepsilon_{zZ45.3}$	$\beta_{zX45.3}, \beta_{zY45.3}$	-
4-5.4	$\delta_{zX45.4}, \delta_{zY45.4}, \delta_{zZ45.4}$ $\varepsilon_{zX45.4}, \varepsilon_{zY45.4}, \varepsilon_{zZ45.4}$	$\beta_{zX45.4}, \beta_{zY45.4}$	-

With the errors in Table.1, the actual POE models of the jaws can be written as [4]:

$$\begin{aligned}
P_{03.1}^e &= e_{PDGE}^{\xi_{01}} e^{\xi_{01}X_{01}} \cdot e_{PIGE}^{\xi_{12}} e_{PDGE}^{\xi_{12}} e^{\xi_{12}\theta_{12}} \cdot e_{PIGE}^{\xi_{23.1}} e_{PDGE}^{\xi_{23.1}} e^{\xi_{23.1}Y_{23.1}} \cdot P_{03.1}(0) \\
P_{03.2}^e &= e_{PDGE}^{\xi_{01}} e^{\xi_{01}X_{01}} \cdot e_{PIGE}^{\xi_{12}} e_{PDGE}^{\xi_{12}} e^{\xi_{12}\theta_{12}} \cdot e_{PIGE}^{\xi_{23.2}} e_{PDGE}^{\xi_{23.2}} e^{\xi_{23.2}Y_{23.2}} \cdot P_{03.2}(0) \\
P_{05.1}^e &= e_{PIGE}^{\xi_{04}} e_{PDGE}^{\xi_{04}} e^{\xi_{04}\theta_{04}} \cdot e_{PIGE}^{\xi_{45.1}} e_{PDGE}^{\xi_{45.1}} e^{\xi_{45.1}Y_{45.1}} \cdot P_{05.1}(0) \\
P_{05.2}^e &= e_{PIGE}^{\xi_{04}} e_{PDGE}^{\xi_{04}} e^{\xi_{04}\theta_{04}} \cdot e_{PIGE}^{\xi_{45.2}} e_{PDGE}^{\xi_{45.2}} e^{-\xi_{45.2}Y_{45.1}} \cdot P_{05.2}(0) \\
P_{05.3}^e &= e_{PIGE}^{\xi_{04}} e_{PDGE}^{\xi_{04}} e^{\xi_{04}\theta_{04}} \cdot e_{PIGE}^{\xi_{45.3}} e_{PDGE}^{\xi_{45.3}} e^{\xi_{45.3}Z_{45.1}} \cdot P_{05.3}(0) \\
P_{05.4}^e &= e_{PIGE}^{\xi_{04}} e_{PDGE}^{\xi_{04}} e^{\xi_{04}\theta_{04}} \cdot e_{PIGE}^{\xi_{45.4}} e_{PDGE}^{\xi_{45.4}} e^{-\xi_{45.4}Z_{45.1}} \cdot P_{05.4}(0)
\end{aligned} \tag{4}$$

Based on the equations (3) and (4), the position error of each jaw can be obtained as:

$$E_{0N} = [E_{xN} \ E_{yN} \ E_{zN} \ 0]^T = P_{0N}^e - P_{0N} \tag{5}$$

where N represents the number of N -th jaw. E_x, E_y, E_z stand for the position errors in each direction. According to equation (5), the concentricity error can be deduced. As for the feeding chuck, a pair of opposite jaws, with the errors $E_{03.1}, E_{03.2}$, clamp the pipe. The concentricity error of the feeding chuck is deduced as:

$$E_{x03} = \frac{E_{x03.1} + E_{x03.2}}{2}; E_{y03} = \frac{E_{y03.1} + E_{y03.2}}{2}; E_{z03} = \frac{E_{z03.1} + E_{z03.2}}{2} \quad (6)$$

As for the main chuck, two pairs of opposite jaws orthogonally, with the errors $E_{05.1}, E_{05.2}, E_{05.3}, E_{05.4}$, clamp the pipe. The tips of the main chuck jaws are bearings, which can not apply constraints in X-direction to the pipe. The concentricity error of the main chuck is:

$$E_{y05} = \frac{E_{y05.1} + E_{y05.2}}{2}; E_{z05} = \frac{E_{z05.3} + E_{z05.4}}{2} \quad (7)$$

According to (6) and (7), the spatial angular deviations of the pipe along Y- and Z-axis can be written as:

$$\theta_y = \frac{E_{z05} - E_{z03}}{L_{04} - L_{23} - L_{01} - X}; \theta_z = \frac{E_{y05} - E_{y03}}{L_{04} - L_{23} - L_{01} - X} \quad (8)$$

Then the actual position of working point is:

$$P'_W = Trans(E_{x03}, E_{y03}, E_{z03}) \cdot e^{\hat{\xi}_z \theta_z} \cdot e^{\hat{\xi}_y \theta_y} \cdot P_W \quad (9)$$

The geometric error model of the laser pipe-cutting system can be formulated as:

$$E = P'_W - P_W = [E_x \quad E_y \quad E_z \quad 0]^T \quad (10)$$

The geometric error model established in this section is the theoretical basis for the sensitivity analysis and error calibration.

3 Error sensitivity analysis based on improved Sobol method

Sobol method is one kind of global sensitivity analysis method, which is suitable for not only linear model but also nonlinear model. In this section, an improved Sobol method based on quasi-Monte Carlo sampling is applied to identify the essential error components for the laser pipe-cutting system. As defined in [11], the computation process can be formulated as:

$$\begin{aligned} V_i &= \frac{1}{k} \sum_{m=1}^k f(B)_m \left(f(A_B^{(i)})_m - f(A)_m \right) \\ V_{Ti} &= \frac{1}{2k} \sum_{m=1}^k \left(f(A)_m - f(A_B^{(i)})_m \right)^2 \\ V_{Ti} &= \frac{1}{2k} \sum_{m=1}^k \left(f(A)_m - f(A_B^{(i)})_m \right)^2 \\ S_i &\approx \frac{V_i}{V} \quad S_{Ti} \approx \frac{V_{Ti}}{V} \end{aligned} \quad (11)$$

where A, B are the sampling matrices needed for the quasi-Monte Carlo algorithm which represent the random input of geometric errors. $f(A \text{ or } B)$ is the output function based on the error model established in Section 2. V_i and V_{Ti} represent the variance of the first-order and total effect of the i -th geometric error component, respectively. S_i and S_{Ti} are the first order and total sensitivity coefficient of the volumetric error model.

The geometric errors are divided into five types, and the sample ranges of each type are given in Table 2. The number of sampling point for each error is set to 20000. Hence, A and B are constructed as 20000×70 sampling matrices. Based on the assumption of small displacement, the sum of first-order sensitivity coefficient is close to 1. For 70 errors, the average sensitivity coefficient is $1/70 \approx 0.015$. In our analysis, we define the essential sensitivity error as the component whose sensitivity coefficient is larger than 0.03.

Table 2. The ranges of all geometric error components

Error types	Error ranges
Linear positioning errors	$[-10\mu\text{m}, 10\mu\text{m}]$
Straightness errors	$[-1\mu\text{m}, 1\mu\text{m}]$
Angular errors	$[-5\mu\text{rad}, 5\mu\text{rad}]$
Squareness errors	$[-5\mu\text{rad}, 5\mu\text{rad}]$
Parallelism errors	$[-5\mu\text{rad}, 5\mu\text{rad}]$

The sensitivity analysis of geometric errors will be discussed under the conditions below. The distances of jaws' motion are set as, $Y_{23.1} = 20, Y_{45.1} = 20, Z_{45.3} = 20$, which represent the jaws clamping the pipe. The DD motors are fixed at the initial degree, $\theta_{12} = \theta_{04} = 0$. The position of feeding truck is varied considered as the feeding motion, $X=100,450,800\text{mm}$.

The results of sensitivity analysis of the feeding system are shown in Fig 2. According to the Fig 2, S_{Ti} of each error component is slightly larger than S_i but almost equal. The result above means that the coupling effect between various errors is small and can be ignored. So, the discussion about sensitivity is based on the analysis of S_i .

The first, second and last three columns show the sensitivity coefficients of each error component along X, Y and Z direction when $X=100,450,800\text{mm}$. For E_X , the linear positioning error of the linear motor plays an absolutely important role which the sensitivity efficient is above 0.40. The reason is that the linear motor controls the feeding along X direction directly, and the main chuck can not apply the constraint to the pipe in axis direction. The straightness errors of feeding-chuck jaws are also essential errors, which represent the clamp error of the feeding chuck.

E_Y and E_Z , which are considered as the radial errors, depend on the distance between the pipe surface and the laser. As the second and last three columns of Fig 2 shown, the linear positioning errors of jaws are the main errors. This is clearly because the pipe pose is depended on the jaws in YoZ plane. Also, the straightness and angular errors of jaws are essential sensitivity errors because the jaws are arranged orthogonally. Compared with E_X , the squareness and parallelism errors of E_Y and E_Z are important factors. As the feeding process continued ($X=100,450,800\text{mm}$), the sensitivity coefficients of linear positioning and angular errors of jaws, the squareness and parallelism errors

between aligned axis are increased. The distance between main chuck and feeding chuck is closer, and higher accuracy of the clamp is needed.

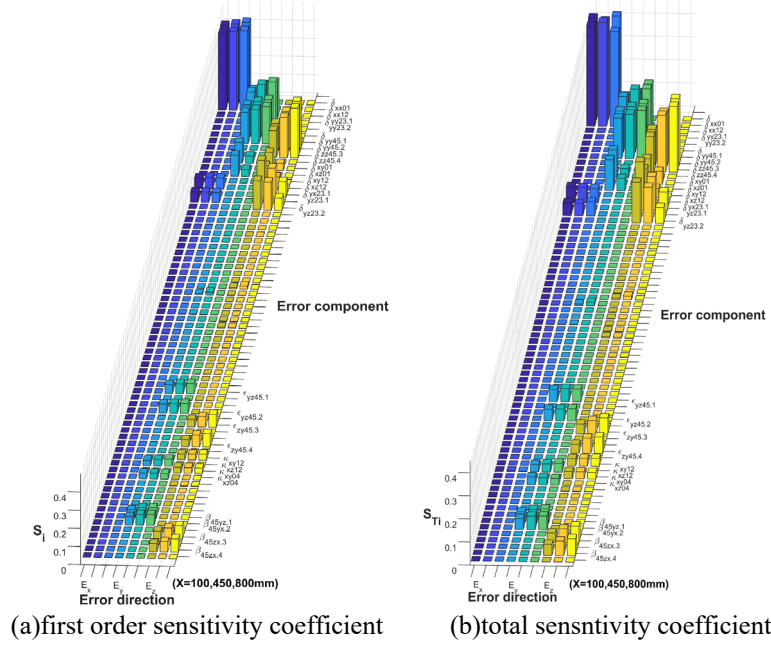


Fig. 2. Sensitivity coefficient of each error component along X direction motion($X=100,450,800\text{mm}$)

Based on the geometric error model and sensitivity analysis, we choose the linear motors with higher feed accuracy. The transmission mechanism of the jaws is redesign, including the feeding accuracy of actuators, the tolerance between the jaws and the slideway. The parts are assembled with multiple adjustments to guarantee the squareness and parallelism errors at a minimum value. As a consequence, the machining quality of the laser cutting system is improved.

4 Conclusion

In this paper, the geometric error model of a laser pipe-cutting system based on Lie group is established. The coupling effect of clamps by two chucks are involved in modeling considered as the angular deviation. To identify the essential sensitivity errors, an improved Sobol method is imposed to compute the sensitivity coefficients of 70 errors along the feeding direction. The results indicate that the concentricity of each chucks caused by jaws play an important part in the position deviation of pipe. The squareness and parallelism errors are also essential which can not be ignored.

Based on the analysis, the accuracy of product is improved. In the future, the optimization of the clamping force and stiffness will become the research topic considering the deformation of pipe.

References

1. Abbaszadeh-Mir Y., Mayer J. R. R., Cloutier G.: Theory and simulation for the identification of the link geometric errors for a five-axis machine tool using a telescoping magnetic ball-bar. *International Journal of Production Research* 40(18),4781-4797 (2002).
2. Zhu S., Ding G., Qin S.: Integrated geometric error modeling, identification and compensation of CNC machine tools. *International Journal of Machine Tools & Manufacture* 52(1),24-29 (2012).
3. Murray R., Sastry S., Li Z.: *A Mathematical Introduction to Robotic Manipulation*. CRC Press, Inc. (1994).
4. Cheng Q., Feng Q., Liu Z.: Sensitivity analysis of machining accuracy of multi-axis machine tool based on POE screw theory and Morris method. *The International Journal of Advanced Manufacturing Technology* 84(9-12),2301-2318 (2016).
5. Xiang S., Altintas Y.: Modeling and compensation of volumetric errors for five-axis machine tools. *International Journal of Machine Tools & Manufacture* 101,65-78 (2016).
6. Sobol I. M.: Sensitivity analysis for non-linear mathematical models. *Mathematical Modeling & Computational Experiment*, 1 (1993).
7. Zou X., Zhao X., Li G.: Sensitivity analysis using a variance-based method for a three-axis diamond turning machine. *International Journal of Advanced Manufacturing Technology* 92(9-12),1-15 (2017).
8. Xia C., Wang S., Sun S.: An identification method for crucial geometric errors of gear form grinding machine tools based on tooth surface posture error model. *Mechanism and Machine Theory* 138, 76-94 (2019).
9. Liu Y., Fei D., Li D.: Machining accuracy improvement for a dual-spindle ultra-precision drum roll lathe based on geometric error analysis and calibration. *Precision Engineering* 66, 401-416 (2020).
10. Dai J.: *Screw Algebra and Kinematic Approaches for Mechanisms and Robotics*, Springer, London (2019).
11. Sobol I.M.: Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates. *Mathematics & Computers in Simulation* 55(1-3), 271-280(2014).