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# Optimal Control of Dengue-Chikungunya Co-infection: A Mathematical Study

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**Abstract.** In the present work, we developed a mathematical model for dengue-Chikungunya co-infection to analyze the disease transmission dynamics and interrelationship. We considered the essence of time dependent optimal control with the help of Pontryagin's maximum principle. The result of diverse combinations of control measures is analyzed and shown graphically.

**Keywords:** Dengue-Chikungunya · Co-infection · Optimal Control

## 1 Introduction

Dengue and chikungunya infections are arboviral diseases which are among the leading public health problem to the entire world [27]. The dengue virus (DENV) infections have been found mainly in Asia, the Pacific, South and Central America and the Caribbean [21]. According to Londhey et al. [16] "Dengue, chikungunya causes a major matter of concern on the health care system. Both the diseases are transmitted through the same female mosquitos *Aedes aegypti* and *Aedes albopictus*. Since both dengue and chikungunya viruses are transmitted through a common vector, they often co-circulate in mosquito and are transmitted to human beings as co-infections following the mosquito bite".

Patients with dengue and chikungunya virus infection generally present with a self-limited febrile disease [20]. Due to the overlap of the symptoms, it is not always easy to distinguish the two infections clinically. The climatic condition

during the rainy season and subsequent months favor mosquito breeding places and thereby amplify the number of mosquitoes as well as a sudden increase of dengue and chikungunya. In recent past, cases of dengue-chikungunya have been confirmed in Africa, Southeast Asia, the Pacific Islands, the Caribbean and Latin America [16]. An effective vaccine for both the diseases are not so frequently available.

The first case of dengue-chikungunya co-infection has been reported in Thailand in 1962 where four co-infected cases among 150 patients was detected with either infected by dengue or by chikungunya; In 1963, three co-infected cases out of 144 infected patients and in 1964, 12 co-infected cases out of 334 infected patients was reported (cf. [10]). In the Southern part of India, the first case of co-infection was reported in 1964 (cf. [5]), where, among the patient with a dengue-like illness, 2% was found to have co-infection. In Myanmar by an active investigation, 36 out of 539 dengue and/or the chikungunya positive patients have been diagnosed to be co-infected in 1970; 8 out of 129 in 1971 and 11 out of 244 in 1972 (c.f. [14]). In the following 30 years, no reported new cases of dengue chikungunya were found, inspite of continuous CHIKV and DENV endemicity in Africa and Asia (cf. [10]).

In 2006, chikungunya-dengue co-infection was found in Malaysia (cf. Nayar et al. [22]), Sri Lanka (cf. Kularatne et al. [15]), Madagaskar (cf. Ratsitorahina et al. [26]) and India (cf. Chahar et al. [7]). Numerous studies have been reported on the co-circulation and co-infection of chikungunya–dengue in Africa (cf. Baba et al. [2] and Caron et al. [6]), Western Pacific region (cf. Chang et al. [8]), South-East Asia (cf. Afreen et al. [1]) and others during 2006-2012.

In the above discussions, it is evident that the spread and severity of the disease have been analyzed by the researchers. But very few articles have been formulated with a mathematical model of dengue chikungunya co-infection. A preliminary mathematical model has been formulated by Isea and Lonngren [11] to analyze the dynamic transmission of dengue chikungunya and zika. Malaria-Schistosomiasis co-infection model has been formulated by Bakare and Nwozo [3]. Mathematical models for HIV/TB Co-infection have been analyzed by Boralin and Omatola [4] and Optimal control theory was applied by Mallela et al. [19]. A co-infection model on Malaria-Cholera model was analyzed by Okosun and Makinde [24].

In this paper, A mathematical model for co-infection of dengue and chikungunya is formulated. Further, the problem is improvised to an optimal control problem and solved using Pontryagin's Maximum Principle [25].

The rest of the paper is arranged as follows: In Section 2, the co-infection model is formulated. A time-dependent optimal control problem is formulated with bed-net as a new time-dependent control measure is obtained at Section 3 and presented the result graphically with the help of numerical experiment in Section 4. The The results are discussed in Section 5 and sensitivity analysis are discussed in detail in Section 6. The paper is concluded in Section 7.

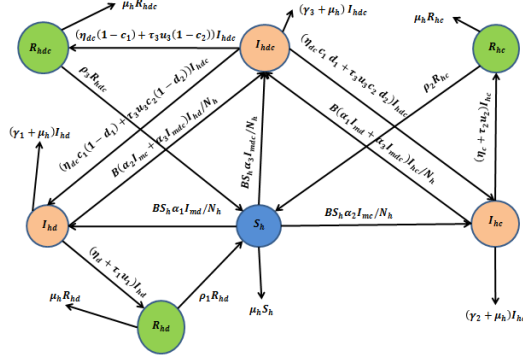
## 2 Formulation of Co-infection Model

The co-infection model sub-divides the total human host population  $N_h$  into sub-classes of susceptible ( $S_h$ ), only dengue infected ( $I_{hd}$ ), only chikungunya infected  $I_{hc}$ , dengue-chikungunya co-infected ( $I_{hdc}$ ), only dengue recovered  $R_{hd}$ , only chikungunya recovered ( $R_{hc}$ ) and dengue-chikungunya co-infection recovered ( $R_{hdc}$ ) human population.

Also the total mosquito vector population  $N_m$  sub-divided into susceptible ( $S_m$ ), dengue infected ( $I_{md}$ ), chikungunya infected ( $I_{mc}$ ) and co-infected  $I_{mdc}$  mosquito population.

The major mosquito vector for the transmission of dengue and chikungunya is the female *Aedes aegypti* species and they can carry and transmit both the disease simultaneously. Initial susceptible population recruitment rate is  $\mu_h N_h$ . The susceptible human population is infected by the dengue infected, chikungunya infected and dengue-chikungunya co-infected mosquito population through biting at a rate  $B$  per day. So a susceptible individual become infected at a rate  $B\alpha_1 \frac{S_h I_{md}}{N_h}$  by dengue infected,  $B\alpha_2 \frac{S_h I_{mc}}{N_h}$  by chikungunya infected and  $B\alpha_3 \frac{S_h I_{mdc}}{N_h}$  by dengue-chikungunya co-infected mosquito. A dengue infected human individual becomes co-infected individual at a rate  $B\alpha_2 \frac{I_{hd} I_{mc}}{N_h}$  by the bite of infected chikungunya infected mosquito and at a rate  $B\alpha_3 \frac{I_{hd} I_{mdc}}{N_h}$  by the bite of dengue-chikungunya co-infected mosquito. Similarly, a chikungunya infected human individual becomes co-infected at a rate  $B\alpha_1 \frac{I_{hc} I_{md}}{N_h}$  and  $B\alpha_3 \frac{I_{hc} I_{mdc}}{N_h}$  by a dengue infected and dengue-chikungunya co-infected mosquito respectively. The  $I_{hd}$ ,  $I_{hc}$  population recovered naturally at a rate  $\eta_1 I_{hd}$  and  $\eta_2 I_{hc}$  and by treatment at a rate  $\tau_1 u_1$  and  $\tau_2 u_2$  respectively. The co-infected human population  $I_{hdc}$  recover totally at a rate  $(\eta_{dc}(1-c_1))I_{hdc}$ , from dengue only at a rate  $(\eta_{dc}c_1 d_1)I_{hdc}$  and from chikungunya only at a rate  $(\eta_{dc}c_1(1-d_1))I_{hdc}$  naturally and totally at a rate  $(\tau_3 u_3(1-c_2))I_{hdc}$ , from dengue only at a rate  $(\tau_3 u_3 c_2 d_2)I_{hdc}$  and from chikungunya only at a rate  $(\tau_3 u_3 c_2(1-d_2))I_{hdc}$  by treatment.  $\gamma_1 I_{hd}$ ,  $\gamma_2 I_{hc}$  and  $\gamma_3 I_{hdc}$  are the disease induced deaths for dengue chikungunya and co-infected classes. The recovered population  $R_{hd}$ ,  $R_{hc}$  and  $R_{hdc}$  again becomes susceptible at a rate  $\rho_1 R_1$ ,  $\rho_2 R_2$  and  $\rho_3 R_{hdc}$ .  $\mu_h$  is the natural death rate of each of the subclasses of human population.

The diseases transmitted to the susceptible mosquito population when they come in the contact of dengue infected, chikungunya infected and dengue-chikungunya co-infected human population through biting. So a susceptible mosquito become infected at a rate  $B\delta_1 \frac{S_m I_{hd}}{N_m}$  by dengue infected,  $B\delta_2 \frac{S_m I_{hc}}{N_m}$  by chikungunya infected and  $B\delta_3 \frac{S_m I_{hdc}}{N_m}$  by dengue-chikungunya co-infected human. Again dengue infected mosquitoes become co-infected at a rate  $B\delta_2 \frac{I_{md} I_{dc}}{N_m}$  by contact of infected chikungunya infected human and at a rate  $B\delta_3 \frac{I_{md} I_{hdc}}{N_m}$  by the contact of dengue-chikungunya co-infected human. Similarly, A chikungunya infected mosquito becomes co-infected at a rate  $B\delta_1 \frac{I_{mc} I_{hd}}{N_m}$  and  $B\delta_3 \frac{I_{mc} I_{hdc}}{N_m}$  by a dengue infected and dengue-chikungunya co-infected human respectively. Mosquitoes die at a rate  $\mu_m$  naturally and  $\tau_4 u_4$  by the use of pesticides. Further,



**Fig. 1.** Pictorial representation of Human Infection

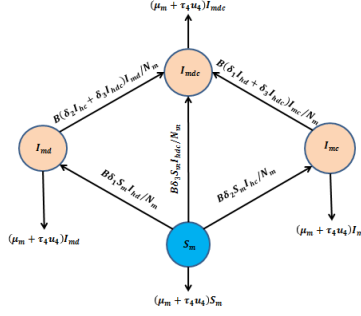
it is assumed that there is no disease induced death in mosquito population. The mathematical formulation is:

$$\left\{ \begin{array}{l}
 \dot{S}_h = \mu_h N_h - \frac{BS_h}{N_h} (\alpha_1 I_{md} + \alpha_2 I_{mc} + \alpha_3 I_{mhc}) - \mu_h S_h + \rho_1 R_{hd} + \rho_2 R_{hc} + \rho_3 R_{hdc} \\
 \dot{I}_{hd} = B\alpha_1 \frac{S_h I_{md}}{N_h} - (\eta_d + \tau_1 u_1 + \gamma_1 + \mu_h) I_{hd} - B(\alpha_2 I_{mc} + \alpha_3 I_{mhc}) \frac{I_{hd}}{N_h} \\
 \quad + (\eta_{dc} c_1 (1 - d_1) + \tau_3 u_3 c_2 (1 - d_2)) I_{hdc} \\
 \dot{I}_{hc} = B\alpha_2 \frac{S_h I_{mc}}{N_h} - (\eta_c + \tau_2 u_2 + \gamma_2 + \mu_h) I_{hc} - B(\alpha_1 I_{md} + \alpha_3 I_{mhc}) \frac{I_{hc}}{N_h} \\
 \quad + (\eta_{dc} c_1 d_1 + \tau_3 u_3 c_2 d_2) I_{hdc} \\
 \dot{I}_{hdc} = B\alpha_3 (S_h + I_{hd} + I_{hc}) \frac{I_{mhc}}{N_h} + \frac{B}{N_h} (\alpha_1 I_{md} I_{hc} + \alpha_2 I_{mc} I_{hd}) \\
 \quad - (\eta_{dc} + \tau_3 u_3 + \gamma_3 + \mu_h) I_{hdc} \\
 \dot{R}_{hd} = (\eta_d + \tau_1 u_1) I_{hd} - (\rho_1 + \mu_h) R_{hd} \\
 \dot{R}_{hc} = (\eta_c + \tau_2 u_2) I_{hc} - (\rho_2 + \mu_h) R_{hc} \\
 \dot{R}_{hdc} = (\eta_{dc} (1 - c_1) + \tau_3 u_3 (1 - c_2)) I_{hdc} - (\rho_3 + \mu_h) R_{hdc} \\
 \dot{S}_m = \mu_m N_m - \frac{BS_m}{N_m} (\delta_1 I_{hd} + \delta_2 I_{hc} + \delta_3 I_{hdc}) - (\mu_m + \tau_4 u_4) S_m \\
 \dot{I}_{md} = B\delta_1 \frac{I_{hd} S_m}{N_m} - (\mu_m + \tau_4 u_4) I_{md} - B(\delta_2 I_{hc} + \delta_3 I_{hdc}) \frac{I_{md}}{N_m} \\
 \dot{I}_{mc} = B\delta_2 \frac{I_{hc} S_m}{N_m} - (\mu_m + \tau_4 u_4) I_{mc} - B(\delta_1 I_{hd} + \delta_3 I_{hdc}) \frac{I_{mc}}{N_m} \\
 \dot{I}_{mhc} = B\delta_3 (S_m + I_{md} + I_{mc}) \frac{I_{hdc}}{N_m} + \frac{B}{N_m} (\delta_1 I_{hd} I_{mc} + \delta_2 I_{hc} I_{md}) - (\tau_4 u_4 + \mu_m) I_{mhc}
 \end{array} \right. \quad (1)$$

with  $S_h(0) \geq 0, I_{hd}(0) \geq 0, I_{hc}(0) \geq 0, I_{hdc}(0) \geq 0, S_m(0) \geq 0, I_{md}(0) \geq 0, I_{mc}(0) \geq 0, I_{mhc}(0) \geq 0$ . Pictorially the above model is presented in Figure 1 and 2.

### 3 Application of optimal control

In this section, optimal control for the dengue-chikungunya co-infection problem is calculated. In place of fixed control, time-dependent optimal control is used. Since the best way to prevent chikungunya is with avoidance of mosquito bites (cf. [18]), in this section, one additional control measure  $u_5(t)$  (namely the use of



**Fig. 2.** Pictorial representation of Mosquito Infection

bed-net or mosquito repellent) is used as a new control variable and the system is reduced to the following:

$$\begin{cases}
 \dot{S}_h = \mu_h N_h - (1 - u_5) \frac{BS_h}{N_h} (\alpha_1 I_{md} + \alpha_2 I_{mc} + \alpha_3 I_{mdc}) - \mu_h S_h + \rho_1 R_{hd} + \rho_2 R_{hc} + \rho_3 R_{hdc} \\
 \dot{I}_{hd} = (1 - u_5) B \alpha_1 \frac{S_h I_{md}}{N_h} - (\eta_d + \tau_1 u_1 + \gamma_1 + \mu_h) I_{hd} - (1 - u_5) B (\alpha_2 I_{mc} + \alpha_3 I_{mdc}) \frac{I_{hd}}{N_h} \\
 \quad + (\eta_{dc} c_1 (1 - d_1) + \tau_3 u_3 c_2 (1 - d_2)) I_{hdc} \\
 \dot{I}_{hc} = (1 - u_5) B \alpha_2 \frac{S_h I_{mc}}{N_h} - (\eta_c + \tau_2 u_2 + \gamma_2 + \mu_h) I_{hc} - (1 - u_5) B (\alpha_1 I_{md} + \alpha_3 I_{mdc}) \frac{I_{hc}}{N_h} \\
 \quad + (\eta_{dc} c_1 d_1 + \tau_3 u_3 c_2 d_2) I_{hdc} \\
 \dot{I}_{hdc} = (1 - u_5) B \alpha_3 (S_h + I_{hd} + I_{hc}) \frac{I_{hdc}}{N_h} + (1 - u_5) \frac{B}{N_h} (\alpha_1 I_{md} I_{hc} + \alpha_2 I_{mc} I_{hd}) \\
 \quad - (\eta_{dc} + \tau_3 u_3 + \gamma_3 + \mu_h) I_{hdc} \\
 \dot{R}_{hd} = (\eta_d + \tau_1 u_1) I_{hd} - (\rho_1 + \mu_h) R_{hd} \\
 \dot{R}_{hc} = (\eta_c + \tau_2 u_2) I_{hc} - (\rho_2 + \mu_h) R_{hc} \\
 \dot{R}_{hdc} = (\eta_{dc} (1 - c_1) + \tau_3 u_3 (1 - c_2)) I_{hdc} - (\rho_3 + \mu_h) R_{hdc} \\
 \dot{S}_m = \mu_m N_m - (1 - u_5) \frac{BS_m}{N_m} (\delta_1 I_{hd} + \delta_2 I_{hc} + \delta_3 I_{hdc}) - (\mu_m + \tau_4 u_4) S_m \\
 \dot{I}_{md} = (1 - u_5) B \delta_1 \frac{I_{hd} S_m}{N_m} - (\mu_m + \tau_4 u_4) I_{md} - (1 - u_5) B (\delta_2 I_{hc} + \delta_3 I_{hdc}) \frac{I_{md}}{N_m} \\
 \dot{I}_{mc} = (1 - u_5) B \delta_2 \frac{I_{hc} S_m}{N_m} - (\mu_m + \tau_4 u_4) I_{mc} - (1 - u_5) B (\delta_1 I_{hd} + \delta_3 I_{hdc}) \frac{I_{mc}}{N_m} \\
 \dot{I}_{mdc} = (1 - u_5) B \delta_3 (S_m + I_{md} + I_{mc}) \frac{I_{hdc}}{N_m} + (1 - u_5) \frac{B}{N_m} (\delta_1 I_{hd} I_{mc} + \delta_2 I_{hc} I_{md}) \\
 \quad - (\tau_4 u_4 + \mu_m) I_{mdc}
 \end{cases} \quad (2)$$

Also, the cost and side effects of applying the control measures are analyzed. Here, the objective is not only to control the epidemics i.e., minimize the total number of infected population, but also to minimize the cost of applying the control and the side effect of applying the controls. In order to minimize both cost and side effect, one needs to minimize the squares of the control variables. Following De et al. [9], the objective function of the present optimal control

problem is given by

$$J = \min_{u_1, u_2, u_3, u_4, u_5} \int_0^T (A_1 I_{hd} + A_2 I_{hc} + A_3 I_{hdc} + A_4 I_{md} + A_5 I_{mc} + A_6 I_{mdc} + B_1 u_1^2 + B_2 u_2^2 + B_3 u_3^2 + B_4 u_4^2 + B_5 u_5^2) dt \quad (3)$$

subject to the system of ODE (2). The objective here is to find the 5-tuples  $(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*)$  such that

$$J(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*) = \min_{u_1, u_2, u_3, u_4, u_5 \in \Theta} J(u_1, u_2, u_3, u_4, u_5). \quad (4)$$

where  $\Theta = \{u : \text{measurable and } 0 \leq u_i(t) \leq 1 \text{ for } t \in [0, T], i = 1, 2, \dots, 5\}$  is the set of controls. Here, the time dependent control variables  $u_1(t)$ ,  $u_2(t)$ ,  $u_3(t)$  are the treatment (medication) on dengue, chikungunya and dengue-chikungunya co-infected human population respectively,  $u_4(t)$  pesticide control on mosquito population and  $u_5(t)$  is the preventive measure control (bed-net, mosquito repellent, etc.) on human population. Also,  $A_1, A_2, A_3, A_4, A_5, A_6$  are the constant weights corresponding to  $I_{hd}, I_{hc}, I_{hdc}, I_{md}, I_{mc}$  and  $I_{mdc}$  respectively. The squares of the controls are taken to consider the side effect and over doses of the controls and  $B_1, B_2, B_3, B_4, B_5$  are respectively, the positive constants associated with the squares of the control to balance the size of the terms.

Now the optimal control problem (3) is solved, subject to the condition (2). The Lagrangian of the problem is given by

$$L = A_1 I_{hd} + A_2 I_{hc} + A_3 I_{hdc} + A_4 I_{md} + A_5 I_{mc} + A_6 I_{mdc} + B_1 u_1^2 + B_2 u_2^2 + B_3 u_3^2 + B_4 u_4^2 + B_5 u_5^2$$

To minimize  $L$ , the Hamiltonian of the problem is formulated as

$$H = L + \lambda_1(t) \frac{dS_h}{dt} + \lambda_2(t) \frac{dI_{hd}}{dt} + \lambda_3(t) \frac{dI_{hc}}{dt} + \lambda_4(t) \frac{dI_{hdc}}{dt} + \lambda_5(t) \frac{dR_{hd}}{dt} + \lambda_6(t) \frac{dR_{hc}}{dt} + \lambda_7(t) \frac{dR_{hdc}}{dt} + \lambda_8(t) \frac{dS_m}{dt} + \lambda_9(t) \frac{dI_{md}}{dt} + \lambda_{10}(t) \frac{dI_{mc}}{dt} + \lambda_{11}(t) \frac{dI_{mdc}}{dt} \quad (5)$$

where  $\lambda_i(t)$ ,  $i = 1, 2, \dots, 11$  are adjoint variables to be determined by the following system of differential equations

$$\begin{aligned} -\frac{d\lambda_1}{dt} &= \frac{\partial H}{\partial S_h}, & -\frac{d\lambda_2}{dt} &= \frac{\partial H}{\partial I_{hd}}, & -\frac{d\lambda_3}{dt} &= \frac{\partial H}{\partial I_{hc}}, & -\frac{d\lambda_4}{dt} &= \frac{\partial H}{\partial I_{hdc}}, & -\frac{d\lambda_5}{dt} &= \frac{\partial H}{\partial R_{hd}} \\ -\frac{d\lambda_6}{dt} &= \frac{\partial H}{\partial R_{hc}}, & -\frac{d\lambda_7}{dt} &= \frac{\partial H}{\partial R_{hdc}}, & -\frac{d\lambda_8}{dt} &= \frac{\partial H}{\partial S_m}, & -\frac{d\lambda_9}{dt} &= \frac{\partial H}{\partial I_{md}}, & -\frac{d\lambda_{10}}{dt} &= \frac{\partial H}{\partial I_{mc}} \\ -\frac{d\lambda_{11}}{dt} &= \frac{\partial H}{\partial I_{mdc}} \end{aligned} \quad (6)$$

satisfying the conditions

$$\lambda_i(T) = 0, \quad i = 1, 2, \dots, 11. \quad (7)$$

We have considered that  $\bar{S}_h, \bar{I}_{hd}, \bar{I}_{hc}, \bar{I}_{hdc}, \bar{S}_m, \bar{I}_{md}, \bar{I}_{mc}, \bar{I}_{mdc}$  are the optimum values of  $\bar{S}_h, \bar{I}_{hd}, \bar{I}_{hc}, \bar{I}_{hdc}, \bar{S}_m, \bar{I}_{md}, \bar{I}_{mc}, \bar{I}_{mdc}$ . Also, let  $\{\bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3, \bar{\lambda}_4, \bar{\lambda}_5, \bar{\lambda}_6, \bar{\lambda}_7, \bar{\lambda}_8, \bar{\lambda}_9, \bar{\lambda}_{10}, \bar{\lambda}_{11}\}$  be the solution of the system (6).

Following Pontryagin et al. [25], the following theorems can be stated.

**Theorem 1.** *There exists optimal controls  $u_1^*, u_2^*, u_3^*, u_4^*, u_5^*$  for  $t \in [0, T]$  such that*

$$J(I_{hd}, I_{hc}, I_{hdc}, I_{md}, I_{mc}, I_{mdc}, u_1^*, u_2^*, u_3^*, u_4^*, u_5^*) = \min_{u_1, u_2, u_3, u_4, u_5} J(I_{hd}, I_{hc}, I_{hdc}, I_{md}, I_{mc}, I_{mdc}, u_1, u_2, u_3, u_4, u_5)$$

subject to the system (2).

**Theorem 2.** *The optimal control 5-tuple  $(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*)$  which minimizes  $J$  over  $\Theta$  is given by*

$$u_1^* = \max\{0, \min(\bar{u}_1, 1)\}, u_2^* = \max\{0, \min(\bar{u}_2, 1)\}, u_3^* = \max\{0, \min(\bar{u}_3, 1)\} \\ u_4^* = \max\{0, \min(\bar{u}_4, 1)\}, u_5^* = \max\{0, \min(\bar{u}_5, 1)\}$$

$$\text{where, } \bar{u}_1 = \frac{(\bar{\lambda}_2 - \bar{\lambda}_5)\tau_1 \bar{I}_{hd}}{2B_1}, \bar{u}_2 = \frac{(\bar{\lambda}_3 - \bar{\lambda}_6)\tau_2 \bar{I}_{hc}}{2B_2}, \\ \bar{u}_3 = \frac{(\bar{\lambda}_4 - \bar{\lambda}_2 c_2(1 - d_2) - \bar{\lambda}_3 c_2 d_2)\tau_3 \bar{I}_{hdc}}{2B_3}, \\ \bar{u}_4 = \frac{(\bar{\lambda}_8 \bar{S}_m + \bar{\lambda}_9 \bar{I}_{md} + \bar{\lambda}_{10} \bar{I}_{mc} + \bar{\lambda}_{11} \bar{I}_{mdc})}{2B_4}, \\ \bar{u}_5 = \frac{1}{2B_5} \left[ \frac{B}{N_h} [\bar{S}_h(\alpha_1 \bar{I}_{md}(\bar{\lambda}_2 - \bar{\lambda}_1) + \alpha_2 \bar{I}_{mc}(\bar{\lambda}_3 - \bar{\lambda}_1) + \alpha_3 \bar{I}_{mdc}(\bar{\lambda}_4 - \bar{\lambda}_1)) \right. \\ \left. + \bar{I}_{hd}(\bar{\lambda}_4 - \bar{\lambda}_2)(\alpha_2 \bar{I}_{mc} + \alpha_3 \bar{I}_{mdc}) + \bar{I}_{hc}(\bar{\lambda}_4 - \bar{\lambda}_3)(\alpha_1 \bar{I}_{md} + \alpha_3 \bar{I}_{mdc}) \right] \\ \left. + \frac{B}{N_m} [\bar{S}_m(\delta_1 \bar{I}_{hd}(\bar{\lambda}_9 - \bar{\lambda}_8) + \delta_2 \bar{I}_{hc}(\bar{\lambda}_{10} - \bar{\lambda}_8) + \delta_3 \bar{I}_{hdc}(\bar{\lambda}_{11} - \bar{\lambda}_8)) \right. \\ \left. + \bar{I}_{md}(\bar{\lambda}_{11} - \bar{\lambda}_9)(\delta_2 \bar{I}_{hc} + \delta_3 \bar{I}_{hdc}) + \bar{I}_{mc}(\bar{\lambda}_{11} - \bar{\lambda}_{10})(\delta_1 \bar{I}_{hd} + \delta_3 \bar{I}_{hdc}) \right]$$

## 4 Numerical Experiment

The following eight possible control strategies can be applied to control the disease. Namely, (i) medicine control only, (ii) insecticide control only, (iii) bed-net or mosquito repellent control only, (iv) medicine and insecticide controls, (v) medicine and bed-net or mosquito repellent controls, (vi) insecticide and bed-net or mosquito repellent controls, (vii) all the three controls and (viii) no control. To graphically analyze the effect of various control strategies, the data set under consideration is shown in Table 1. All the input and output data are in appropriate units. The results are shown graphically in Figures 3-8.



**Table 1.** Input Data for Optimal control problem

$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$B_1$	$B_2$	$B_3$	$B_4$	$B_5$
$\alpha_1$	$\alpha_2$	$\alpha_3$	$B$	$c_1$	$c_2$	$d_1$	$d_2$	$\eta_d$	$\eta_c$	$\eta_{dc}$
$\delta_1$	$\delta_2$	$\delta_3$	$\gamma_1$	$\gamma_2$	$\gamma_3$	$\mu_h$	$\mu_m$	$N_h$	$N_m$	
$\tau_1$	$\tau_2$	$\tau_3$	$\tau_4$	$\rho_1$	$\rho_2$	$\rho_3$				
$S_h(0)$	$I_{hd}(0)$	$I_{hc}(0)$	$I_{hdc}(0)$	$R_{hd}(0)$	$R_{hc}(0)$	$R_{hdc}(0)$	$S_m(0)$	$I_{md}(0)$	$I_{mc}(0)$	$S_{mdc}(0)$
1.0	1.0	1.0	1.0	1.0	1.0	0.1	0.1	0.1	0.1	0.1
0.001	0.001	0.001	.1	0.5	0.5	0.5	0.5	0.01	0.01	0.01
0.1	0.1	0.1	0.01	0.01	0.01	0.01	0.00004	0.02	500000	1500000
0.5	0.5	0.5	0.5	0.5	0.5	0.5				
450000	15000	15000	7500	5000	5000	2500	1200000	100000	100000	10000

## 5 Discussion

The Optimal control problem (3) is solved in Section 4, Numerical Experiment using fourth order Runge-Kutta method in the time interval  $[0, T]$ . Using the transversality condition given by (3) and the optimality condition (cf. [9], [12], [13]), the state variables and the adjoint variables are solved simultaneously using Runge-Kutta forward and backward methods respectively. The set of parameters under consideration is presented in Table1 as input data. Here we consider the parameters  $A_1 = A_2 = A_3 = A_4 = A_5 = A_6 = 1$  in order to minimize the total number of the infected population. The solution curves of different types of population for different control strategies are presented in Figures 3-4. Time series plot for the bed-net/mosquito repellent control is given in 7. From Figures 3, we can relate the effect of different control strategies on Co-Infected human population ( $I_{hdc}$ ). Similarly, from Figures 4 the effect of different controls on Co-Infected mosquito population( $I_{mdc}$ ) can be compared.

It is observed from Figure 3 that the effect of all possible control is same as effect of medicine and bed-net control on human population. Also, it can be noticed that if we do not apply any control strategy, then the infected populations increase with an exponential nature from the initial point of consideration. In Figure 4, it is found that the effect of all possible control is same as effect of pesticide and bed-net control on mosquito population and like human population infected mosquito population also grows exponentially if no control is applied to the system.

Further, it can be observed from the solution graph of the co-infected human population that only treatment control on infected human can control only human infection but not the co-infected mosquito population (cf. Figure-3). Again, the pesticide control on co-infected mosquito population can control co-infected mosquito only, but not infected human population (cf. Figure 4). The role of the control strategies with both treatment and pesticide control is quite significant.

Only the bed-net control shows not much significant effect in controlling or removing the disease from the system, though when ever the bed-net control is combined with either medicine or treatment or both, it plays a significant role on the system.

Therefore, for simultaneous eradication or control of the co-infection of dengue and chikungunya in a system with human and mosquito population, all possible controls have to be applied.

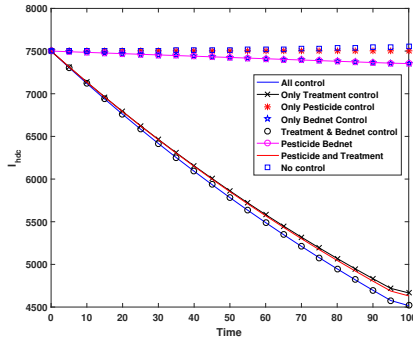


Fig. 3. Co-infected human population

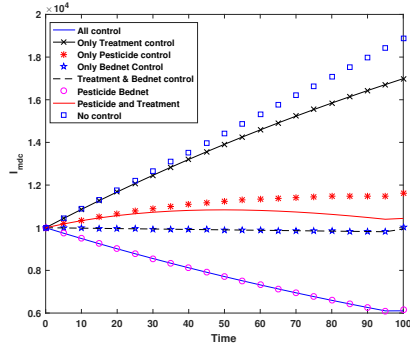


Fig. 4. Co-infected mosquito population

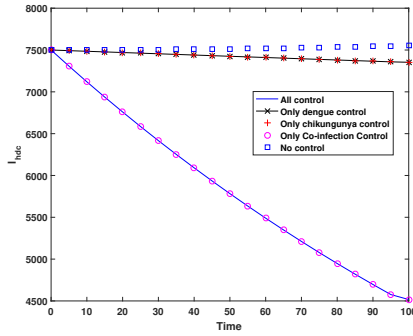


Fig. 5. Impact of different control on Co-infected human population

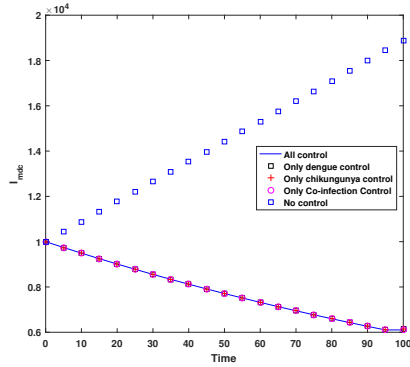


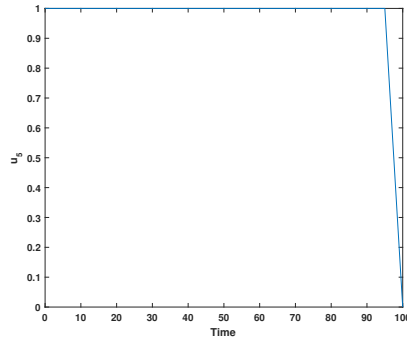
Fig. 6. Impact of different control on Co-infected mosquito population

In Figure 7 it can be observed that the control variables  $u_5$  is bounded between  $[0,1]$ . Similar type of figures can be obtained to  $u_1, u_2, u_3$  and  $u_4$ . Figures 5-6 analyze the impact of no-control, single disease controls (only dengue control or only chikungunya control), only co-infection control and all control on co-infected human and co-infected mosquito population only.

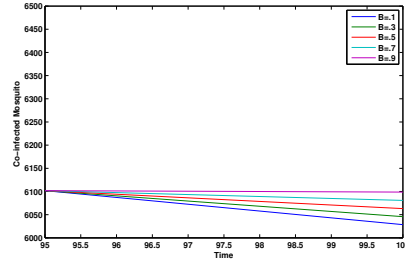
Here, it can be noticed that single disease control measure fails to control the growth of co-infection on human and effectiveness of co-infection control is same as that of applying all control. So, in order to control co-infection, controlling co-infection is very important.

## 6 Sensitivity Analysis

The contact rate between human and mosquito is represented by the parameter  $B$ . From the optimum result of numerical experiment in 4, it is observed that, to control the infected mosquito insecticide control is more dominating in the con-



**Fig. 7.** Mosquito repellent and/or bed-net control for mosquito



**Fig. 8.** Sensitivity analysis of  $B$  on co-infected mosquito population

trol of the dengue chikungunya co-infection. If the contact rate between human and mosquito be regulated, then the disease can be removed completely from the system. From Figure 8, it can be observed that the infected population for each species is monotonically increasing with the increase of  $B$ .

## 7 Conclusion

In this paper, a dengue-chikungunya co-infection model is formulated mathematically and analyzed with three different types of control measures, namely treatment control on the human population, pesticide control on the mosquito population and bed-net control of the human population. Initially, for single infection, the basic reproduction numbers in each of dengue and chikungunya disease for the fixed control are obtained and analyzed. In the optimal control part, bed-net or mosquito repellent is introduced as new control and analyzed the effect of time-dependent control on the co-infected human mosquito system. The graphical solutions are obtained and compared the solutions with different control strategies. It is depicted by the figures that if there is no control, then the disease spreads exponentially with time from the initial state. It can be observed that a single control measure can not eradicate or control the infection completely from the system. Also, it can be noticed that the effect of all the three different controls is the same as the effect of medication and bed-net control of the human population. Again, the effect of all the three different controls is the same as the effect of pesticide and bed-net control on mosquito populations. Medication control only does not have any significant impact on the infected mosquito population. Similarly, pesticide control does not have any impact on the infected human population. So, for simultaneous controlling of co-infection with both human and mosquito population, simultaneous application of all the three types of control measures are needed. It is obtained that all the control measures are bounded. From the sensitivity analysis of the model it is observed

that the contact rate between the human and the mosquito population plays significant role in amplification of infection.

This model can suggest the policymakers to take appropriate strategies to control when both the disease strikes at once to a population.

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