

Stability and accuracy of Runge-Kutta based split-explicit time-stepping algorithms for free-surface ocean models

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Runge-Kutta methods

Advantages of RK methods:

- ▶ 2-time-level schemes (no computational mode)
- ▶ Good stability properties for advection and diffusion
- ▶ Abundant literature (e.g. RK-IMEX, RK-SSP, DIRK, etc)
- ▶ Easy to implement (convex combination of Euler steps)

Main questions:

- ▶ Many variants of RK schemes
- ▶ **RK with mode-splitting ?**

Existing work:

- ▶ Many RK-based atmospheric models (e.g. WRF, Cosmo, Homme-NH, etc)
- ▶ « *Pseudo-second order Runge-Kutta time stepping scheme* » in HIM/MOM6 [Hallberg, 1997]

Runge-Kutta methods in the oceanic context

► Many variants of RK schemes

- Low-storage and « order=stage » method for advection [Wicker & Skamarock, 2002]

$$\begin{aligned} \partial_t \phi = \mathcal{F}(\phi, t) \quad \longrightarrow \quad \begin{aligned} \phi^{n+1/3} &= \phi^n + \frac{\Delta t}{3} \mathcal{F}(\phi^n) \\ \phi^{n+1/2} &= \phi^n + \frac{\Delta t}{2} \mathcal{F}(\phi^{n+1/3}) \\ \phi^{n+1} &= \phi^n + \Delta t \mathcal{F}(\phi^{n+1/2}) \end{aligned} \end{aligned}$$

- Introduce Shuman averaging for internal gravity waves (« order=stage-1 »)

► Additional difficulty inherent to oceanic models

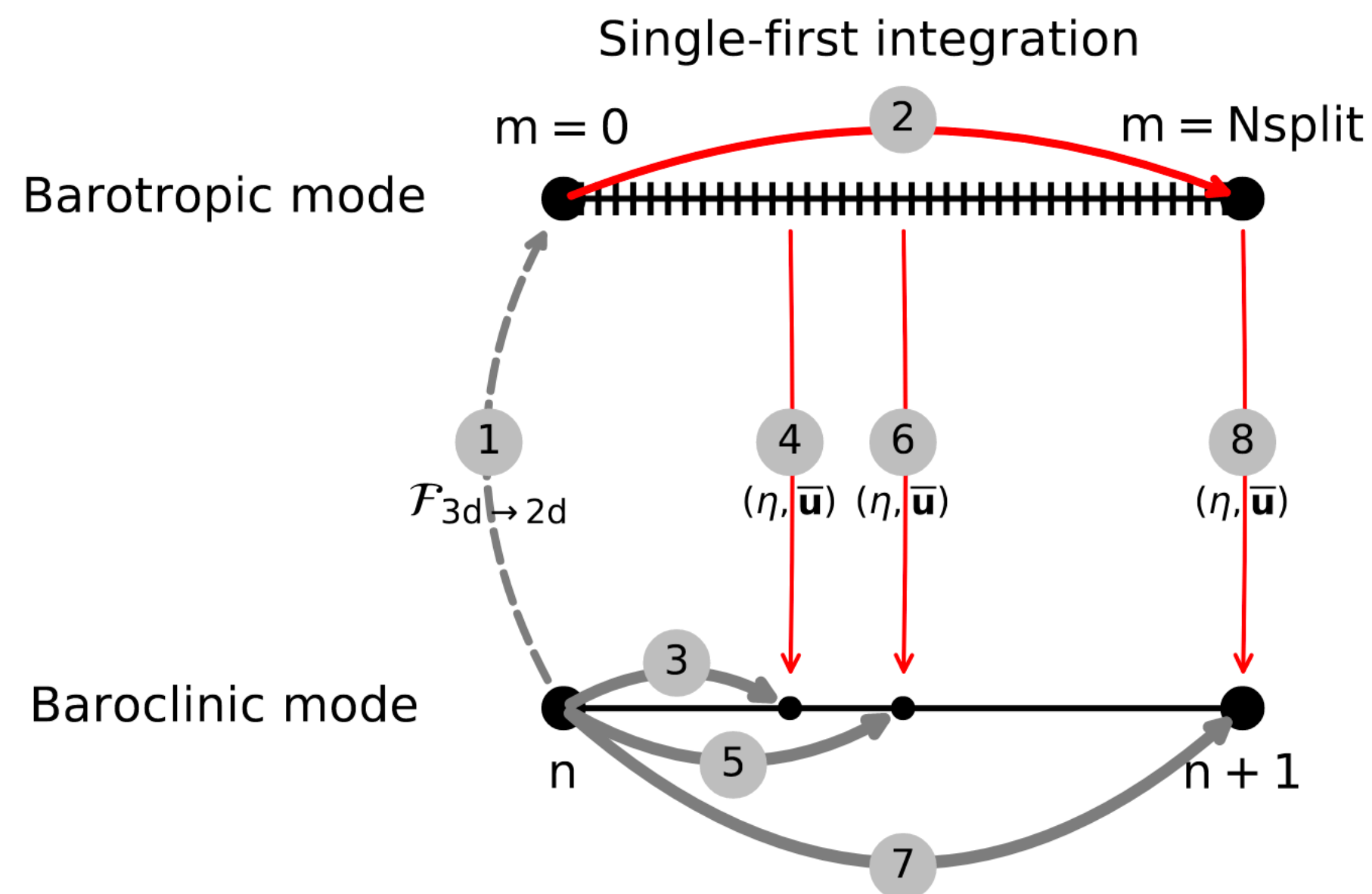
- 2D barotropic / 3D baroclinic mode splitting to improve computational efficiency (fast and slow dynamics are split into separate subproblems)
- Separation of slow and fast modes is non-orthogonal (depth-independent barotropic mode assumption)

➡ Some form of filtering is required to stabilize the splitting

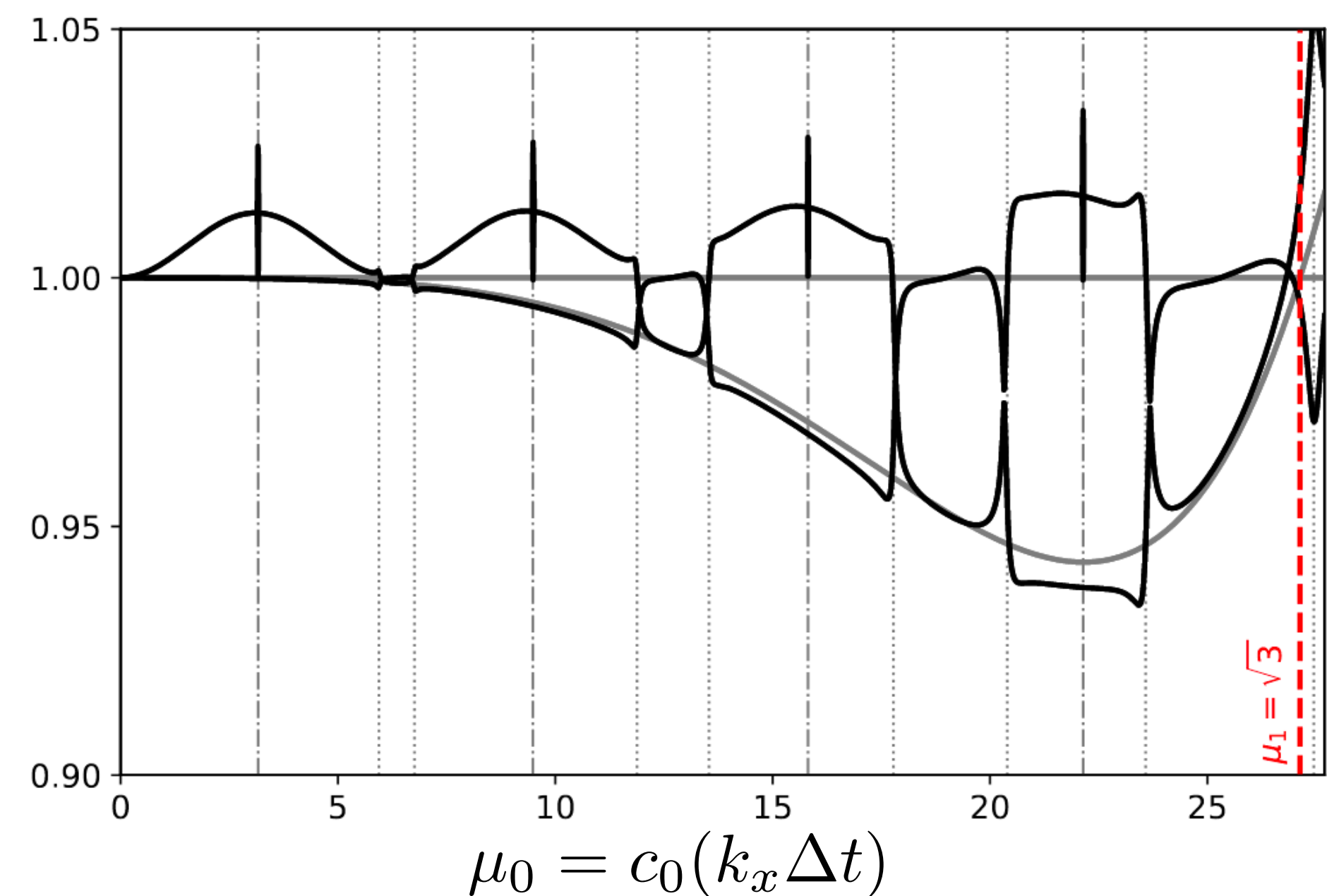
$$\begin{aligned} \partial_t \bar{u} &= -g \partial_x \left(\eta + \frac{1}{\rho_0 g H} \int_{-H}^0 p_h \, dz \right) \\ \partial_t \eta &= -H \partial_x \bar{u} \end{aligned}$$

Robust mode-splitting method based on RK3

Peculiarity of oceanic models: 2D barotropic / 3D baroclinic mode splitting to improve computational efficiency



Gravity waves integration (without dissipation in 2D mode)



Single-first approach:

- Only one integration of the 2D within the RK3 time-step
- Split-explicit approach with a dissipative time-stepping for the barotropic mode
- Linear stability analysis can be conducted following [Demange et al. \(2019\)](#)

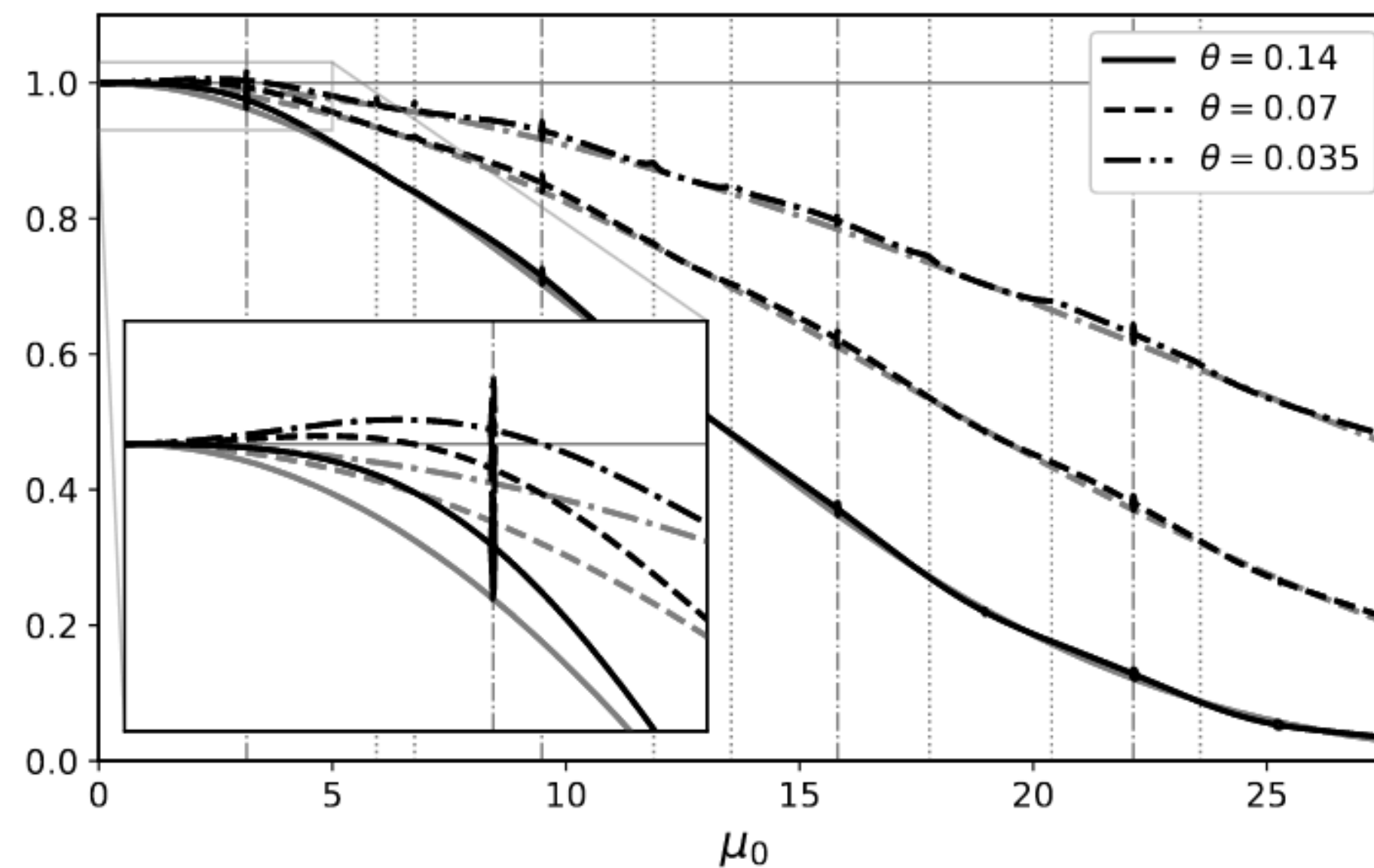
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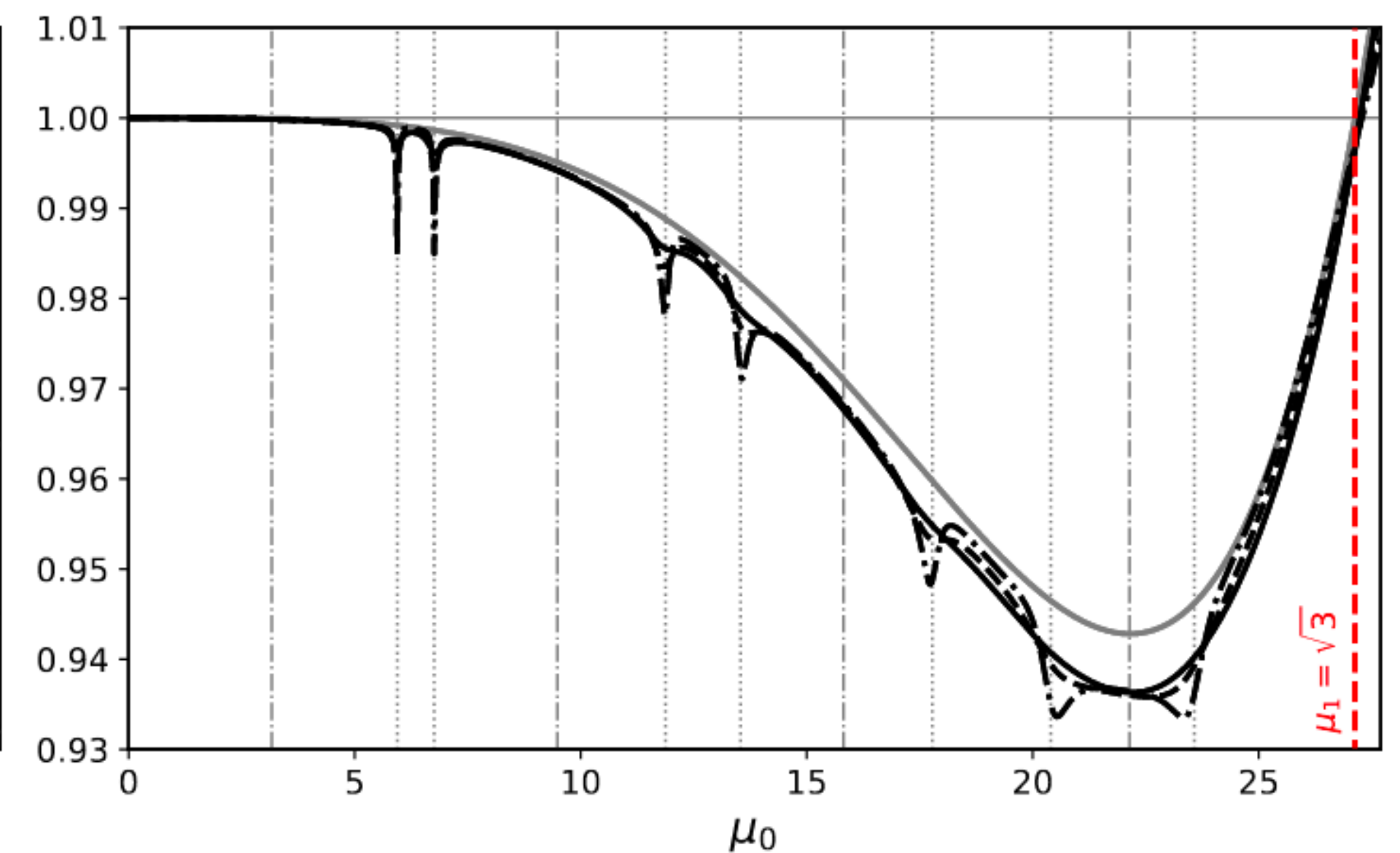
$$\bar{u}^{n+1} = \bar{u}^n - g\Delta t_0 \partial_x \eta^n$$

$$\eta^{n+1} = \eta^n - H\Delta t_0 \partial_x \left((1+\theta)\bar{u}^{n+1} - \theta\bar{u}^n \right)$$

(a) Barotropic mode amplification



(b) Baroclinic mode amplification



- Linear stability analysis provides the minimal dissipation to stabilize the splitting
- Stability constrained by barotropic mode and not by baroclinic ones

Conclusion and future work

Summary:

- ➡ Design of an RK3 based time-stepping algorithm for split-explicit free-surface ocean models
- ▶ The single-first strategy appears to be more robust than other alternatives
- ▶ Thought for a quasi-Eulerian vertical coordinate
- ▶ A non-linear z^* 2D x - z model has been developed to check results from stability analysis

Ongoing work:

- ▶ Implementation in NEMO (see next talk)
- ▶ Accuracy analysis and consequences of mode-splitting

◆ Ducousso, N., Lemarié, F., Debreu, L., Madec, G.: *Stability and accuracy of Runge-Kutta-based time-stepping algorithms for split-explicit free-surface ocean models*. In preparation for J. Adv. Model. Earth Syst.