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# Switching Kalman Filter for damage estimation in the presence of sensor faults

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## Abstract

Bayesian filtering based approaches for diagnosis of structural damage have been widely employed in structural health monitoring (SHM) research. The approach however may lead to an inaccurate alarm/decision due to the presence of faulty sensor/s. Nevertheless, sensor faults are inevitable during real field SHM in which sensor may malfunction or get detached from the structural surface, registering completely irrelevant information as measurement. Eventually, such erroneous information induce error in the estimation which leads to an inaccurate, sometimes divergent and impractical solution. The current study deals with Bayesian filtering based structural damage detection in the presence of one or multiple (consecutive) sensor faults. The damage detection is addressed with joint state-parameter estimation approach while a switching filtering strategy is employed for sensor fault detection. Switching approach employs multiple possible sensor fault models which are subsequently integrated to the measurement model of the joint estimation approach. The selection of the competent model (/switching between model ensembles) is undertaken recursively based on their likelihood against measured response. The proposed approach is tested on a numerical lumped mass model of a shear frame building, followed by a laboratory experiment on a cantilever beam. It has been perceived that estimation of health for structures measured with faulty sensors can actually lead to a false (positive and negative) alarm which can, however, be avoided by the employment of the proposed approach. The performance of the proposed approach is further established for healthy and damaged system with pre-existing and sudden sensor faults.

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## 1. Introduction

The deterioration of a structure over time has always been a cause of concern. A structure may face a premature member failure due to rusting in reinforcements, water seepage, damage induced by an earthquake, wind and wave, etc. Maintaining large and complex structures within adequate serviceability has therefore become a necessity. To maintain the safety and serviceability or to determine the remaining life of the

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12 structure of interest, monitoring its health, commonly denoted as structural health monitoring (SHM), has  
13 therefore become important. To achieve uninterrupted health monitoring, the structure is equipped with a  
14 network of sensors in order to collect the response data. Structural health is further interpreted through  
15 analysing these responses. Model based SHM approaches have been extensively employed assuming the  
16 availability of a sufficiently accurate model, parameterized with a set of location based health parameters  
17 or indices (**HI**s). The model-predicted response is then compared to the real measurements in frequency or  
18 time domain and the feedback, in terms of prediction error, is employed within a model-updating pipeline  
19 to update the model parameters (i.e. **HI**s).

20 To ensure damage detection in a structure before any catastrophic failure, several SHM approaches have  
21 been employed [1] over the last few decades. Vibration based SHM techniques have been very well devel-  
22 oped to detect, localize and quantify damage present in the structure. Generally these techniques require  
23 sufficiently accurate measurement data (acceleration/strain, etc.) from sensors placed on the structure. Typ-  
24 ical deterministic approaches take basis on an idealistic perfect model and clean measurement conditions.  
25 However in real-life, measuring the characteristics of a structure faces several uncertainties that originate  
26 from unavoidable model inaccuracies, sensor noises, unknown external disturbances, discrepancy in the data  
27 recorded by the sensor, etc. Eventually, uncertainties of different kinds and of unknown magnitude creep  
28 in the model estimation process to jeopardize the **HI** estimation process. Disregarding the existence of  
29 such uncertainties, a deterministic approach is not sufficient to deal with realistic conditions in structural  
30 monitoring.

31 Bayesian filtering provides an efficient approach in this regard, which recursively estimates the param-  
32 eters in time while handling uncertainties due to model inaccuracies and measurement noise. It should  
33 be mentioned here that parameter estimation with Bayesian filtering approach is a nonlinear problem (ex-  
34 plained in Section 2), for which a traditional Kalman filtering approach is not applicable. Eventually, for  
35 such problems, resorting to nonlinear filter variants (e.g., Extended or Unscented Kalman filter (EKF/UKF),  
36 Ensemble Kalman filter (EnKF) and Particle Filter (PF)) becomes imperative. Regardless of the filter vari-  
37 ant employed for such problems, there is a need of a sufficiently accurate process and measurement model  
38 in order to ensure precision in the estimation. A process model defines the system dynamics through time  
39 evolution of a set of unobserved variables, denoted as states. A measurement model further demonstrates the  
40 mapping of such hidden states to the corresponding available measurement that can be measured through  
41 the sensors. An optimal state estimate eventually necessitates the inaccuracies in these models to be mini-  
42 mal, unbiased and ideally Gaussian so that the error distribution can be modelled through a stationary white  
43 Gaussian noise (SWG N) model of known statistics. Thankfully, for most SHM problems, the uncertainties  
44 related to the process model due to unknown forcing and subtle model inaccuracies can be modelled using an  
45 SWGN model while the measurement noise can also be assumed as a stochastic process of known statistics.

46 However, sensor faults are inevitable in real field SHM problems which further introduce another kind of

47 uncertainty in the estimation process and as such can not be modelled through an SWGN model. Typical  
48 sensor faults originate due to electronic malfunction and/or detachment of the sensors from the structural  
49 surface. Faults are more probable in extreme service conditions. Most filtering-based approaches rely on  
50 a predefined measurement model that is invalidated under such conditions. Eventually, for such scenarios,  
51 estimating the system parameters by comparing the measurement recorded by a faulty sensor with a non-  
52 compatible measurement model can result in a divergence of the filtering solution, causing a false alarm,  
53 positive or negative.

54 This article addresses the aforementioned problem by employing a Switching Extended Kalman filter  
55 (SEKF), wherein the system response and health parameters are estimated within a Joint Extended Kalman  
56 filter (JEKF) environment. The proposed approach considers a switching strategy based on multiple mea-  
57 surement models (candidates) instead of a single measurement model. The selection of an appropriate  
58 measurement model among the possible candidates is done by employing both a switch variable, defined as  
59 a Markovian process with a predefined transition probability, and the likelihood of the model prediction.  
60 The Switching Kalman filter (SKF) has previously been successfully employed for the Remaining Usable  
61 Life (RUL) detection under the assumption of time invariance of the measurement model. This article  
62 further extends this approach for systems that are undergoing changes due to both damage and multiple  
63 sensor faults. Thereby, the proposed approach ensures the performance of the employed damage detection  
64 procedure even in the cases of singular or multiple sensor faults.

65 The following section discusses the existing knowledge on the topics of SKF based system health estima-  
66 tion and prognosis. Further, the specific application of SKF and its modifications pertinent to health and  
67 RUL estimation is discussed. The SKF approach is, then, detailed with its adaption to the proposed SEKF  
68 environment. The proposal is numerically tested on an eight degrees of freedom (*dofs*) lumped mass model  
69 of a shear frame building followed by a laboratory experiment on a two-dimensional (2-D) cantilever beam.

## 70 **2. Literature Review**

71 Bayesian filtering [2–9] has proved its efficiency to accommodate the inherent uncertainties present in  
72 modeling real-life SHM problems. This approach defines the system dynamics and the response measure-  
73 ments in a state-space form with the help of two probabilistic models, namely the process (/state propaga-  
74 tion) and the measurement (/observation) models. With the process model, the unobserved state is updated  
75 over time using a belief propagation strategy, following the Chapman-Kolmogorov equation. Subsequently,  
76 the propagated state is observed through the measurement model incorporating the measurement uncer-  
77 tainty. With Bayesian filters, the propagated state estimate is corrected based on the innovation, comparing  
78 the predicted measurement with the actual measurement. A recursion of this procedure using the response  
79 data received at each sampling instant ensures the state estimation error to be refined in an optimal sense.

80 To further estimate damage in the structure, health parameters/indices (**HI**s), corresponding to the loca-  
81 tions of interest, are defined and incorporated in the so-called augmented state, to be estimated in time  
82 alongside the states [10, 11] in a joint state-parameter estimation formulation. However, incorporation of  
83 the parameters into the state variables, renders the otherwise linear problem to become a nonlinear one  
84 since bi-linear formulation is induced by augmenting parameters in the state vector and the parameters can  
85 be estimated through the measurements only by employing a nonlinear mapping (system model, usually a  
86 Finite Element model (FEM)).

87 Typically, for a linear system defined with linear models, Kalman filter (KF) [12] can be considered as  
88 a righteous choice for state estimation. However, for nonlinear models where nonlinearity is induced in the  
89 process and/or the measurement relations, the application of KF is invalidated. To address nonlinearity  
90 in the system model, several modifications in KF have been introduced, such as, EKF [13], UKF [14, 15],  
91 EnKF [16] and PF [17]. Focusing on joint state-parameter estimation, JEKF has been established as an  
92 efficient method to estimate the state and the augmented health parameters (**HI**s) simultaneously. At the  
93 expense of a higher computational cost, this method has been improved upon by decoupling the estimation  
94 approaches [7, 18–20], but traditional joint estimation approach is still impelling for handling systems of  
95 moderate dimension, because of its excellent computational performance.

96 The above mentioned methods, although efficient and robust to handle usual model and measurement  
97 uncertainties, do not account for anomalies in the measured data due to faulty sensors. SHM may face  
98 challenges where a sensor may come off the surface, starts malfunctioning or simply encounters a connection  
99 failure. Eventually, for all such cases, the sensors usually either stop recording all together or may register  
100 an electrical noise. Consequently, unaware of such discrepancies, an otherwise robust algorithm may fail to  
101 estimate the state and the health parameters. An efficient, stand-alone and autonomous SHM algorithm  
102 should therefore be designed for localizing and isolating a faulty sensor at its time of occurrence and still  
103 continues with the damage estimation, undeterred by the fault. Faulty sensor data may also arise due to  
104 the presence of outliers in the measured data, for which outlier probability for detection can be utilized  
105 [21, 22]. However present study focuses on faulty measurement data obtained following before-mentioned  
106 reasons only.

107 Several neural network-based sensor fault detection and isolation (FDI) techniques have been developed  
108 for application in aerospace engineering as well as structural health monitoring [23]. Most of these stud-  
109 ies limit themselves to one faulty sensor at a time. A few methodologies have been able to detect multiple  
110 consecutive faults [23] with autonomous detection and isolation [24]. However neural network-based method-  
111 ologies require large database for training the network. Similar requirements have been observed in statistical  
112 hypothesis test model-based sensor fault diagnosis [25]. Detection and isolation of multi-sensor faults have  
113 been achieved using null subspace-based approach [26]. Kalman filter has also been employed in a two-step  
114 approach, where fault detection has been handled by detecting the change in mean value/variance of the

115 residuals, and fault isolation is achieved by dual sensor redundancy [27]. However, none of these methods  
 116 consider the presence of damage in the structure while detecting and isolating faulty sensors.

117 In this regard SKF [28] can be utilized. SKF defines multiple individual models corresponding to various  
 118 workability conditions (for example, fault in sensor 1, 2, 3... or RUL option 1, 2, 3...). From the defined  
 119 models, the model most suitable/close to current situation (i.e., fault in sensor  $x$  in reality or RUL  $x$ ) is  
 120 then selected by SKF. SKF has been successfully used in condition assessment to predict RUL of various  
 121 structures [29–31]. The most probable state for each time step is identified by Bayesian estimation [29].  
 122 Based on the selected model the predicted remaining useful life varies.

123 This paper, first, estimates state and parameters (with JEKF), corresponding to all applicable models,  
 124 where each model corresponds to a scenario of workability of accelerometers with no or single faulty sensor.  
 125 Further SKF has been used to select the most probable model corresponding to the measurement model  
 126 with maximum likelihood. The estimated state and health parameters are then chosen accordingly.

### 127 3. System model

128 Typically a state-space model of the system is required to replicate the structural dynamics for Bayesian  
 129 filtering-based SHM approaches. Moreover, in order to estimate the system health indices in parallel to the  
 130 states, they need to be integrated within the state-space model. Usually, the physical system is replicated  
 131 using numerical modeling techniques (e.g. FEM) as an  $n^{th}$  order linear time varying (LTV) system with  
 132 time invariant mass,  $\mathbf{M}_{n \times n}$ , and time-varying stiffness,  $\mathbf{K}(t)_{n \times n}$ , and damping,  $\mathbf{C}(t)_{n \times n}$ , matrices. The  
 133 non-stationarity in both the stiffness and the damping is due to the time-varying **HI**s, represented with the  
 134 parameter vector  $\boldsymbol{\theta}(t)_{N_\theta \times 1}$ . An explicit description of the functional dependence of the system matrices on  
 135  $\boldsymbol{\theta}(t)$  is however avoided here for the sake of readability.

136 The governing differential equation for such a system subjected to ambient excitation can be represented  
 137 as [18, 19],

$$\dot{\mathbf{x}}(t) = \mathbf{F}_c(t)\mathbf{x}(t) + \mathbf{B}_c\mathbf{u}(t) + \mathbf{v}(t) \quad (1)$$

138 with  $\mathbf{F}_c(t) = \begin{bmatrix} \mathbf{0}_{n \times n} & \mathbf{I}_{n \times n} \\ -\mathbf{M}^{-1}\mathbf{K}(t) & -\mathbf{M}^{-1}\mathbf{C}(t) \end{bmatrix}_{2n \times 2n}$  and  $\mathbf{B}_c = \begin{bmatrix} \mathbf{0}_{n \times m} \\ \mathbf{M}^{-1}\boldsymbol{\tau} \end{bmatrix}_{2n \times m}$ .

139  $\mathbf{x}(t) = \begin{bmatrix} \mathbf{q}(t) & \dot{\mathbf{q}}(t) \end{bmatrix}_{2n \times 1}^T$ , denotes the state variable with  $\mathbf{q}(t)$  and  $\dot{\mathbf{q}}(t)$  being the displacement and velocity  
 140 responses, respectively.  $\mathbf{0}$  and  $\mathbf{I}$  denotes the null and identity matrices, respectively and  $\mathbf{u}(t)_{m \times 1}$  represents  
 141 the ambient force.  $\mathbf{v}(t)_{2n \times 1}$  is the process noise accounting for the inherent model inaccuracies.  $\boldsymbol{\tau}_{n \times m}$  is  
 142 a Boolean matrix that maps the effect of  $m \times 1$  order input force to  $n \times 1$  order *dofs*.  $\mathbf{u}(t)$  and  $\mathbf{v}(t)$  can  
 143 be assumed to have known statistics of SWGN processes with covariance  $\mathbf{Q}_{m \times m}^u$  and  $\mathbf{Q}_{2n \times 2n}^v$  collectively

144 inducing the uncertainty in model prediction in the state propagation model. The state variable  $\mathbf{x}(t)$  can  
 145 be mapped to the acceleration measurement  $\mathbf{y}(t)$  as

$$\mathbf{y}(t) = \mathfrak{L} [\mathbf{H}(t)\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)] + \mathbf{w}(t) \quad (2)$$

146  $\mathbf{H}(t)_{n \times 2n}$  denotes the LTV measurement model correlating  $\mathbf{x}(t)$  to  $\mathbf{y}(t)$  while  $\mathbf{D}_{n \times m}$  is the direct transmission  
 147 matrix that maps the impact of  $\mathbf{u}(t)$  to  $\mathbf{y}(t)$ .

$$\begin{aligned} \mathbf{H}(t) &= \begin{bmatrix} -\mathbf{M}^{-1}\mathbf{K}(t) & -\mathbf{M}^{-1}\mathbf{C}(t) \end{bmatrix} \\ \mathbf{D} &= \mathbf{M}^{-1}\boldsymbol{\tau} \end{aligned} \quad (3)$$

148 Considering only  $p$  out of  $n$  *dofs* are instrumented with accelerometers, the location matrix  $\mathfrak{L}_{p \times n}$  selects  
 149 the responses from those  $p(\subseteq n)$  measured *dofs* and  $\mathbf{w}(t)_{p \times 1}$  is the corresponding measurement noise, again  
 150 assumed as an SWGN process of covariance  $\mathbf{R}_{p \times p}$ .

151 The discrete time representation of the above continuous time definition can further be obtained by  
 152 employing *zero-order-hold* (ZOH) technique [18], with  $\mathbf{F}_k$ ,  $\mathbf{B}_k$ ,  $\mathbf{D}_k$ ,  $\mathbf{E}_k$ ,  $\mathbf{H}_k$ ,  $\mathbf{x}_k$ ,  $\mathbf{y}_k$ , and  $\mathbf{w}_k$ , as the discrete  
 153 time counterparts of the corresponding continuous time entities described above,

$$\begin{aligned} \mathbf{x}_k &= \mathbf{F}_k\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{v}_k; \\ \mathbf{y}_k &= \mathbf{H}_k\mathbf{x}_k + \mathbf{D}_k\mathbf{u}_k + \mathbf{w}_k \end{aligned} \quad (4)$$

154  $\mathfrak{L}$  has been included within  $\mathbf{H}_k$  and  $\mathbf{D}_k$  for notational simplicity. The employed time indexing scheme is  
 155 used as per [7, 32]. The system is defined with the hidden state  $\mathbf{x}_k$  and an unknown LTV state propagation  
 156 matrix  $\mathbf{F}_k$ . Moreover, a sudden fault in a sensor invalidates the employment of a predefined measurement  
 157 matrix  $\mathbf{H}_k$  throughout the surveillance. In this article, the estimation of LTV  $\mathbf{F}_k$  is approached with a  
 158 joint state-parameter estimation approach (JEKF) in which  $\mathbf{F}_k$  is parameterized with  $\boldsymbol{\theta}_k$  which are then  
 159 estimated alongside  $\mathbf{x}_k$ . For sensor fault detection and isolation, the present study proposes an SKF based  
 160 approach detailed later in this article.

#### 161 4. Joint state-parameter estimation

162 The present study focuses on detecting structural damage in the presence of sensor faults. Eventually,  
 163 the state propagation matrix  $\mathbf{F}_k$  is time varying and therefore needs to be estimated online. Clearly, such  
 164 problem can be posed as a joint state-parameter estimation problem in which time evolution of certain  
 165 location-based  $\mathbf{H}\mathbf{I}$ s is defined with  $\boldsymbol{\theta}_k$  and estimated jointly in time.

166 With the joint estimation approach, the system is defined with a joint state-space formulation and  
 167 estimated using traditional filtering approaches, such as EKF [33–35], denoted JEKF. The joint estimation

168 approach adds up the parameters  $\boldsymbol{\theta}_k$  in the vector  $\mathbf{x}_k$  to yield a new formalism for the state dynamics, defined  
 169 with an augmented state vector  $\mathbf{X}_k = \{\mathbf{x}_k; \boldsymbol{\theta}_k\}$  of larger dimension. Accordingly, the Chapman-Kolmogorov  
 170 based belief propagation can be defined as

$$\mathbf{P}(\mathbf{X}_k | \mathbf{y}_{1:k-1}) = \sum_{\mathbf{X}_{k-1}} \mathbf{P}(\mathbf{X}_k | \mathbf{X}_{k-1}) \mathbf{P}(\mathbf{X}_{k-1} | \mathbf{y}_{1:k-1}) \quad (5)$$

171  $\mathbf{X}_{k|k-1}$  is obtained as the mean of such distribution,  $\mathbf{E}(\mathbf{X}_k | \mathbf{y}_{1:k-1})$ , where  $\mathbf{y}_{1:k-1}$  collectively represents  
 172 all observations including and up to time step  $(k-1)$ . From Equation (1), the process and input uncertainties  
 173 for  $\mathbf{x}_k$  are known (i.e.,  $\mathbf{Q}^v$  and  $\mathbf{Q}^u$ ). The same relation for  $\boldsymbol{\theta}_k$  requires the associated state transition model  
 174 to be defined. For this,  $\boldsymbol{\theta}_k$  is defined as a random variable and its transition is achieved by adding a  
 175 perturbation  $\mathbf{v}_k^\theta$  at each time step,

$$\boldsymbol{\theta}_k = \boldsymbol{\theta}_{k-1} + \mathbf{v}_k^\theta \quad (6)$$

176  $\mathbf{v}_k^\theta$  can be safely assumed to follow an SWGN model of covariance  $\mathbf{Q}_{N_\theta \times N_\theta}^\theta$ , where  $N_\theta$  is the number of  
 177 parameters. Finally, Equation (5) can be represented through an algebraic equation,

$$\mathbf{X}_k = \mathcal{F}_k(\mathbf{X}_{k-1}) + \mathcal{B}_k \mathbf{u}_k + \mathbf{v}_k^{\mathbf{X}}; \quad (7)$$

178 where  $\mathcal{F}_k(\mathbf{X}_{k-1})$  defines the new nonlinear state transition function for both response as well as parameter  
 179 states. The input matrix  $\mathcal{B}_k$  can be obtained as  $\mathcal{B}_k = \begin{bmatrix} \mathbf{B}_k \\ \mathbf{0} \end{bmatrix}$ . The covariances of the overall process noise,  
 180  $\mathbf{v}_k^{\mathbf{X}}$ , and the overall input noise can be obtained as,  $\mathcal{Q}^{\mathbf{X}} = \begin{bmatrix} \mathbf{Q}^v & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}^\theta \end{bmatrix}$  and  $\mathcal{Q} = \begin{bmatrix} \mathbf{Q}^u & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}^\theta \end{bmatrix}$ , respectively.  
 181 The prior estimate (i.e.,  $\mathbf{X}_{k|k-1}$ )<sup>1</sup> can be further be updated to the corresponding posterior (i.e.,  $\mathbf{X}_{k|k}$ ), using  
 182 the Bayesian approach conditioned on the current measurement data, i.e.,  $\mathbf{y}_k$ , as

$$\mathbf{P}(\mathbf{X}_k | \mathbf{y}_{1:k}) \propto \mathbf{P}(\mathbf{y}_k | \mathbf{X}_k) \mathbf{P}(\mathbf{X}_k | \mathbf{y}_{1:k-1}) \quad (8)$$

183 where  $\mathbf{X}_{k|k}$  is obtained as its mean. The data likelihood  $\mathbf{P}(\mathbf{y}_k | \mathbf{X}_k)$  however needs a mapping of the estimated  
 184 states to the corresponding measurements (i.e., a measurement model). For a linear system, the mapping  
 185 of  $\mathbf{x}_k$  is straight forward, while  $\boldsymbol{\theta}_k$  requires a nonlinear mapping (typically with an FE model) forcing the  
 186 measurement model to be nonlinear.

$$\tilde{\mathbf{y}}_k = \mathcal{H}_k(\mathbf{X}_k) + \mathbf{D}_k \mathbf{u}_k + \mathbf{w}_k \quad (9)$$

---

<sup>1</sup>An estimate  $x_{i|j}$  denotes the estimate for random variable  $x$  at  $i^{th}$  time instant using information including and up to  $j^{th}$  time instant along the sample path  $\mathbf{y}_{1:k}$ .

187 Upon mapping the states to the predicted measurements  $\tilde{\mathbf{y}}_k$ , the data likelihood can be obtained using a  
188 likelihood function with the departure of prediction from the real measurement  $\mathbf{y}_k$  as its argument.

189 EKF has been efficiently employed with such joint estimation modeling for several joint state-parameter  
190 estimation problems (JEKF). In these approaches, the nonlinear state and measurement functions are re-  
191 placed with the corresponding Jacobians around the current state estimates obtained through a first order  
192 Taylor series expansion. It should be acknowledged here that despite its wide spread application as a pa-  
193 rameter estimator, JEKF has also been criticized for not having a required analytical form and for not  
194 being ergodic [36]. JEKF has often been observed to cause divergence in the solution owing to the increased  
195 dimension in the state variables [37]. Still, JEKF has been established as the cheapest nonlinear estimator,  
196 in terms of computational cost, for moderate-size systems and its wide application also supports its use for  
197 systems that are not strongly nonlinear. With prompt detection of structural faults in our focus, JEKF  
198 therefore can be identified as an efficient candidate that can localize and quantify structural damage quickly  
199 at a reduced computational effort. However, other Bayesian estimation and monitoring approaches can  
200 replace JEKF, which will not hurt the applicability of the switching method proposed in this article.

## 201 5. SKF approach

202 With the system estimation framework defined in Section 4, the system model should either be param-  
203 eterized and estimated along with state or have several filters to compute with the different system models.  
204 However, the model can not always be parameterized correctly, especially for cases where sudden changes  
205 have to be considered. For the current study, due to the possibility of a sensor fault, the measurement  
206 model may change abruptly and this change may be difficult to track through parameterization of the sys-  
207 tem model, if the tracking is done through some adaptive algorithm. Eventually, one needs to employ several  
208 acceptable measurement models in parallel which will lead to an exponential growth in the belief state, in  
209 the order of  $n^t$ , where  $t$  is the number of steps.

210 In this endeavor, SKF can be considered to address detecting and isolating sensor faults in the presence  
211 of the usual system model inaccuracies and measurement uncertainties due to sensor noises. Like any other  
212 typical Bayesian filter, SKF can also be posed as a Dynamic Bayesian Network with multiple possible system  
213 models at each time instant. For each of the system model candidates, a dedicated state estimator (say  
214 Kalman filter) can be employed. In the current paper, where sensor fault is considered, the models differ  
215 by the measurement modeling. The different models are  $\{S^{m_i}; m_i = 1, 2, \dots, m\}$ , where  $m$  denotes the  
216 number of measurement model possible, which of course depends on the assumption on maximum possible  
217 sensor faults ( $d_s$ ) out of  $p$  sensors as  $m = \sum_{i=0}^{d_s} C_i^p$ . At each time step, the current model belongs to this  
218 model set. An additional discrete Markovian variable  $S_k$ , termed as switch, is employed to describe the  
219 current model at a particular time instant  $k$ . Accordingly,  $S_k$  is a discrete random variable with values in  
220  $(1, 2, \dots, m)$ . It denotes the index of the measurement model at time instant  $k$ .

221 Eventually, in each time step, the switch can be positioned at any arbitrary measurement model (say  $i$ )  
 222 with  $n_o$  options for transition to another measurement model (say  $j$ ). The stationary transition probability  
 223 corresponding to transition from the  $i^{th}$  model, at time instant  $(k-1)$ , to the  $j^{th}$  model, at time instant  $k$ ,  
 224 is denoted as  $\mathbf{Z}^{ij}$ . The current study focuses on sensor fault detection and the switch is employed to select  
 225 among the possible measurement models while the state propagation model is assumed as an LTV model  
 226 and its estimation is dealt separately with the JEKF approach detailed in Section 4. Notice that a variation  
 227 in the model parameter values due to damage does not change  $S_k$ , only a sensor fault can. The challenge  
 228 for the algorithm is to discriminate between sensor faults and damage.

229 Eventually, for such system definition, the objective is to estimate the model switch probability for an  
 230 ensemble of measurement models and thereby enabling the measurement/state propagation modeling to  
 231 switch between different sensor fault conditions. With JEKF in force, the linearized system equation can  
 232 be represented as

$$\begin{aligned}\mathbf{X}_k &= \bar{\mathcal{F}}_k \mathbf{X}_{k-1} + \mathcal{B}_k \mathbf{u}_k + \mathbf{v}_k^{\mathbf{X}} \\ \tilde{\mathbf{y}}_k &= \bar{\mathcal{H}}_k^j \mathbf{X}_k + \mathbf{D}_k^j \mathbf{u}_k + \mathbf{w}_k^j\end{aligned}\quad (10)$$

233  $\bar{\mathcal{F}}_k$  and  $\bar{\mathcal{H}}_k$  denote the locally linearized state transition and measurement models. Superscript  $j$  in  $\bar{\mathcal{H}}_k^j$   
 234 associates the notation to the  $j^{th}$  measurement model and  $\mathbf{D}_k^j$  is the associated direct transmission matrix.  
 235  $\mathbf{w}_k^j$  is a realization from an SWGN process modeling the measurement noise for the  $j^{th}$  noise model and  
 236 assumed to be an SWGN process of covariance  $\mathbf{R}^j$ . Additionally,  $S_k^{ij}$  defines the probability of the model  
 237 to be  $\mathcal{S}^i$  at time instant  $(k-1)$  and to be  $\mathcal{S}^j$  at time instant  $k$ , given the state of observations  $\mathbf{y}_{1:k}$ .

238 For each  $j^{th}$  model ( $j \in (1, \dots, n_o)$ ), a standalone state filter can be employed to propagate and update  
 239 the prior estimate obtained from  $i^{th}$  filter ( $i \in (1, \dots, n_o)$ ), thereby resulting in  $n_o$  filters running in parallel:  
 240 the covariance  $\mathbf{P}_{k|k}$  (resp.  $\mathbf{P}_{k|k-1}$ ) is the variance of the state estimate given the observations  $\mathbf{y}_{1:k}$  (resp.  
 241  $\mathbf{y}_{1:k-1}$ ) [38]. For  $i^{th}$  filter, the prior estimates for  $\mathbf{X}_{k-1|k-1}^i$  and  $\mathbf{P}_{k-1|k-1}^i$  can be evolved in time and later  
 242 observed through the  $j^{th}$  model as

$$\begin{aligned}\text{State propagation} & \quad \mathbf{X}_{k|k-1}^i = \bar{\mathcal{F}}_k \mathbf{X}_{k-1|k-1}^i \\ \text{Covariance propagation} & \quad \mathbf{P}_{k|k-1}^i = \bar{\mathcal{F}}_k \mathbf{P}_{k-1|k-1}^i \bar{\mathcal{F}}_k^T + \mathcal{B}_k \mathcal{Q}^{\mathbf{X}} \mathcal{B}_k^T + \mathcal{Q} \\ \text{Innovation} & \quad \mathbf{i}_k^{ij} = \mathbf{y}_k - \bar{\mathcal{H}}_k^j \mathbf{X}_{k|k-1}^i\end{aligned}\quad (11)$$

243 In the following, the state estimates can be updated considering correlated noise in process and measurement  
 244 model as

$$\begin{aligned}\text{Kalman gain} & \quad \mathbf{K}^{ij} = (\mathbf{P}_{k|k-1}^i \bar{\mathcal{H}}_k^{jT} + \mathcal{B}_k \mathcal{Q}^{\mathbf{X}} \mathbf{D}_k^{jT}) \mathcal{J}_k^{j-1} \\ \text{State update} & \quad \mathbf{X}_{k|k}^{ij} = \mathbf{X}_{k|k-1}^i + \mathbf{K}^{ij} \mathbf{i}_k^{ij} \\ \text{Covariance update} & \quad \mathbf{P}_{k|k}^{ij} = (\mathbf{I} - \mathbf{K}^{ij} \bar{\mathcal{H}}_k^j) \mathbf{P}_{k|k-1}^i\end{aligned}\quad (12)$$

245 where, innovation error covariance,  $\mathcal{J}_k^j = \bar{\mathcal{H}}_k^j \mathbf{P}_{k|k-1}^i \bar{\mathcal{H}}_k^{jT} + \mathbf{D}_k^j \mathcal{Q}^X \mathcal{B}_k^T \bar{\mathcal{H}}_k^{jT} + \bar{\mathcal{H}}_k^j \mathcal{B}_k \mathcal{Q}^X \mathbf{D}_k^{jT} + \mathbf{D}_k^j \mathcal{Q}^X \mathbf{D}_k^{jT} + \mathbf{R}_k^j$ .

246 Subsequently, the estimates obtained from all the filters under different possible model transition sce-  
 247 narios have to be weighted and combined. The switch variable, in this regard, helps to obtain a weighted  
 248 sum of the posterior estimates for state and covariance based on their individual likelihood  $\mathbf{L}_k^{ij}$ , that can be  
 249 obtained as:

$$\mathbf{L}_k^{ij} = |\mathbf{G}_k^{ij}|^{-0.5} \exp^{-0.5 \delta_k^{ij} \mathbf{G}_k^{ij} \delta_k^{ijT}} \quad (13)$$

250 with  $\delta_k^{ij} = \mathbf{y}_k - \bar{\mathcal{H}}_k^j \mathbf{X}_{k|k}^{ij}$  and  $\mathbf{G}_k^{ij} = \bar{\mathcal{H}}_k^j \mathbf{P}_{k|k}^{ij} \bar{\mathcal{H}}_k^{jT} + \mathbf{R}_k^j$  being the residual error and its covariance estimate,  
 251 respectively, for the model transition scenario from  $i^{th}$  to  $j^{th}$  measurement model. In that sense the  $n_o$   
 252 values of  $j$  span a subset of the whole model collection depending on the current model  $i$ .

253 The probability of the model to switch from  $i^{th}$  to  $j^{th}$  model, i.e.,  $S_k^{ij}$ , can be obtained using a prede-  
 254 fined (and time invariant) transition probability  $\mathbf{Z}^{ij}$ , and the likelihood estimate  $\mathbf{L}_k^{ij}$ . Each element in the  
 255 transition probability matrix  $\mathbf{Z}^{ij}$  signifies that the model switches between  $i$  and  $j$  as defined by the user.  
 256  $S_k^{ij}$  can be obtained for all  $\{i,j\}^2$

$$S_k^{ij} = \mathbf{P}(S_k = j | \mathbf{y}_{1:k} \text{ and } S_{k-1} = i) = \frac{\mathbf{L}_k^{ij} \mathbf{Z}^{ij}}{\sum_{j=1}^{n_o} \mathbf{L}_k^{ij} \mathbf{Z}^{ij}} \quad (14)$$

257 Corresponding to the state transition from any of the  $S^i$ 's to  $S^j$ ,  $\mathbf{X}_{k|k}^{ij}$  denotes the corresponding posterior  
 258 estimates in which measurement model  $j$  has been employed. There are  $n_o$  state estimates  $\{\mathbf{X}_k^{ij}; j =$   
 259  $1, \dots, n_o\}$  with their individual normalized weights  $w^{ij}$  as  $S_k^{ij}$ .

260 Using this collection of weights ( $w^{ij}$ ), eventually, for the a priori measurement model  $i$ , the weighted  
 261 estimate for the state can be obtained as:

$$\hat{\mathbf{X}}_{k|k}^i = \sum_{j=1}^{n_o} w_k^{ij} \mathbf{X}_{k|k}^{ij} \quad (15)$$

262 Subsequently, for each state estimator, the state error covariance associated to  $\hat{\mathbf{X}}_{k|k}^i$  can be defined as:

$$\begin{aligned} \hat{\mathbf{P}}_{k|k}^i &= \mathbf{cov}_{|\mathbf{y}_{1:k}}(\hat{\mathbf{X}}_{k|k}^i - \mathbf{X}_k) \\ &= \sum_{j=1}^{n_o} w_k^{ij} \{ \mathbf{cov}_{|\mathbf{y}_{1:k}}(\mathbf{X}_{k|k}^{ij} - \mathbf{X}_k) + \mathbf{cov}_{|\mathbf{y}_{1:k}}(\hat{\mathbf{X}}_{k|k}^i - \mathbf{X}_{k|k}^{ij}) + 2\mathbf{E}[(\mathbf{X}_{k|k}^{ij} - \mathbf{X}_k)(\hat{\mathbf{X}}_{k|k}^i - \mathbf{X}_{k|k}^{ij})^T | \mathbf{y}_{1:k}] \} \\ &= \sum_{j=1}^{n_o} w_k^{ij} (\mathbf{P}_{k|k}^{ij} + \epsilon_k^{ij} \epsilon_k^{ijT}) + 2\mathbf{E}[(\mathbf{X}_{k|k}^{ij} - \mathbf{X}_k) | \mathbf{y}_{1:k}] (\hat{\mathbf{X}}_{k|k}^i - \mathbf{X}_{k|k}^{ij})^T \end{aligned} \quad (16)$$

263 where the residual error  $\epsilon_k^{ij} = \hat{\mathbf{X}}_{k|k}^i - \mathbf{X}_{k|k}^{ij}$  is the departure of  $\{\mathbf{X}_k^{ij}; i = 1, \dots, n_o\}$  from the corresponding  
 264 weighted sum, i.e.,  $\hat{\mathbf{X}}_{k|k}^i$ .  $\mathbf{X}_{k|k}^{ij}$  being the optimal unbiased estimate for  $\mathbf{X}_k$ , the expectation of the departure  
 265 is ideally zero. Finally, the residual error covariance for  $k^{th}$  time step can be obtained as:

---

<sup>2</sup> $\{ij\}$  is a list of all possible transitions at a given time step, in accordance to the decision tree (cf. Figure 1, Section 5.1) denoting transition between  $i^{th}$  and  $j^{th}$  measurement models.

$$\hat{\mathbf{P}}_{k|k}^i = \sum_{j=1}^{n_o} w_k^{ij} (\mathbf{P}_{k|k}^{ij} + \epsilon_k^{ij} \epsilon_k^{ijT}) \quad (17)$$

266 *5.1. Candidate measurement model selection*

267 In this section, the adopted measurement model and assumed transition probability between one model  
 268 to another has been detailed. Assuming  $p$  ( $p \leq n$ ) *dofs* are instrumented with sensors,  $m$  measurement  
 269 models are defined to be possible. The possible model number is however conditioned on the assumption  
 270 on possible sensor faults being detected by the algorithm. In this study, a maximum of two sensor faults  
 271 are considered. Under each sensor fault condition, the next possible sensor fault models can be formulated  
 272 using a decision tree as given in Figure 1.

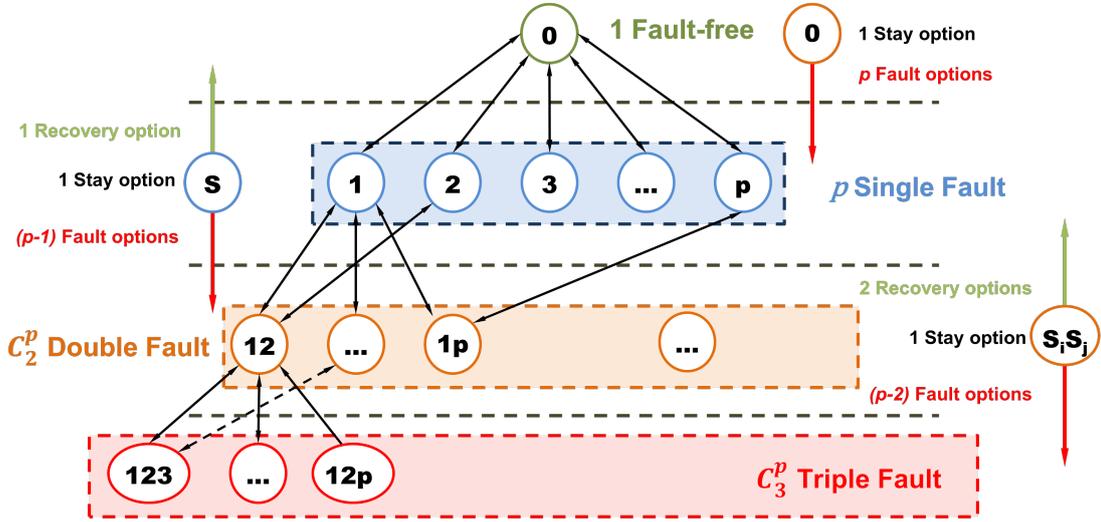


Figure 1: Model transition tree schematic

273 Evidently, each candidate model is defined with the sensor working states (i.e. working (1) or faulty (0))  
 274 with three transition possibilities: 1. the system continues with the same fault model (Stay), 2. get induced  
 275 with one more sensor fault (Fault Progression) and lastly, 3. a faulty sensor starts working again. The last  
 276 scenario is obviously rare but still needs consideration in the model ensemble to appreciate the minuscule  
 277 transition probability for a sensor to be working again. This option also allows to compensate for false  
 278 alarms allowing the algorithm to return to a steady state after a wrongly assumed false alarm. Clearly, with  
 279 this renewed definition for the candidate models, the ensemble becomes capable of capturing a multi-sensor  
 280 fault scenario.

281 Accordingly, the starting model can be assumed to be fault free, meaning all sensors are working fine.  
 282 This model got  $(p + 1)$  options for model transition: 1 option to stay or move to  $p$  possible sensor fault  
 283 situation (Fault Progression) involving fault at a particular sensor. It has however been assumed that no

284 two sensors are getting faulty or repaired at the same instant during the estimation. However it no way  
 285 restricts those faults to occur simultaneously in reality. This strategy is adopted to simplify the estimation  
 286 process. On premise of this assumption, only one fault-free and  $p$  single fault models are possible. With a  
 287 fault already confirmed, each of the  $p$  single fault model has therefore one option to stay,  $(p - 1)$  options to  
 288 get induced to another fault in the remaining  $(p - 1)$  working sensors or a minuscule probability of getting  
 289 repaired. Recall that  $C_2^p$  denotes the number of possible combination with 2 out of  $p$  sensors. Eventually,  
 290 there can be a total number of  $C_2^p$  possible fault models, with each having again one stay, two repair, and  
 291  $(p - 2)$  fault progression options.

292 To elaborate the candidate models further, the fault models pertinent to a simple three sensor system  
 293 can be explained. Given that, the estimation will be considering maximum of two faulty sensors, the possible  
 294 models are 1. all sensors working, 2. three single fault models with faults at  $1/2/3^{rd}$  sensor, 3. three double  
 295 fault scenarios with faults at  $(1 - 2)/(2 - 3)/(3 - 1)$  sensors. Note that with three fault condition no relevant  
 296 information is available from which the system can be estimated. However for a sensor network with more  
 297 sensors, three or more sensor fault models can always be considered. Following the same strategy detailed  
 298 before, the ensemble can be bigger with consideration of three fault sensor as well. The possible model  
 299 transition will follow the path as given in the decision tree (or fault propagation tree) which will clearly  
 300 prevent a fault model of double fault at sensors 3 and 1 to transit to no fault or single fault at 2, but surely  
 301 allow to have options like single fault at 3 or 1 or moving to triple fault at 1, 2 and 3.

302 Among these, the first model ( $\mathcal{S}^1$  of order  $1 \times p$ ) denotes a no-fault condition in which each of the sensors  
 303 is intact and working perfectly while the remaining models denote a sensor-fault scenario involving one or  
 304 two sensor faults. Eventually the model ensemble can be represented as,

$$\begin{aligned}
 \mathcal{S} &= \left[ \mathcal{S}^1; \mathcal{S}^2; \mathcal{S}^3; \dots; \mathcal{S}^m \right]_{(1+p+C_2^p) \times p} \\
 &= \begin{array}{|c|} \hline \begin{array}{|c|} \hline \begin{array}{|c|} \hline 1 & 1 & 1 & \cdots & 1 & 1 \\ \hline \end{array} \\ \hline \begin{array}{|c|} \hline 0 & 1 & 1 & \cdots & 1 & 1 \\ \hline \end{array} \\ \hline \begin{array}{|c|} \hline 1 & 0 & 1 & \cdots & 1 & 1 \\ \hline \end{array} \\ \hline \begin{array}{|c|} \hline \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \hline \end{array} \\ \hline \begin{array}{|c|} \hline 1 & 1 & 1 & \cdots & 1 & 0 \\ \hline \end{array} \\ \hline \begin{array}{|c|} \hline 0 & 0 & 1 & \cdots & 1 & 1 \\ \hline \end{array} \\ \hline \begin{array}{|c|} \hline \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \hline \end{array} \\ \hline \begin{array}{|c|} \hline 1 & 1 & 1 & \cdots & 0 & 0 \\ \hline \end{array} \\ \hline \end{array} \\ \hline \end{array} \tag{18}
 \end{aligned}$$

305 The green, blue and red blocks denote fault-free, single fault and double fault models respectively. 1 denotes  
 306 a working sensor that is recording the structural response, contaminated by the sensor noise, while 0 denotes  
 307 that the sensor at that position is faulty and gives away only sensor noise.

308 Of course, this approach theoretically prevents the detection of sensor faults occurring at the same time  
 309 instant which however can be considered not abundantly happening in reality. Nevertheless, such scenarios  
 310 can be encompassed through further modification of  $\mathcal{S}$ , yet not considered in this paper for simplicity.  
 311 Alternate modeling for the fault scenarios and their occurrence can be envisioned. They will generalize  
 312 this paper without refuting the basics of this approach. An empirical robustness test of this assumption is  
 313 performed in the numerical simulations.

314 For the SKF approach, the candidate measurement models, i.e.  $\bar{\mathcal{H}}_k^i$  can then be obtained as

$$\bar{\mathcal{H}}_k^i = \mathcal{S}^i \cdot \bar{\mathcal{H}}_k \quad (19)$$

315 wherein  $a \cdot b$  denotes the inner product between  $a$  and  $b$ . Finally, a suitable assumption has to be made on  
 316 the model state transition probability  $\mathbf{Z}^{ij}$  which is defined as

$$\mathbf{Z} = \begin{bmatrix} \tilde{Z}_0 & \beta & \beta \cdots & \beta & \beta \\ \tilde{Z}_1 & \delta & \cdots & \beta & \beta \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \tilde{Z}_1 & \beta & \cdots & \beta & \delta \\ \tilde{Z}_2 & \delta & \delta \cdots & \beta & \beta \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \tilde{Z}_2 & \beta & \beta \cdots & \delta & \delta \end{bmatrix} \quad (20)$$

317 In order to interpret this model transition probability matrix  $\mathbf{Z}$ , the following has to be considered.  
 318 The green, blue and red blocks are the transition probabilities of each of the models already presented  
 319 with  $\mathcal{S}$  matrix (c.f. Equation 18) in the respective blocks. Accordingly, the transition probabilities for any  
 320 arbitrary  $i^{th}$  sensor fault model listed in the  $i^{th}$  row of the  $\mathcal{S}$  matrix are given in the  $i^{th}$  row of the  $\mathbf{Z}$   
 321 matrix. Accordingly,  $\mathbf{Z}$  is a  $(1 + p + C_2^p) \times (p + 1)$  order matrix depicting transition probabilities for  $(p + 1)$   
 322 transition options corresponding to each model. Among the entries, the first entry is always maintained to  
 323 be the probability of staying at the current model while the remaining are for fault propagation scenarios,  
 324 except for the entry  $\delta$  that denotes the probability of repair.

325 Here,  $\beta$  denotes the probability of getting induced with another sensor fault. Since at any given time, all  
 326 the sensors are equally likely to get faulty, the probability of switching from the first model to other models  
 327 (or any model corresponding to fault progression), is considered to be the same, i.e.  $\beta$ . Further,  $\delta$  denotes  
 328 minuscule probability of repair and accordingly a small value has been assumed for it implying that a faulty  
 329 sensor has a minuscule chance of going back to its perfect working condition again. This also allows the  
 330 fault detection to recover from a possible false alarm.

331  $\tilde{Z}_0$ ,  $\tilde{Z}_1$  and  $\tilde{Z}_2$  are the probabilities of continuing with the same fault-free, single fault or double fault

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**Algorithm 1** Proposed algorithm.

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1: procedure SKF( $\mathbf{y}_k, \mathbf{Z}, \mathbf{Q}^u, \mathbf{Q}^v, \mathbf{Q}^\theta, \mathbf{R}$ ) ▷ Input, process and measurement noise covariances
2:   At  $k = 0$  initialize  $\{\mathbf{X}_{0|0}^i\}$  and  $\{\mathbf{P}_{0|0}^i\}$  with  $i = 1$  corresponding to no sensor fault model ▷ Initialization
3:   for <each  $k^{th}$  measurement  $\mathbf{y}_k$  with source switch as  $\mathcal{S}^i$  at  $k - 1^{th}$  time step> do
4:     procedure JEKF( $\mathbf{X}_{k-1|k-1}^i, \mathbf{P}_{k-1|k-1}^i$ )
5:       Prediction: Propagate state and state error covariance,  $\mathbf{X}_{k|k-1}^i, \mathbf{P}_{k|k-1}^i$  ▷ see Equation (11)
6:       Selection of all possible model transitions  $\{ij\}$  (i.e.  $\mathcal{S}^i$  to  $\mathcal{S}^j$ ) from the model transition tree ▷ see Figure 1
7:       for <each possible model transition  $\in \{ij\}$ > do
8:         Innovation and gain: Compute innovation,  $\mathbf{i}_k^{ij}$ , and Kalman gain,  $\mathbf{K}^{ij}$  ▷ see Equations (11) and (12)
9:         Measurement Update: Update state ( $\mathbf{X}_{k|k}^{ij}$ ) and state error covariance ( $\mathbf{P}_{k|k}^{ij}$ ) ▷ see Equation (12)
10:        Likelihood estimation: Estimate the likelihood of the transition mode  $ij$ ,  $\mathbf{L}_k^{ij}$  ▷ see Equation (13)
11:      end for
12:    end procedure
13:    Obtain the probability for each transition mode  $\in \{ij\}$ ,  $S_k^{ij}$  (also  $w^{ij}$ ) ▷ see Equation (14)
14:    Compute weighted estimates for states and its error covariance ( $\tilde{\mathbf{X}}_{k|k}^i$  and  $\hat{\mathbf{P}}_{k|k}^i$ ) ▷ see Equations (15) and (17)
15:    For time step  $k$ , position the switch at  $j$  corresponding to  $\max_{1 < j < n_o} S_k^{ij}$ 
16:    Reset the destination switch  $\mathcal{S}^j$  as source switch  $\mathcal{S}^i$  and go back to line 3 for time instant  $k + 1$ .
17:  end for
18: end procedure

```

---

332 models respectively. Further, ensuring the total sum of probabilities to be unity, the values for these entries  
333 can be obtained as the complimentary to the other transition probabilities, i.e.,

$$\begin{aligned}
\tilde{Z}_0 &= 1 - p\beta \\
\tilde{Z}_1 &= 1 - \delta - (p - 1)\beta \\
\tilde{Z}_2 &= 1 - 2\delta - (p - 2)\beta
\end{aligned} \tag{21}$$

334 With this formalism,  $n_o$  in Equation (14) is equal to  $p + 1$ .

335 The proposed SKF algorithm for simultaneous sensor fault and damage detection is detailed in a pseudo-  
336 code in Algorithm 1.

## 337 6. Numerical experiment

338 Numerical experiments on a simplified building model have been performed to check the efficiency of  
339 the proposed algorithm. An 8-story building is simplified as a lumped-mass model with 8 *dofs*, all in the  
340 x-direction, cf. Figure 2. Each floor mass is assumed to be equal, i.e.,  $m_1 = m_2 = \dots = m_8 = 14KN$ .  
341 Similarly, all story stiffness values are also assumed to be equal, i.e.,  $k_1 = k_2 = \dots = k_8 = 3000KN/m$ .  
342 The Rayleigh damping model [39] is availed to determine the damping corresponding to the mass and  
343 stiffness matrices. The damping ratio is assumed to be 0.02. The external force is modelled as an SWGN  
344 of unit variance,  $N(0, 1)$   $N$ , on all its *dofs*. To obtain the necessary measurement data (acceleration)  
345 for the numerical validation, the mechanical system is simulated for 50 *s* with a sampling frequency of  
346 50 *Hz*, for all of the forthcoming cases, unless mentioned otherwise. The numerical response is subsequently  
347 contaminated with a 1% SNR (signal-to-noise ratio) SWGN process to mimic the real life measurement noise  
348 contamination. It is assumed that electrical noises are always present in the sensor, irrespective of sensor's

349 intended placement. Hence a fault in the sensor (data) is created by replacing the original simulated data  
 350 with Gaussian noise equivalent to the measurement noise.

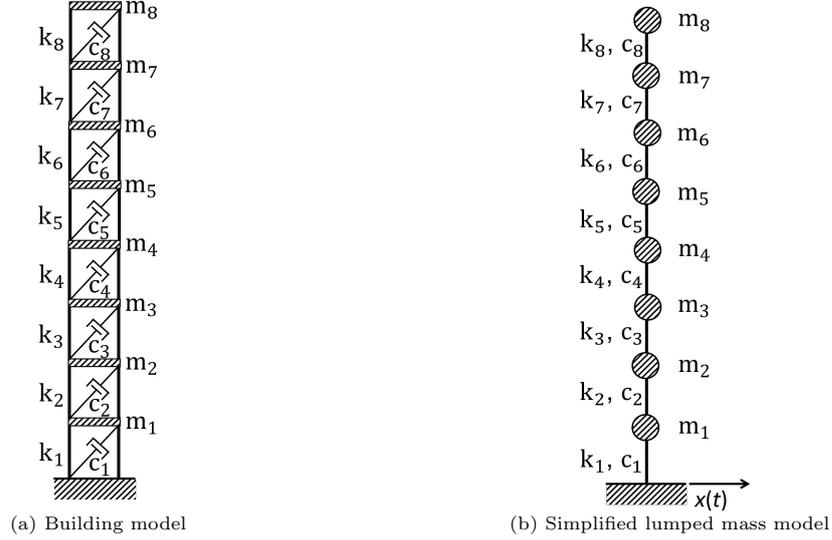
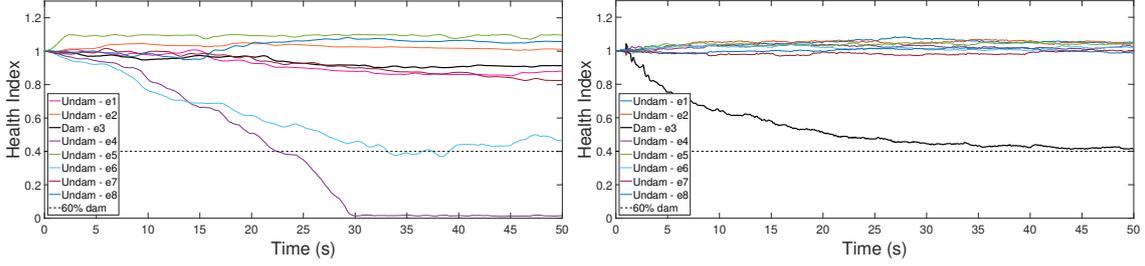


Figure 2: Building model for numerical experiments.

### 351 6.1. Justification of the requirement of the proposed approach

352 There exist sufficient studies that demonstrate the capability of JEKF approach in estimating/tracking  
 353 structural health using a set of measurements. However, the inherent assumption on the quality of measure-  
 354 ment is often considered to be too obvious. For cases with faulty sensors, the performance of JEKF can be  
 355 substantially poor. A numerical experiment is performed on the simplified model (cf. Figure 2), wherein a  
 356 damage is induced in the third element by a 60% reduction in its stiffness value. Meanwhile, the fifth sensor  
 357 corresponding to the fifth node of the model is considered faulty, which registers only a Gaussian noise.  
 358 From past experience with JEKF [40], the state and parameter covariances are selected as,  $\mathbf{Q}^v = 10^{10} \times \mathbf{I}$ ,  
 359 and  $\mathbf{Q}^\theta = 10^3 \times \mathbf{I}$ , and have been maintained throughout the article unless mentioned otherwise. However  
 360 existing methods [41–43] that identify and quantify  $\mathbf{Q}^u$ ,  $\mathbf{Q}^v$ , and  $\mathbf{Q}^\theta$  can also be opted.

361 Figure 3a shows that the JEKF algorithm could not detect the presence of damage in element 3 and  
 362 wrongly identified damage in fourth and sixth elements which were adjacent to the fifth node/element, where  
 363 the faulty sensor is assumed. Had the absence of such faulty sensors been supplied to the algorithm, the  
 364 algorithm could have correctly detected the damage, cf. Figure 3b. Real-life experiments are not immune  
 365 to such possibility of sensor/s getting faulty during experimentation which justifies the need for a robust  
 366 approach that can efficiently isolate a faulty sensor and its mal-effects. It should be noted that in the  
 367 proposed algorithm, JEKF can be substituted by any other damage detection algorithm available, and there  
 368 should not be any variation in the outcome.



(a) Health Index estimation in the presence of faulty sensor - *faulty\_sens = 5; dam\_mem = 3*. (b) Health Index estimation in the absence of faulty sensor - *faulty\_sens = none; dam\_mem = 3*.

Figure 3: Effect of the presence of faulty sensor on Health Index estimation - JEKF.

369 The proposed method is further investigated for several cases: i) presence of single faulty sensor (Section  
 370 6.2), ii) presence of multiple (consecutive) faulty sensors (Section 6.3), and iii) presence of damage in the  
 371 structure as well as faulty sensor/s (Section 6.4). For each case, it is assumed during estimation that no  
 372 two sensors/accelerometers can suffer fault occurrence at the same time instant, meaning, the multi-sensor  
 373 fault situations are realised by assuming the sensors getting faulty in sequence only. Nevertheless, this  
 374 does not restrict the algorithm to be employed for simultaneous multi-sensor fault scenarios. Under this  
 375 assumption, the model state transition probability is chosen as per the structure provided in Equation (21),  
 376 with  $\beta = 0.05$ ,  $\delta = 0.005$ , and  $\tilde{Z}_0$ , and  $\tilde{Z}_1$ , calculated as per Section 5.1. The number of sensors available  
 377 for measurement (acceleration) has been assumed to be,  $p = 8$ . Accordingly,  $m = (1 + 8 + 28) = 37$  (one no  
 378 fault, eight single fault and 28 double fault models) has been assumed as the total number of measurement  
 379 models employed, with  $(p + 1) = 9$  options ( $n_o$ ) available for each model to transit to others (cf. Figure 1).  
 380 The proposed approach is subsequently employed on the simulated (contaminated) measurements and the  
 381 most likely model candidates and their transitions are tracked online.

382 Note that in the following sections the faulty models have been identified and plotted in recurrence.  
 383 However, the faulty models have been represented based on the information about the faulty sensors. Thus  
 384 instead of indicating switch position among  $m$  numbers of models, the plots present the health state of the  
 385 sensors by  $(p + 1)$  sensor health states (0 corresponding to no-fault and subsequent number  $\in 1, p$  as damage  
 386 at that corresponding sensor).

### 387 6.2. Detection of faulty sensor

388 In real-life SHM, a sensor may stop functioning adequately midway into the experiment or right from  
 389 the beginning. Since SHM utilizes a lot of instrumentation, it is difficult to keep track of such discrepancies.  
 390 Hence the proposed algorithm is aimed to detect such faulty sensors promptly so that a faulty measurement  
 391 does not jeopardize the overall monitoring. To mimic the aforementioned scenarios, two cases are simulated,  
 392 i) a sensor is faulty from the beginning of the simulation (cf. Figure 4a), and ii) a sensor becomes faulty  
 393 10 s after the start of the simulation (cf. Figure 4b). While the case study with fault at the beginning

394 investigates how robust the fault detection is with the proposed method, the second case study investigates  
 395 a sudden sensor fault scenario and the proposed method's promptness in isolating it.

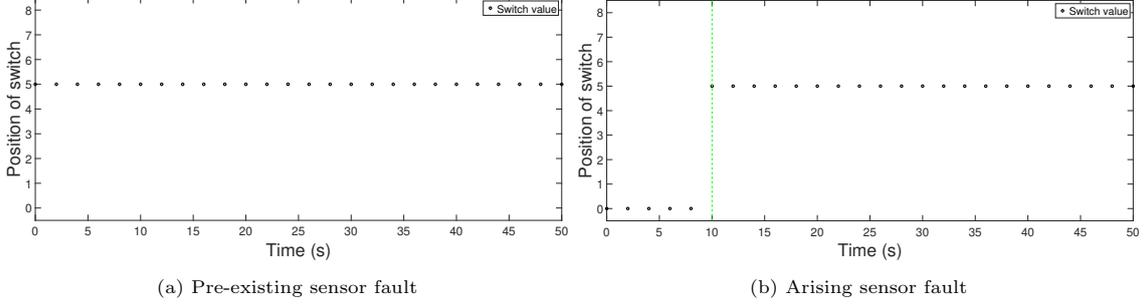


Figure 4: Detection of sensor fault - *faulty sens = 5*.

396 Figure 4 demonstrates the capability of the proposed algorithm in detecting existing as well as sudden  
 397 faults in the sensors. The detection is perceived to be prompt while being precise with no experience of false  
 398 alarm (cf. Figure 4b). With the proposed algorithm, the lack of meaningful measurement from the faulty  
 399 sensor is avoided by selecting suitable measurement model, and thereby estimating corresponding state and  
 400 parameter. From the estimated states, acceleration data corresponding to the faulty sensor is reconstructed,  
 401 which had previously registered only noise ( $\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k$ , following Equation(4)). Figure 5 demonstrates  
 402 seemingly accurate acceleration data estimate.

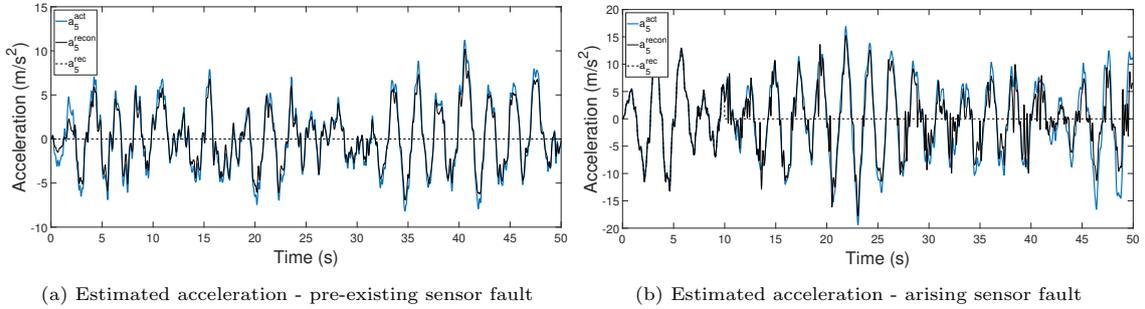


Figure 5: Estimated acceleration corresponding to the faulty sensor - *faulty sens = 5*,  
 where,  $a^{act}$ ,  $a^{recon}$ , and  $a^{rec}$  are the actual, reconstructed, and recorded acceleration, respectively, corresponding to faulty  
 sensor 5.

### 403 6.3. Multi-sensor fault detection

404 To avoid the non-functioning of sensor/s to affect the damage estimation, a general practice is to thor-  
 405 oughly check the correct working of each sensor at the beginning of the monitoring, manually or through  
 406 an algorithm similar to the procedure detailed in the previous section. Most of the faulty sensors can thus  
 407 be detected and isolated. Subsequently rectification can be performed before the inspection or the faulty  
 408 measurement can be allowed to be corrected by the proposed approach. However, it is not rare to find that  
 409 a second sensor goes faulty even in the presence of a pre-existing faulty sensor. This scenario has already

410 been discussed previously and can be corrected by altering the candidate model, cf. Section 5.1. The same  
 411 approach has been employed here for a case study with multi-sensor faults.

412 For this, the lumped-mass model is simulated with the third sensor getting faulty at 10<sup>th</sup> s after the start  
 413 of the simulation. Next, the fifth sensor also gets faulty starting at 20<sup>th</sup> s of the simulation and thereafter  
 414 the acquisition system records faulty measurement data from the mentioned sensors till the end of the  
 415 simulation. Figure 6 shows the capability of the proposed algorithm in detecting multiple consecutive faults  
 416 in different sensors. With the presumption made within the model switch transition that the probability of  
 417 a faulty sensor to work again is very low (Section 5.1), the switch transition should be interpreted keeping  
 418 prior switch position into consideration. The timing in switching the model also describes that the model  
 419 selection with the proposed algorithm is significantly prompt.

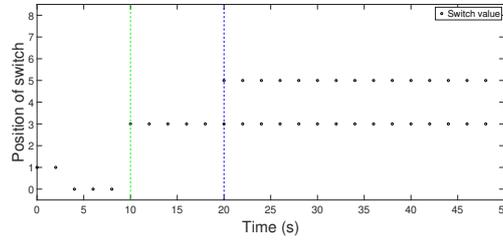


Figure 6: Identification of multiple faulty sensors - *faulty sens = 3 & 5*.

420 The algorithm has further been successful to reconstruct the acceleration data corresponding to both the  
 421 faulty sensors 3 and 5, with minimum deviation from the actual acceleration data (except at the crests and  
 422 troughs) (cf. Figure 7).

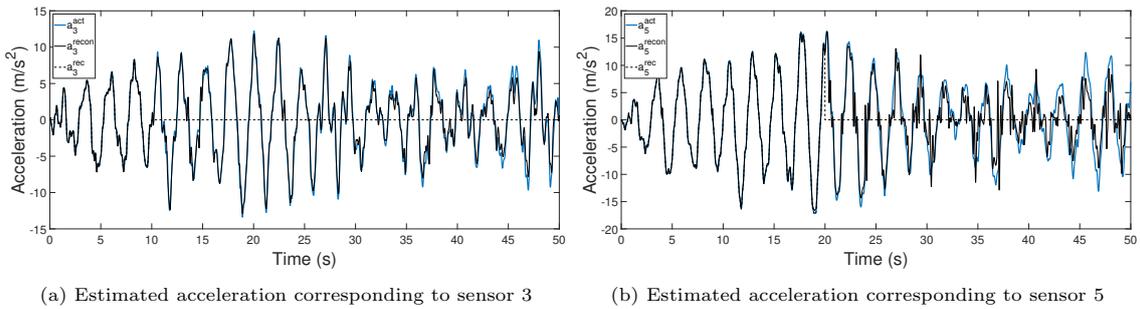


Figure 7: Estimated acceleration corresponding to the faulty sensor - *faulty sens = 3 & 5*,  
 where,  $a^{act}$ ,  $a^{recon}$ , and  $a^{rec}$  are the actual, reconstructed, and recorded acceleration, respectively, corresponding to faulty  
 sensors 3 & 5.

#### 423 6.4. Structural damage identification in the presence of faulty sensor/s

424 The major objective of this study is to ensure that the presence of some faulty sensor/s does not hinder  
 425 the functioning of the employed SHM approach. The detection of structural anomalies should ideally be  
 426 equally seamless regardless of the presence or absence of faults in certain sensors in a network of sensors.

427 Sensor fault isolation with the proposed algorithm has already been discussed in the previous section. Its  
 428 efficiency in prompt detection and estimation of structural damage is investigated next. Damage is quantified  
 429 with the help of **HI**s that quantify location health with values ranging from 0 (complete damage) to 1 (no  
 430 damage). The model is now simulated for 80 s at 50 Hz sampling frequency with an induced damage and  
 431 the response is recorded assuming the presence of faulty sensors. A 60% damage is induced in the 3<sup>rd</sup>  
 432 member of the lumped-mass model by reducing its stiffness to 40% of its original value. A fault in sensor 5  
 433 is induced 1 s after the start of the simulation. The proposed algorithm is subsequently employed with the  
 434 recorded response to isolate sensor faults while identifying the damage in the structure.

435 Figure 8 shows that the proposed algorithm is capable of detecting and quantifying damage in the  
 436 structure as well as identifying and localizing the faulty sensor. The algorithm has correctly identified the  
 437 presence of fault in sensor 5 immediately after its occurrence (cf. Figure 8a), safeguarding the damage  
 438 estimation algorithm. An **HI** of about 0.4 is estimated for the third stiffness element that corresponds to  
 439 the 60% damage induced during the simulation. The proposed algorithm is found to be prompt, precise and  
 440 efficient in detecting damage in the model (cf. Figure 8b) in the presence of sensor fault.

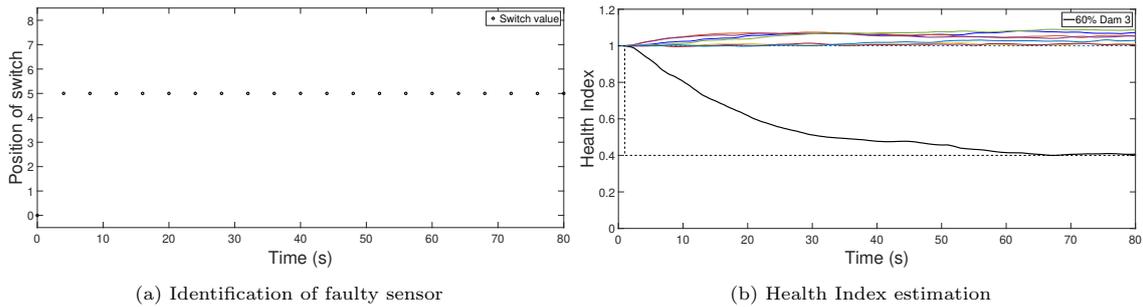


Figure 8: Detection of faulty sensor and structural damage - *faulty sens = 5; dam mem = 3*.

441 Further investigation on the performance of the proposed algorithm is undertaken by introducing a 60%  
 442 damage in the second member with successive faulty sensors. It is assumed that any faulty sensor stays  
 443 defunct throughout the simulation (cf. Section 5.1). Accordingly, faults in sensors 3 and 5 are introduced  
 444 10 s and 20 s after the start of the simulation, respectively. The proposed algorithm is able to detect,  
 445 localize and estimate the extent of damage present in the structure while promptly isolating both the faulty  
 446 sensors, cf. Figure 9.

447 In real-time SHM, a sudden member damage may lead to sensor faults in its vicinity. The proposed  
 448 algorithm is further tested for its efficacy under simultaneous initiation of sensor fault and member damage.  
 449 A fault is introduced in sensor 5 while a 60% damage is induced in the 3<sup>rd</sup> member, after 1 s of simulation.  
 450 The system is simulated for 120 s. The proposed algorithm (cf. Figure 10) is able to accurately detect  
 451 presence of sensor fault as well as damage in member. While the sensor fault detection was prompt, the  
 452 promptness for damage detection decreased when compared to that observed in Figure 8b, where sensor

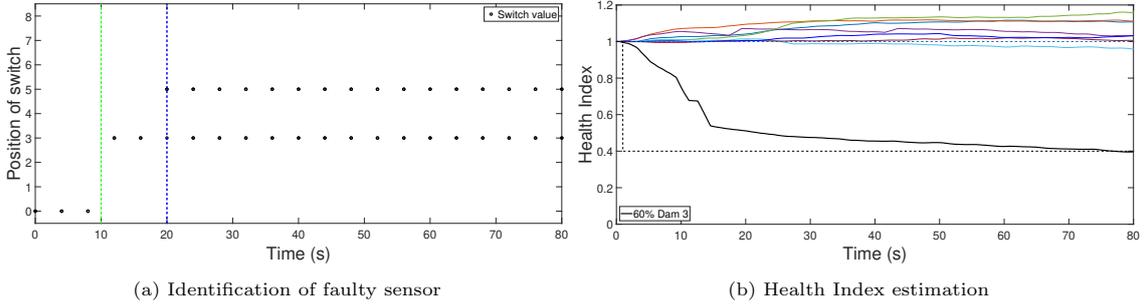


Figure 9: Detection of faulty sensors and structural damage -  $faulty\ sens = 3 \ \& \ 5$ ;  $dam\ mem = 3$ .

453 fault and member damage were initiated at different time instants.

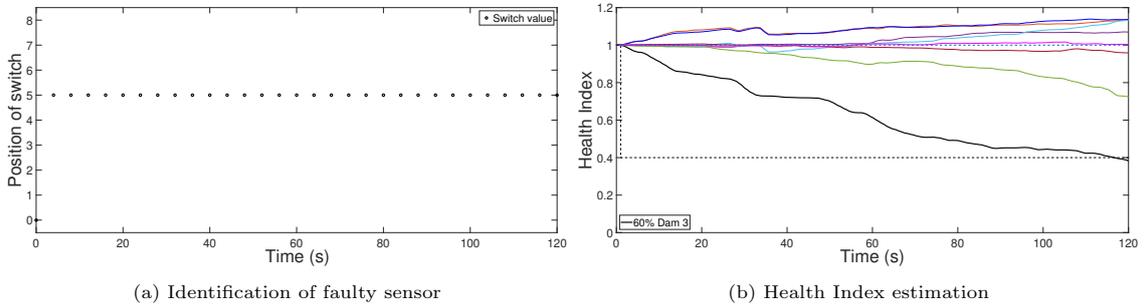


Figure 10: Detection of simultaneous sensor fault and structural damage -  $faulty\ sens = 5$ ;  $dam\ mem = 3$ .

454 The proposed algorithm's efficiency is checked under simultaneous initiation of multiple sensor faults  
 455 (3 and 5) and member damage (60% damage in member 4). The faults and the damage start 1 s into  
 456 the simulation. Instead of utilizing double fault model from the beginning (Equation (18)), the algorithm  
 457 follows the same approach as followed in multiple consecutive faults (cf. Figure 9) where the algorithm  
 458 is started by considering no fault is present, followed by detection of a single fault. If a single fault is  
 459 detected consistently for certain iterations (here, 1000), the pertinent double fault model is introduced in  
 460 the algorithm. The iteration number has been arbitrarily chosen, however it is suggested that a minimum of  
 461 500 iterations - depending on sampling rate (not more than a few seconds) - maybe considered so that the  
 462 presence of single fault is ascertained. It is a trade-off to be adjusted by the user. Figure 11a shows that the  
 463 algorithm detected the simultaneous faults promptly and accurately. Meanwhile damage in member 4 has  
 464 also been accurately identified. It has also been observed that the promptness in detection of damage with  
 465 simultaneous multiple fault is slower than that presented in before mentioned cases (Figures 9b and 10b).

466 To understand the effect of SNR, sensor number and location on the proposed approach, several case  
 467 studies have been undertaken with a summary of them listed in Table 1. Under these investigations, the  
 468 third member has been damaged by 30% after 1 s of the start of the simulation and a sensor fault is induced  
 469 after 15 s. For C.No. 1-4, measurement noise sensitivity studies for noise contamination levels of 1/2/5/10%

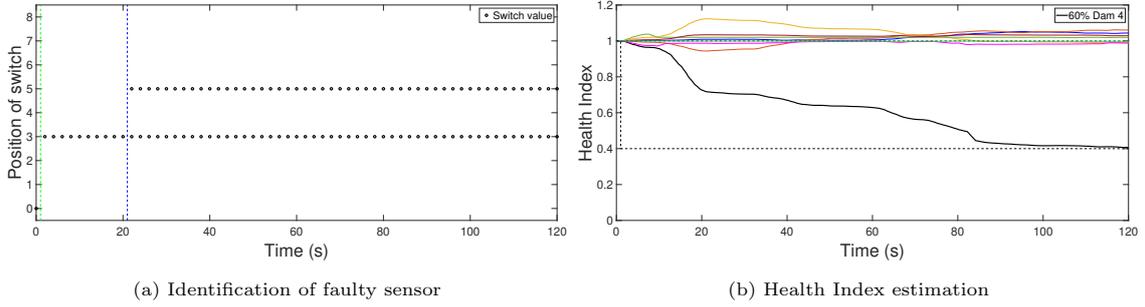


Figure 11: Detection of simultaneous sensor faults and structural damage -  $faulty\ sens = 3 \ \& \ 5$ ;  $dam\ mem = 4$ .

470 SNR have been undertaken. While the isolation of the faulty sensors has been perceived to be prompt, the  
 471 accuracy in damage detection has been observed to deteriorate with increasing noise levels.

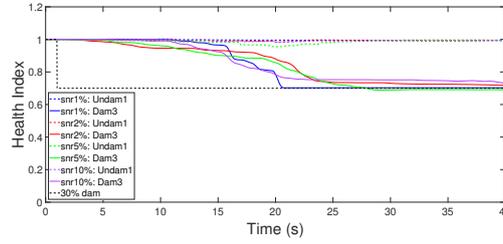


Figure 12: Measurement noise sensitivity -  $faulty\ sens = 5$ ;  $dam\ mem = 3$ .

472 Similar measurement noise sensitivity study has been done with a varying number of sensors (cf. Table 1).  
 473 The accuracy in damage detection has been observed to be dependent on the sensor density besides the noise  
 474 levels. A few false positives (for member damage) have also been observed for higher SNR% and lower sensor  
 475 densities. The algorithm fails to correctly determine the damaged member for cases C.No. 11-13. However  
 476 it should be noted that this inefficiency should be attributed to the intrinsic performance of the damage  
 477 detection algorithm, here JEKF. Therefore, keeping the complexity level of the problem into consideration,  
 478 the investigator must choose the proper filter-based damage detection algorithm to couple with the proposed  
 479 SKF approach.

Table 1: Signal-to-noise ratio, sensor number and location sensitivity.

C.No.	No. of sensors	Sensor fault location - detection	SNR (%)	Damage location - detection	Accuracy (%)	False positive	
						Sensor fault	Member damage
1	8 - {1:8}	5 - ✓	1	3 - ✓	99.61	×	×
2			97.60		×	×	
3			98.10		×	×	
4			95.28		×	✓	
5			99.24		×	×	
6	6 - {1:6}	5 - ✓	1	3 - ✓	97.12	×	×
7			97.54		×	✓	
8			94.85		×	✓	
9			98.00		×	×	
10	4 - {1:4}	4 - ✓	2	3 - ×	96.41	×	×
11			—		—	—	
12			—		—	—	
13	3 - {1:3}	2 - ✓	1	3 - ×	—	—	—

480 **7. Experimental validation - Cantilever beam**

481 To investigate the performance of the proposed algorithm on real-life structures, a laboratory experiment  
 482 has been conducted on a cantilever beam (cf. Figure 13a) replicated with a numerical 2D Euler-Bernoulli  
 483 beam for the proposed model-based damage detection approach. The length of the beam is 750 mm with  
 484 a cross-sectional area of 8 mm × 8 mm. Further, the numerical model is calibrated (detailed later) for  
 485 its material properties in order to ensure perfect replication of the real beam. Details of geometric and  
 486 calibrated material properties are presented in Table 2. The schema of the numerical model compared to  
 487 the real experiment is presented in Figure 13b.

Table 2: Properties of the cantilever beam.

Length of beam	750 mm
Cross section of beam	8 mm × 8 mm
Calibrated modulus of elasticity	135 GPa
Calibrated density	7890 Kg/m <sup>3</sup>
Damping ratio	0.006

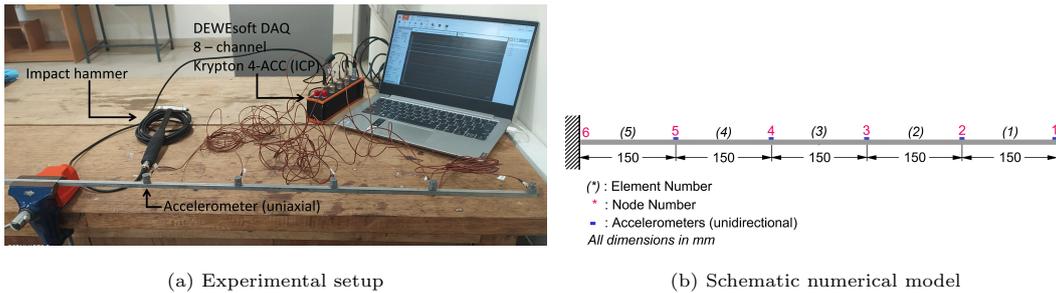
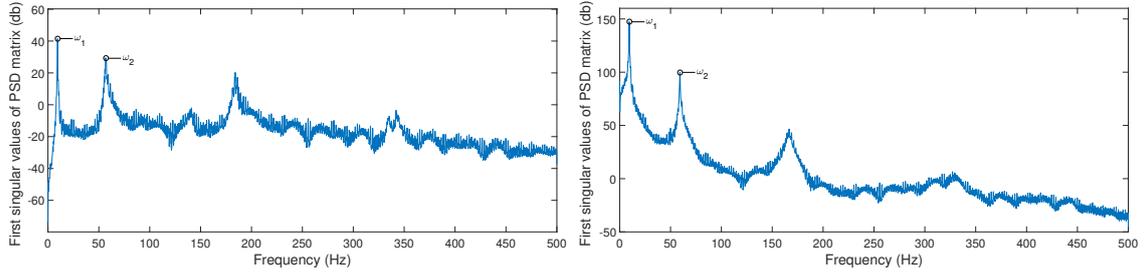


Figure 13: Experimental setup - cantilever beam.

488 *7.1. Calibration of the numerical model*

489 The experimental beam is segmented into 5 elements and the accelerometers are placed on each of the 5  
 490 free nodes (cf. Figure 13b). By parameterizing each of the health elements, isolation of the damage location  
 491 is possible. The accelerometers are sampled at a fixed sampling rate of 1000 Hz for 3 s. The cantilever beam  
 492 is subjected to an arbitrary input force by an impact hammer. The measured signal is further decomposed  
 493 in frequency domain using the frequency domain decomposition (FDD) approach. Figure 14a highlights  
 494 the first two natural frequencies obtained from the undamaged cantilever beam, (i.e.,  $\omega_1 = 9.40 Hz$ , and  
 495  $\omega_2 = 56.64 Hz$ ). The associated frequency response function (FRF) is presented in Figure 14a. The sensors  
 496 from which the signal for the FDD computation was sampled were all functioning perfectly.

497 Simultaneously, the supporting numerical FEM model is developed with similar spatial discretization  
 498 and each of the elements is defined as a 2D-Euler-Bernoulli beam element. The modulus of elasticity ( $E$ )  
 499 and density ( $\rho$ ) are firstly assumed for the model which is further manually calibrated to obtain a set of

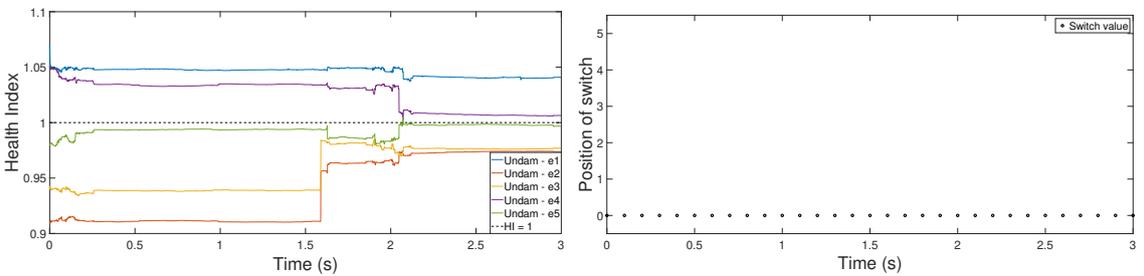


(a) FDD of acceleration data obtained from the experiment; (b) FDD of acceleration data obtained from calibrated numerical model;  $\omega_1 = 9.40 \text{ Hz}$ , and  $\omega_2 = 56.64 \text{ Hz}$   $\omega_1 = 9.40 \text{ Hz}$ , and  $\omega_2 = 57.33 \text{ Hz}$

Figure 14: Calibration of the 2-D cantilever beam by comparing frequencies obtained from Frequency Domain Decomposition (FDD);  $\omega_1$ , and  $\omega_2$  are the first, and second frequency of the structure, respectively.

500  $E$  and  $\rho$  that results in a set of natural frequencies that are marginally close (within 1-2%) to the ones  
 501 obtained experimentally. The calibrated material properties are presented in Table 2. The FRF and the  
 502 natural frequency peaks for both experimental beam and numerical model are presented in Figure 14. It  
 503 can be verified that the calibrated numerical model closely replicates the reality. Still, this model assumes  
 504 the same material properties for all its elements which is not practical for the present purpose.

505 In the following, the measured response, corresponding to the undamaged cantilever beam and no sensor  
 506 faults, is processed through the proposed algorithm. Five **HI**s corresponding to the five elements are  
 507 defined and selected randomly around 1 and further the proposed algorithm is employed to converge to their  
 508 corresponding best estimates. The model state transition probability is chosen such that  $\beta = 0.01$ , and  
 509  $\delta = 0.001$ , with  $\tilde{Z}_0$ , and  $\tilde{Z}_1$ , calculated following Section 5.1,  $m = (1 + 6 + 15) = 22$  and  $n_o = 6$ . Further,  
 510 the state and parameter covariances have been tuned to  $\mathbf{Q}^y = 10^{-11} \times \mathbf{I}$ , and  $\mathbf{Q}^\theta = 10^{-4} \times \mathbf{I}$  for the current  
 511 and the following experiments. The convergence of the **HI**s to their respective best estimates are presented  
 512 in Figure 15a and corresponding elasticity values (pertaining to individual members) are calculated. These  
 513 values are then selected as the initial healthy state of the respective members for experiments performed  
 514 in Section 7.2. Meanwhile, the algorithm additionally identified the absence of any faulty sensor, with the  
 515 switch position consistently at zero for the entire time (cf. Figure 15b). Consequently, this updated model  
 516 is used as the undamaged predictor model in the proposed algorithm from here on.



(a) Health Index estimation of the elements, e1-e5. (b) Identification of faulty sensor by the proposed algorithm.

Figure 15: Bench-marking health indices with respect to undamaged 2-D cantilever beam.

517 7.2. Experimental verification of the proposed approach

518 Further experiments are performed on the cantilever beam with sensor faults and damage. In practice,  
 519 it has been experienced that a sensor fault can be caused by several reasons: sensor detachment, tearing  
 520 or connection faults in the sensor/data acquisition system (DAS) ends, hardware faults in the response  
 521 recording channels, etc. Among them, two types of sensor fault scenarios have been considered: detaching  
 522 the second sensor from its position and loosening the connection of wire of the second sensor to the DAS. It  
 523 has been observed that for sensor fault scenarios, DAS records only a sensor noise from which measurement  
 524 noise covariance has been estimated. Similar to the calibration study, the acceleration data is recorded for  
 525 3 s at a sampling frequency of 1000 Hz.

526 A fault is introduced in sensor 2 for the entire duration of the experiment, whereas no damage is inflicted  
 527 on the cantilever beam. Figure 16a shows that the algorithm is able to detect the faulty sensor (sensor 2)  
 528 with no false positives. Furthermore, the algorithm correctly established an absence of any damage with  
 529 the **HI**s for all the members being estimated around their nominal value, i.e.  $\approx 1$  (cf. Figure 16b).

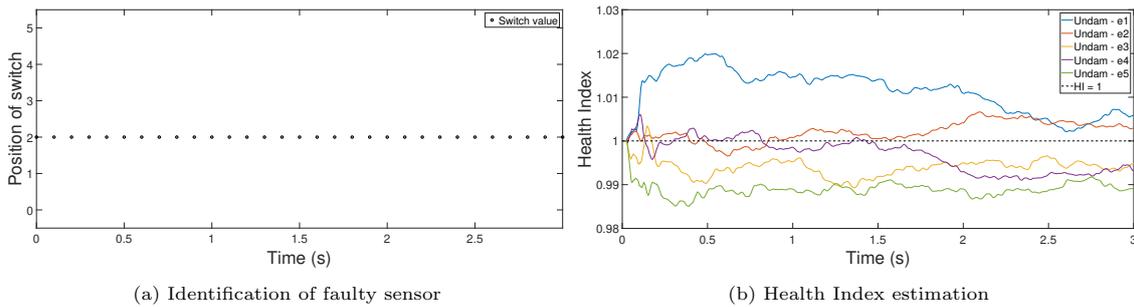


Figure 16: Detection of faulty sensors and structural damage - *faulty sens = 2; dam mem = none*.

530 For logistical reasons, the structure was not allowed to be damaged. Therefore, to establish the efficiency  
 531 of the proposed method in estimating **HI**s, one of the initial **HI**s, i.e., **HI** corresponding to e1 (element 1),  
 532 is set to 1 with an increased modulus of elasticity,  $E = 180 \text{ GPa}$ . This value is 1.33 times its bench-marked  
 533 value of 135 GPa, mimicking a 25% damage in element, e1, which is subsequently estimated using the  
 534 proposed method. A fault in sensor 2 is established for the entire duration. Figure 17 substantiates that  
 535 the proposed algorithm is capable of identifying the faulty sensor (Figure 17a) as well as detecting damage  
 536 in the structure in the presence of faulty sensor data (Figure 17b).

537 **8. Conclusion**

538 Sensor faults being inevitable during a real-life structural health monitoring endeavor, it is practical to  
 539 make the employed SHM approaches robust against such potential sensor faults. In an ideal sense, it is  
 540 desired that a sudden sensor fault is isolated as soon as it occurs in order to avoid the effects of erroneous  
 541 measurements which might lead to inaccurate and/or divergent estimates. To fulfil this objective, this paper

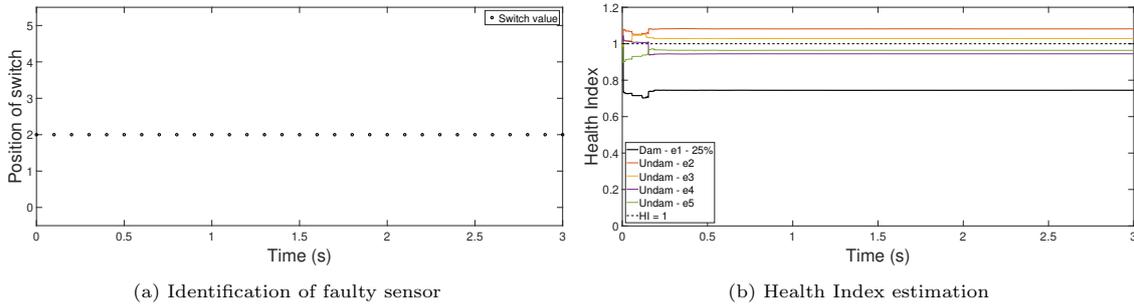


Figure 17: Detection of faulty sensors and structural damage -  $faulty\ sens = 2$ ;  $dam\ mem = 1$ .

542 proposes an Joint Extended Kalman filtering (JEKF) approach coupled with a switching filtering strategy.  
 543 The switching strategy relies on multiple measurement models corresponding to all possible single fault  
 544 scenarios, and a switch variable is defined to select the most suitable candidate among all the models. This  
 545 strategy ensures immediate removal of the measurement sampled at the faulty sensors in order to ensure  
 546 stable estimation. The capabilities of the algorithm are illustrated through numerical and real experiments  
 547 in which the proposed algorithm has been perceived to be prompt, accurate, and sufficiently robust to  
 548 possible false alarms.

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 550 through grant file no. ECR/2018/001464.

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