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Second-order cone programming for rolling friction contact mechanics

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GENERAL FRICTION CONTACT PROBLEM

\[ Mv + f - H^T r = 0 \]
\[ Hv + w = u \]
\[ F^* \ni u + \Phi(u) \perp r \in F \]

where
- Parameters: \( n \) No. of contact points, \( m \) No. of degrees of freedom.
- Given data: \( 0 < M = M^T \in \mathbb{R}^{m \times m}, H \in \mathbb{R}^{m \times m}, f \in \mathbb{R}^m, w \in \mathbb{R}^m. \)
- Convex problem: \( \Phi(u) \) is fixed.
- Variables: global velocity \( v \in \mathbb{R}^m \), velocity \( u \in \mathbb{R}^m \), reaction \( r \in \mathbb{R}^m. \)

NESTEROV & TOBB (NT) SCALING

To get a symmetric and non-singular system, we need a change of variables at each iteration of the Interior Point algorithm. Define a vector \( p \) such that
\[
\hat{p} := Q_p^p, y = \hat{y}
\]

where \( Q_p \) is quadratic representation of \( p \).

\[
\begin{align*}
\min_{v, u} & \quad \frac{1}{2} v^T Mv + \frac{1}{2} f^T v \\
\text{s.t.} & \quad Hv + w = u
\end{align*}
\]

\[
\begin{align*}
\max_{v, r} & \quad -\frac{1}{2} r^T Mv - w^T r \\
\text{s.t.} & \quad Mv + f - H^T r = 0 \\
& \quad r \in F
\end{align*}
\]

SOLVER

The perturbed KKT system is
\[
Mv + f - H^T KQ^p \hat{y} = 0,
\]
\[
JQ^p \hat{z} - Hv + w = 0,
\]
\[
\hat{z} \circ \hat{y} = 2pe,
\]
\[
(\hat{z}, \hat{y}) \in \text{int}(L^2).
\]

The linear system to solve at each iteration is of the form
\[
\begin{bmatrix}
M & -H^T \\
-H & 0
\end{bmatrix}
\begin{bmatrix}
d^p \\\
d^e
\end{bmatrix} =
\begin{bmatrix}
Mv + f - H^T \hat{r} \\
J\hat{z} - Hv + w
\end{bmatrix}.
\]

One difficulty is that \( Q_p \) is badly conditioned near the optimal solution.

EFFICIENCY & ROBUSTNESS

\[ PA(n) \]
\[ CPU \]

Figure 3. Performance profile

THEORETICAL RESULT FOR FRICTION CONTACT MECHANICS WITHOUT ROLLING FRICTION

- For the case without rolling friction, the original friction cone \( F \) is the Lorentz cone \( L \). If (P) is feasible, then it has a unique optimal solution \((v^*, u^*)\). It is not the case for (D), because \( H \) is rank deficient.

- If the Slater’s conditions and the strict complementarity hold, there are 3 possible cases:

  \[ u = 0, r \in \text{int}(L) \]

  \[ u \in \text{int}(L), r = 0 \]

  \[ u \in bd(L), r \in bd(L) \]

Then, the optimal solution \( r \) will converge to the analytic center of the optimal set \( \hat{R} \) which is the unique optimal solution of the problem
\[
\min_{r \in \text{int}(R)} \psi(r) := - \sum_{i \in B} \log \det r_i - \sum_{i \in R} \log r_{i,0},
\]

where \( (B, N, R) \) is a partition of \( \{1, \ldots, n\} \).

- This result can be extended to the case of rolling friction.