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Data-Driven Solutions for the Newsvendor Problem: A Systematic Literature Review

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Abstract. The newsvendor problem captures the trade-off between ordering decisions, stocking costs and customer service level when the demand distribution is known. Nonetheless, in real case scenarios, it is unlikely that the decision maker knows the true demand distribution and its parameters, encouraging the use of datasets for empirical solutions that will achieve more precise results and reduce misleading decisions. Motivated by the availability of large amount of quality datasets, advances in machine learning algorithms and enhancement of computational power, the development of data-driven approaches has been emerging over the recent years. However, it is still unclear in which settings these data-driven solutions outperform the traditional model-based methods. In this paper, a systematic literature review is conducted for the descriptive analysis and classification of the most relevant studies that addressed the newsvendor problem and its variations under the data-driven approaches. The methods developed to solve the problems with unknown demand distribution are categorized and assessed. For each category, our paper discusses the relevant publications in detail and how they evidence the data-driven performance better. By identifying the gaps in the available literature, the future research directions are suggested.

Keywords: Newsvendor, Distribution-Free, Nonparametric Methods, Data-Driven, Inventory Optimization, Systematic Literature Review.

1 Introduction

Decision making under uncertainty is a major inventory management challenge that has been addressed in the literature over the past decades. Within this topic, the newsvendor problem (NVP) is a well-known inventory model that captures the decision maker trade-offs between overstocking and understocking of goods. In practice, it can be employed in a variety of industries, with different cost structures and uncertainty levels, such as travel tickets, fashion goods, textbooks, and bakery products.

Traditionally, according to [15], the retailer aims to satisfy a stochastic demand d for a single product in a single period. A cost of C is incurred at the end of this period, which is comprised of an overage cost o for unsold products, and an underage cost u for stockouts. The NVP intends to minimize the total expected cost, as depicted in (1), where $a^+ := \max\{a, 0\}$.

$$\min_{q \geq 0} C(q), \text{ where } C(q) \triangleq E [u(d - q)^+ + o(q - d)^+] \quad (1)$$

The stochastic demand d is represented by a cumulative distribution function $F(\cdot)$. The optimal order quantity q^* can be achieved by calculating (2).

$$q^* \triangleq \inf \left\{ q : F(q) \geq \frac{u}{u+o} \right\} \quad (2)$$

This representation holds for when the demand distribution is known, which is unlikely to happen in real life scenarios. If $F(\cdot)$ is unknown, then the optimal order quantity q^* cannot be directly evaluated [15]. Estimating a probability distribution is challenging and may result in misleading solutions, which is enhanced in settings with very little demand data to estimate market response to new products, such as the launch of medical devices or equipment by a start-up company [10].

In this context, there are two large groups of methods to solve the unknown demand distribution in inventory decisions: the parametric and nonparametric approaches. The first assumes that the distribution pertains to a parametric family of distributions, but the values of its parameters are unknown [12]. Within this group, the Bayesian approach corresponds to the earliest solutions that were developed. Readers may refer to [25] as the prominent study. Besides that, Operational Statistics is a parametric approach that was designed to perform demand estimation and inventory optimization simultaneously, readers may refer to [16].

In contrast, the nonparametric family of methods requires no assumptions regarding the demand distribution or its parameters and might be referred as data-driven in the literature since they rely on empirical information instead of assumptions to reach the solutions [6]. These methods can be executed in a single stage or in a separated parameters estimation and optimization steps, and can use contextual information, named data features, to enhance the predictive analytics models.

There is a recent surge on the interest in developing data-driven solutions, highlighting the novelty and early-stages of this topic. However, it could not yet be concluded to what extent the data-driven approaches are more accurate and applicable than their model-based counterparts, and in what scenarios the single-stage solutions outperform the two-steps methods [1, 11].

Therefore, our focus will be on the data-driven nonparametric solutions. A systematic literature review filters the publications with the aim to answer the following questions: (RQ 1) What are the data-driven approaches developed to solve the NVP in distribution-free settings? (RQ 2) How do these methods outperform the model-based approaches? (RQ 3) What are the advantages of single stage over two-steps solutions? (RQ 4) What are the research directions that remain unexplored?

The remainder of the paper is structured as follows. Section 2 details the steps executed in the review. Section 3 provides a descriptive analysis and classification of the selected papers with further evaluation and discussion of the findings, highlighting the future research directions. Lastly, Section 4 presents the conclusions of this review.

2 Methodology of the Systematic Literature Review

This research is based on the Systematic Literature Review method proposed by [26]. The procedure generally consists of a) identification of the need for a review and formulation of a research question, which was described in Section 1; b) elaboration of a research protocol, with establishment of search strings and selection criteria, which is illustrated in Table 1; and c) selection, assessment of relevant studies and discussion of findings as it is detailed in Section 3.

Table 1. Research Protocol for the Systematic Literature Review

Variable	Description
Databases	Scopus and Web of Science
Publication type	Peer-review journal articles, book chapters and technical reports
Language	Only articles written in English
Date range	All papers published until June 2021
Search fields	Advanced search in titles, abstracts, and keywords
Search terms	("data driven" OR "distribution free" OR "demand uncertainty" OR "nonparametric") AND ("newsvendor" OR "single period inventory" OR "stochastic inventory")
Deselection criterion I: Semantic relevance	Title, abstract, keywords or scanning the full paper was executed to determine the fitting of the paper to the topic
Deselection criterion II: Relevance to data-driven NVP	Full text was reviewed to determine the relevance of the paper to data-driven solutions in the NVP

Scopus showed 306 results, whereas Web of Science provided 357. Those studies that did not focus on the NVP demand uncertainty or adopted parametric methods were removed from the list. The significant papers that did not appear in the results but were frequently cited by the relevant studies were considered. Overall, 24 papers were thoroughly analyzed and will be discussed in Section 3.

3 Analysis and Discussion of the Findings

3.1 Descriptive Analysis of Selected Studies

Tables 2 and 3 present the selected studies that are relevant to the review, and facilitate the paper categorization and research trend identification. The categorization was made in terms of the main methods applied to solve the problem of unknown demand distribution. The publications were divided in four groups of methods that were largely adopted and one class for the miscellaneous techniques.

Table 2. Main Topics Addressed on the Selected Studies

Paper	Main topic
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Scarf (1958) [24]	Min-Max solution for the demand uncertainty in a single period NVP with partial information available
Gallego & Moon (1993) [7]	Compact proof of [24] and extension to several settings
Godfrey & Powell (2001) [8]	Nonparametric adaptive algorithm with censored demand considering a sequence of piecewise linear functions
Powell et al. (2004) [20]	Extension of [8] algorithm that achieves an asymptotically optimal solution
Bertsimas & Thiele (2005) [2]	Data-driven optimization for the NVP and its extensions
Levi et al. (2007) [14]	Bounds in the sampling-based policies for solving the single and multi-period NVP
Perakis and Roels (2008) [19]	Min-Max regret solution in the NVP with partial demand information available
Huh et al. (2011) [12]	Nonparametric adaptive data-driven inventory control with censored demand based on Kaplan-Meier estimator
Lee et al. (2012) [13]	Newsvendor-type models with empirical distributions used as the quantile estimator
Beutel & Minner (2012) [4]	Single-step linear programming model for the NVP
Sachs & Minner (2014) [23]	Extension of [4] in a censored demand distribution and price-dependent settings
Levi et al. (2015) [15]	Extension of [14] with tighter bounds on the relative regret of the sampling-based solutions of the NVP
Wang et al. (2016) [27]	Use of likelihood function to build the distribution uncertainty set in data-driven robust optimization
Methan & Thiele (2016) [17]	Correction term to account for rare events in data-driven robust optimization
Ban & Rudin (2019) [1]	Empirical risk minimization and kernel-weights optimization as machine learning solutions for the NVP
Cheung & Simchi-Levi (2019) [6]	Polynomial time approximation scheme for the sample average approximation solution of the data-driven NVP
Hu et al. (2019) [10]	Functionally robust optimization as a data-driven approach for pricing and ordering decisions
Huber et al. (2019) [11]	Machine learning algorithms along with quantile regression and sample average approximation to solve the NVP
Cao & Shen (2019) [5]	Neural network model to forecast quantiles of stochastic inventory models
Oroojlooyjadid et al. (2020) [18]	Deep learning algorithm for the single-step optimization of the multi-feature NVP
Halman (2020) [9]	Development of approximation schemes for the non-linear and sample based NVP
Punia et al. (2020) [21]	Machine learning algorithm along with quantile regression to solve the NVP and a heuristics for capacity constraint

Qiu et al. (2020) [22]	Support vector clustering-based data-driven robust optimization to solve the multi-product NVP
Bertsimas and Koduri (2021) [3]	Global machine learning algorithms for approximating the function and the optimizer based on kernel Hilbert spaces

Table 3. Overview of Journals and Methods Applied on the Selected Studies

Paper	Journal	RDO	SAA	QR	ML	Other
Scarf (1958)	Book chapter	✓				
Gallego & Moon (1993)	J. Oper. Res. Soc.	✓				
Godfrey & Powell (2001)	Manag. Sci.					✓
Powell et al. (2004)	Math. Oper. Res.					✓
Bertsimas & Thiele (2005)	Technical Report	✓				
Levi et al. (2007)	Math. Oper. Res.		✓			
Perakis & Roels (2008)	Oper. Res.	✓				
Huh et al. (2011)	Oper. Res.					✓
Lee et al. (2012)	Math. Meth. Oper. Res.			✓		
Beutel & Minner (2012)	Int. J. of Prod. Econ.				✓	
Sachs & Minner (2014)	Int. J. of Prod. Econ.				✓	
Levi et al. (2015)	Oper. Res.		✓			
Wang et al. (2016)	Comput. Manag. Sci.	✓				
Methan & Thiele (2016)	Comput. Manag. Sci.	✓				
Ban & Rudin (2019)	Oper. Res.			✓	✓	
Cheung & Simchi-Levi (2019)	Math. Oper. Res.		✓			
Hu et al. (2019)	Oper. Res.	✓				
Huber et al. (2019)	Eur. J. Oper. Res.		✓	✓	✓	
Cao & Shen (2019)	Oper. Res. Lett.			✓	✓	
Oroojlooyjadid et al. (2020)	IISE Trans.				✓	
Halman (2020)	IJOC		✓			
Punia et al. (2020)	Decis. Support Syst.			✓	✓	
Qiu et al. (2020)	Soft Comput.	✓				
Bertsimas and Koduri (2021)	Oper. Res.				✓	

RDO: Robust and Data-Driven Optimization; *SAA*: Sample Average Approximation; *QR*: Quantile Regression; *ML*: Machine Learning

3.2 Evaluation and Findings

Robust and Data-Driven Optimization (RDO)

Scarf [24] was the pioneer in developing a solution for the NVP when the demand information is uncertain. The author established a Min-Max approach to the single product and single period NVP with knowledge about the mean and standard deviation of the distribution. Gallego and Moon [7] extended this model to the recourse, fixed ordering cost, random yields, and multi-item cases.

Bertsimas and Thiele [2] were the first to investigate how to use demand observations as a direct input in data-driven optimization instead of assuming knowledge of the mean and standard deviation. Similarly to [7], the authors studied the NVP with several extensions and showed that these data-driven models could be reformulated as linear programming problems. Perakis and Roels [19] proposed an algorithm to minimize the NVP maximum regret when there is partial demand information available.

Wang et al. [27] addressed the drawbacks of the parametric and Distributionally Robust Optimization approaches by elaborating a new method named Likelihood Robust Optimization that chooses a function in a way that the observed data in the distribution achieve a certain level of likelihood. Hu et al. [10] proposed a modification named Functionally Robust Optimization that achieves a joint pricing and ordering decision under function form uncertainty to address the problem of model misspecification.

Moreover, Methan and Thiele [17] highlighted the weakness of data-driven approaches that rely solely on empirical demand distributions, since they may not consider rare occurrences. Their solution was the first to merge empirical distributions and range forecasts in robust optimization. To account for these tail events, a correction term was aggregated to the solution of the NVP. Recently, Qiu et al. [22] solved a multi-product NVP adopting a Support Vector Clustering based data-driven robust optimization method, which yields less conservative and better performance solutions than the traditional box and ellipsoid uncertainty sets.

Sample Average Approximation (SAA)

Levi et al. [14] considered the single and multi-period NVP to analyze the precision of the SAA method and to establish probabilistic bounds on the number of observations required to achieve a near-optimal solution without considering data features. By using the relative regret, Levi et al. [15] extended this procedure and derived a tighter bound for the probability that the solution exceeds a limit.

Similarly, Cheung and Simchi-Levi [6] evaluated the SAA performance by establishing an upper bound on the number of samples required to achieve a near-optimal solution that is independent of the demand distribution. They proposed a polynomial-time approximation scheme and established the sample lower bounds comparable to that by [15] to solve both single period and multi period NVP.

The SAA was one of the methods executed in Huber et al. [11] to demonstrate that data-driven approaches outperform model-based methods in most of the NVP settings. Halman [9] complemented the results from [6, 14, 15] by extending the SAA to set bounds on the number of observations required in nonlinear cost functions.

Quantile Regression (QR)

It is shown in a large body of literature that the solution for newsvendor-like problems is given by a particular quantile of the cumulative demand distribution [13]. By using an empirical distribution as estimator and QR for reaching optimal decisions in the NVP, Lee et al. [13] showed that erroneous decisions are made if the decision maker adopts an inappropriate model that overlooks or incorrectly considers the dependence of distributions on decisions.

Cao and Shen [5] were the first in the data-driven literature to handle an unknown form general autoregressive demand process by developing a single-step nonparametric method for quantile forecasting. Their model does not need previous quantile values as input and can deal with both stationary and nonstationary time series, outperforming the present neural network-based solutions for quantile prediction.

QR was also one of the methods executed in Huber et al. [11], Ban and Rudin [1] and Punia et al. [21]. The authors demonstrated that their proposed single-step data-driven NVP solution is equivalent to a high-dimensional QR and yields the satisfactory results when there is a large amount of data available.

Machine Learning (ML)

Beutel and Minner [4] designed the optimal inventory levels in a single-step procedure as the decision variables of a linear programming. The data-driven approach was compared with other benchmark methods and underperformed when the data sample was small. Sachs and Minner [23] extended this approach by studying the censored demand and price-dependent scenarios.

Ban and Rudin [1] highlighted that the demand estimation and optimization in separated steps are problematic in high-dimensional settings since it mainly relies on the performance of the demand estimation specifications in the first step. If there is an error, it will be amplified in the optimization. The authors developed algorithms based on the Empirical Risk Minimization principle and Kernel-weights Optimization.

Huber et al. [11] executed several combinations of single and two-steps methods with different target service levels to reach the optimal solution for the multi-feature NVP. Artificial Neural Networks and Decision Trees were adopted as ML forecasting methods. They concluded that in the two-steps method, the choice of the estimation procedure is a key decision to produce optimal results. They highlighted that the data-driven methods outperform because they could identify patterns across products and stores from contextual data, but they present the drawback that the reliable outcomes depend on the availability of a large amount of data.

Next, Cao and Shen [5] complemented the current neural network literature by developing the Double Parallel Feedforward Network-Based QR, a method capable of dealing with nonstationary time series that does not need past quantile values as input.

Oroojlooyjadid et al. [18] addressed the issue mentioned in [11] regarding the necessity of large quantities of historical data by developing a single-step Deep Learning solution for the multi-feature NVP. The model indeed outperforms other ML approaches and provides the solutions that could be achieved even with a small number of data points or high fluctuations in demand. Punia et al. [21] presented a study similar to [11], but with the novelty of addressing a multi-item NVP with a capacity constraint. They developed a heuristics that considers the hierarchies of the retail products, adopted Random Forest and Deep Neural Networks as forecasting methods and proposed a ML based QR for the single-step non-linear optimization.

Bertsimas and Koduri [3] reproduced a kernel Hilbert space to propose a global ML method to predict the objective and optimizer. Global ML predicts by choosing a func-

tional form of the prediction that minimizes the loss function, whereas local ML predicts by measuring closeness to the existing data. They were the first to develop a general and asymptotically optimal approach based on loss function minimizing.

Other Approaches

Some of the studies were concentrated in censored demands settings, which means that there is only sales data available instead of the actual demand information. In this scenario, Godfrey and Powell [8] developed the Concave Adaptive Value Estimation algorithm to approximate the NVP cost function with a sequence of piecewise linear functions but did not provide convergence proof. Powell et al. [20] extended this procedure and demonstrated that an asymptotically optimal solution is achieved.

Huh et al. [12] proposed the first nonparametric adaptive data-driven policy for stochastic inventory models based on the product-limit form of the Kaplan-Meier estimator. Their proposed KM-myopic policies converge to the set of optimal solutions in the case of discrete demand distributions.

3.3 Research Directions

The data-driven methods for solving the NVP and other stochastic inventory models are a recent and active field of research that can be extended in several directions. It is noticed a surge in Machine Learning based approaches for uncertainties in inventory. It is suggested to improve and extend their applications in the development of tractable and accurate solutions in hyper-parameter and higher service level scenarios.

A meaningful downside of ML is the model with lack of interpretability. Building interpretable black-box models is a very important and applicable research direction. In addition, incorporating contextual information from both supply and demand sides will assist in understanding their influence in sales and customer behavior.

For the separated parameter estimation and inventory optimization solutions, exploring different newsvendor situations that have little or no historical sales data will require new approaches for forecasting such as adoption of hybrid demand estimation methods. In the case of single-step methods, Reinforcement (Deep) Learning has shown to be capable of dealing with higher degrees of uncertainty and processing larger datasets, hence this potential can be further extended to solve complex multi-echelon problems or evaluate the relationships in multi-period and multi-product settings.

Another opportunity to investigate is modifications of the loss function in Quantile Regression-Machine Learning based solutions for considering the impact of costs in problems with substitution, capacity, time, space, or budget constraints. Moreover, data-driven robust optimization might be further studied for developing assertive solutions that are protected from rare occurrences without being overly conservative.

4 Conclusions

This review identified and discussed about the major data-driven approaches for the distribution-free newsvendor problem, which have been developed over the past years. The main advantage of data-driven methods over their model-based counterparts is

their adaptability in solving complex models with non-linear parameters and processing larger amounts of contextual information along with the demand. However, fully data-driven solutions present the disadvantages of being vulnerable to unusual events, model overfitting if not carefully tuned, lack of interpretability, or requiring large datasets.

RQ 1 in Section 1 was addressed in Subsection 3.1, with five groups of methods that were largely applied in the NVP. The studies discussed in Subsection 3.2, especially the ones that developed a Machine Learning-based solution, showed how the data-driven methods outperform model-based techniques, and the advantages of single-step over the two-step solutions, answering both RQ 2 and 3. The possible research directions that were suggested in Subsection 3.3 answered RQ 4.

This study achieved its objectives by executing a systematic literature review about the studies in data-driven solutions to uncertainties in inventory decisions, more specifically in a Newsvendor Problem setting, which is a recent and promising research area. With the discussion of the major methodologies, their performance and identification of the gaps in the literature, it was possible to suggest the future research directions.

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