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**Abstract:** For mechanical system structural health monitoring, a new residual generation method is proposed in this paper, inspired by a recent result on subspace system identification. It improves statistical properties of the existing subspace residual, which has been naturally derived from the standard subspace system identification method. Replacing the monitored system state-space model by the Kalman filter one-step ahead predictor is the key element of the improvement in statistical properties, as originally proposed by Verhaegen and Hansson in the design of a new subspace system identification method.

*Keywords:* Structural health monitoring, damage detection, fault diagnosis, residual design, subspace system identification, vibration analysis.

## 1. INTRODUCTION

The detection of damages based on vibration analysis has been widely applied to structural health monitoring (SHM) (Farrar and Worden, 2007). In this approach, early sign of damages are modeled as changes in the parameters of the underlying mechanical system, then damage detection amounts to parameter change detection. A particular difficulty for such vibration analysis-based SHM is due to the absence of known system inputs, since the structural excitations are usually only caused by natural ambient disturbances. In this sense the considered vibration analysis-based SHM is an *output-only* monitoring problem.

Among model-based methods for damage detection (Carden and Fanning, 2004), it is quite natural to perform parameter change detection by comparing model parameters estimated from data collected at different dates. Nevertheless, due to the difficulty for distinguishing the intrinsic characteristics of the monitored structure and the characteristics of unknown natural excitations, both being present in estimated model parameters, it is difficult to automatize such methods. Alternatively, other methods based on direct model-data matching are more suitable for automated damage detection. In this approach, current measurement data are directly confronted to a reference model. For instance, such methods include non-parametric change detection based on novelty detection (Worden et al., 2000), whiteness tests on Kalman filter innovations (Bernal, 2013), and the local asymptotic approach to change detection (Benveniste et al., 1987).

The two approaches, based on repeated model parameter estimation and on direct model-data matching, are closely related. In particular, for each parameter estimation method based on the minimization of some error criterion, it is quite straightforward to design its counterpart for direct model-data matching following the local asymptotic approach to change detection (Benveniste et al., 1987; Basseville and Nikiforov, 1993). More recently, a model-data matching method has been inspired by the *subspace system identification* method, which is a state-space model estimation method with no direct connection to an error criterion minimization, originally developed in the automatic control community (Van Overschee and De Moor, 1996; Ljung, 1999). The essential element of this model-data matching method is the design of a residual derived from the subspace system identification method (Basseville et al., 2000). Based on this residual and its associated statistical evaluation as in (Basseville et al., 2000), extensions have been proposed to consider changing process noise properties (Döhler and Mevel, 2013; Döhler et al., 2014b) or uncertainties related to the reference model (Viefhues et al., 2022), as well as damage localization and quantification (corresponding to fault isolation and estimation) with the proposed residual (Döhler et al., 2016; Allahdadian et al., 2019). Successful applications to field data analysis have been reported e.g. in (Döhler et al., 2014a; Gres et al., 2017).

The purpose of the present paper is to propose a *new residual* related to the *subspace system identification* method, with an important improvement in its statistical properties, compared to the residual proposed in (Basseville et al., 2000). This improvement will simplify residual evaluation for damage detection.

In most methods for fault diagnosis, residuals are computed from sensor signals such that their behaviors reflect the occurrence of the monitored faults (Blanke et al., 2006; Volosencu, 2018; Escobet et al., 2019). The subspace residual proposed in (Basseville et al., 2000) has a zero mean when the monitored system is in its nominal state, and a non zero mean when some damage appears. These properties are useful for damage detection. However, the computation of this residual mixes noises of different time instants, leading to complex statistical properties. Even under the assumptions that the noises involved in the system are white, the statistical properties of this residual remain complex. The same complexity is also known in the error terms of the subspace system identification method (Van Overschee and De Moor, 1996).

The main idea in the new residual design of this paper is to replace the state-space model of the monitored system by its Kalman one-step ahead predictor, while the other steps of this design remain similar to those of (Basseville et al., 2000). This new design has been inspired by a recent subspace system identification method (Verhaegen and Hansson, 2016) which avoids mixing noises of different time instants in its error terms. Though this subspace system identification method relies on a non convex optimization, for the damage detection problem considered in the present paper, the related subspace residual computation remains as simple as in (Basseville et al., 2000).

This paper is organized as follows. The considered damage detection problem is stated in Section 2. The standard subspace residual is recalled in Section 3. The new subspace residual is presented in Section 4. The evaluation of the new residual is considered in Section 5. A simulation example is presented in Section 6. The paper is then concluded by Section 7.

## 2. THE CONSIDERED DAMAGE DETECTION PROBLEM

The vibration behavior of mechanical structures subject to unknown ambient excitations can be described by the differential equation

$$\mathcal{M}\ddot{\mathcal{X}}(t) + \mathcal{C}\dot{\mathcal{X}}(t) + \mathcal{K}\mathcal{X}(t) = f(t), \tag{1}$$

where $t \in \mathbb{R}$ denotes the time; $\mathcal{M}, \mathcal{C}, \mathcal{K} \in \mathbb{R}^{m\times m}$ are mass, damping, and stiffness matrices, respectively; $\mathcal{X}(t) \in \mathbb{R}^m$ is the displacement vector of the $m$ degrees of freedom of the structure; and $f(t) \in \mathbb{R}^{m\times m}$ is the vector containing the external unmeasured forces, treated as random disturbances. It is assumed that the vibration amplitude is small so that the linear differential equation (1) is accurate enough.

Observed at $r$ sensor positions by displacement, velocity or acceleration sensors at discrete time instants $t = k\tau$ (with sampling period $\tau$), system (1) is then described by a discrete-time state space system model (Juang, 1994)

$$\begin{cases} x_{k+1} = Ax_k + w_k \\ y_k = Cx_k + v_k \end{cases}, \tag{2}$$

where the state vector $x_k = \left[\mathcal{X}(k\tau)^T \; \dot{\mathcal{X}}(k\tau)^T\right]^T \in \mathbb{R}^n$ with $n = 2m$, the measured output vector $y_k \in \mathbb{R}^r$ and the system matrices

$$A = \exp(A^c\tau) \in \mathbb{R}^{n\times n}, \quad A^c = \begin{bmatrix} 0 & I \\ -\mathcal{M}^{-1}\mathcal{K} & -\mathcal{M}^{-1}\mathcal{C} \end{bmatrix}, \tag{3}$$

$$C = \begin{bmatrix} L_d - L_a\mathcal{M}^{-1}\mathcal{K} & L_v - L_a\mathcal{M}^{-1}\mathcal{C} \end{bmatrix} \in \mathbb{R}^{r\times n}, \tag{4}$$

with selection matrices $L_d, L_v, L_a \in \{0,1\}^{r\times m}$ indicating the positions of displacement, velocity or acceleration sensors, respectively. The state noise $w_k \in \mathbb{R}^n$ and output noise $v_k \in \mathbb{R}^r$ are unmeasured and assumed to be of zero-mean, white and Gaussian.

In this paper, the considered damages are changes in the structural stiffness properties of system (1), corresponding to changes related to the parameters of structural elements. A new subspace residual will be designed with an associated residual evaluation method for damage detection.

## 3. THE STANDARD SUBSPACE RESIDUAL

Based on the well known subspace method for system identification (Van Overschee and De Moor, 1996), a subspace residual has been proposed in (Basseville et al., 2000) and succesfully applied to mechanical system damage detection (Döhler and Mevel, 2013; Döhler et al., 2014b, 2016; Viefhues et al., 2022). Let us first shortly recall this "natural" subspace residual, in order to motivate the new residual proposed in the next section.

To design a residual based on the state-space model (2), the unknown state vector $x_k$ has to be handled somehow. The subspace approach eliminates $x_k$. For this purpose, it follows from (2) that

$$\begin{cases} y_k = Cx_k + v_k \\ y_{k+1} = CAx_k + Cw_k + v_{k+1} \\ \quad\vdots \\ y_{k+l} = CA^lx_k + CA^{l-1}w_k + \cdots + Cw_{k+l-1} + v_{k+l} \end{cases} \tag{5}$$

for any integer $l > 0$. Then, for some integer $s > n$,

$$\begin{bmatrix} y_k \\ y_{k+1} \\ \vdots \\ y_{k+s-1} \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{s-1} \end{bmatrix} x_k + \begin{bmatrix} v_k \\ Cw_k + v_{k+1} \\ \vdots \\ C\sum_{j=1}^{s-1} A^{s-1-j}w_{k+j-1} + v_{k+s-1} \end{bmatrix}. \tag{6}$$

Given a data set containing collected sensor measurements $y_1, y_2, \ldots, y_N$, define the Hankel matrices

$$Y = \begin{bmatrix} y_1 & y_2 & \cdots & y_{N-s+1} \\ y_2 & y_3 & & \vdots \\ \vdots & & \ddots & \vdots \\ y_s & y_{s+1} & \cdots & y_N \end{bmatrix}, \tag{7}$$

$$W = \begin{bmatrix} w_1 & w_2 & \cdots & w_{N-s+1} \\ w_2 & w_3 & & \vdots \\ \vdots & & \ddots & \vdots \\ w_s & w_{s+1} & \cdots & w_N \end{bmatrix}, \tag{8}$$

$$V = \begin{bmatrix} v_1 & v_2 & \cdots & v_{N-s+1} \\ v_2 & v_3 & & \vdots \\ \vdots & & \ddots & \vdots \\ v_s & v_{s+1} & \cdots & v_N \end{bmatrix}, \tag{9}$$

the Toeplitz matrix

$$T = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C & 0 & & 0 \\ \vdots & & \ddots & \\ CA^{s-2} & \cdots & C & 0 \end{bmatrix}, \tag{10}$$

the extended observability matrix

$$O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{s-1} \end{bmatrix}, \tag{11}$$

and the state sequence stack matrix

$$X = [x_1 \; x_2 \; \cdots \; x_{N-s+1}]. \tag{12}$$

Apply (6) to each column of $Y$, then, the results are appropriately arranged as

$$Y = OX + TW + V. \tag{13}$$

This equation shows that, if the noises terms $TW+V$ were neglected, the matrix $Y \in \mathbb{R}^{(sr)\times(N-s+1)}$ would have a rank *not larger* than those of $O$ and $X$. Moreover, without the noises terms, each column of $Y$ would belong to the *subspace* spanned by the columns of $O$. This is the key element behind the subspace system identification method.

Assume that the rank of the extended observability matrix $O \in \mathbb{R}^{sr\times n}$ is $n$ (otherwise the size $n$ of the state vector $x_k$ would have to be reduced). Let $S \in \mathbb{R}^{sr\times(sr-n)}$ be the left kernel matrix of $O$, *i.e.*, a full column rank matrix such that $S^TO = 0$. Given $O$, such an $S$ is typically computed through the singular value decomposition (SVD) of $O$. Then, left multiply both sides of (13) by $S^T$ cancels out the term $OX$, yielding

$$S^TY = S^TTW + S^TV. \tag{14}$$

In (Basseville et al., 2000), a residual for damage detection has been proposed as follows.

A reference data set is first collected, from which a nominal state-space model is estimated by a subspace identification method (Van Overschee and De Moor, 1996), yielding an extended observability matrix $O_0$ and the associated left kernel matrix $S_0$, with the subscript "0" indicating the **nominal** case.

Given a new data set, including a Hankel matrix $Y$ as defined in (7) and *another Hankel matrix* $Y_-$ similar to $Y$ but filled with past $y_k$, so that $Y_-$ is uncorrelated with the noise matrices $W$ and $V$ defined in (8) and (9), which contain respectively noises entries $w_k$ and $v_k$ with $k > 0$. Then the subspace residual is defined as

$$Z \triangleq \frac{1}{\sqrt{N}} S_0^T Y {Y_-}^T, \tag{15}$$

where $N$ is the data sample size, $Y_-$ serves as an "instrumental variable" (Soderstrom and Stoica, 1989; Benveniste and Mevel, 2007) matrix attenuating the noise terms $W$ and $V$ in $Y$ as expressed in (13), due to the fact that $Y_-$ is uncorrelated with $W$ and $V$.

According to (Basseville et al., 2000), when $N \to +\infty$, the limiting distribution of $Z$ is Gaussian, of *zero mean* if the system remains in the nominal state when the data matrices $Y$ and $Y_-$ are collected, or of *non-zero mean* otherwise. This result is based on a central limit theorem, for which the denominator $\sqrt{N}$ in (15) is necessary (Basseville et al., 2000).

Now let us have a close view of the term $TW$ in (13). Each column of this matrix product is expressed in the last term of (6), mixing the noises terms $w_k$ of different time index $k$. Though it is assumed that $w_k$ and $v_k$ are white noises, the mixing effect of the Toeplitz matrix $T$ "colorizes" the noise terms in (13).

The left hand side of (14) may also serve as a residual, because the term $OX$ containing the unknown states $OX$ has been canceled out by $S^T$. However, the remaining noise terms in (14) are complex due to mixing effect of the Toeplitz matrix $T$. Instead of analyzing the complex noises terms, the "instrumental variable" matrix $Y_-$ has been introduced, and the limiting distribution of the residual $Z$ is used for hypothesis testing, based on a central limit theorem (Basseville et al., 2000).

Another issue of this subspace residual is that the Toeplitz matrix $T$ and the extended observability matrix $O$ involve powers of the matrix $A$ up to $A^{s-1}$. If the considered system has unstable modes, these powers may involve large numerical values.

In order to simplify residual evaluation, the next section will propose a new subspace residual avoiding mixed noise terms. Simpler statistical analysis will become possible, without resorting to a central limit theorem.

## 4. THE KALMAN PREDICTOR SUBSPACE RESIDUAL

Now let us recall the stationary Kalman filter applied to the state-space system (2):

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k}, \tag{16a}$$

$$e_{k+1} = y_{k+1} - C\hat{x}_{k+1|k}, \tag{16b}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + Ke_{k+1}, \tag{16c}$$

where $\hat{x}_{k+1|k}$ and $\hat{x}_{k+1|k}$ are respectively the one-step ahead state prediction and the updated state estimation, $e_k$ is the *Kalman innovation*, $K$ is the stationary Kalman gain, usually computed by solving an algebraic Riccati equation.

Combine (16a) and (16c) to yield

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k-1} + AK(y_k - C\hat{x}_{k|k-1}) \tag{17}$$

$$= A(I_n - KC)\hat{x}_{k|k-1} + AKy_k, \tag{18}$$

and rewrite (16b) as

$$y_k = C\hat{x}_{k|k-1} + e_k. \tag{19}$$

The following results will be based on the "state-space model"

$$\hat{x}_{k+1|k} = A(I_n - KC)\hat{x}_{k|k-1} + AKy_k \tag{20a}$$

$$y_k = C\hat{x}_{k|k-1} + e_k \tag{20b}$$

by treating $AKy_k$ as an "input" term.

Define

$$\bar{A} \triangleq A(I_n - KC), \tag{21}$$

$$\bar{O} \triangleq \begin{bmatrix} C \\ C\bar{A} \\ \vdots \\ C\bar{A}^{s-1} \end{bmatrix}, \tag{22}$$

$$\bar{X} \triangleq \begin{bmatrix} \hat{x}_{1|0} & \hat{x}_{2|1} & \cdots & \hat{x}_{N-s+1|N-s} \end{bmatrix}, \tag{23}$$

$$\bar{T} \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 \\ CAK & 0 & & 0 \\ \vdots & & \ddots & \\ C\bar{A}^{s-2}AK & \cdots & CAK & 0 \end{bmatrix}, \tag{24}$$

$$E \triangleq \begin{bmatrix} e_1 & e_2 & \cdots & e_{N-s+1} \\ e_2 & e_3 & & \vdots \\ \vdots & & \ddots & \vdots \\ e_s & e_{s+1} & \cdots & e_N \end{bmatrix}. \tag{25}$$

Following the same procedure deriving (13) from (2), now the following equation is derived from (20):

$$Y = \bar{O}\bar{X} + \bar{T}Y + E, \tag{26}$$

where $Y$ is still as defined in (7).

Remarkably, the noise term $E$ in (26) as defined in (25) contains simple entries $e_k$, unlike the mixed noise term $TW$ in (13). It is well known that the Kalman innovation sequence $e_k$, as defined in (16b), is a white noise under standard assumptions, as those made in this paper. This simple noise term $E$ in (26) is *clearly an advantage compared to the terms* $TW + V$ in (13), with the Toeplitz matrix $T$ mixing $w_k$ as detailed in the last term of (6). The simple noise term $E$ will be helpful for residual evaluation.

For residual generation, like in the case of the previous section, a reference data set is first collected, from which a nominal state-space model is estimated by a subspace identification method (Van Overschee and De Moor, 1996), and the associated stationary Kalman gain $K$ is computed. The corresponding matrix $\bar{O}$ is then denoted by $\bar{O}_0$ to indicate the nominal case. Accordingly, the left kernel matrix of $\bar{O}_0$, denoted by $\bar{S}_0$, is computed (typically through the SVD of $\bar{O}_0$), so that $\bar{S}_0^T\bar{O}_0 = 0$.

A new residual is then defined as

$$\Xi \triangleq \bar{S}_0^T(I_{sr} - \bar{T})Y. \tag{27}$$

If the data matrix $Y$ has been collected when the monitored system remains in its nominal conditions, then (26) holds with $\bar{O} = \bar{O}_0$. In this case, the analysis form of the new residual writes

$$\Xi = \bar{S}_0^T(Y - \bar{T}Y) \tag{28}$$

$$= \bar{S}_0^T(\bar{O}_0\bar{X} + E) \tag{29}$$

$$= \bar{S}_0^T E. \tag{30}$$

## 5. EVALUATION OF THE NEW RESIDUAL

The full statistical analysis of the residual $\Xi$ should take into account the Hankel matrix structure of $E$ as detailed in (25). For simplicity, the following residual analysis will be adopted in this work.

Notice in (25) that each column of $E$ is correlated with its $s-1$ neighboring columns at each side, but not with the other columns. The left multiplication of $E$ with $\bar{S}_0^T$ does not mix the columns of $E$, hence each column of $\Xi = \bar{S}_0^T E$ is not correlated to the columns of $\Xi$ other than the $s-1$ neighboring columns. This property of $\Xi$ implies that "downsampling" $\Xi$ by keeping one column out of every $s$ columns will result in a residual matrix with uncorrelated columns.

Let $\Xi_{s,1}$ denote the matrix containing the 1st, $(s+1)$-th, $(2s+1)$-th,..., $(n_c s+1)$-th columns of $\Xi$, and $\Xi_{s,2}$ denote the matrix containing the 2nd, $(s+2)$-th, $(2s+2)$-th,..., $(n_c s+2)$-th columns of $\Xi$, and so on, with $n_c = \lfloor N/s \rfloor$.

Similarly, the "downsampled" notations $E_{s,1}$, $E_{s,2}$, ... are also defined.

Then $\mathrm{vec}(E_{s,1}) = [e_1 \quad e_2 \quad \dots \quad e_{sn_c}]^T$ is the vector containing the innovation sequence until sample $s \cdot n_c$, and

$$\mathrm{vec}(\Xi_{s,1}) \triangleq (I_{n_c} \otimes \bar{S}_0^T)\mathrm{vec}(E_{s,1}). \tag{31}$$

To compute the $\chi^2$ test

$$\mathcal{T}_{s,1} = \mathrm{vec}(\Xi_{s,1})^T \mathrm{cov}(\mathrm{vec}(\Xi_{s,1}))^{-1}\mathrm{vec}(\Xi_{s,1}), \tag{32}$$

we need

$$\mathrm{cov}(\mathrm{vec}(E_{s,1})) = I_{n_c s} \otimes \Sigma = I_{n_c} \otimes (I_s \otimes \Sigma), \tag{33}$$

where $\Sigma = \mathrm{cov}(e_k)$ is the innovation covariance (computed as part of the Kalman filter), which is invertible. Then

$$\mathrm{cov}(\mathrm{vec}(\Xi_{s,1})) = I_{n_c} \otimes \bar{S}_0^T(I_s \otimes \Sigma)\bar{S}_0. \tag{34}$$

Define $\tilde{\Sigma} \triangleq \bar{S}_0^T(I_s \otimes \Sigma)\bar{S}_0$. Since $\bar{S}_0$ is of full column rank and $\Sigma$ is invertible, it follows that $\tilde{\Sigma}$ is invertible. Hence,

$$\mathrm{cov}(\mathrm{vec}(\Xi_{s,1}))^{-1} = I_{n_c} \otimes \tilde{\Sigma}^{-1}, \tag{35}$$

and the $\chi^2$ test is computed as

$$\mathcal{T}_{s,1} = \mathrm{vec}(\Xi_{s,1})^T(I_{n_c} \otimes \tilde{\Sigma}^{-1})\mathrm{vec}(\Xi_{s,1}). \tag{36}$$

Similarly, $\mathcal{T}_{s,2}, \dots, \mathcal{T}_{s,s}$ are computed from $\Xi_{s,2}, \dots, \Xi_{s,s}$, then

$$\mathcal{T}_s \triangleq \frac{1}{s}(\Xi_{s,1} + \Xi_{s,2} + \cdots + \Xi_{s,s}) \tag{37}$$

is the test over all the columns of $\Xi$.

## 6. SIMULATION EXAMPLE

In this example, a simulated mass-spring chain with eight elements (Fig. 1) is considered for damage detection. The mass, damping and stiffness matrices of the nominal structural model are defined based on the masses $m_1 = m_3 = m_5 = m_7 = 1, m_2 = m_4 = m_6 = m_8 = 2$, stiffnesses $k_1 = k_3 = k_5 = k_7 = 1000, k_2 = k_4 = k_6 = k_8 = 500$ and a damping ratio of 2% for all modes, leading to a state-space model with model order 16. A fault, i.e., damage, is introduced by decreasing stiffness in the second spring by 7.5% and 15%, respectively.

Output signals are computed at four sensor coordinates at masses 1, 3, 5 and 7 from white noise excitation at the same coordinates with time step $\tau = 0.05$ s. Datasets are simulated for different structural states and for different number of samples $N$. White measurement noise is added with a magnitude of 5% of each generated output signal.

For each structural state, namely the nominal state and the two faulty (damaged) states with 7.5% and 15%

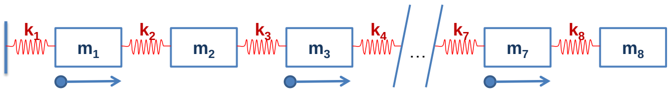


Fig. 1. Mass-spring chain with four sensors.

stiffness decrease in spring 2, 1000 datasets of length $N = 10{,}000$ are simulated and the test statistic $\mathcal{T}_s$ defined in (37) is computed for $s = 10$. The test values are shown in the respective histograms in Fig. 2. With $N = 10{,}000$, $s = 10$, model order $n = 16$ and $m = 4$ sensors, matrix $\bar{S}_0$ has $m \cdot s - n = 24$ columns, and matrix $\Xi_s$ has 999 columns. The histograms in the faulty state are well separated with increasing fault size.

The test values of the nominal state are used to set up an empirical threshold to decide between hypotheses $H_0$ and $H_1$. For a 1% type I error, the probability of detection for the 7.5% damage is 94.8%, and for the 15% damage the probability of detection is 100%.

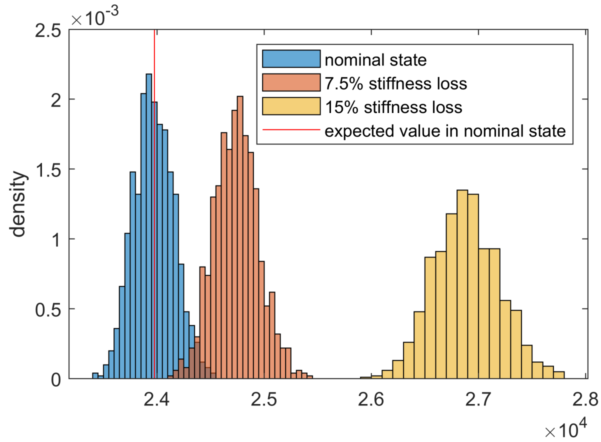


Fig. 2. Histogram of $\chi^2$ statistic values for the mass-spring chain in the nominal state, and 7.5% and 15% damage in spring 2.

Next, the influence of different sample lengths $N$ on the test are examined. Based on 1000 simulated datasets in the nominal and in the 7.5% damage state for each considered $N$, the non-centrality parameter of the test is evaluated and shown in Fig. 3. As expected, the non-centrality parameter appears to be linear in $N$. The resulting probability of detection assuming a 1% type I error for each case is shown in Fig. 4.

## 7. CONCLUSIONS

In this paper, based on a recent result on subspace system identification, a new residual generation method is proposed in order to improve the statistical properties of the existing subspace residual naturally derived from the standard subspace system identification method. By replacing the monitored system state-space model with the Kalman filter one-step ahead predictor, the new subspace residual avoids mixing noise terms.

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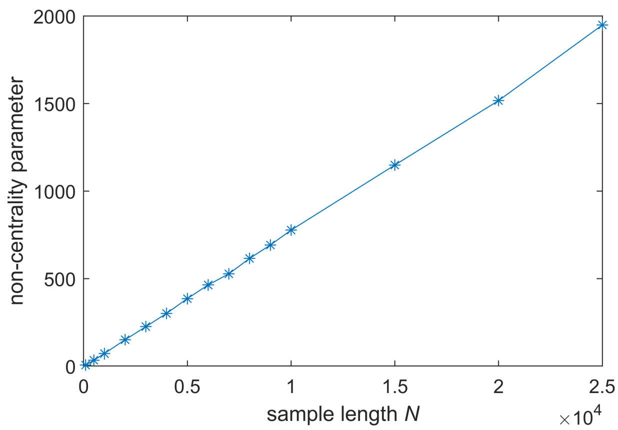


Fig. 3. Estimated non-centrality parameter of the test for the 7.5% damage case in spring 2 for different sample lengths $N$.

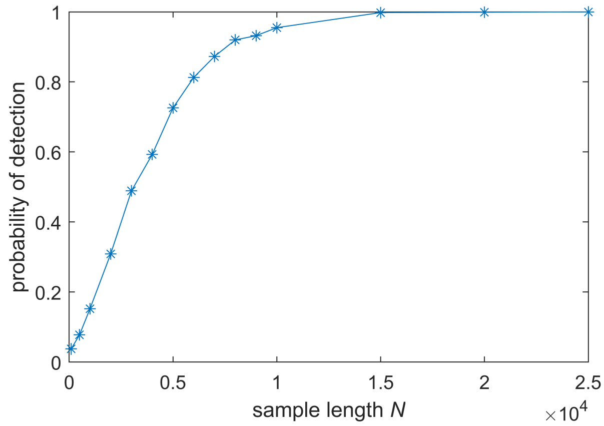


Fig. 4. Probability of detection of the 7.5% damage case in spring 2 for different sample lengths $N$.

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