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# A Robust Optimization Model for A Community Healthcare Service Network Design Problem

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**Abstract.** In this paper, we present a robust model for the community healthcare service network design problem. The community healthcare service network can determine the location of central hospitals and the allocation of the community medical service stations in order to minimize that cost of the whole network. Beside, we employ the Box+ellipsoidal set to deal with the objective which obtain the uncertain parameter. In addition, we reformulate the proposed model into a computationally tractable robust counterpart form under Box+ellipsoidal set. Finally, we verify the validity of the model by a case study.

**Keywords:** 2-service network · Community healthcare service network · Robust optimization.

## 1 Introduction

In recent years, community health service networks have become an increasing research hot spot for scholars. It is necessary to establish a reasonable and efficient community health care network to meet the health care needs of the community. The community healthcare service network is a typical facility location problem. [1] first proposed the study of location-allocation in the field of healthcare facilities, and many scholars have since conducted extensive research on the issue of healthcare facilities different perspectives [2–4]. [5] put forward that some healthcare facility location models are based on p-median model. Based on the 2-allocation p-hub model proposed in [6], this paper proposes a community medical service network model with 2-service structure. The community healthcare network with 2-service structure that needs to decided  $p$  central hospitals and each community medical service station is served by exactly two central hospitals. We conduct a 2-service community healthcare network that can effectively immunize against unforeseen circumstances and reduce the risk of uncertainty. For example, the emergence of covid-19 is an unexpected and unforeseen event that requires specialized designated hospital to test. Our proposed 2-service community healthcare network that indicates a designated hospital is served by two central hospitals has a stronger ability to cope with the outbreak of the epidemic.

Our primary concern with this study also contains uncertain factors in the service network. In recent years, the research on uncertainty in medical field has attracted more and more attention from scholars. [7] studied a stochastic optimization of primary health care network to deal with random variables in the network, and proposed a random model of integer nonlinear programming to improve the service level of the whole service network. In practice, the precise probability of the realization of uncertain variables is difficult to obtain. Distributionally robust optimization approach is an alternative to address this defect. Distributionally robust optimization [8–10] is an emerging approach to deal with uncertainty by limiting uncertain variables through an uncertainty set with certain characteristics. [11] proposed a new hybrid robust possibility programming for hierarchical health network design based on input parameter uncertainty. In this paper, we propose a distributionally robust model for the proposed 2-service network problem and use Box+ellipsoid uncertain set to deal with the uncertain demand in medical network.

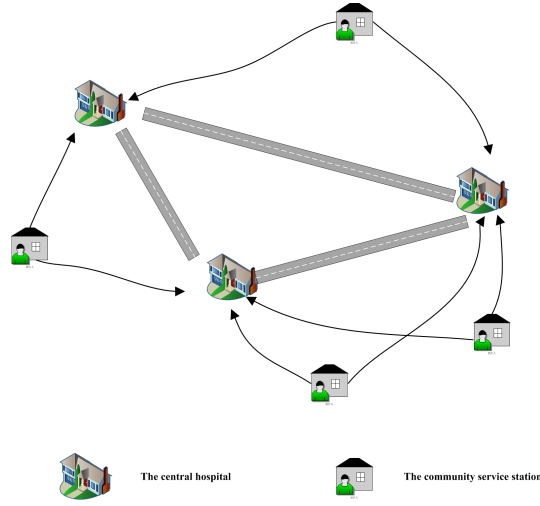
## 2 Problem statement of CHSN

In this section, we introduce structure of the community health care service network network. As shown in Figure 1, the community health care service network design problem includes a fixed central hospitals, a group of community medical service stations and the allocation structure between them. In particular, we assumed that any community health service station was assigned to exactly two central hospitals for service. We believe that the identified central hospitals serve the community medical service stations allocated to them and that medical resources are shared among the central hospitals.

## 3 The robust healthcare service network model

In this section, we introduce the robust model for the community health care service network.

$$\begin{aligned}
 & \min_{\mathbf{h}, \mathbf{Y}, \mathbf{x}} \sum_{k \in [N]} F_k h_k + \sum_{i, k \in [N]} C_{ik} Y_{ik} + \\
 & \max_{\mathbb{P} \in \mathcal{F}} \mathbf{E}_{\mathbf{u} \sim \mathbb{P}} \left[ \sum_{i, k \in [N]} u_i x_{ik} d_{ik} \beta + \sum_{k \in [N]} P_k \left( \sum_{i \in N} u_i x_{ik} - Q_k \right) \right]
 \end{aligned} \tag{1}$$



**Fig. 1.** The network of community medical service design problem

$$\text{s. t. } \sum_{k \in [N]} Y_{ik} + (2-p)h_i = 2 \quad \forall i \in [N] \quad (2)$$

$$Y_{kk} = h_k \quad \forall k \in [N] \quad (3)$$

$$\sum_{k \in [N]} h_k = p \quad (4)$$

$$Y_{ik} \leq h_k \quad \forall i \in [N], k \in [N] \quad (5)$$

$$m_{kl} \leq h_k \quad \forall k \in [N], l \in [N] \quad (6)$$

$$m_{kl} \leq h_l \quad \forall l \in [N], k \in [N] \quad (7)$$

$$m_{kl} \geq h_l + h_k - 1 \quad \forall l \in [N], k \in [N] \quad (8)$$

$$x_{ik} \leq Y_{ik} \quad \forall i \in [N], k \neq i \in [N] \quad (9)$$

$$\sum_{k \in [N]} x_{ik} = 1 \quad \forall i \in [N] \quad (10)$$

$$x_{ik} \geq 0, h_k, Y_{ik}, m_{kl} \in \{0, 1\} \quad \forall i \in [N], k \in [N], l \in [N] \quad (11)$$

The purpose of objective (1) is to minimize all the fixed cost and the maximum of the expected operating cost by limiting the distributions of the uncertain variables to Box+ellipsoidal sets. Constraint (2) indicates that one community medical service station is assigned to two central hospitals. Constraint (3) implies that community medical service stations can only be allocated to central

hospitals. Constraint (4) ensures that  $P$  central hospitals are selected. Constraint (5) shows that a community medical service station can only be assigned to an open central hospital. Constraints (6)-(8) indicate the contact of the central hospital. Constraint (9) implies that community service stations can only be assigned to open links. Constraint (10) requires that all demands be served by the central hospital. Constraint (11) is standard binary and non-negative integral constraint.

For the objective with uncertain parameters, we use the Box+ellipsoidal uncertain set to process. We assume that the uncertain parameter  $E_{\mathbb{P}}[u]$  has the following form:  $E_{\mathbb{P}}[u] = E_{\mathbb{P}}^0[u] + \sum_{l \in L} \zeta_l E_{\mathbb{P}}^l[u]$ . In this study, we assume that the first moment  $E_{\mathbb{P}}[u]$  is subject to a Box+ellipsoidal set, which can be depicted by the following distributional set  $\mathcal{F}$ :  $\mathcal{F} = \{\zeta_l \in \mathbf{R}^l : -1 \leq \zeta_l \leq 1, l \leq L, \sqrt{\sum_{l \in [L]} \zeta_l^2 / \sigma_l^2} \leq \Omega, l \in L\}$ , where  $\sigma_l$  is the given parameter, and  $\Omega$  is the adjustable safe parameter controlling the size of the distributional set. Finally, we obtain the computationally processable form of the robust community healthcare service network.

$$\begin{aligned} \min \quad & \sum_{k \in [N]} F_k h_k + \sum_{i, k \in [N]} C_{ik} Y_{ik} + \sum_{k \in [N]} P_k Q_k + 2t \\ \text{s. t.} \quad & \sum_{l \in [L]} |z_l| + \Omega \sqrt{\sum_{l \in [L]} w_l^2 \sigma_l^2} \leq t - (E_p^0[u])^T \mathbf{x} \mathbf{d}^T \beta \\ & z_l + w_l = -(E_p^l[u])^T \mathbf{x} \mathbf{P}^T \quad \forall l \in [L] \\ & \sum_{l \in [L]} |s_l| + \Omega \sqrt{\sum_{l \in [L]} v_l^2 \sigma_l^2} \leq t - (E_p^0[u])^T \mathbf{x} \mathbf{d}^T \beta \\ & s_l + v_l = -(E_p^l[u])^T \mathbf{x} \mathbf{P}^T \quad \forall l \in [L] \end{aligned} \tag{2)-(11)}$$

where  $t, z, w, s, v$  are auxiliary decision variables.

## 4 Case study

In this section, we apply our model to a small-scale case. We selected ten community medical service stations as shown in Figure 2 for experimental study. Figure 3 and Table 1 show our experimental results. As shown in Figure 3, the optimal solution of the robust model selects community medical service stations 2, 7 and 9 as central hospitals, and gives the corresponding demand allocation. The topology of the robust model shows that the demand of each district service station is served by two central hospitals, thus improving the overall service level of the medical service network.



**Fig. 2.** The network of community medical service design problem

**Table 1.** The result of robust model

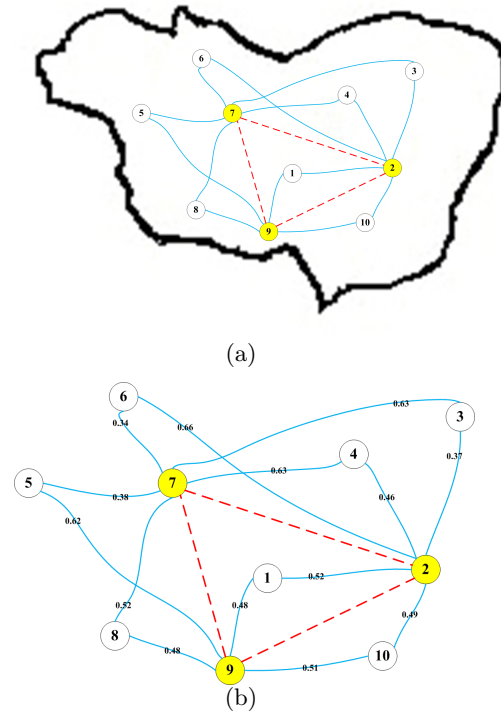
	Optimal value	Hubs	Routes
<i>RobustModel</i>	863939.2	2,7,9 3,4,6	$\rightarrow (2,7)$ 1,10 $\rightarrow (2,9)$ 5,8 $\rightarrow (7,9)$

## 5 Conclusions

In this paper, we aimed to minimize the total cost of service network, and proposed a community healthcare network system to enable markers to make an effective location-allocations decision. For the uncertain parameters in the community healthcare network, we use robust optimization to deal with the objective with uncertain parameters, and obtain a exact model which can be processed. We also provide an illustrative example and highlight the impact of uncertain parameters on the model. The results show that the robust community healthcare network model proposed by this paper can well resist the uncertainty of the uncertain parameters.

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**Fig. 3.** Topology analysis of the robust model.

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