Stability and partial instability of multi-class retrial queues

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1 Introduction

It is desirable that queueing systems and networks operate with small queues and not overloaded servers. This is the topic of stability analysis [17, 20, 18, 24]. We would also like to note that the methods of stability analysis are used in control [25] and performance evaluation [19].

In most standard queueing systems, in the case of overload, customers are either lost or experience very long or infinite delays. However, in many real systems, rejected or significantly delayed customers leave the queue and then return later. This natural phenomenon motivates the development of *retrial queues* [15, 3]. In retrial queueing models, once a customer is rejected, he or she goes into the *orbit* and retries from there. This significantly increases the complexity of the system and makes the analysis of retrial queues challenging. There are two main types of retrial: independent retrials and constant rate retrials. In the former case the retrying customers are in "competition" and retry from the orbit independently. In the latter case, retrying customers wait in the orbit, according to the FIFO principle, and retry only when they are at the front of the orbit queue. Under fairly general assumptions, the stability condition for queueing models with independent retrials is the customary condition on the system load $\rho < 1$ (for the single-server case), see e.g., [15, 2, 21]. The stability conditions for models with a constant retrial rate are more complicated [16, 12, 4, 10, 5].

It is also quite natural to consider retrial queueing models with several classes of customers, where each class has its own orbit [9, 22]. Such multi-class retrial systems are useful to model call centers [23] as well as wireless and access control systems [9, 14]. An important feature of multi-class retrial systems with a constant retrial rate was observed in [9]: even if one class is very aggressive, the other classes are protected by the nature of the constant retrial rate. This also manifests itself in the very interesting phenomenon of *partial* or *local* stability [8, 1], when some components (orbits) of the

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system can go to infinity in probability or almost surely, whereas the other components are tight [8] or converge component-wise in distribution [1].

2 Problem statement

Consider a single-server K-class retrial queueing system with constant retrial rates. The system has K Poisson inputs with rates λ_k and service times with general distributions F_k and means $1/\mu_k$, $k = 1,...,K$. Customers cannot queue at the server, only within one of K orbits. Define a basic $K+1$-dimensional process $\mathscr{X}(t) = (N(t), X^{(1)}(t),...,X^{(K)}(t))$, $t \geq 0$, where $N(t) = 1$ if the server is busy ($N(t) = 0$ otherwise) and $X^{(k)}(t)$ is the size of orbit k. If a new customer of class-k comes to the system and sees that the server is busy, he or she goes to the k-th orbit. The class-k customers retry from orbit k, according to FIFO, with exponential retrial times with rate α_k. Let $\rho = \lambda_1/\mu_1 + ... + \lambda_K/\mu_K$.

Conjecture. The multi-class retrial queue with constant retrial rates is stable (the zero-delayed regenerative process $\mathscr{X}(t)$ with the regeneration state $(0,0,...,0)$ is positive recurrent) if and only if

$$\rho < \min_{k=1,...,K} \frac{\alpha_k}{\lambda_k + \alpha_k}.$$

In addition to the above conjecture, there are other interesting open research directions: What are the stability conditions for more general arrival processes and retrial times? In [8,6] some sufficient conditions for partial stability have been proposed. It is already clear that those conditions can be strengthened. Also, it is interesting to consider control and game-theoretic formulations in the multi-class retrial queues.

3 Discussion

The conjecture was proven using an algebraic approach in [9] for two classes with exponentially distributed service times and $\mu_1 = \mu_2$. Recently, in [6] the conjecture was proven for two classes and general, non-identical service time distributions, using a combination of Lyapunov functions for random walks [17] and a regenerative approach [24]. It is actually not too difficult to prove the “only if” part of the conjecture for any number of classes, which was done in [7]. In [8], sufficient conditions, which are not necessary in general, have been established for the case of any number of classes. In fact, from the analysis in [8], it follows that the conjecture holds for the case of homogeneous classes.

Of course, there is an impressive arsenal of tools to establish stability conditions: Lyapunov functions [20], fluid approach [27,13], saturation rule [11], regenerative approach [24] to name just a few. From [17,6] it is clear that in order to obtain tight conditions, the polynomial Lyapunov functions need to have cross-terms. The standard fluid approach is not applicable as the system is not work-conserving. An approach based on dominating systems and monotonicity [26,28] seems natural for this model. The dominating systems constructed in [8,23] lead to sufficient conditions, which are not necessary. Of course, this does not exclude the construction of a more suitable dominating system. Monotonicity based techniques were also very useful in dealing with partial instability [8,6].

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