Validation mécanique des codes d'informatique graphique

Mickaël Ly (PhD)
Abdullah Haroon Rasheed (PhD)
Raphaël Charrondière (PhD)
Florence Bertails-Descoubes (cherch.)

INRIA
Univ. Grenoble-Alpes

Victor Romero (postdoc)

Arnaud Lazarus (ens-cherch.)
Sébastien Neukirch (cherch.)

CNRS
Sorbonne-Université

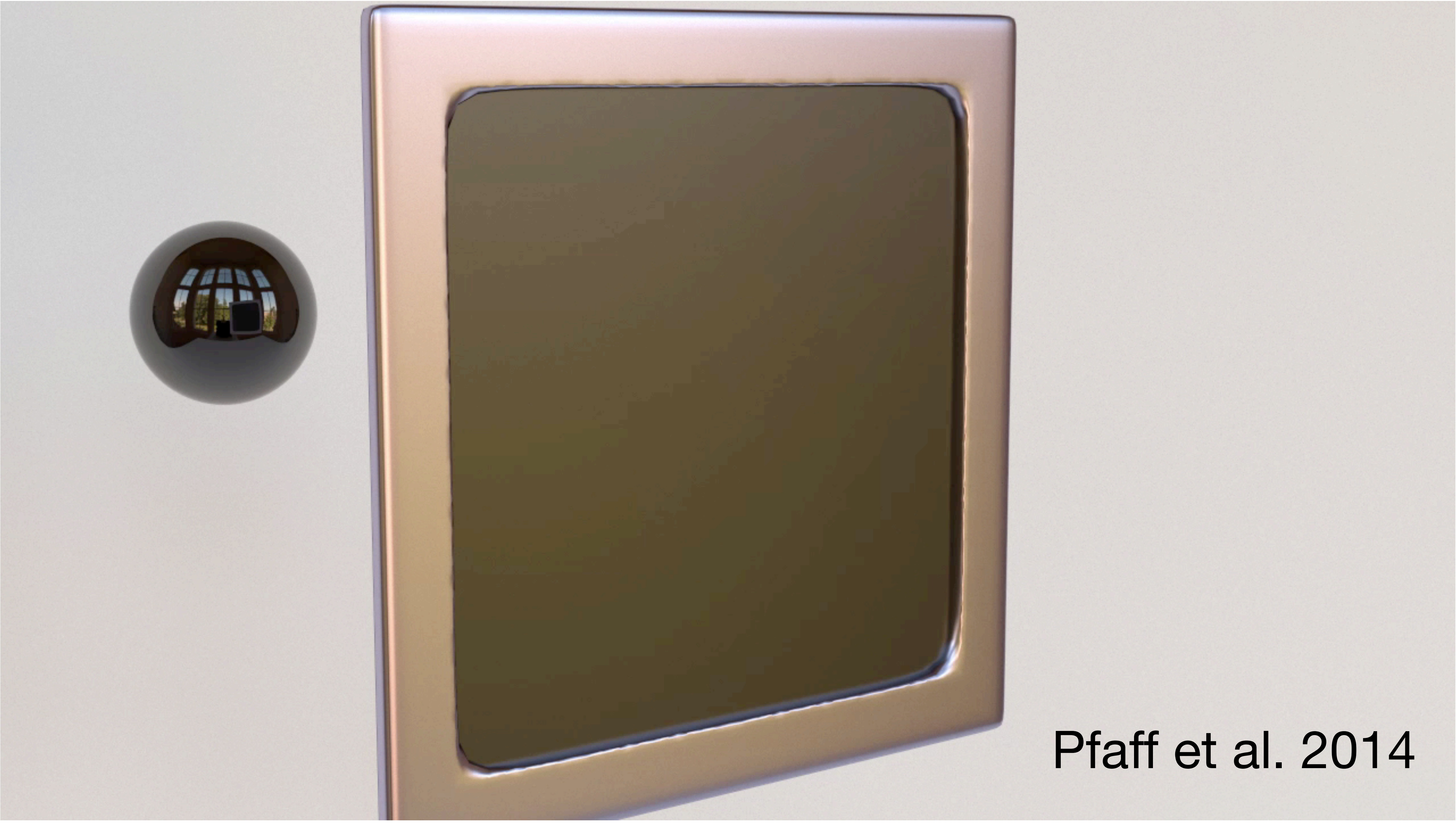
V. Romero, et al. ACM Transactions on Graphics, 40(4) 2021

Motivation

Computer Graphics Simulators
are used in the
Animated Video Industry

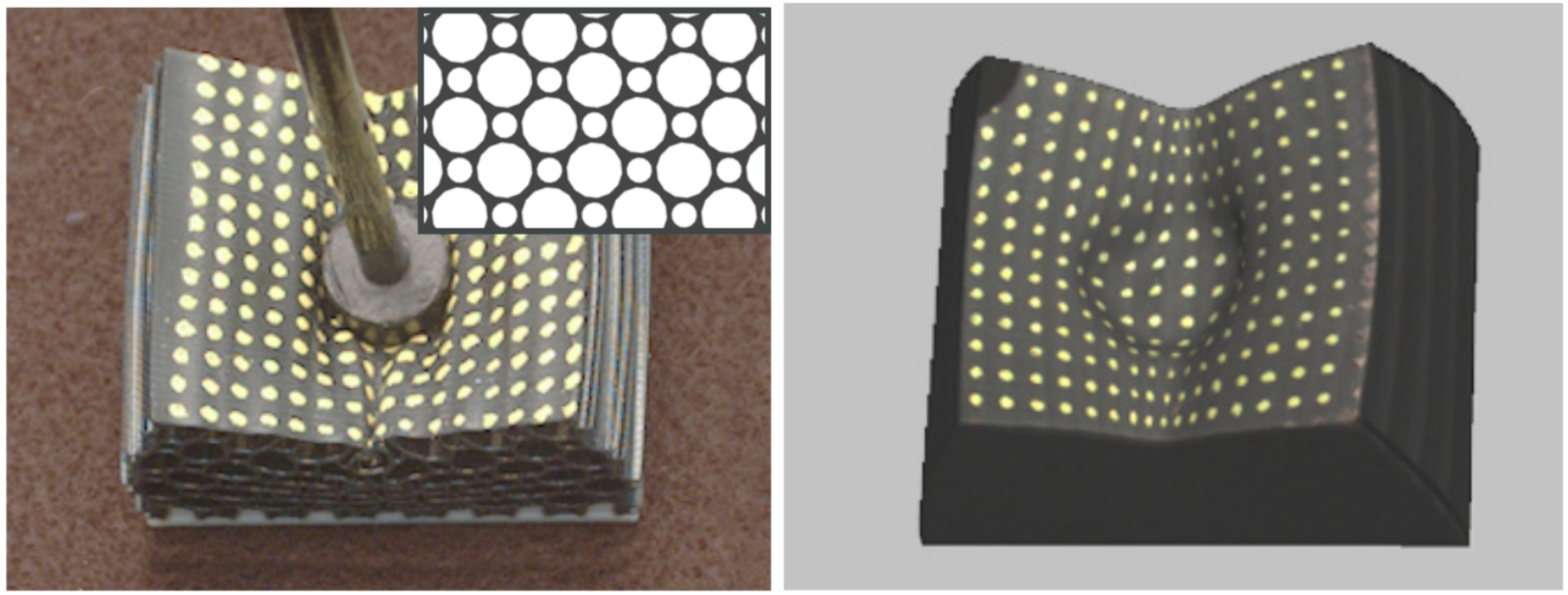
Buffet et al. SIGGRAPH, 2019

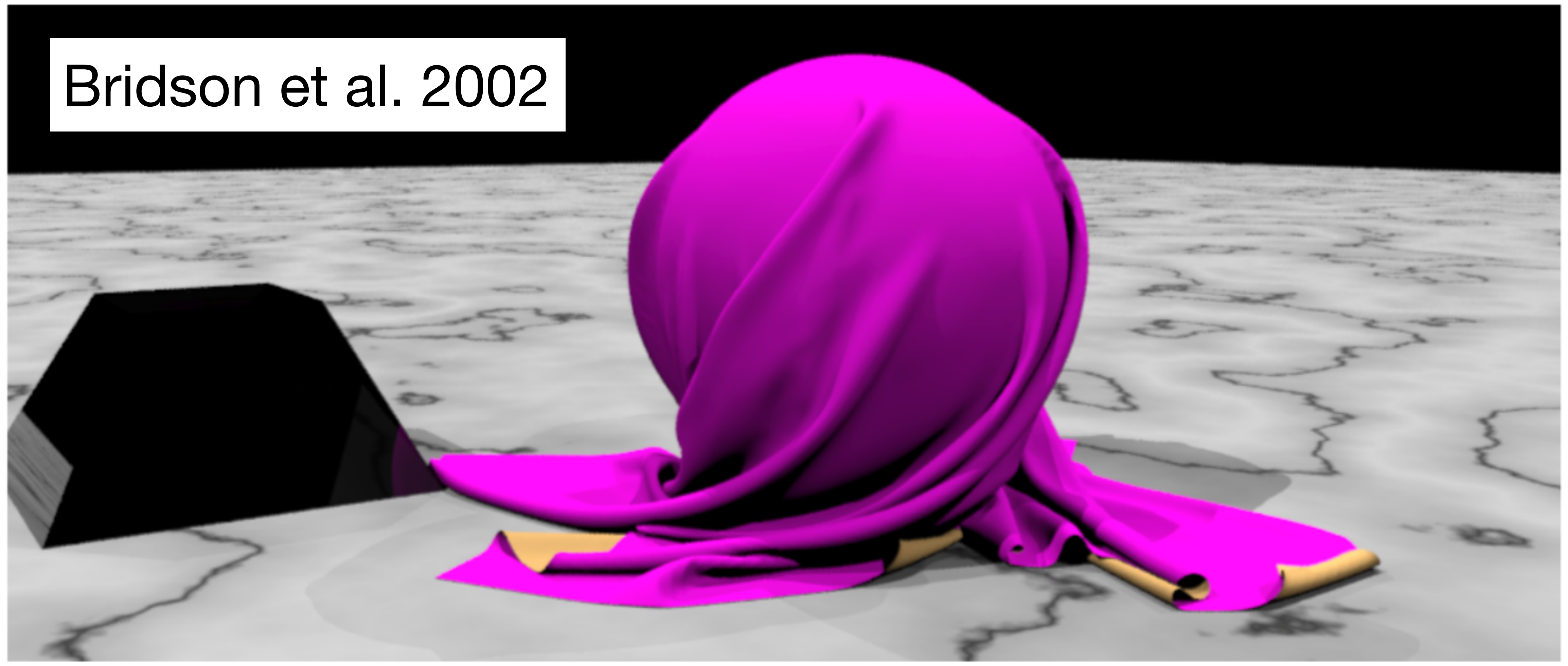
Impressive but qualitative scenarios



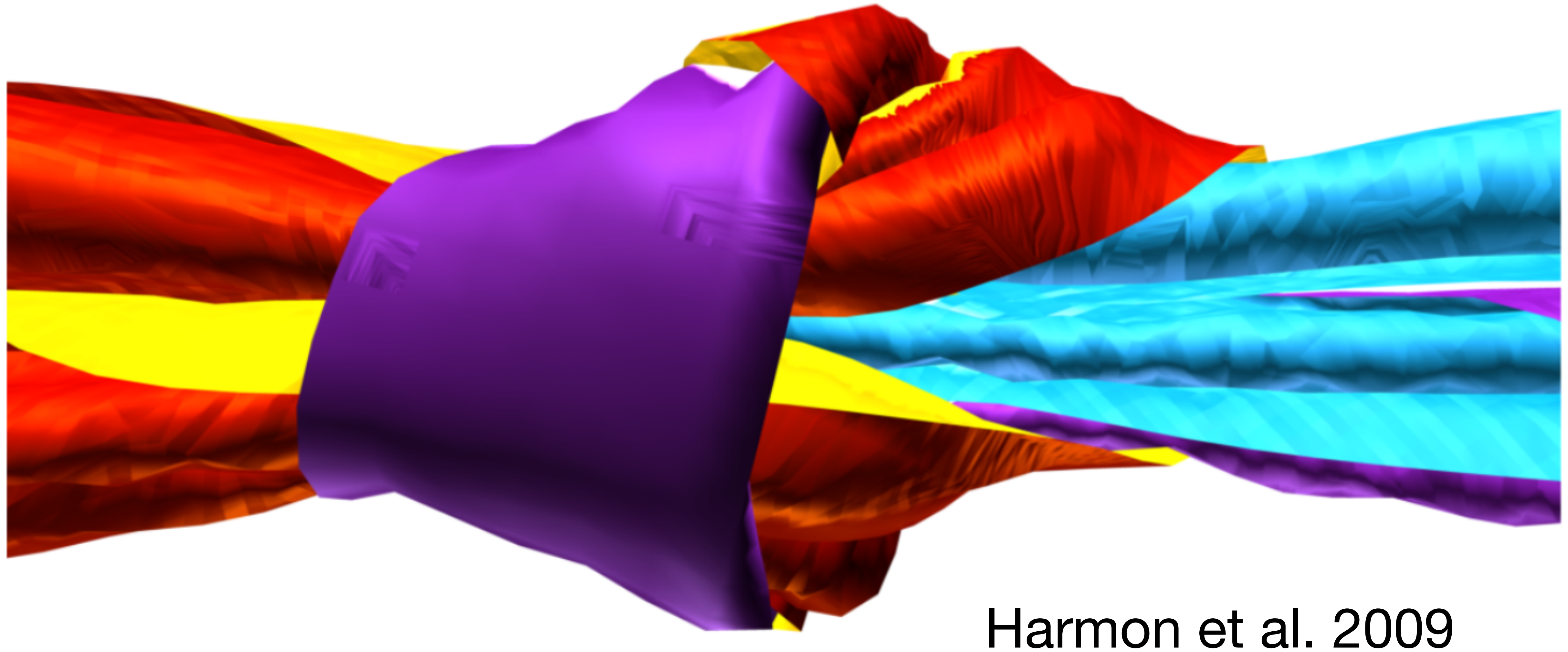
Pfaff et al. 2014

Bickel et al. 10





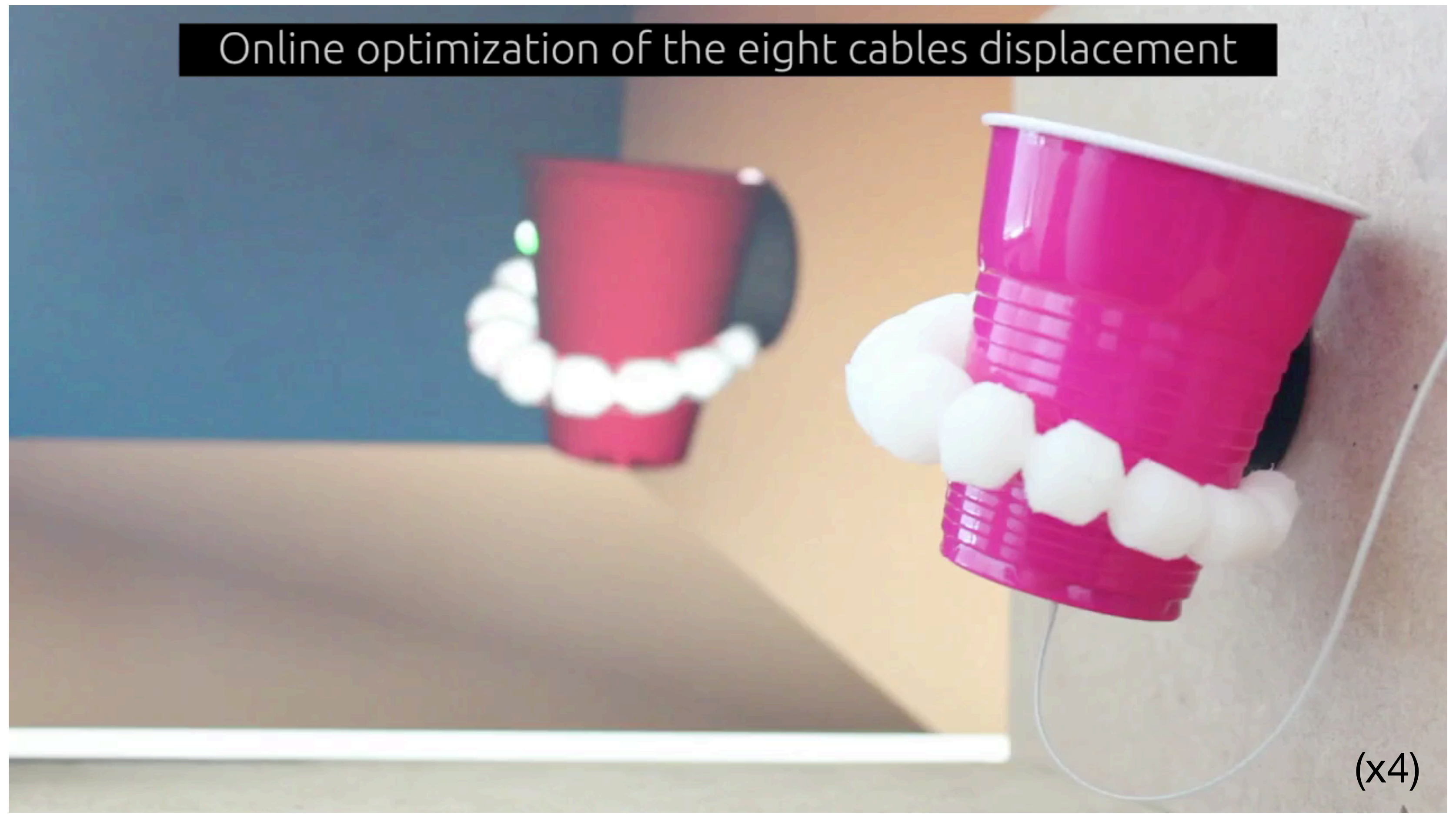
Bridson et al. 2002

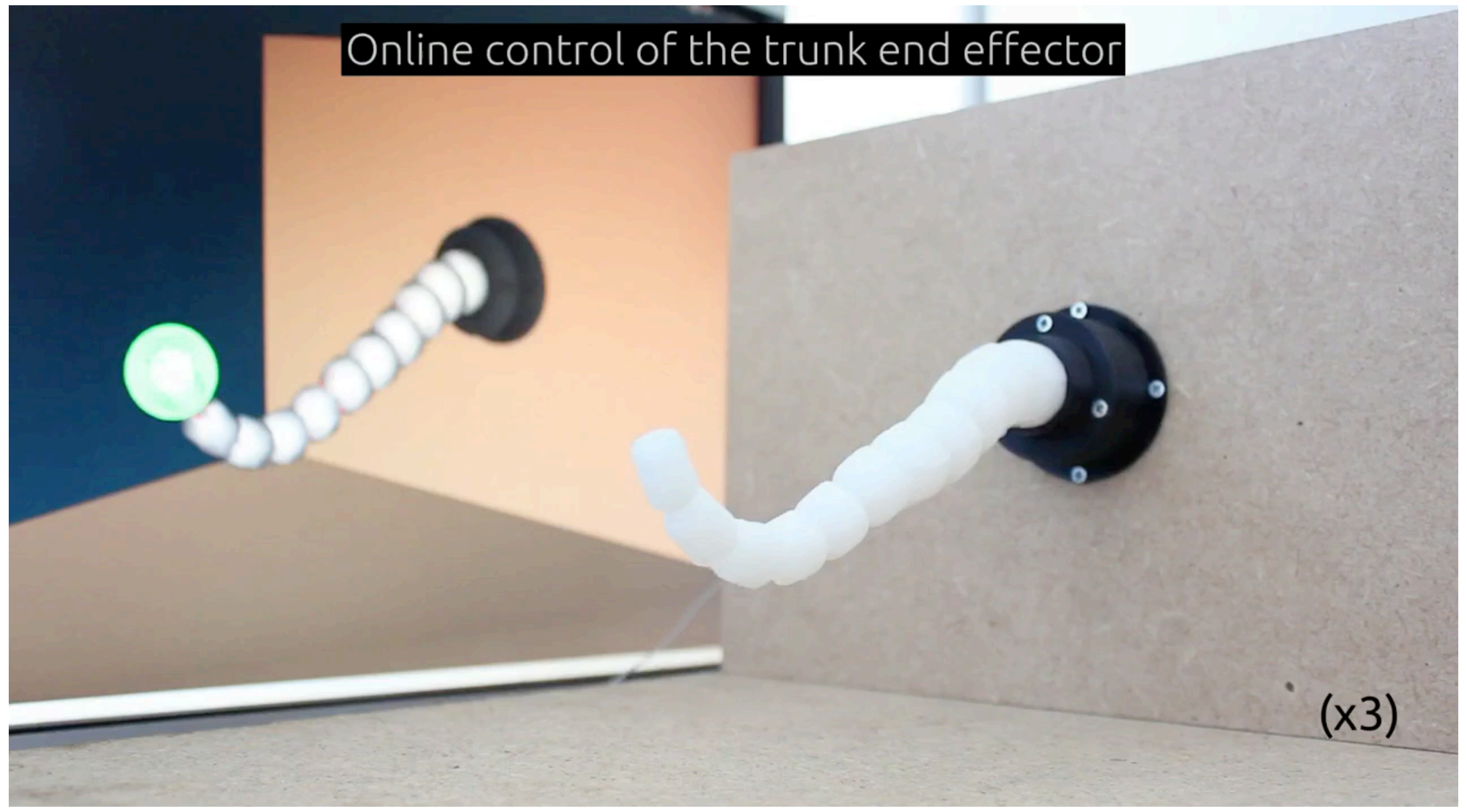


Harmon et al. 2009

Engineering applications

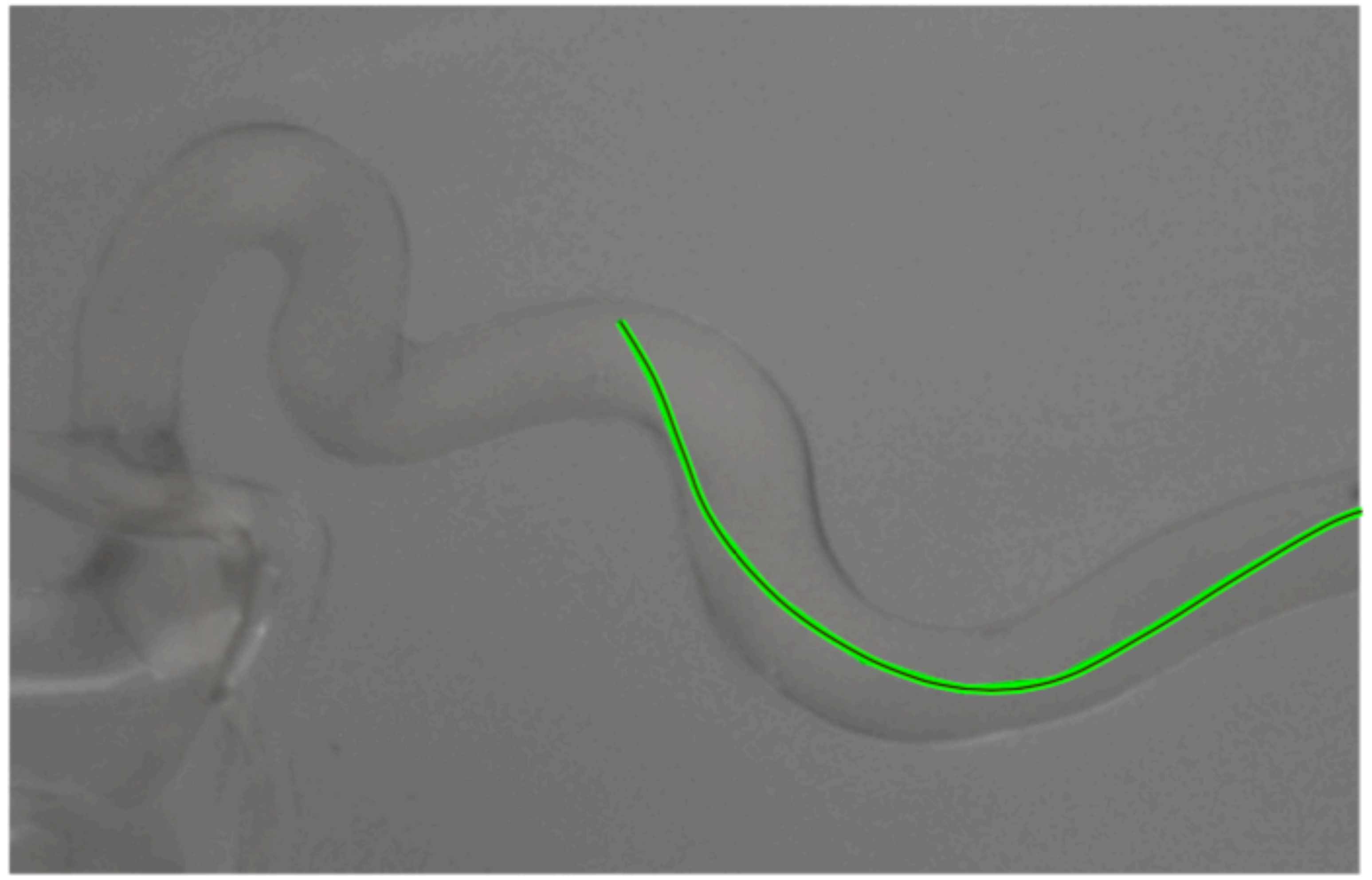
Soft Robotics





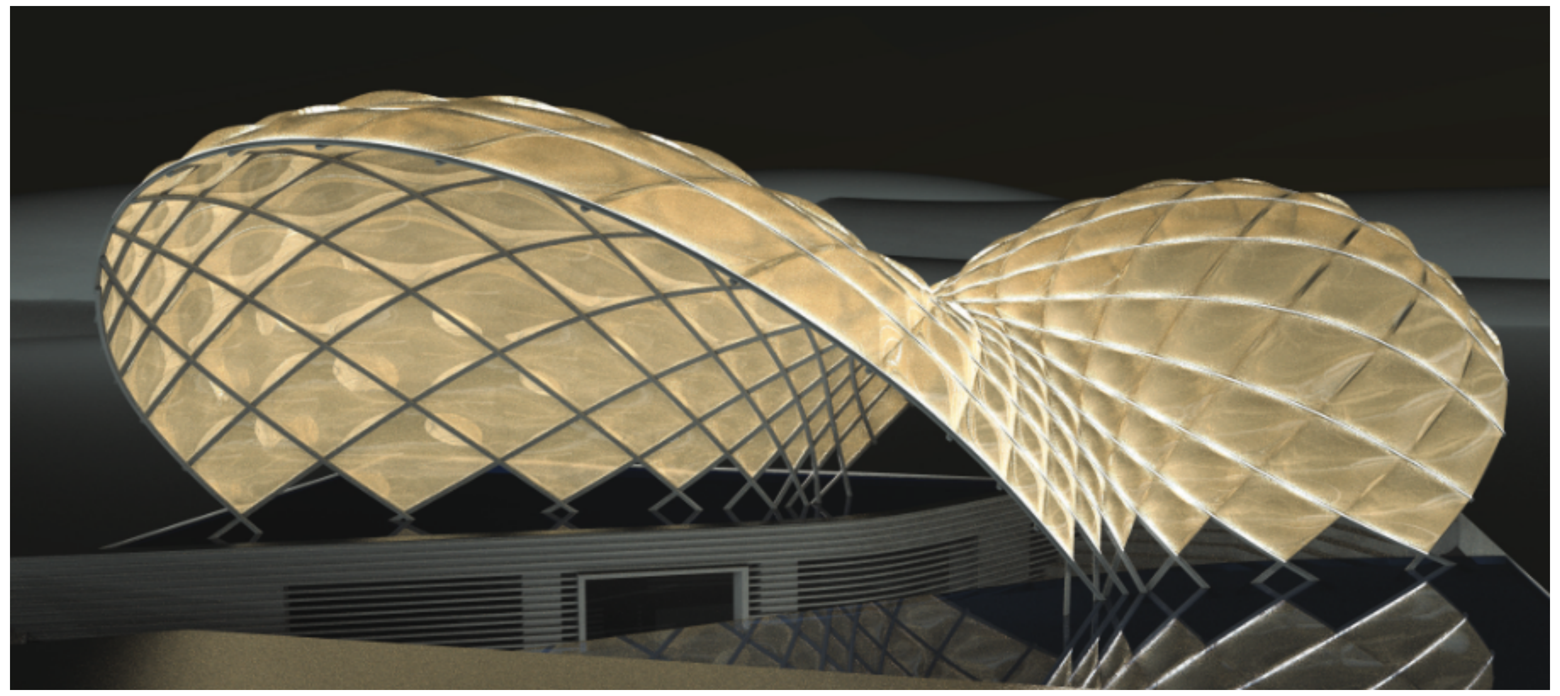
Coevoet et al. Robosoft, 2019

Medicine



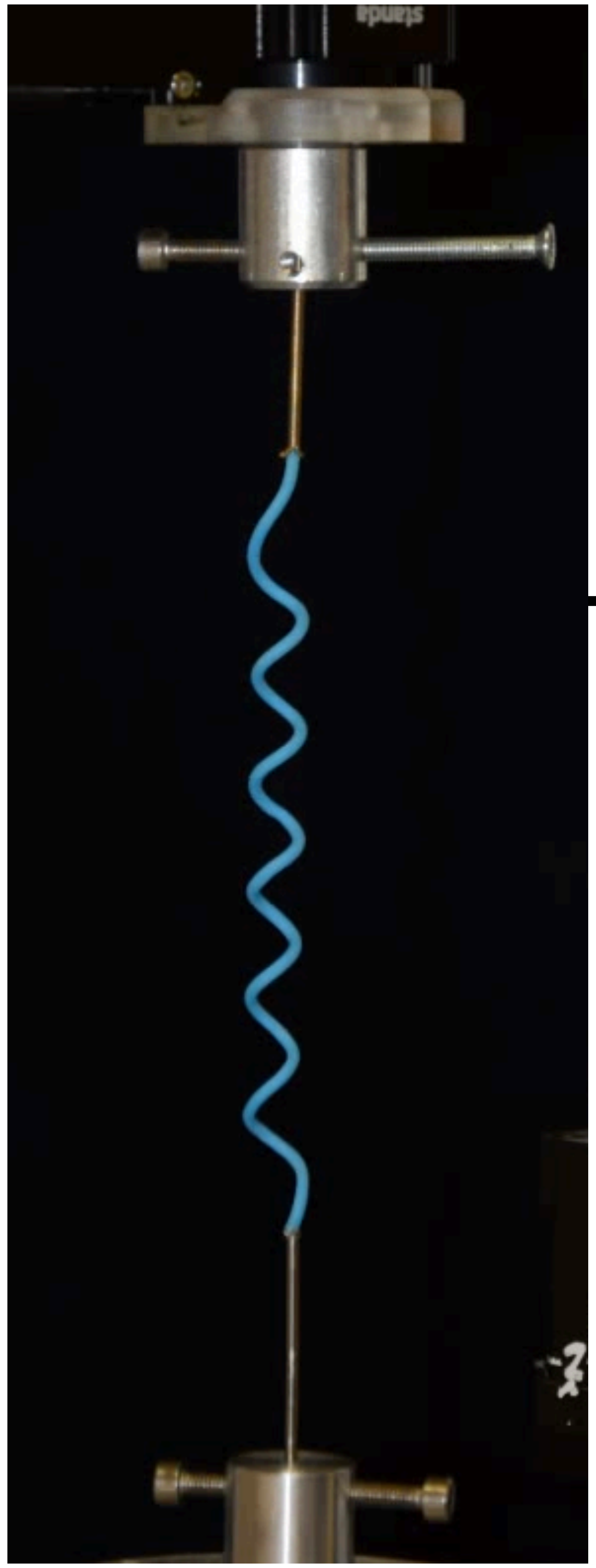
Trivisonne et al., Comp. Medical Imaging and Graphics, 2020

Architecture



Panetta et al. Siggraph, 2019

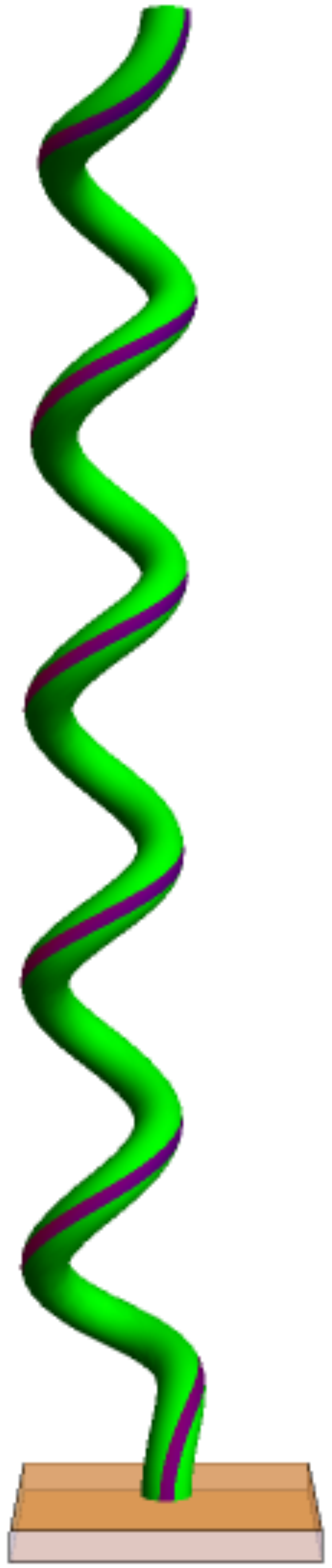
Traditional validation work flow



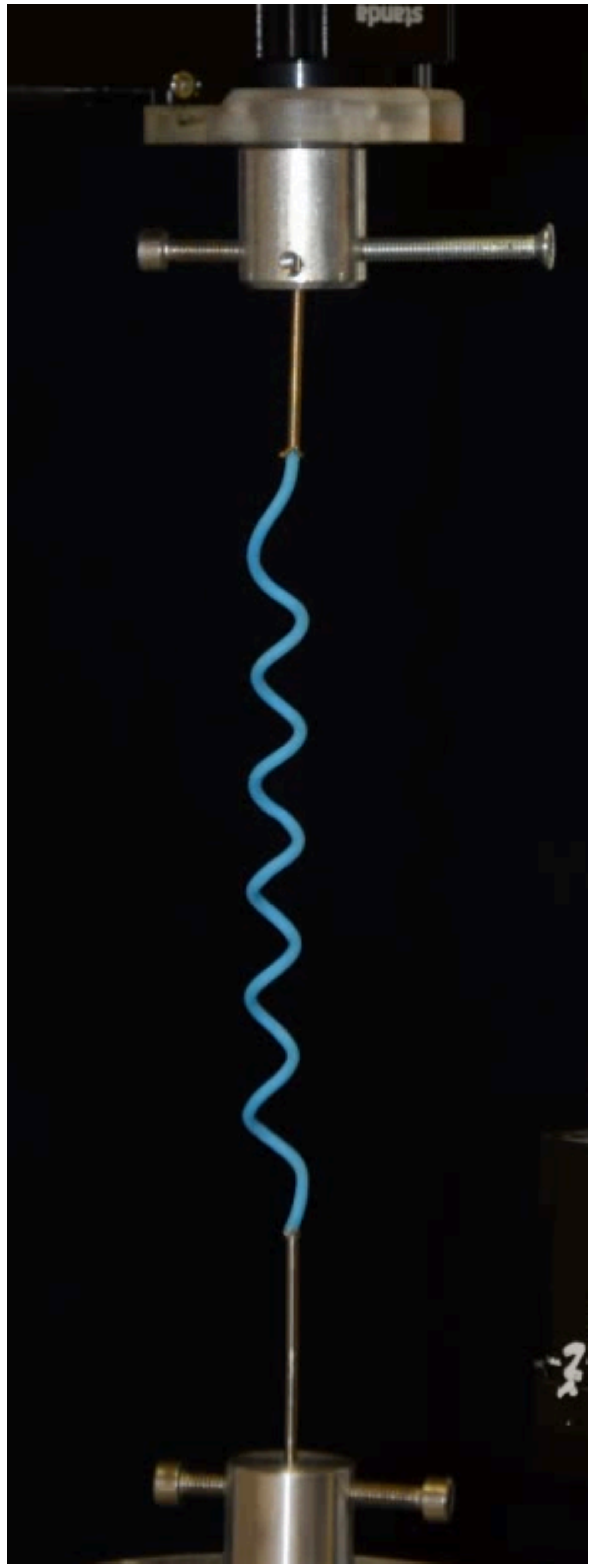
Mechanical Modeling → $\operatorname{div} \sigma = 0$ → Discretisation →

Physical validation

Numerical verification

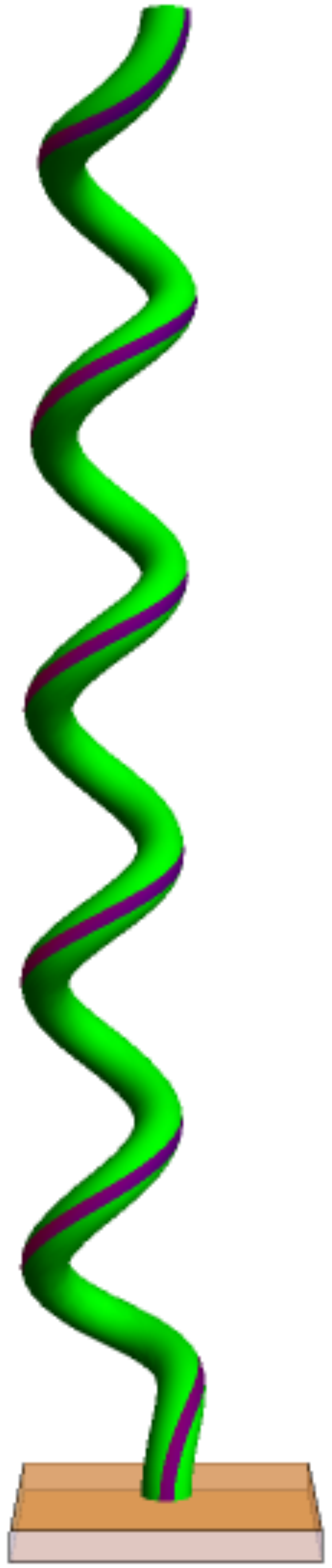


Work flow in Computer Graphics

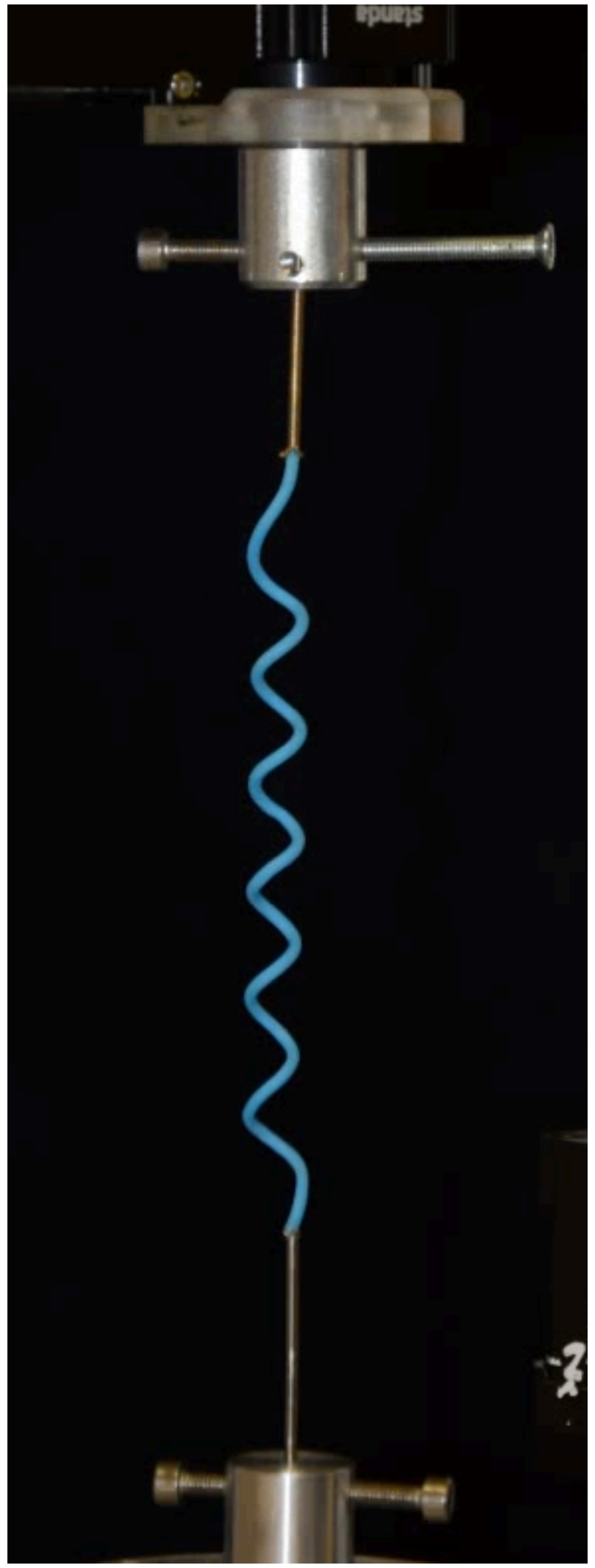


Discretisation

Numerical verification



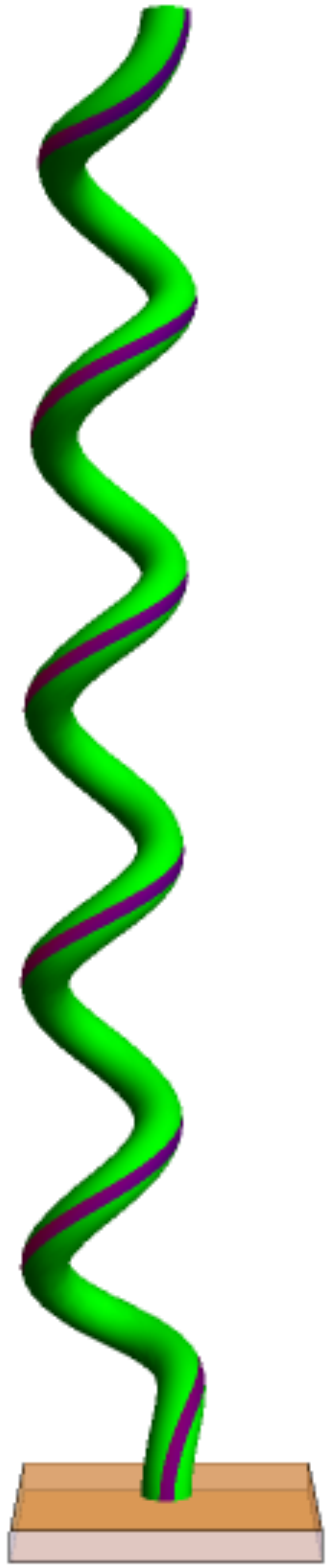
Work flow in Computer Graphics



Discretisation

‘Qualitative’ validation

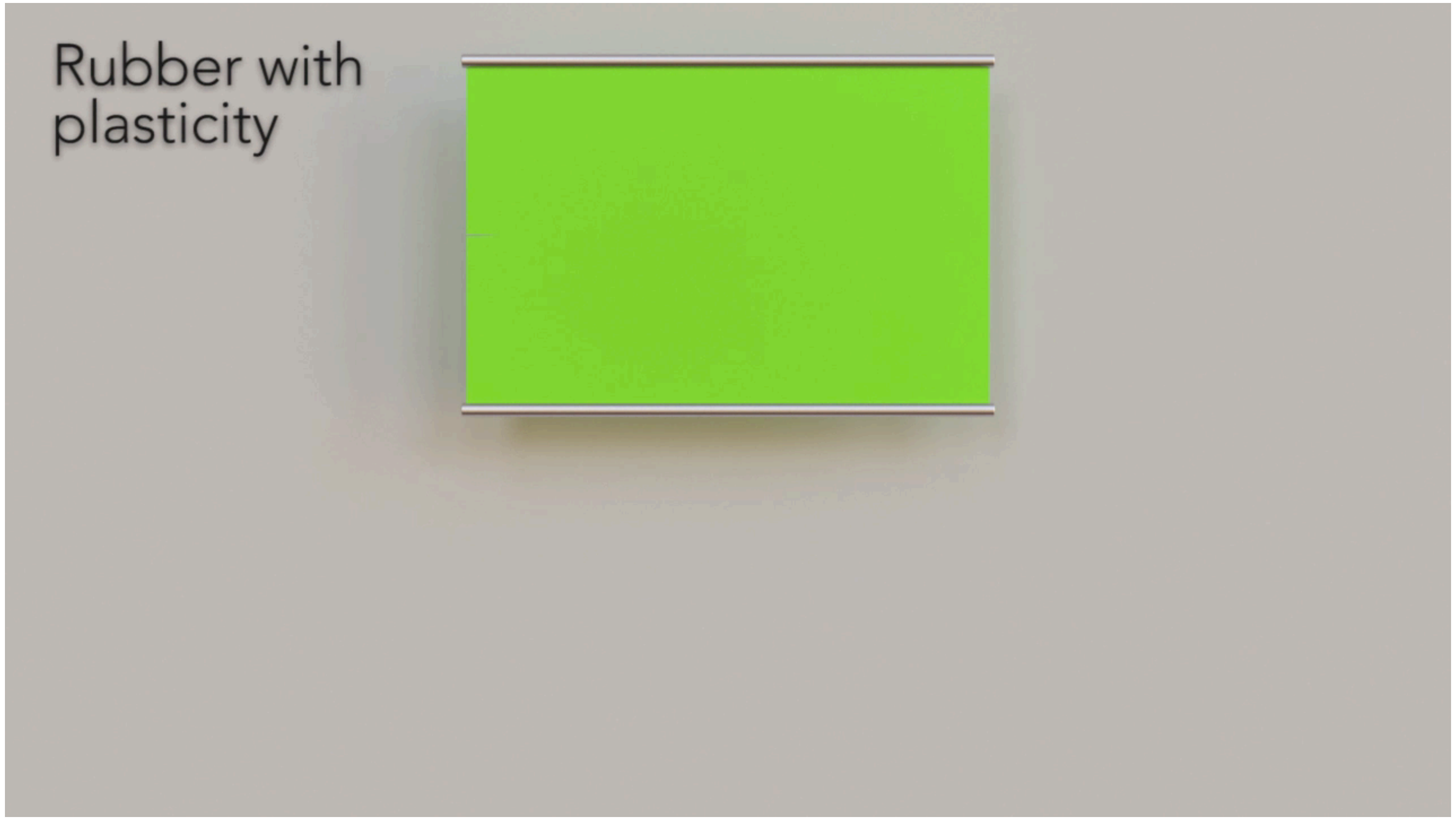
Wang et al. 2011
Smith et al. 2012
Bergou et al. 2008, 2010



Taking advantage of numerical simulators
… for exploring complex physical systems

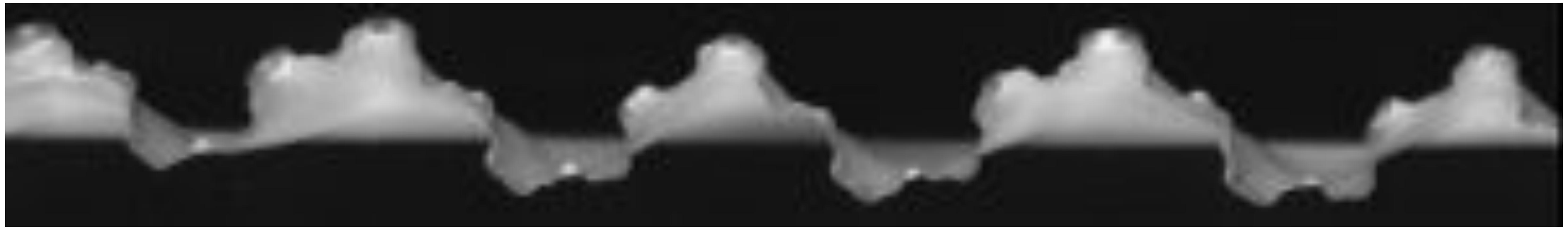
Pfaff et al.
ACM Transaction on Graphics
Vol. 33, No. 4 (2014)

(numerics)

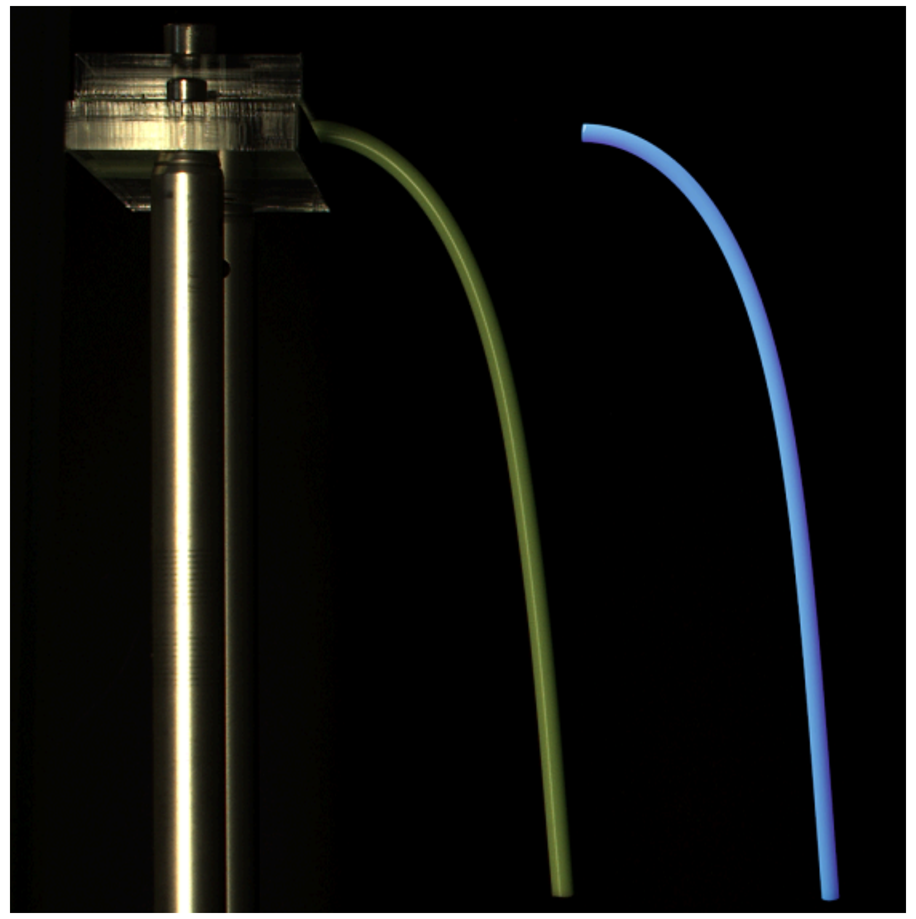


Sharon et al. Nature (2002)

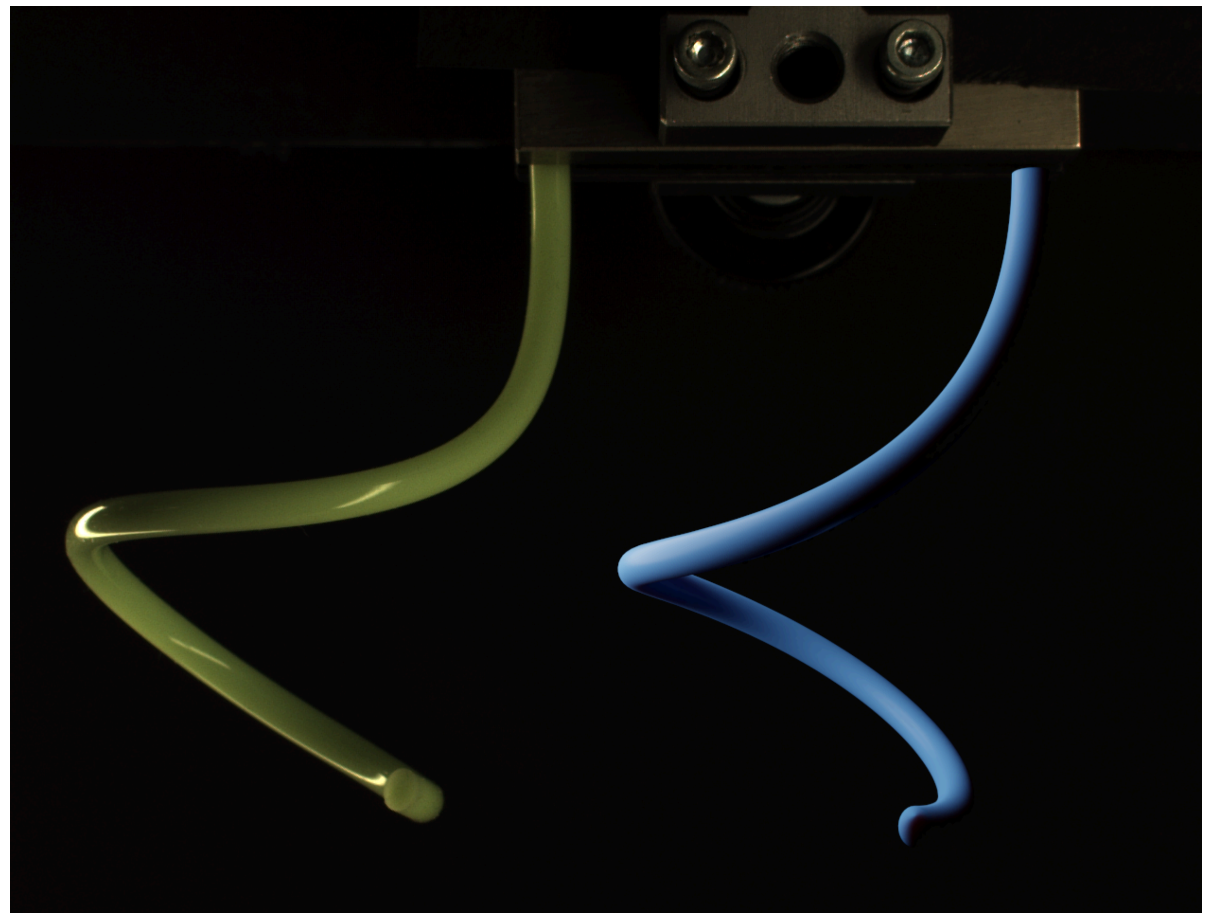
(Physics experiments)



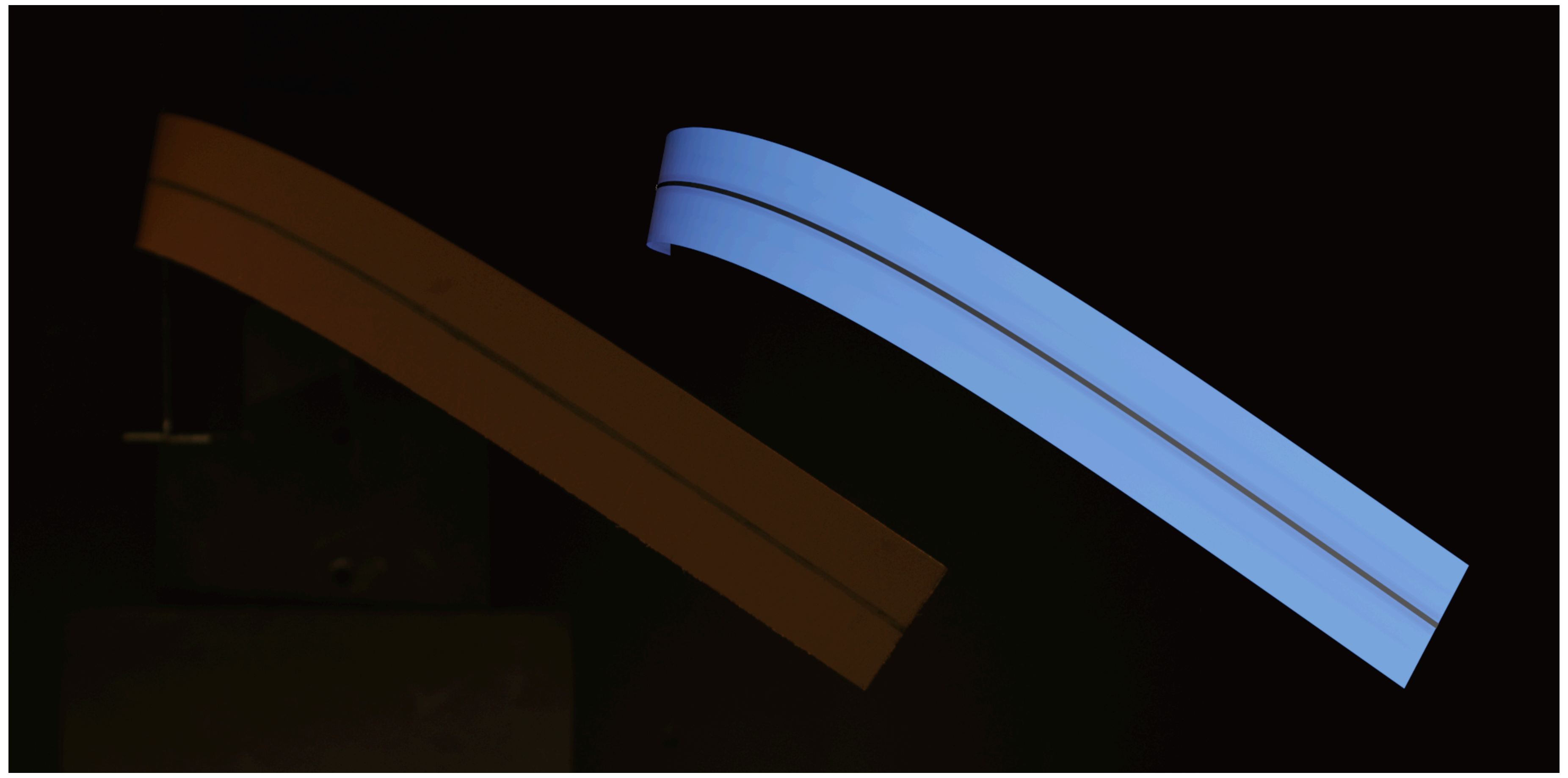
The 4 tests



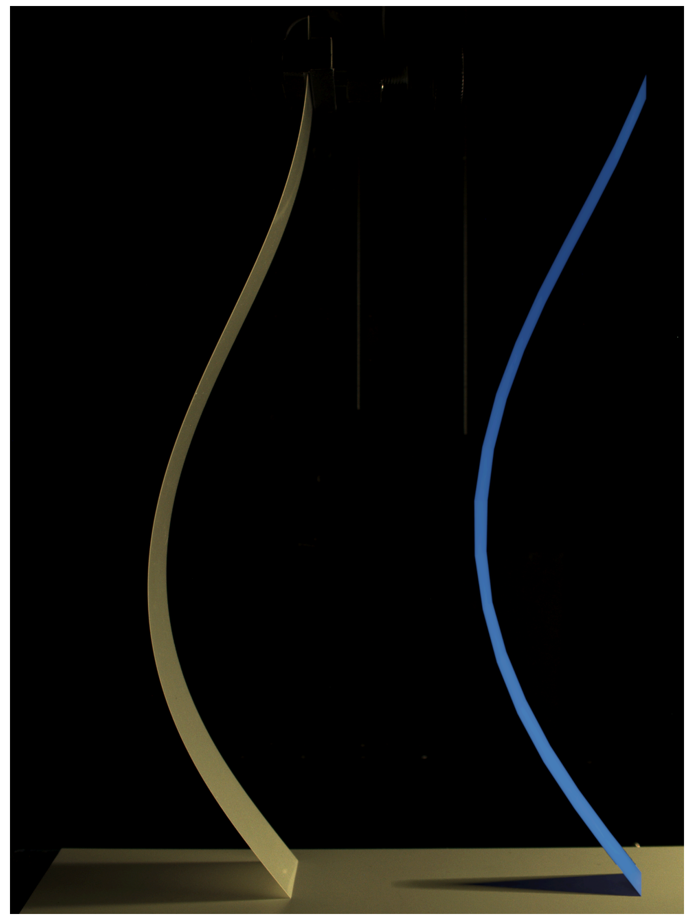
Cantilever



Bend-Twist



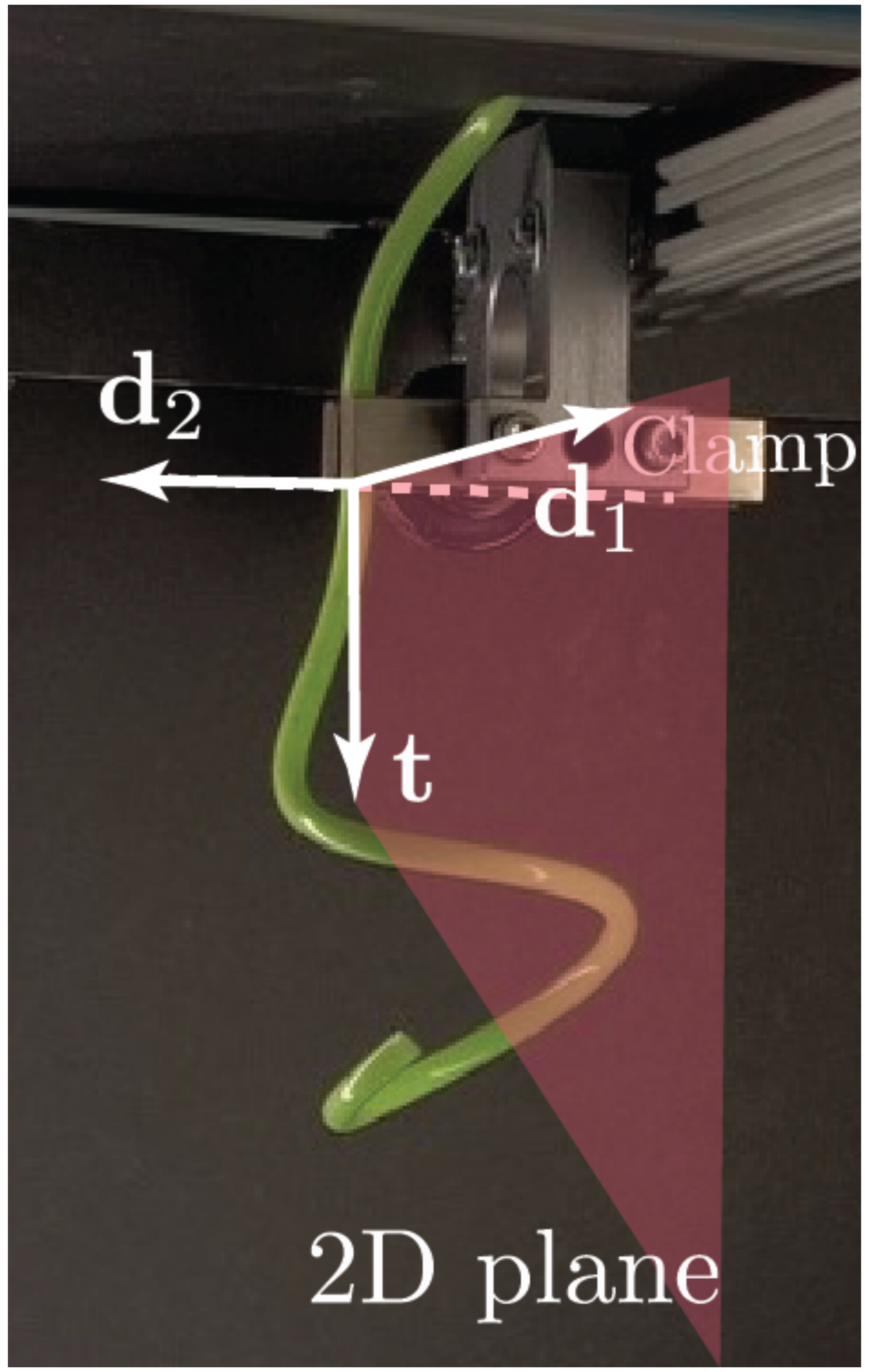
Lateral Buckling



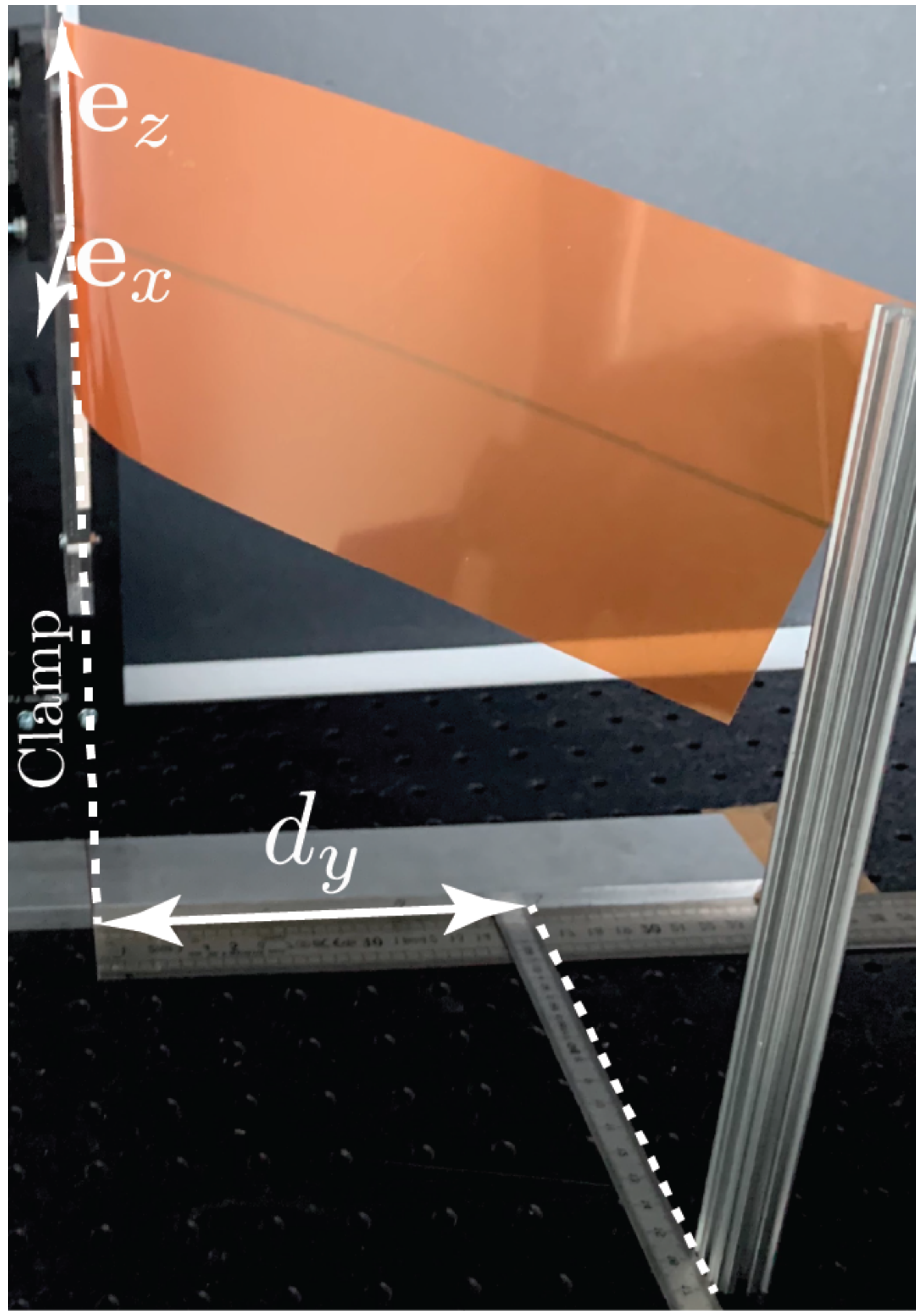
Stick-Slip

Experimental setups

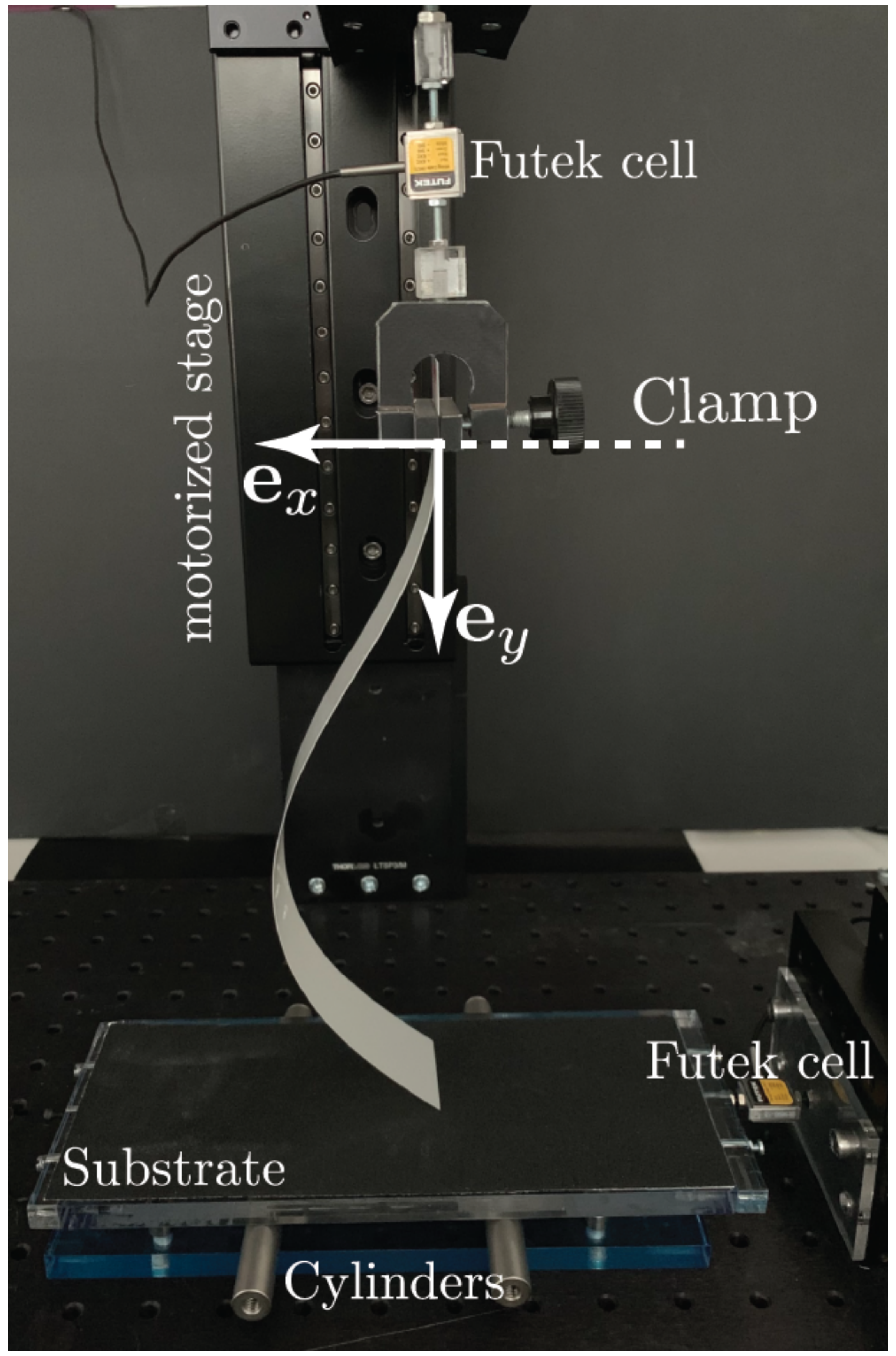
Cantilever



Bend-Twist

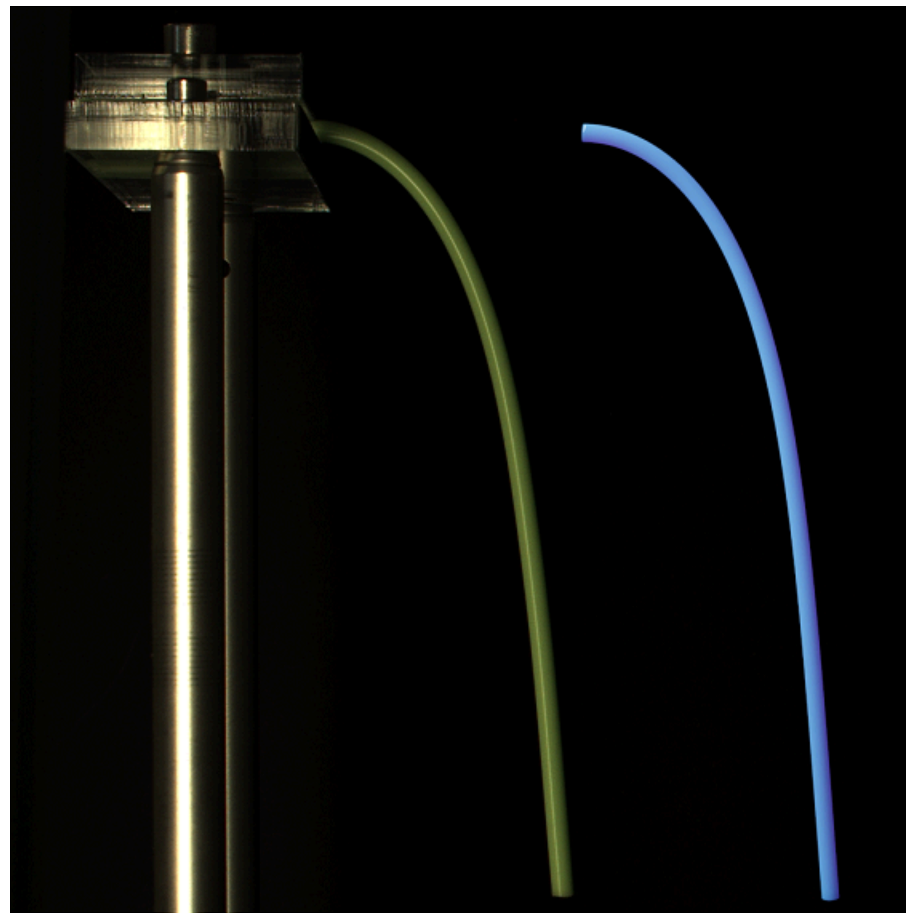


Lateral Buckling

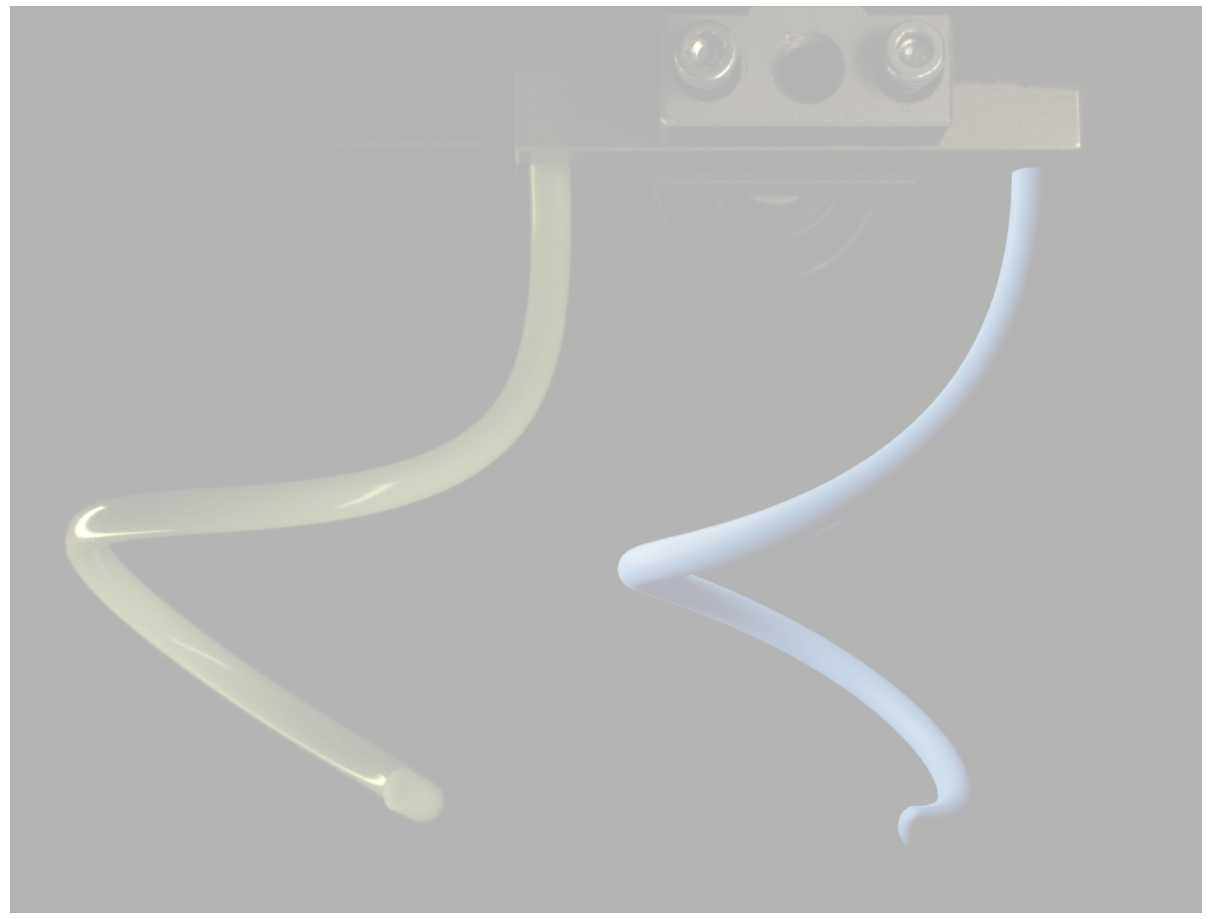


Stick-Slip

The 4 tests

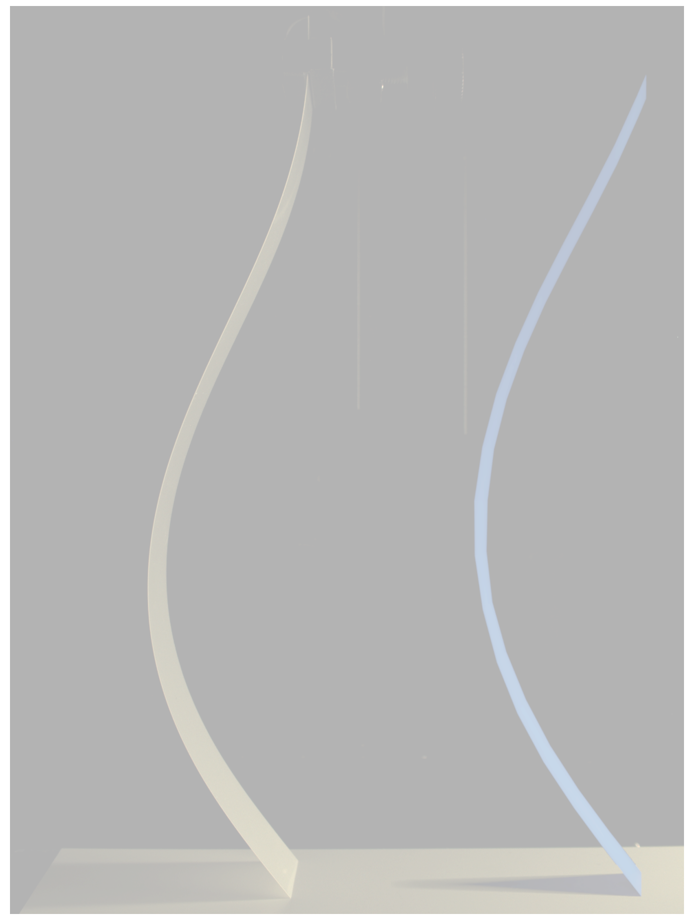


Cantilever



Bend-Twist

Lateral Buckling



Stick-Slip

Cantilever test

Well-known in Engineering and Computer Graphics (here, finite rotations)

Bending rigidity	EI
Length	L
Weight	Mg

The Magic behind non-dimensionalization

SI Units

M Kg Sec Amp.

here: statics, and
no electric charges

M Kg

2 units, which can be chosen *arbitrarily*

Length Force

Our Units (adim units)

We choose

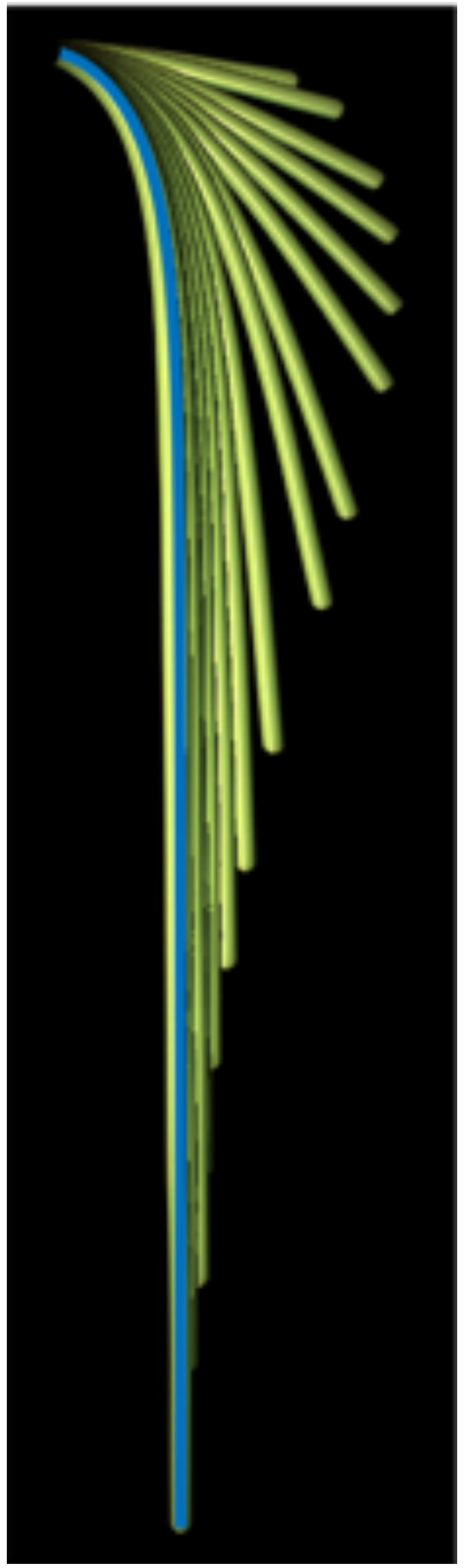
$L \quad EI/L^2$

Equiv. to
formally write

$L=1 \quad EI=1$

no loss of generality!

Cantilever test



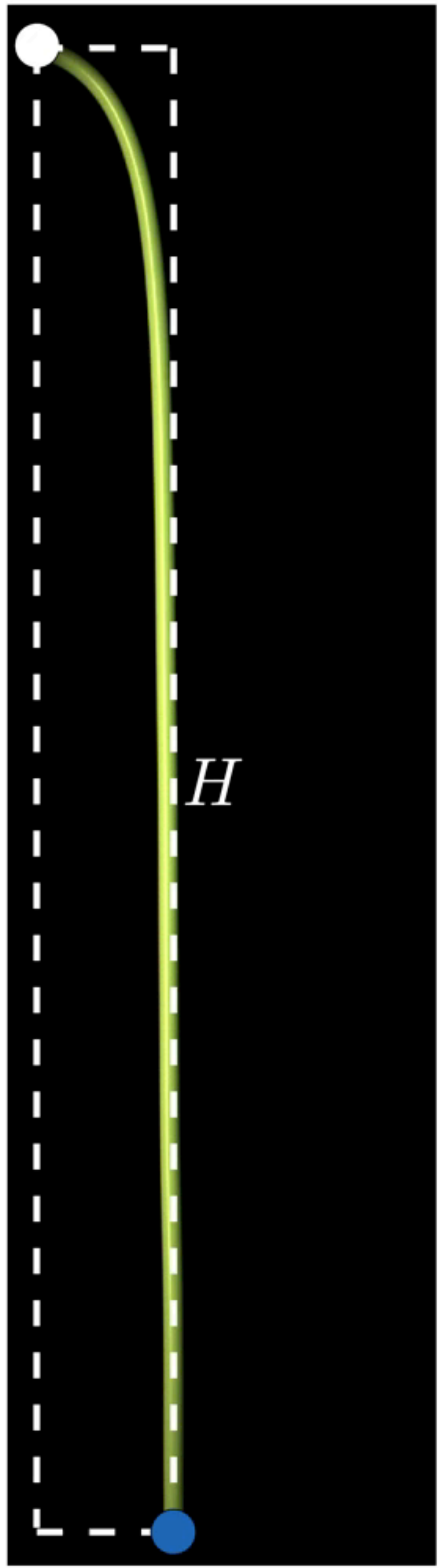
Well-known in Engineering and Computer Graphics (here, finite rotations)

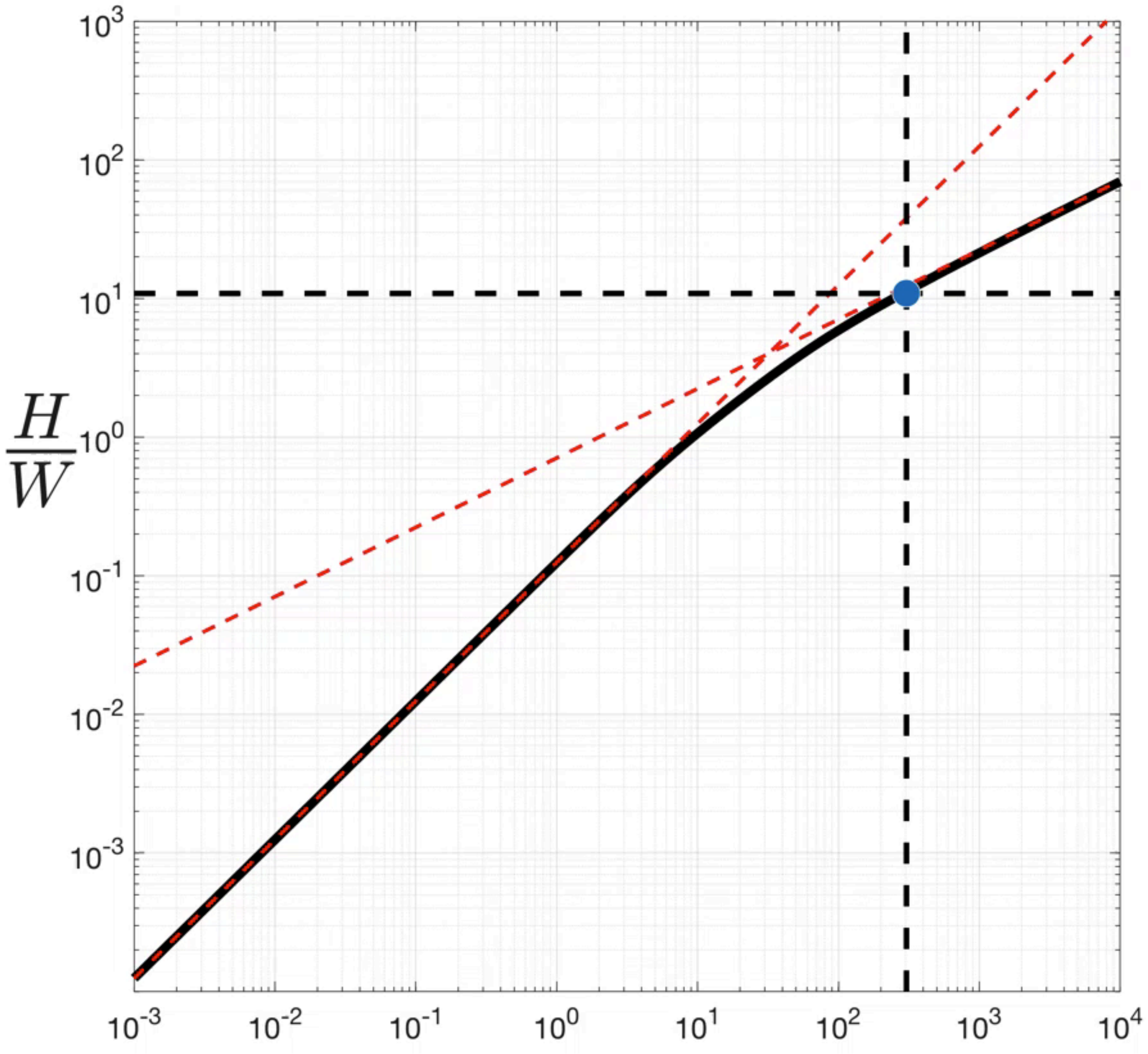
$$\begin{array}{ll} \text{Bending rigidity} & EI \\ \text{Length} & L \\ \text{Weight} & Mg \end{array}$$

$$\downarrow$$

Adim weight: $\Gamma = \dfrac{MgL^2}{EI}$

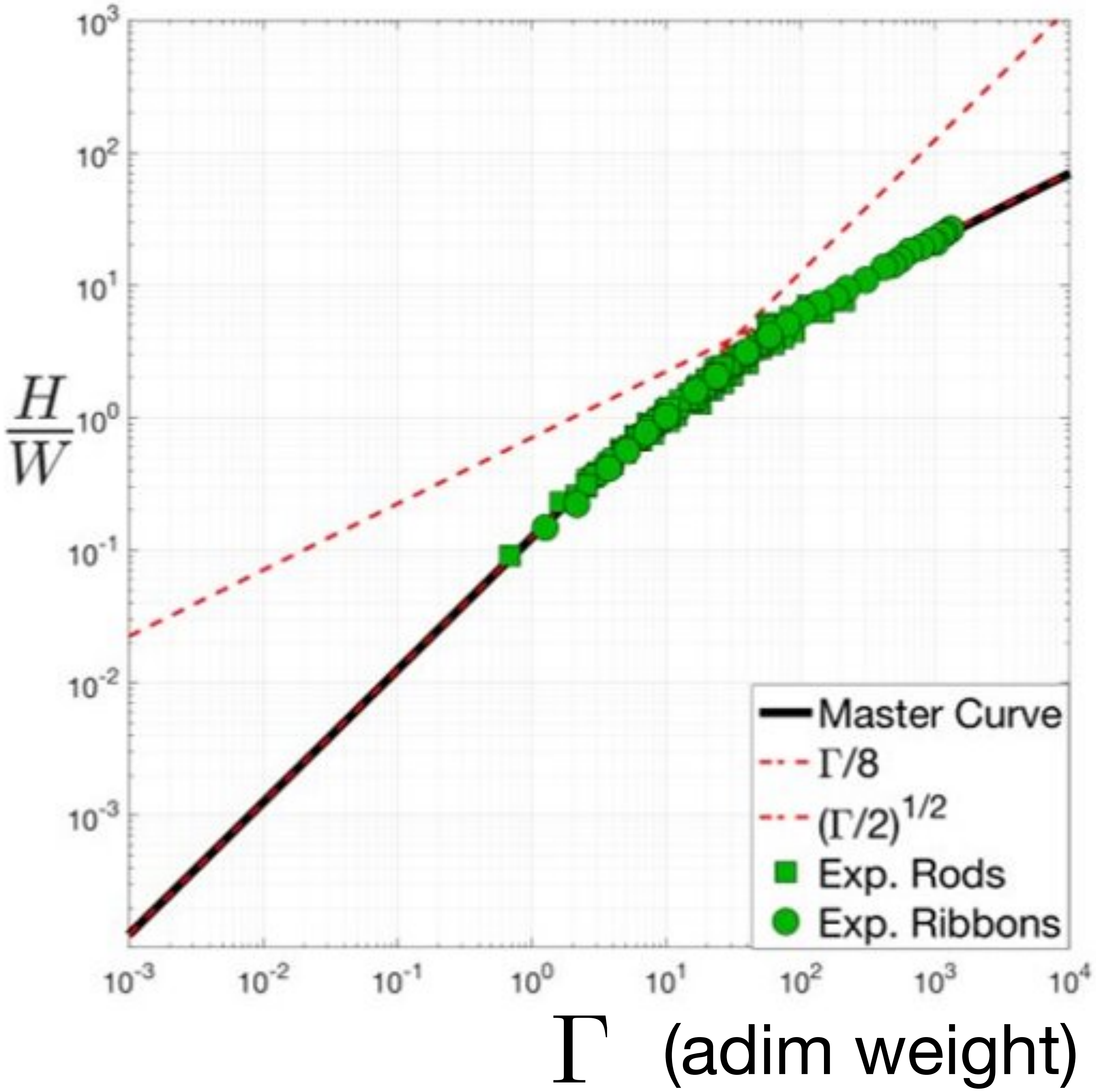
Cantilever test – experimental validation





$$\Gamma = \frac{MgL^2}{EI}$$ (adim weight)

Cantilever test – experimental validation

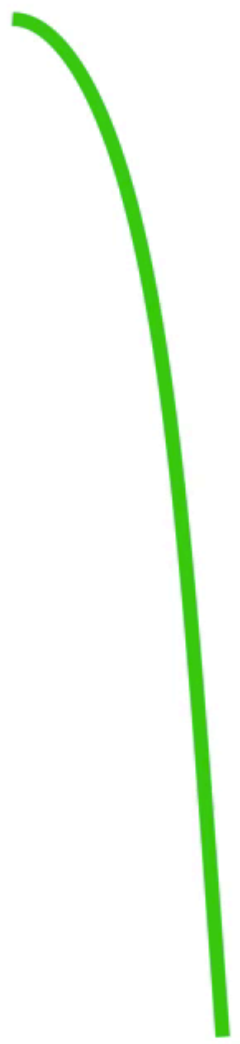


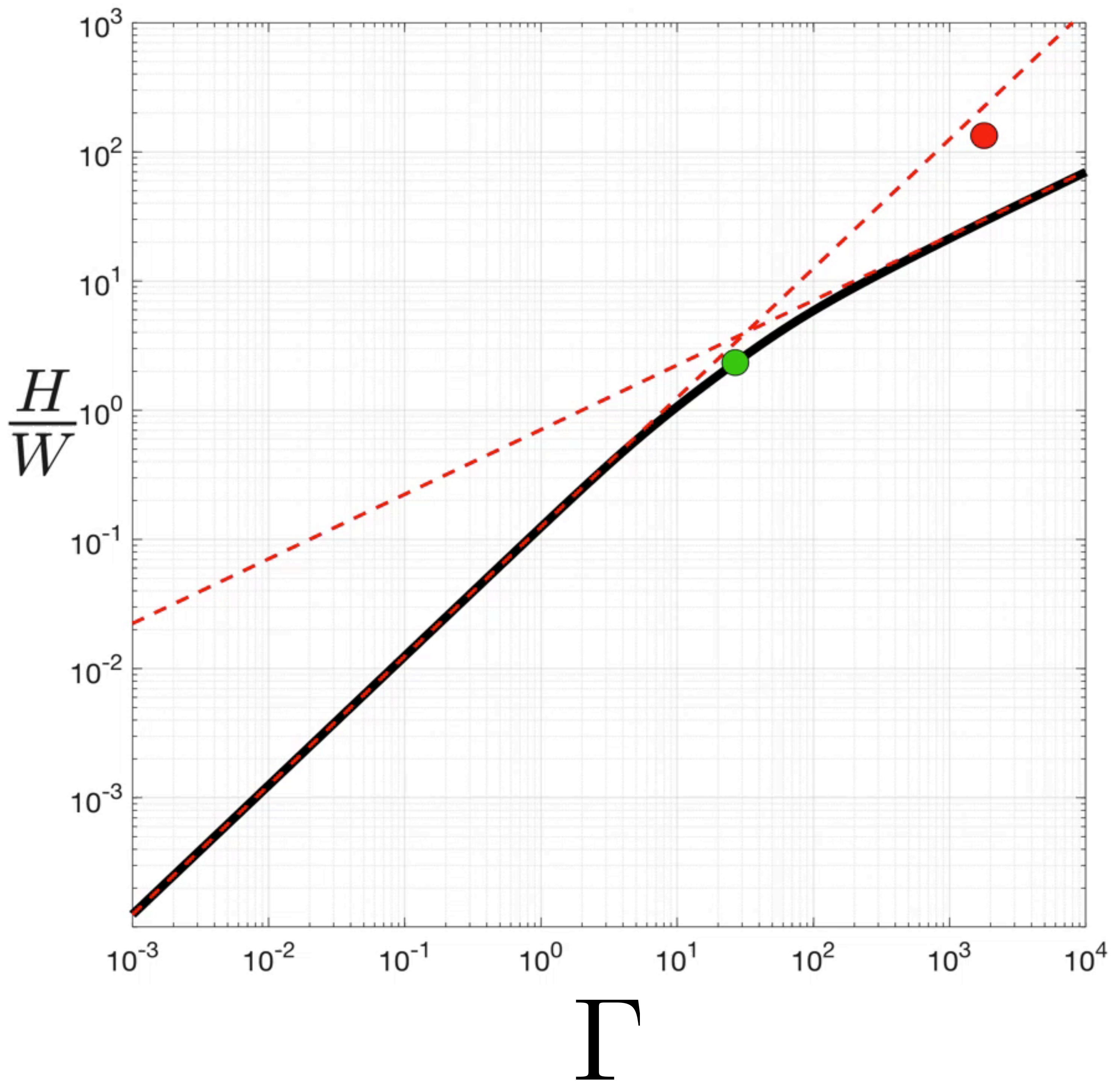
Results

methodology

the simulator output must match the master curve
over a range of physical parameters

==> OK or KO





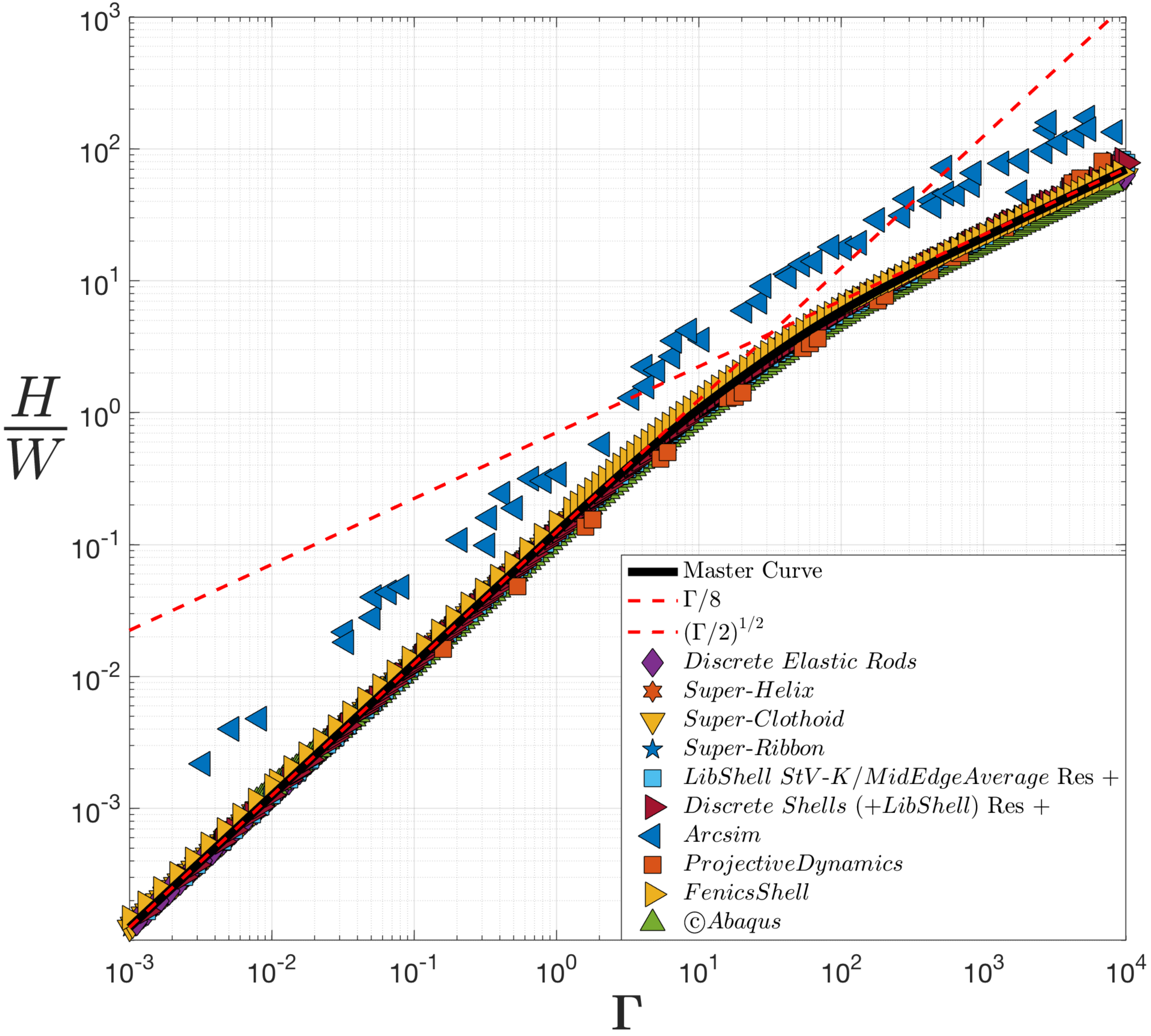
OK KO

Results

Cantilever test

Tested Code	Cantilever	Bend-Twist	Lateral Buckling	Stick-Slip
Rod				
Discrete Elastic Rod [Bergou et al. 2010]	OK (300 elts)	OK (200 elts)	–	–
Super-Helix [Bertails et al. 2006]	OK (50 elts)	OK (30 elts)	–	–
Super-Clothoid [Casati and Bertails-Descoubes 2013]	OK (20 elts)	OK (25 elts)	–	–
Ribbon				
Super-Ribbon [Charrondière et al. 2020]	OK (20 elts)	–	KO	–
Plate				
LibShell [Chen et al. 2018]	OK (Res 0)	–	OK (Res +)	–
Discrete Shell (+ LibShell) [Grinspun et al. 2003]	OK (Res +)	–	KO	–
ArcSim [Narain et al. 2012]	KO	–	×	–
Discrete Shell + ArcSim (**tentative fix** of ArcSim)	KO	–	×	–
Projective Dynamics [Bouaziz et al. 2014] (**fit**)	KO	–	×	–
Contact & friction				
Viscous Friction (+ Super-Helix 2D)	–	–	–	KO
So-Bogus [Daviet et al. 2011] (+Super-Helix 2D)	–	–	–	OK (dt=0.5 ms, tol $= 10^{-13}$ N)
Argus (≈ ArcSim + So-Bogus) [Li et al. 2018]	–	–	–	KO
Argus Non Adaptive (**fix** of Argus)	–	–	–	OK (dt=0.5 ms, tol $= 10^{-13}$ N)
Bridson-Harmon [Bridson et al. 2002; Harmon et al. 2008] (+ ArcSim)	–	–	–	KO
Projective Friction [Ly et al. 2020]	–	–	–	KO
Reference codes in Mechanical Engineering				
FenicsShell [Hale et al. 2018]	OK (Res 0, $P_{2,3}$ elts)	–	OK (Res 0, $P_{2,3}$ elts)	–
©Abaqus	OK (200 P_2 elts)	KO	KO	OK (dt=9 μs)

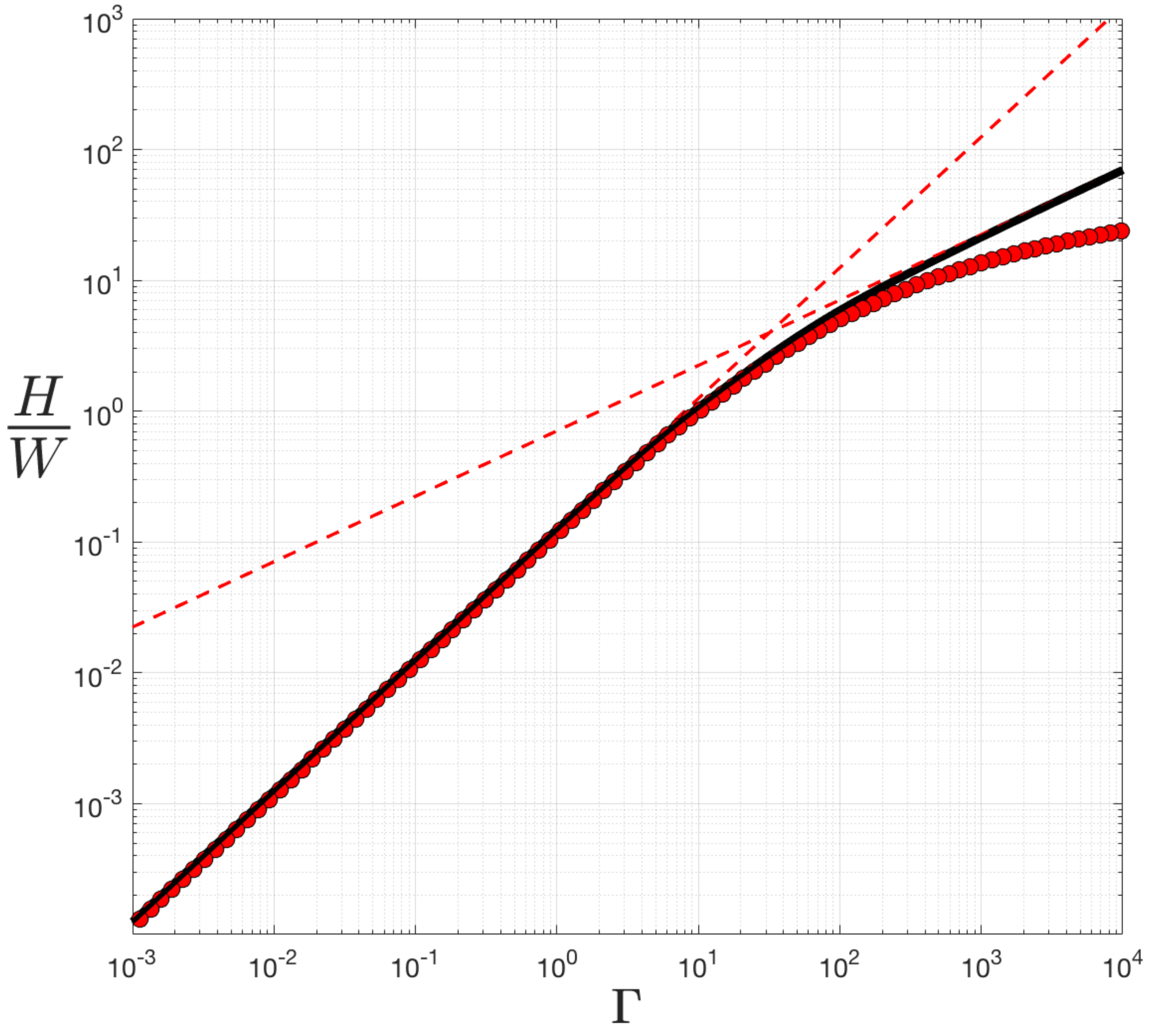
Results
Cantilever test



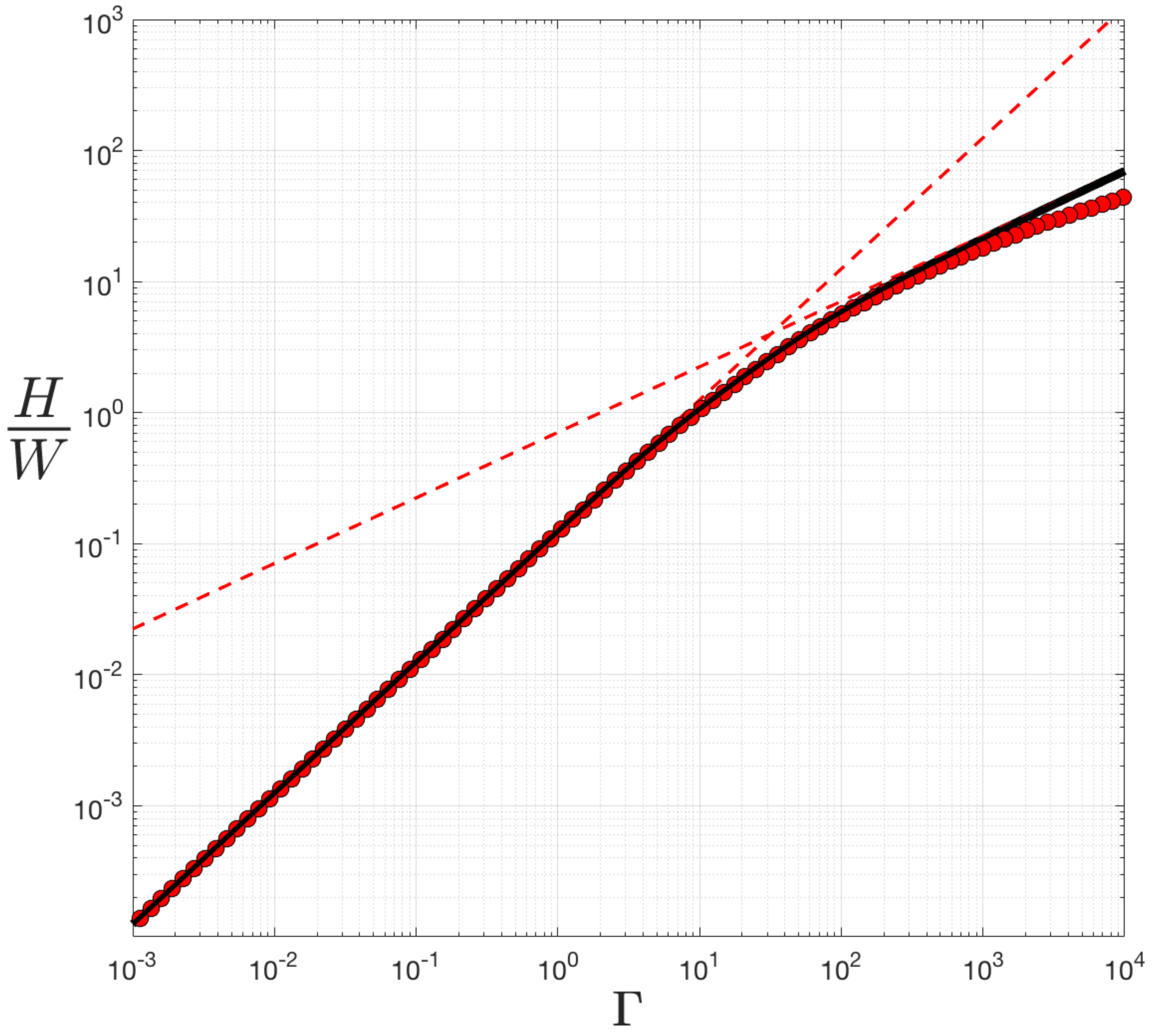
Results

Cantilever test: High values of Γ

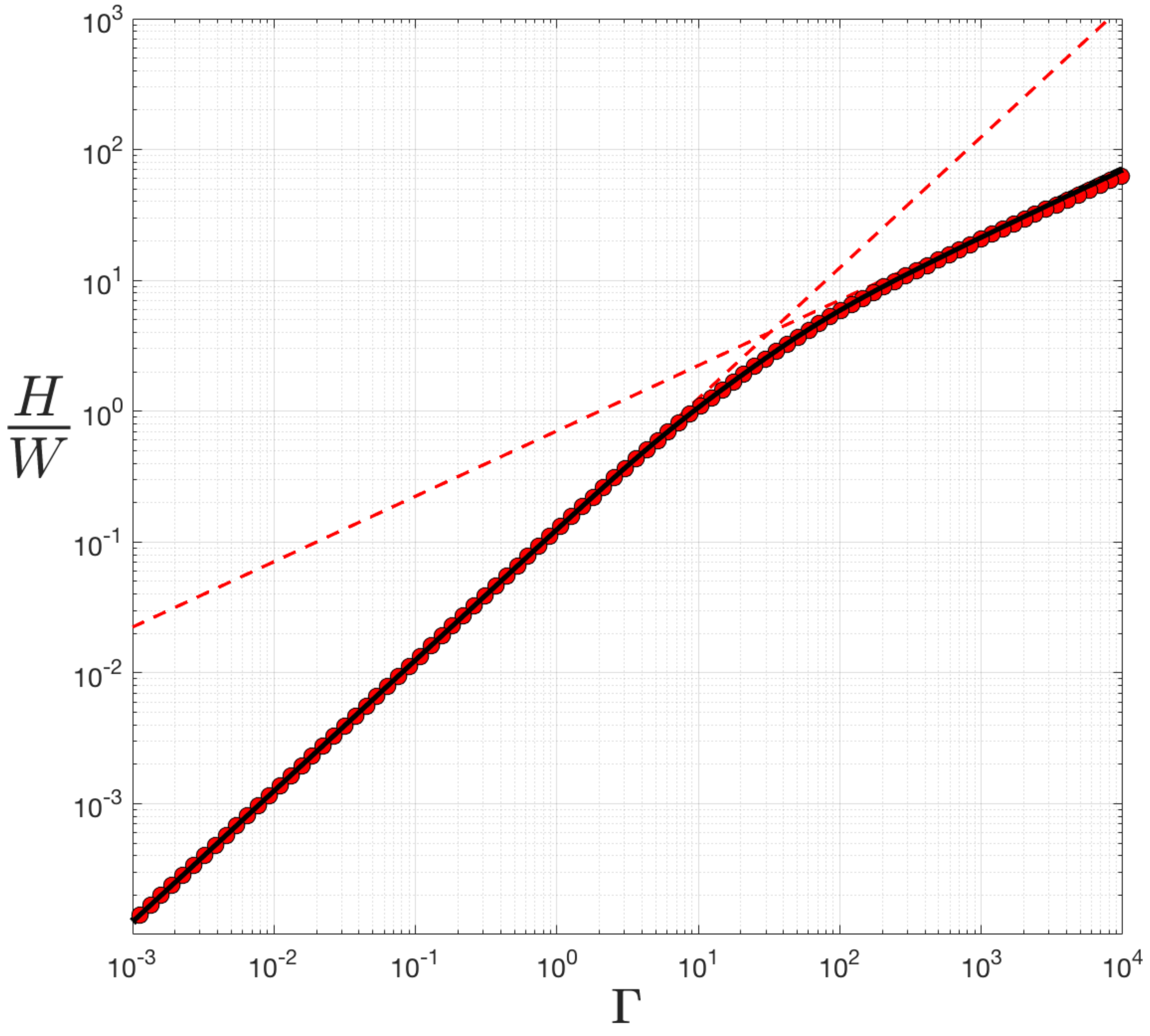
OK



30 elements



80 elements



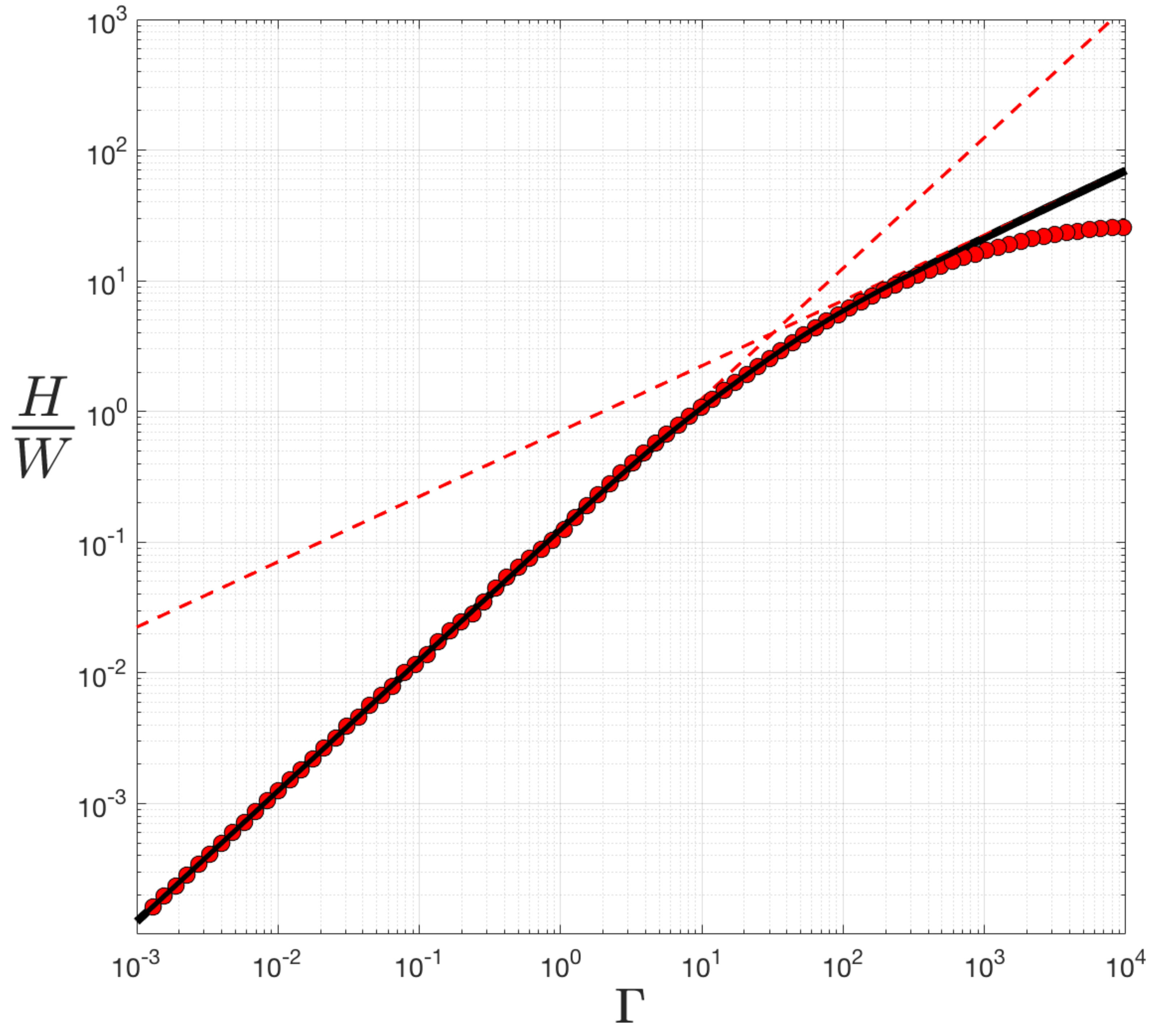
300 elements

Discrete Elastic Rod [M. Bergou et al. ACM Transactions on Graphics, 27 #3 (2008), 1–12]

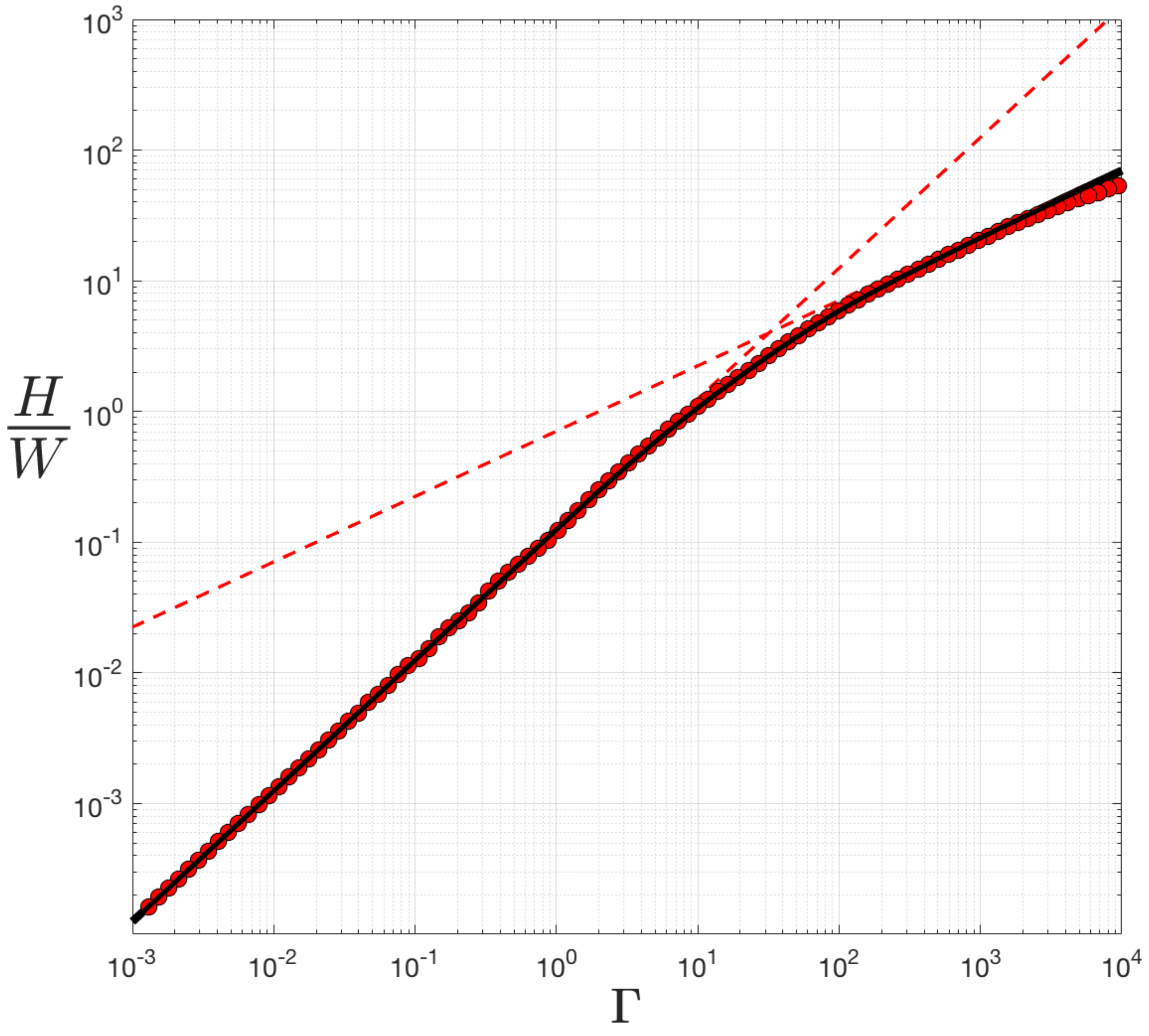
Results

Cantilever test: High values of Γ

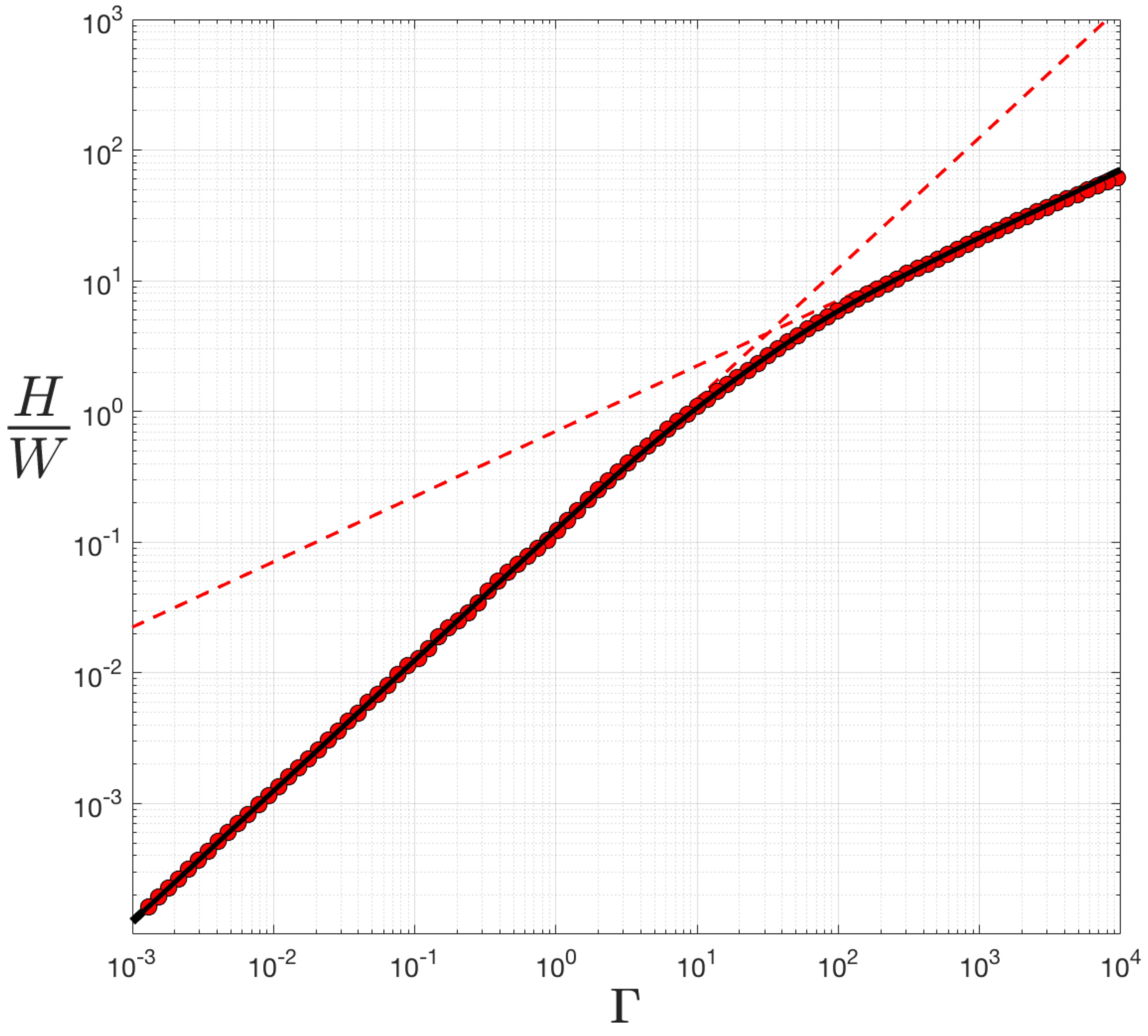
OK



10 elements



30 elements

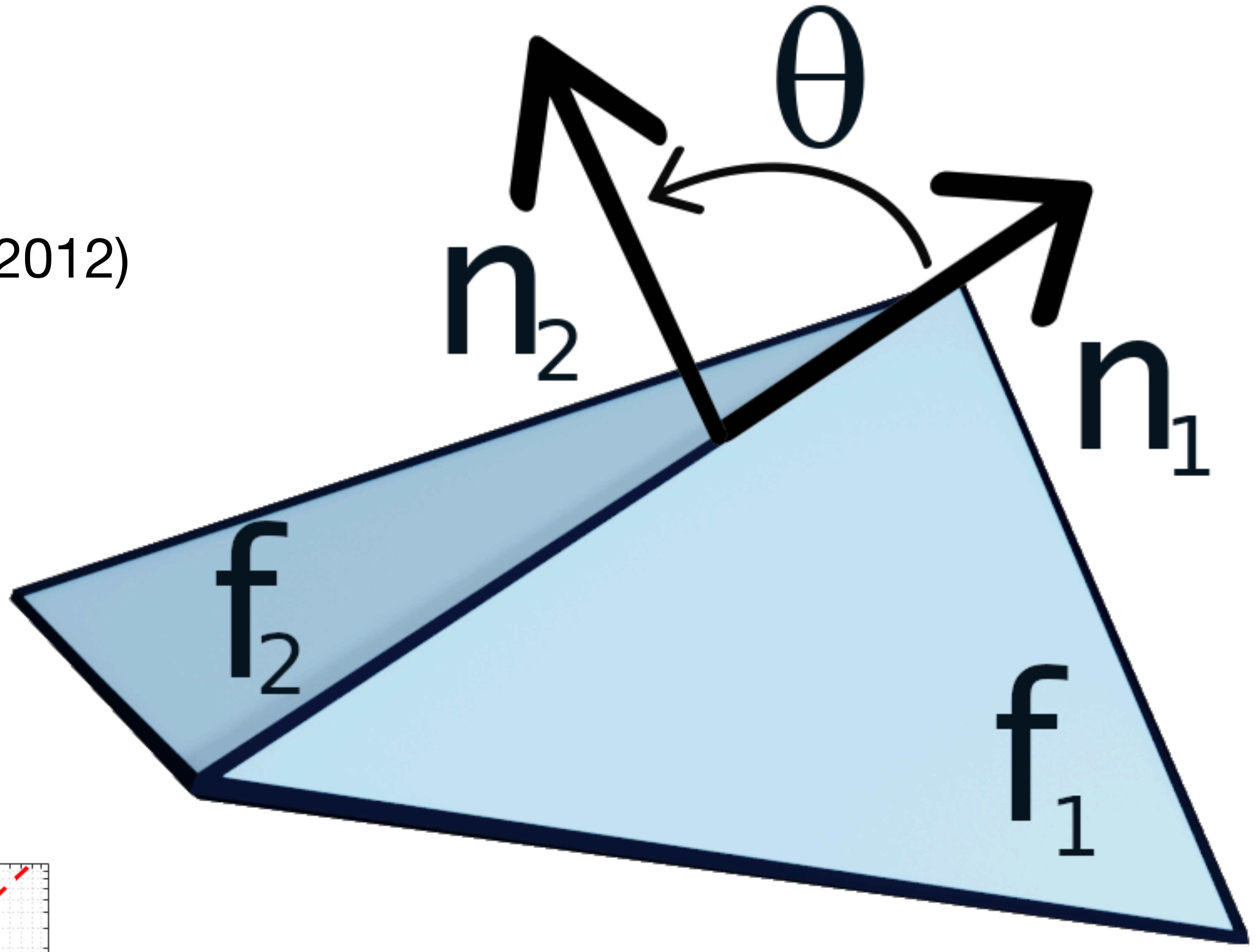


50 elements

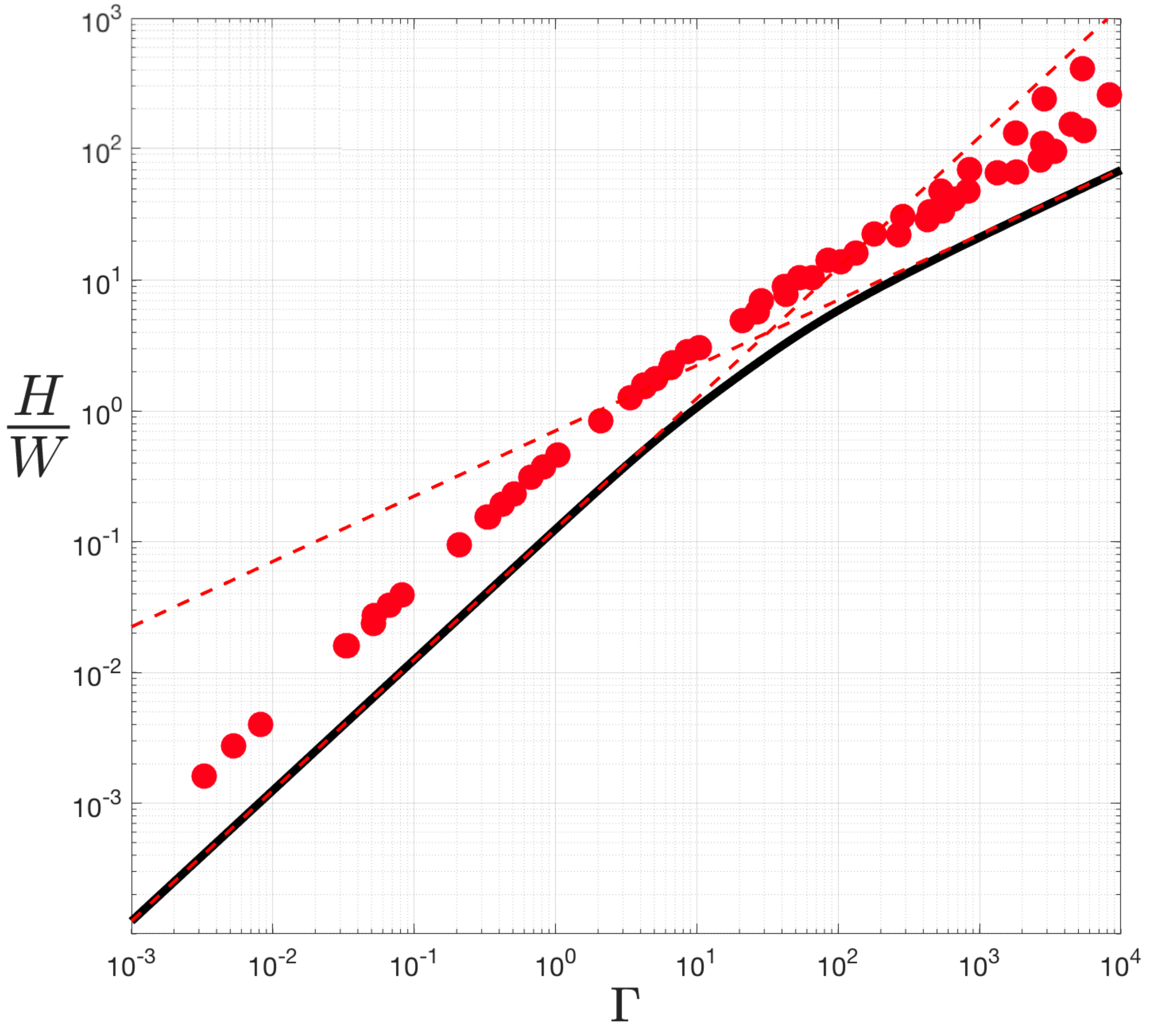
Super-Helix *[F. Bertails et al. ACM Transactions on Graphics 25 #3 (2006) 1180–1187]*

Results

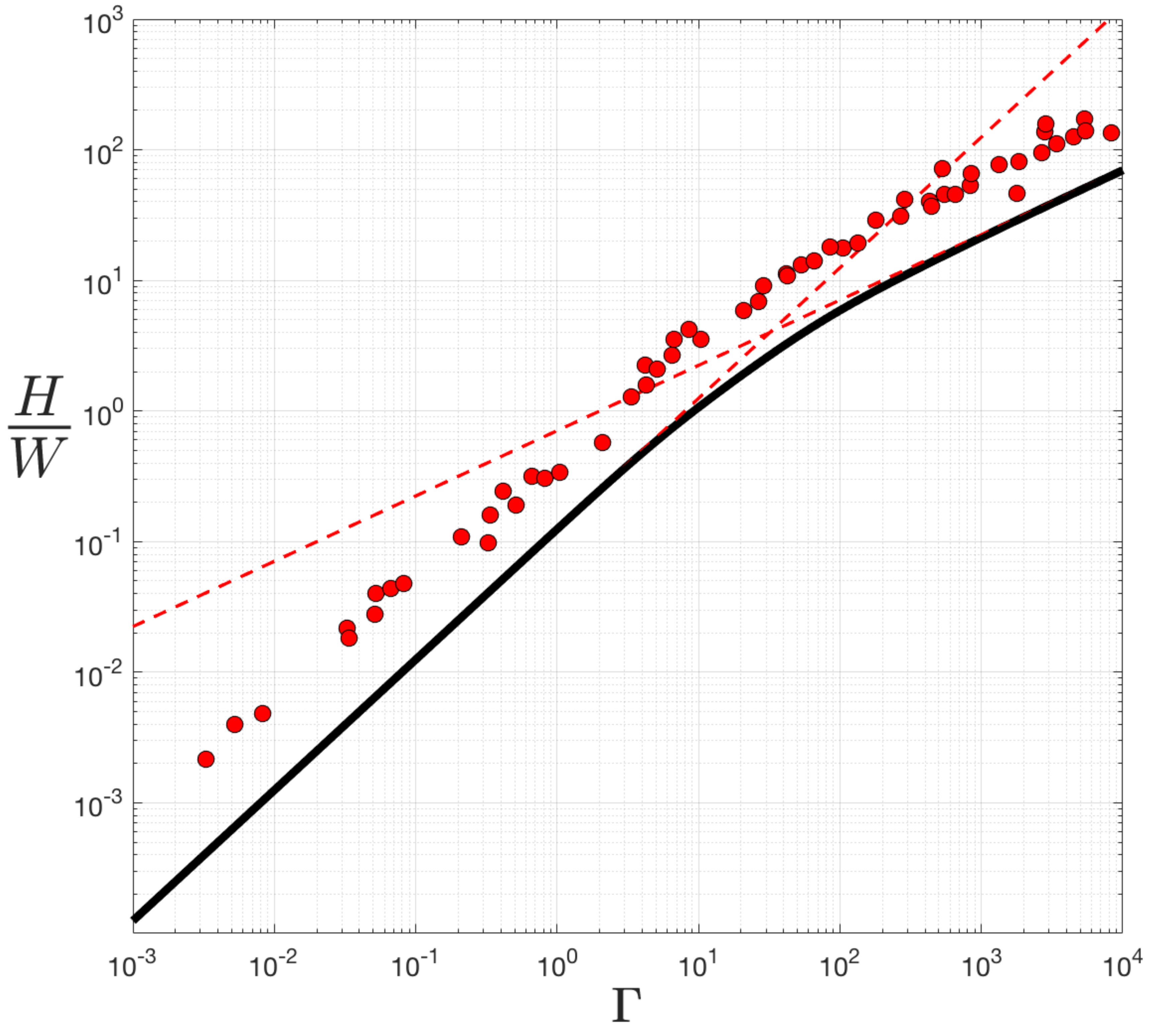
Cantilever test: ArcSim Narain et al. ACM Transactions on Graphics (2012)



based on a discrete bending energy:



No remeshing



Adaptive remeshing

Shifted and scattered

KO

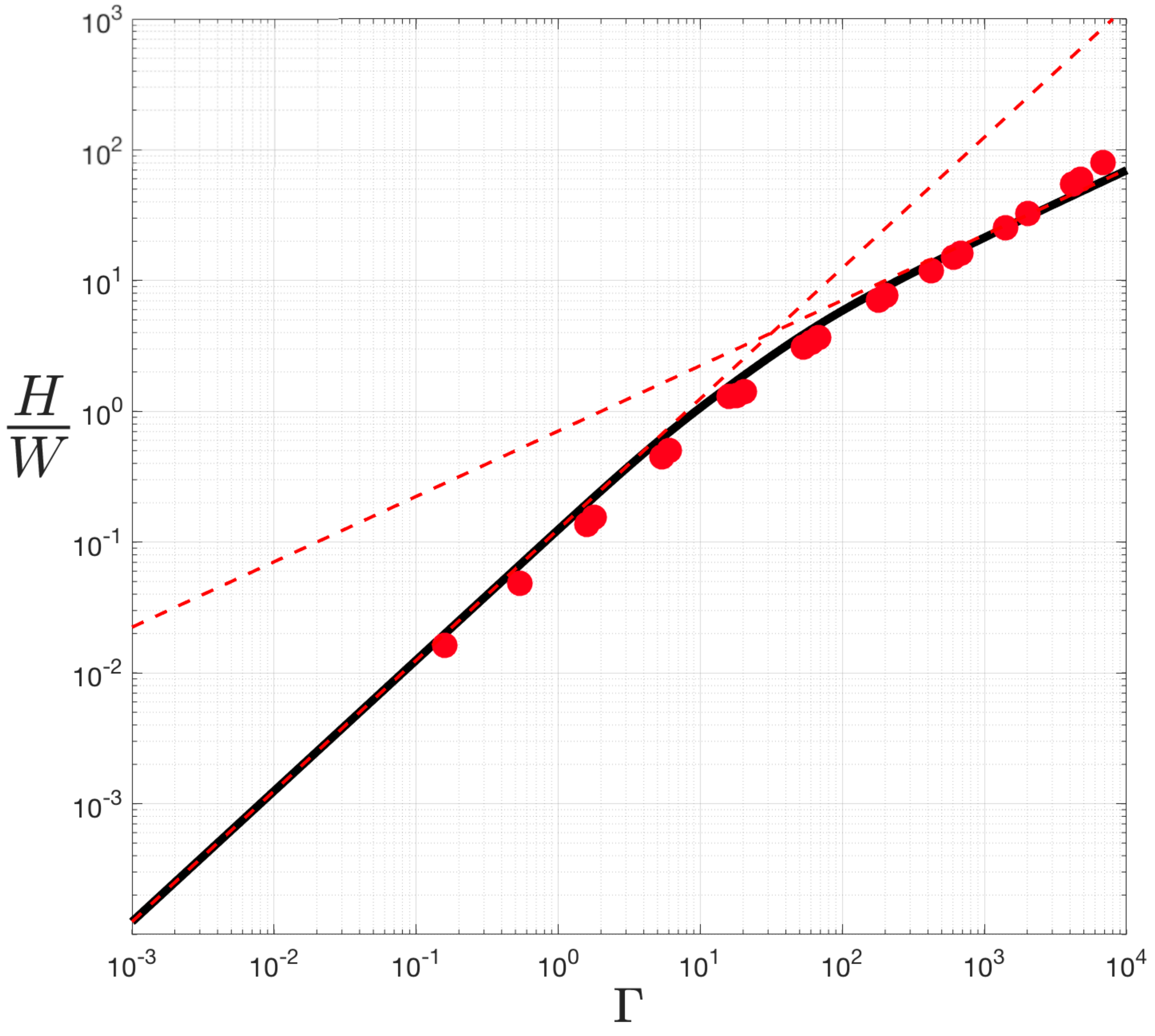
Results

Cantilever test: Projective Dynamics

Bouaziz et al. ACM Transaction on Graphics (2014)

Fitting the bending coefficient

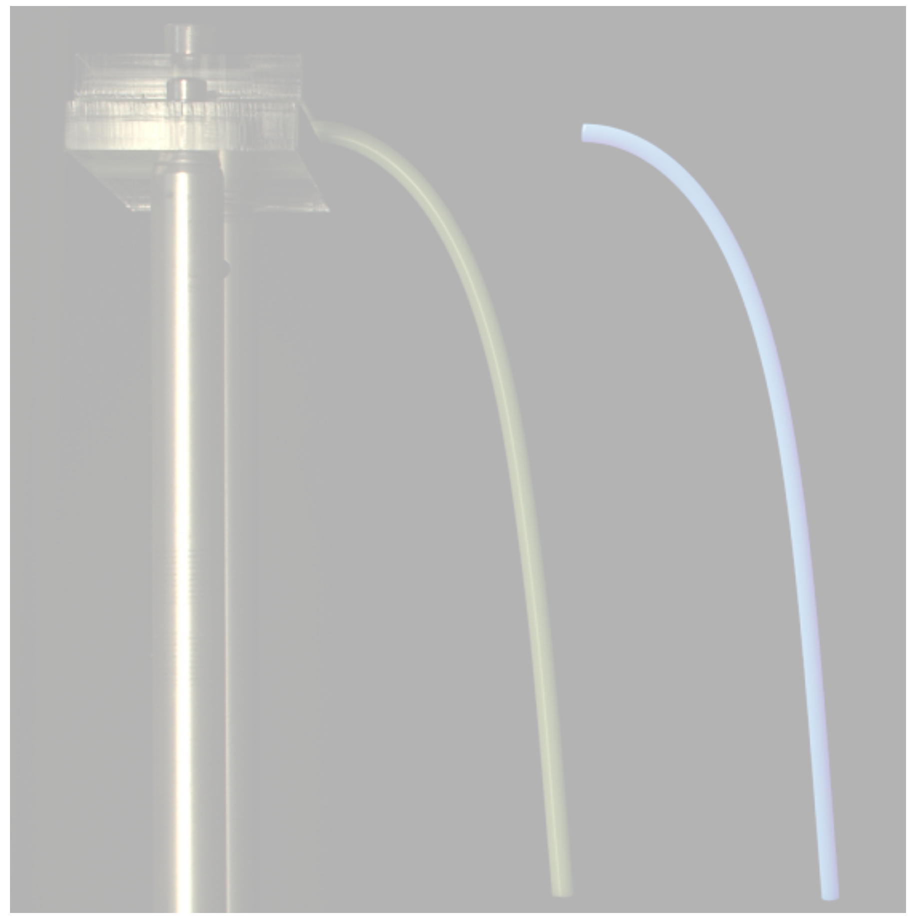
Bending modulus $B = \frac{Eh^3}{12(1-\nu^2)} \longrightarrow$ Fitted coefficient $k_B = 0.25B$



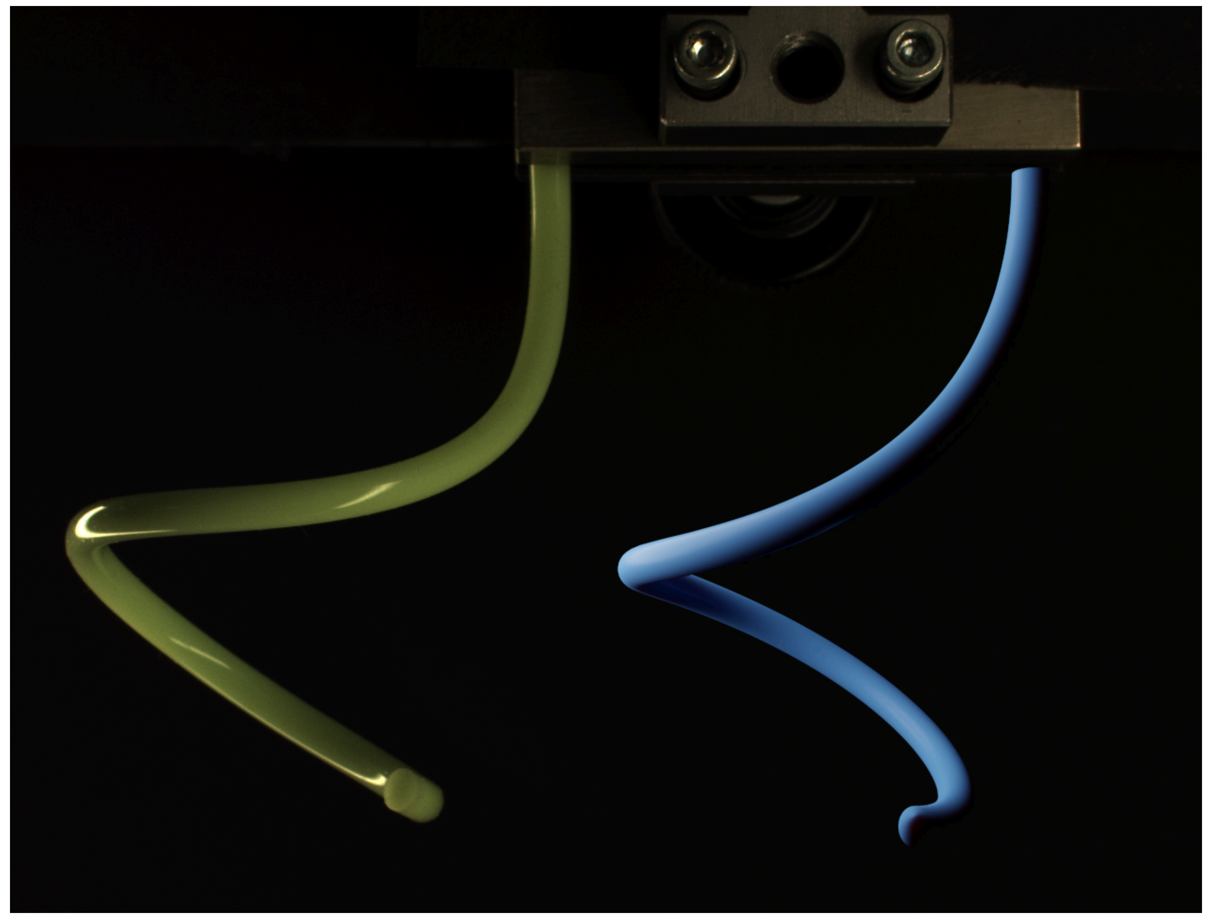
But *sparse results*:
Could not find reasonable solver parameters to converge for *all* the parameter space

KO

The 4 tests

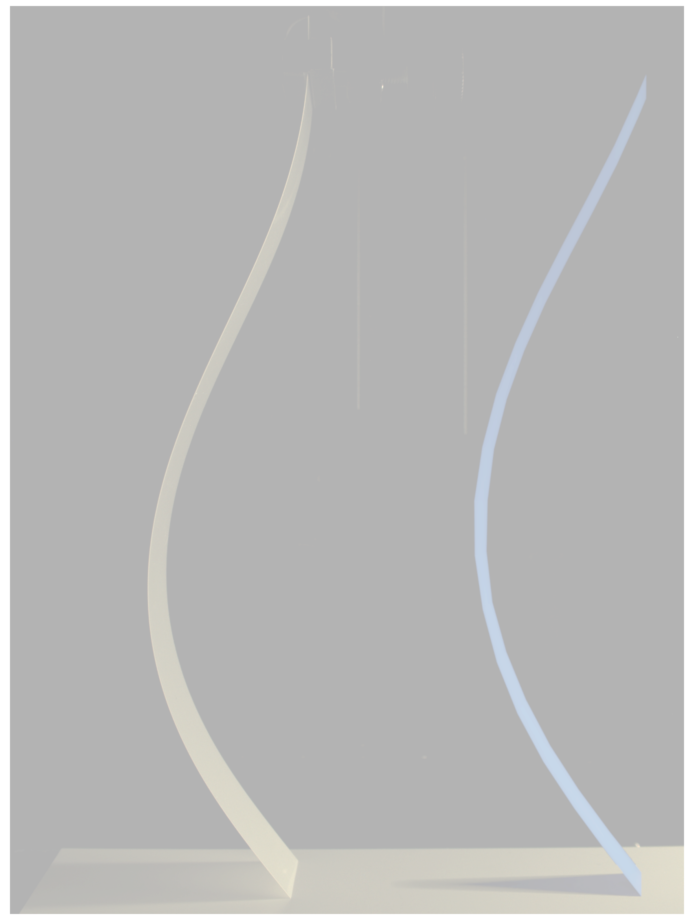


Cantilever



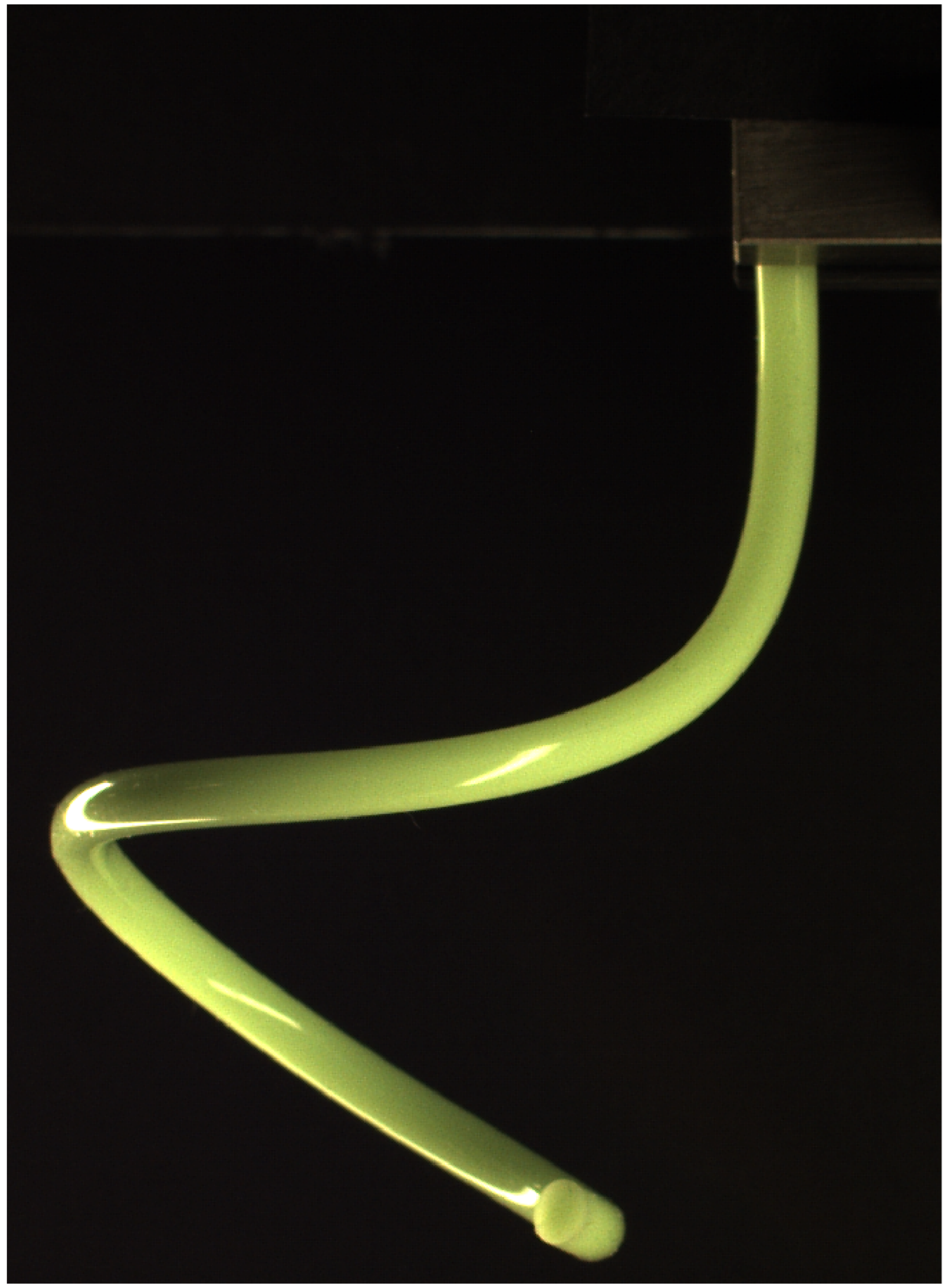
Bend-Twist

Lateral Buckling



Stick-Slip

Bend-Twist test (inspired from Miller et al. Phys Rev Lett. 2014)



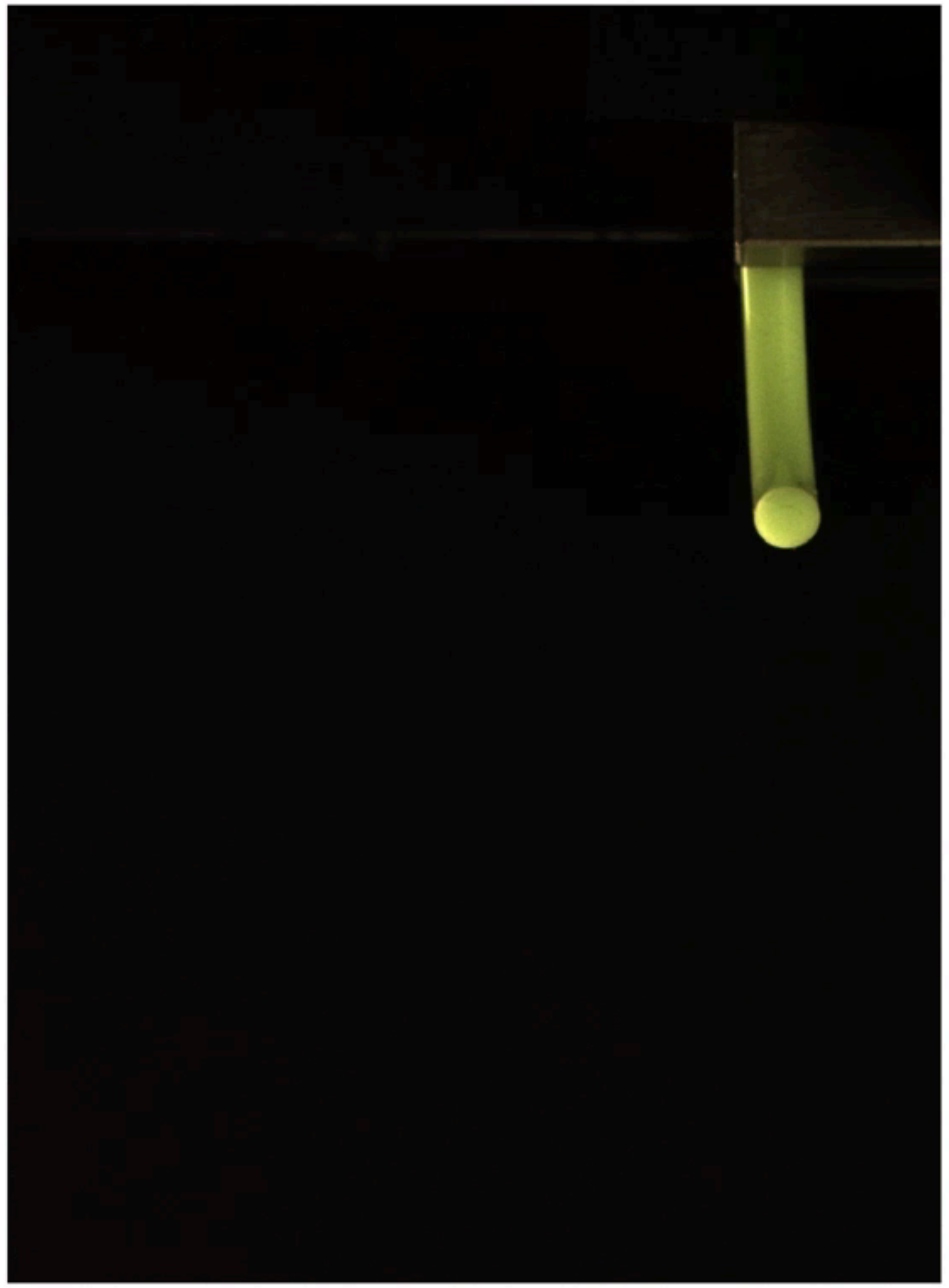
Poisson's ratio	$\nu = 0.5$
Twist rigidity	$GJ = \frac{2}{3}\,EI$

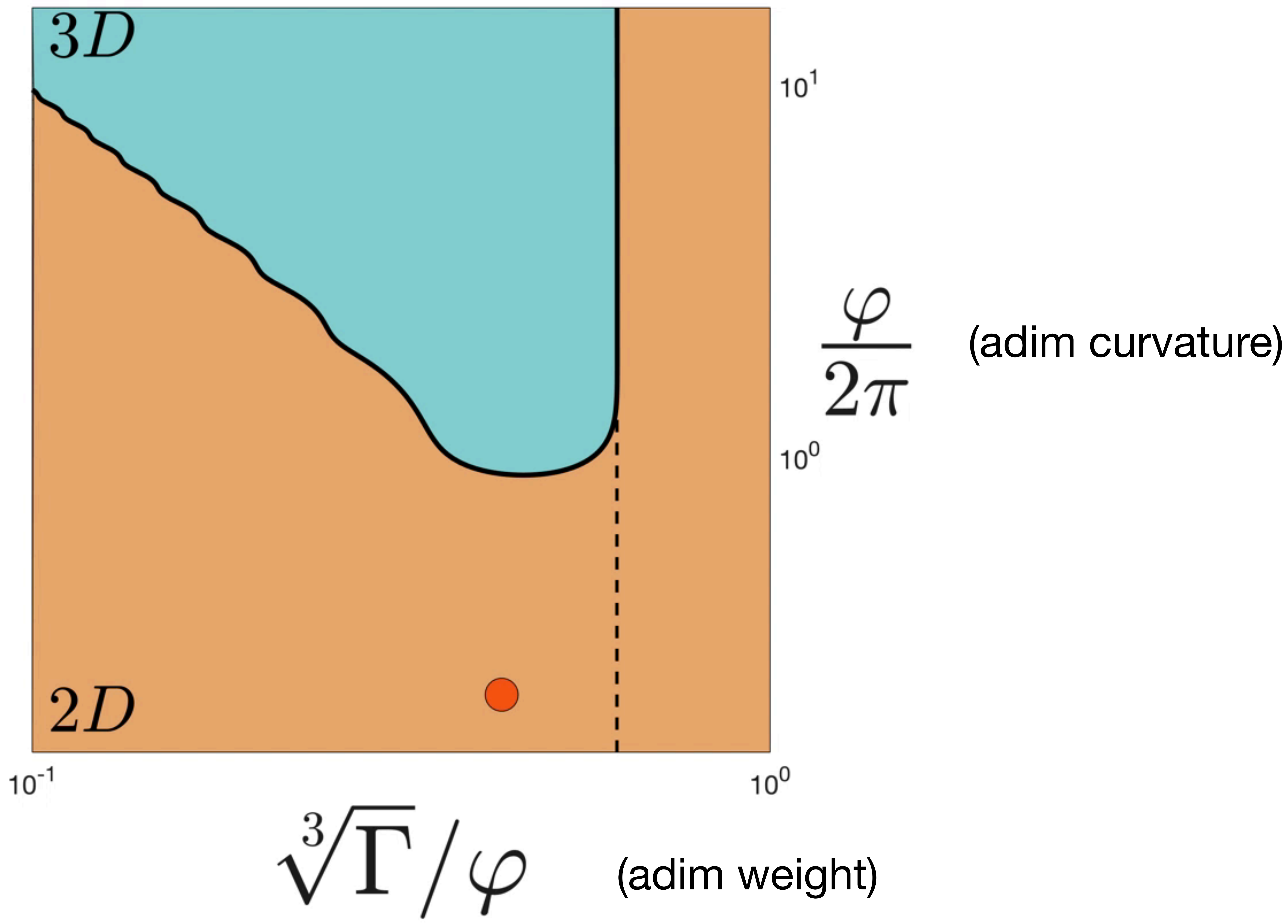
Bending rigidity	EI
Length	L
Weight	Mg
Natural Curvature	κ_0

$$\Gamma = \frac{MgL^2}{EI} \text{ and } \varphi = L \times \kappa_0$$

Bend-Twist test (inspired from Miller et al. Phys Rev Lett. 2014)

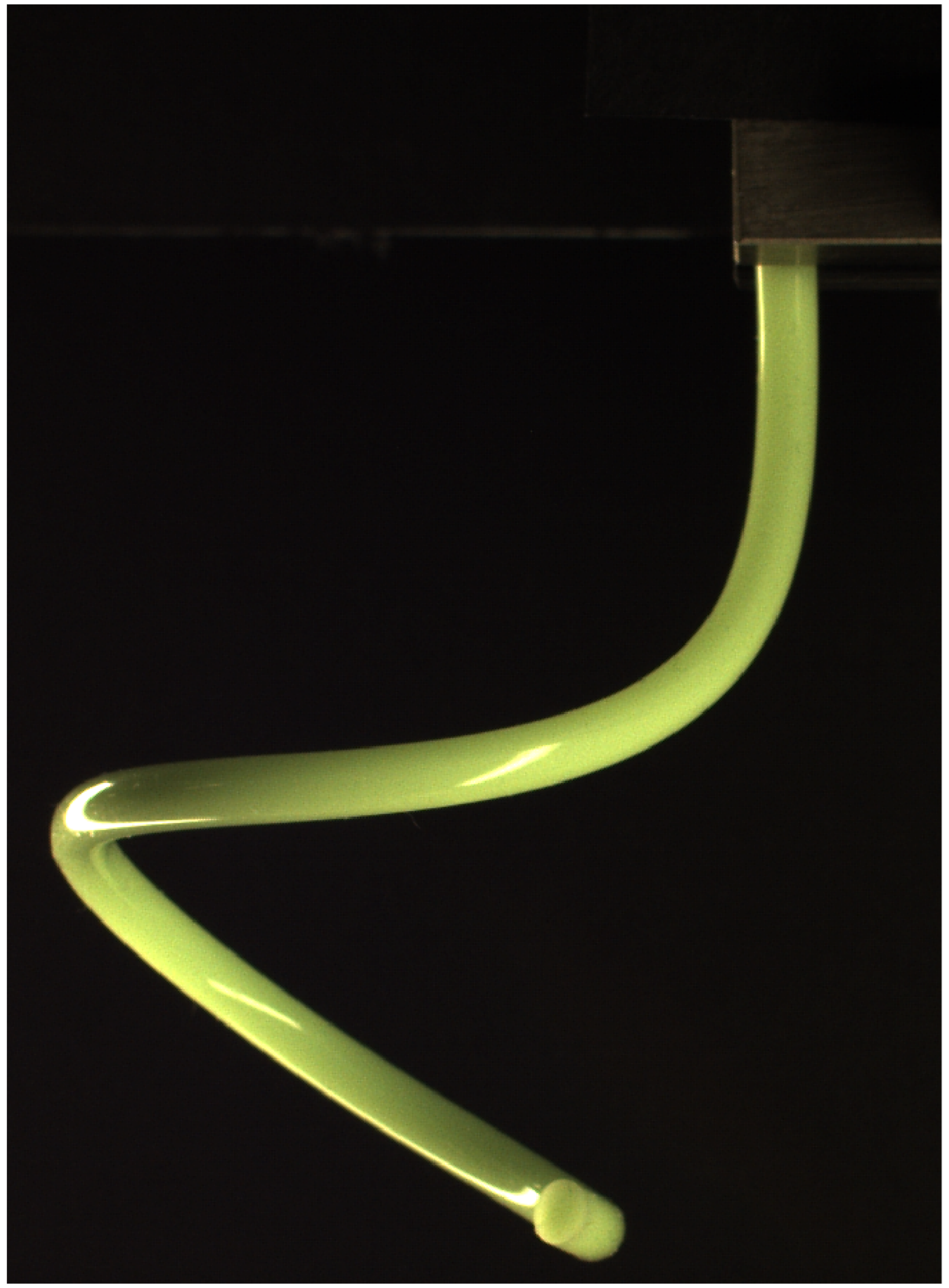
Experimental validation

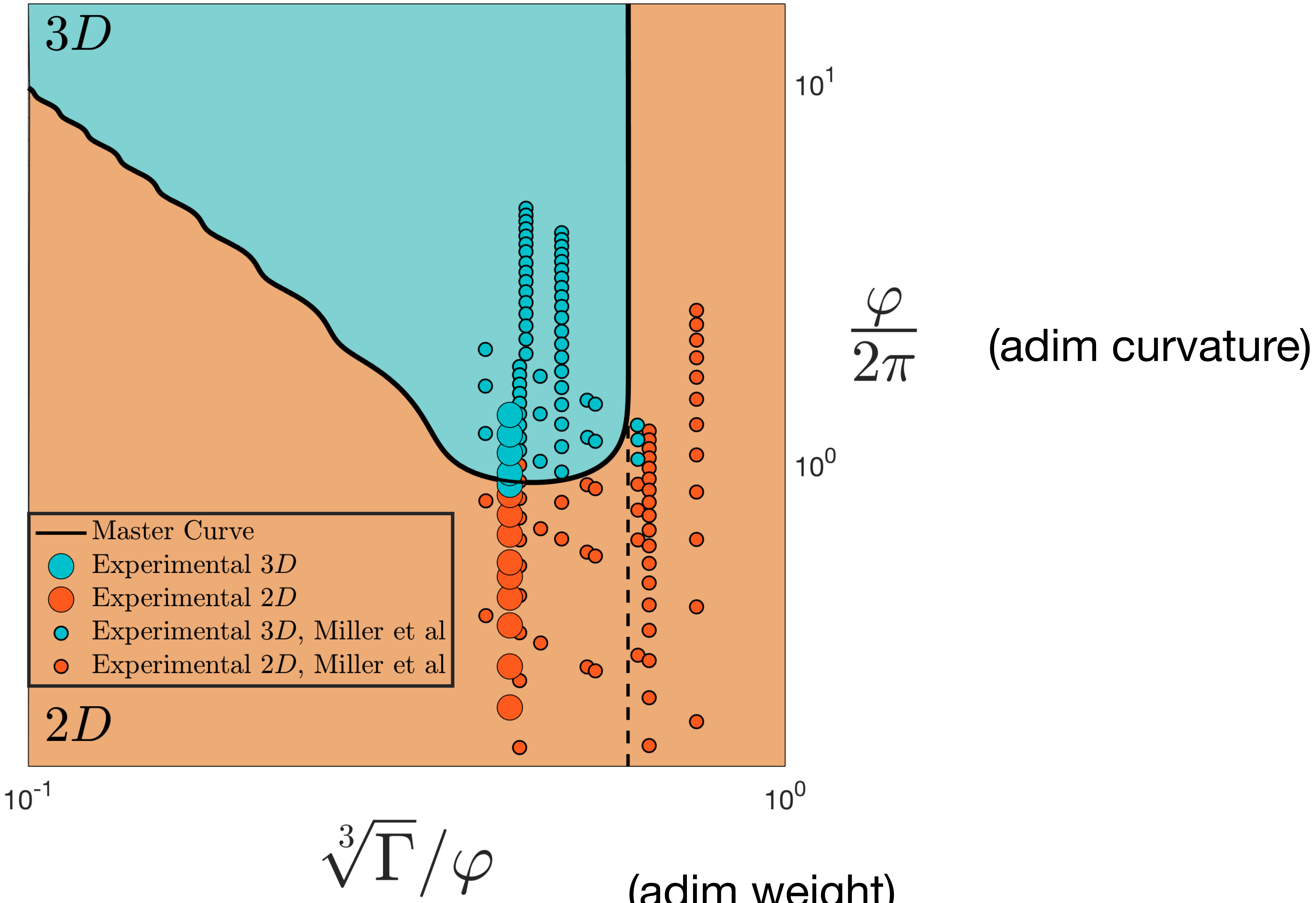




Bend-Twist test (inspired from Miller et al. Phys Rev Lett. 2014)

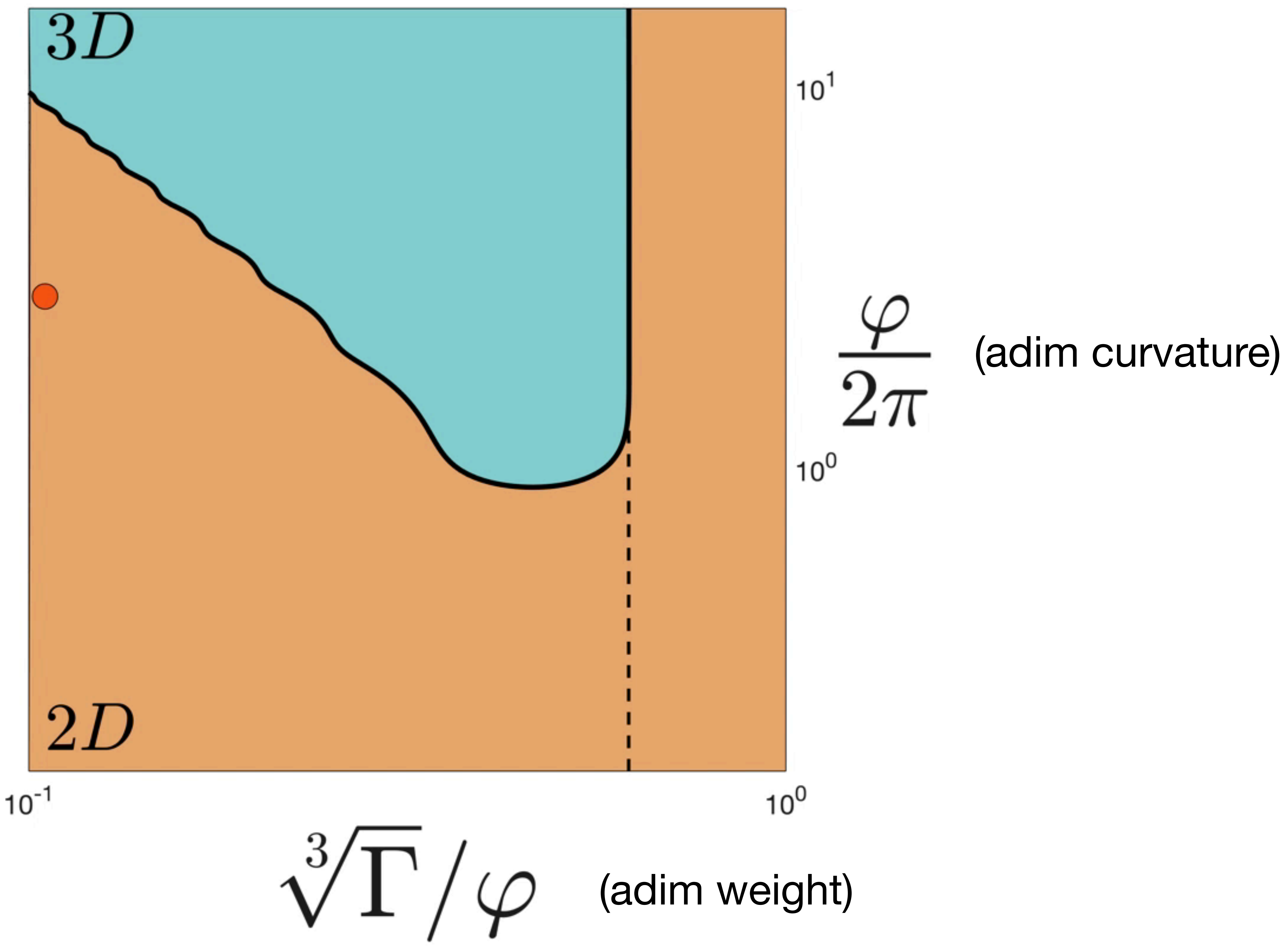
Experimental validation



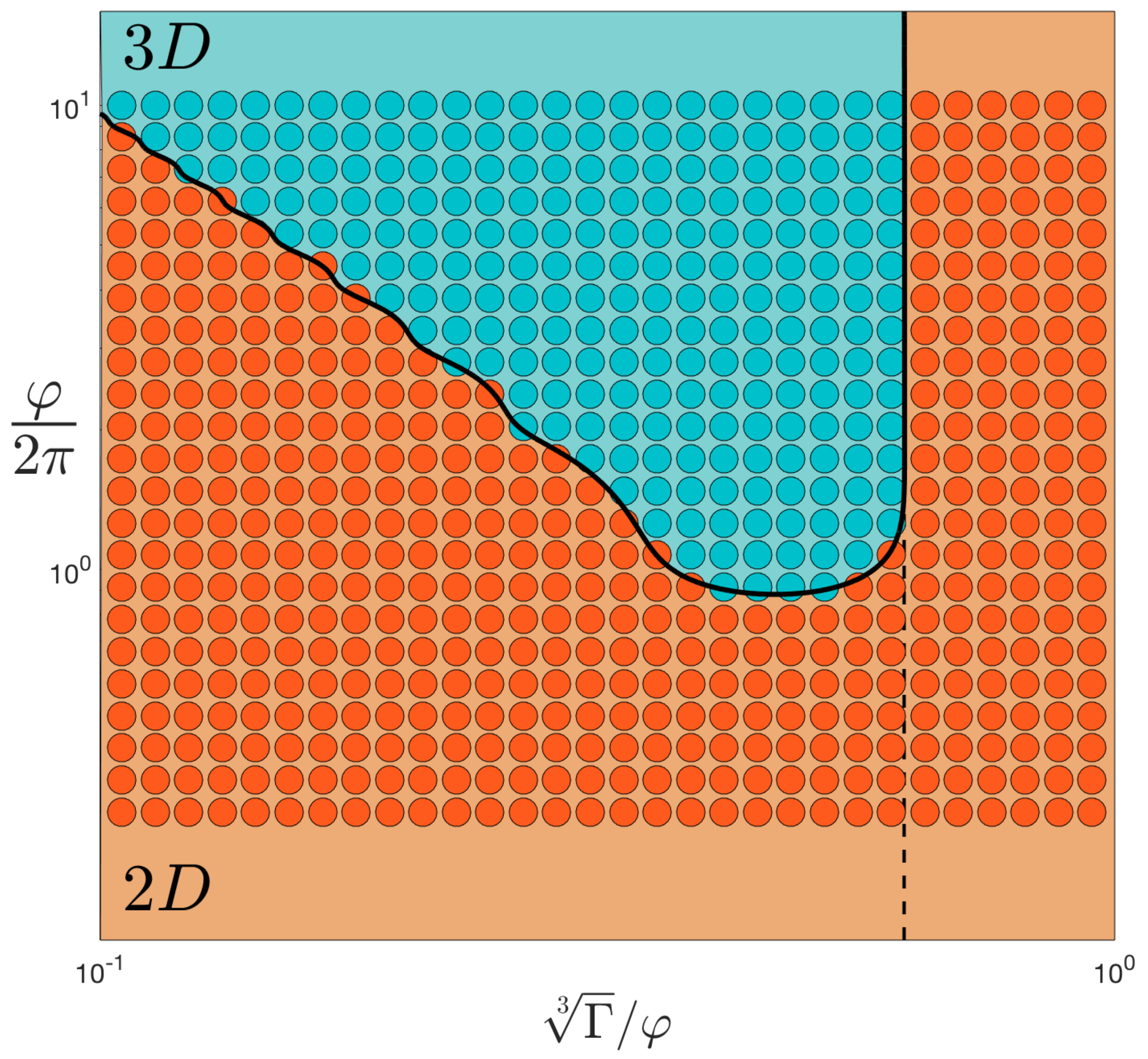


Bend-twist test: numerical results

equilibrium
computed
with increasing
Γ values



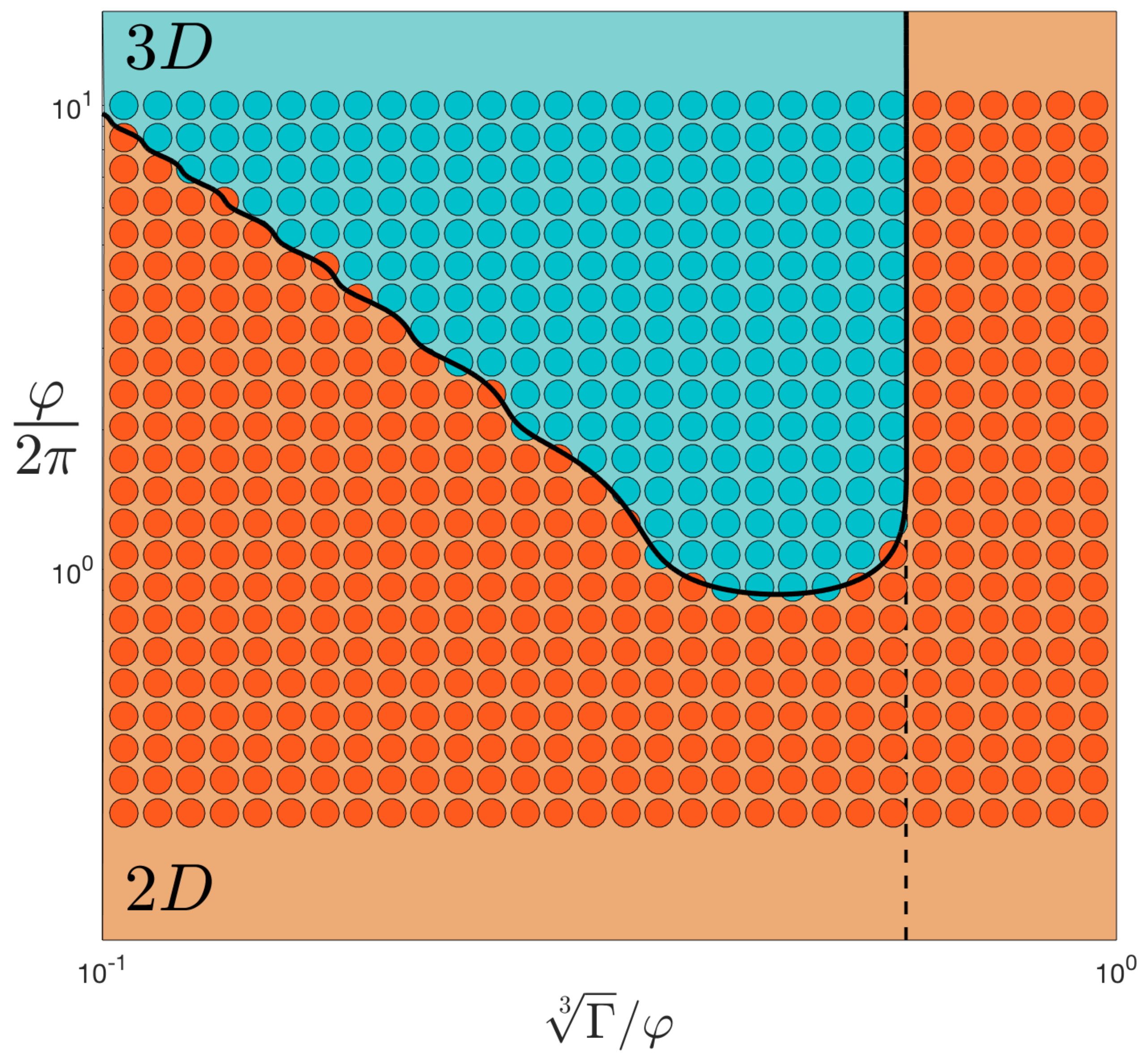
Bend-twist test: numerical results



Discrete Elastic Rods

OK

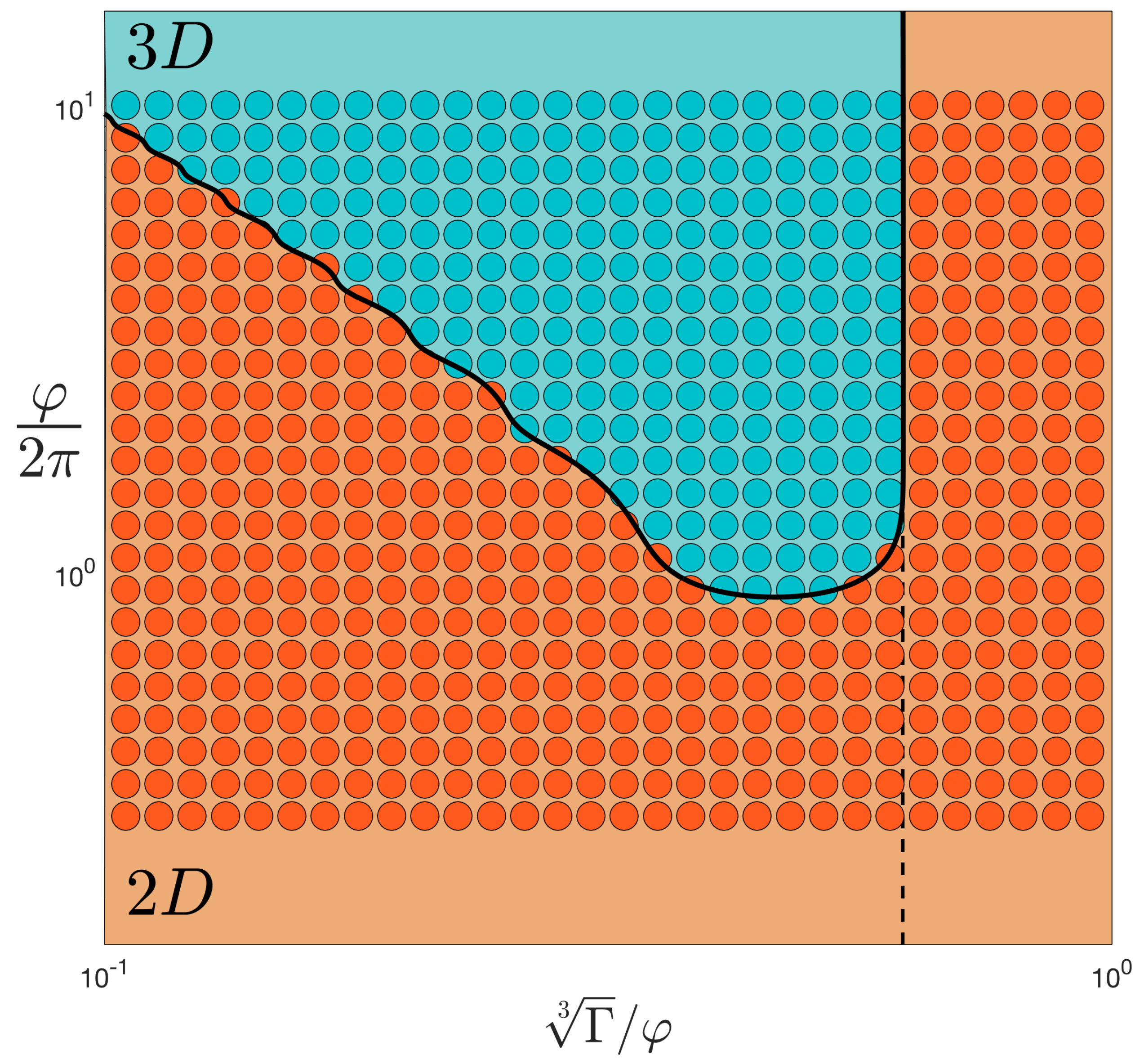
Bergou et al.
ACM Transactions on Graphics
27 #3 (2008)



Super-Helix

OK

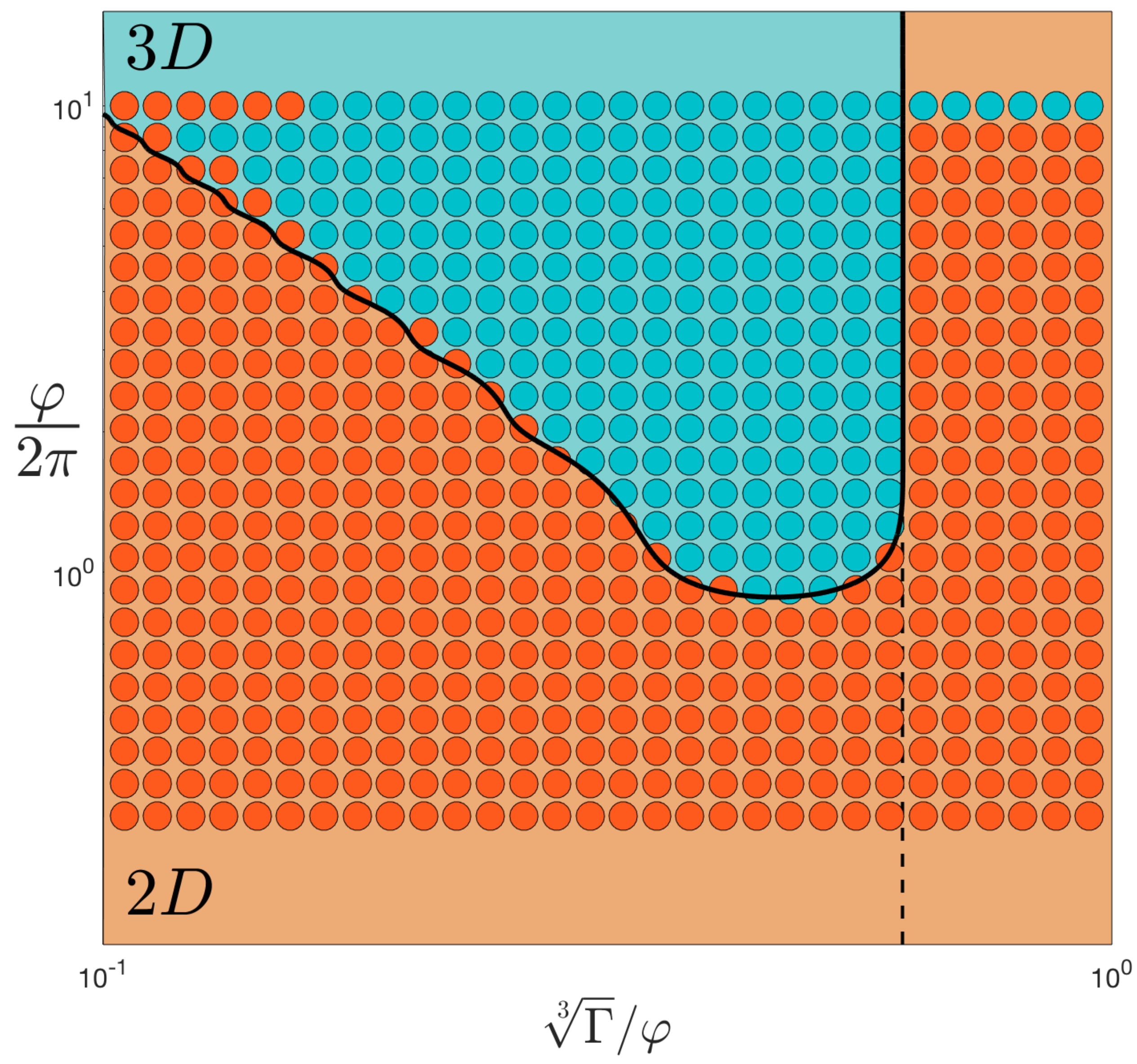
Bertails et al.
ACM Transactions on Graphics
25 #3 (2006)



Super-Clothoid

OK

Casati et al.
ACM Transactions on Graphics
32 #4 (2013)



©Abaqus

KO

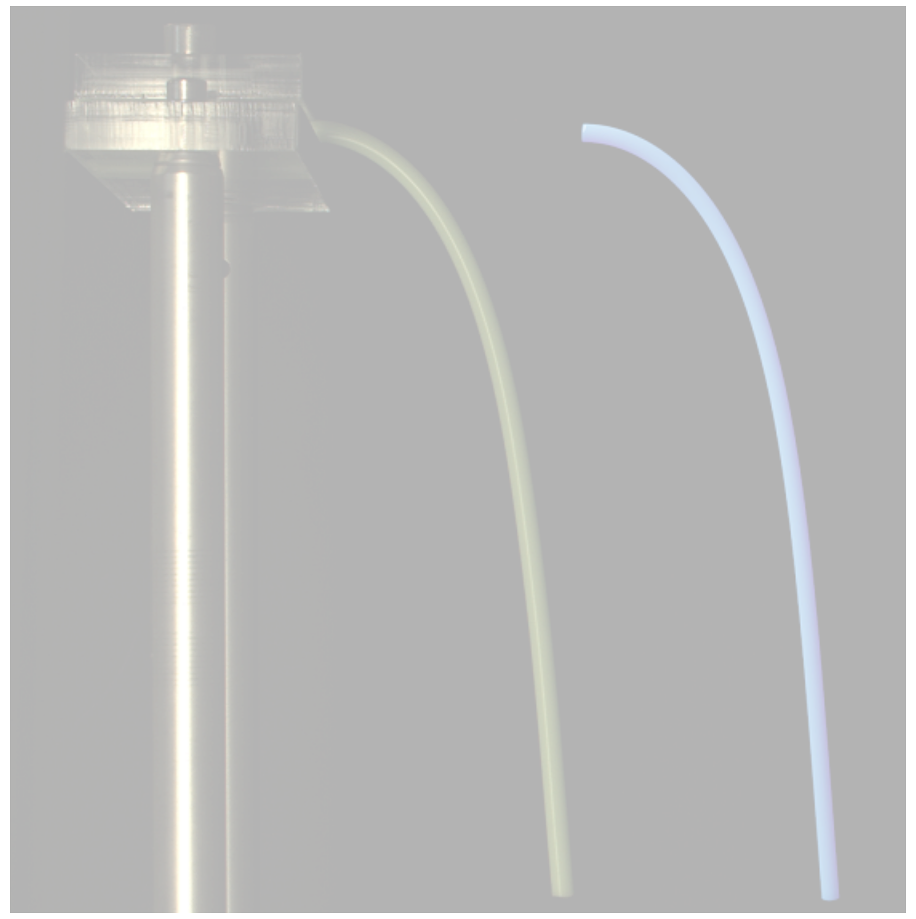
785 elements
Stabilized Newton routine
Warm-start from previous

Bend-twist test: mesh-size dependence

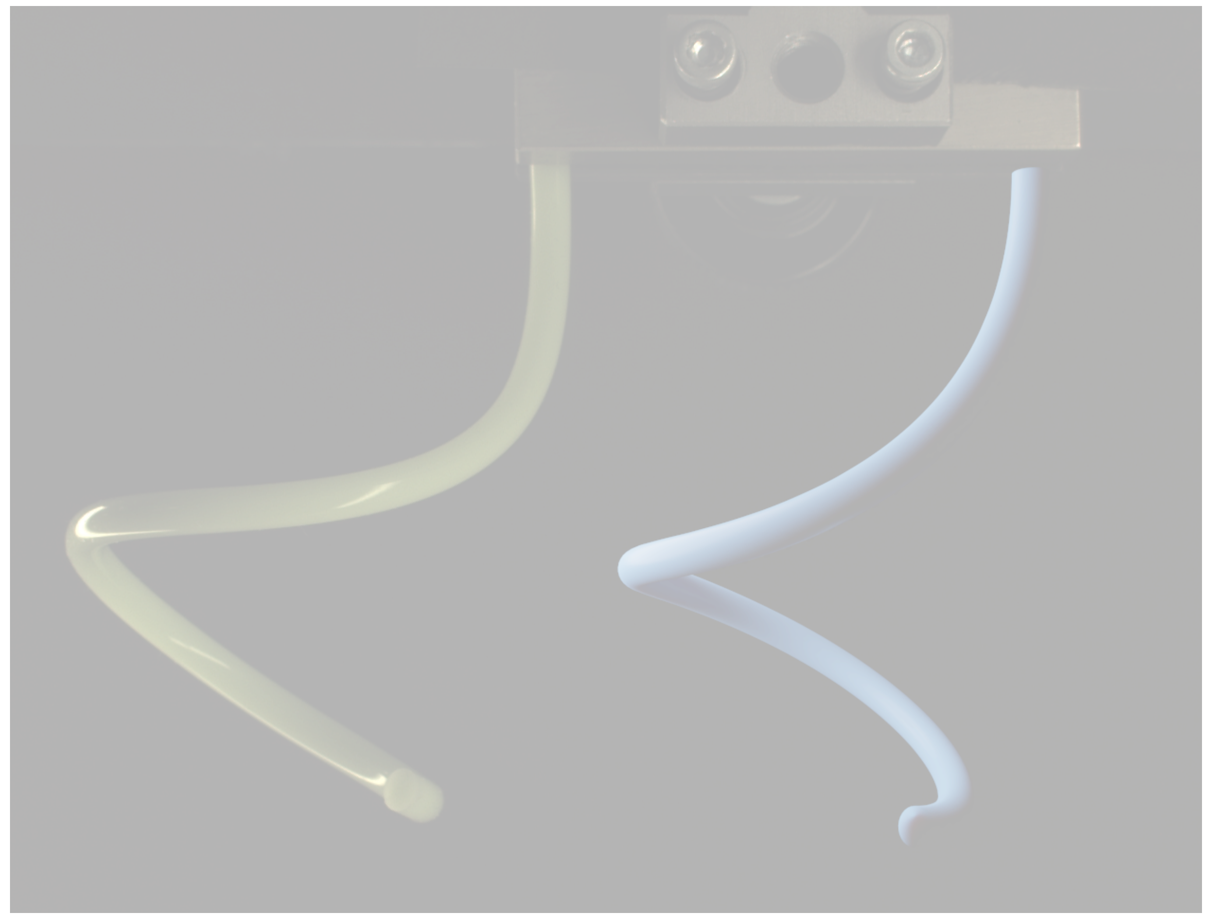
Results
Bend-twist test

Tested Code	Cantilever	Bend-Twist	Lateral Buckling	Stick-Slip
Rod				
Discrete Elastic Rod [Bergou et al. 2010]	OK (300 elts)	OK (200 elts)	–	–
Super-Helix [Bertails et al. 2006]	OK (50 elts)	OK (30 elts)	–	–
Super-Clothoid [Casati and Bertails-Descoubes 2013]	OK (20 elts)	OK (25 elts)	–	–
Ribbon				
Super-Ribbon [Charrondière et al. 2020]	OK (20 elts)	–	KO	–
Plate				
LibShell [Chen et al. 2018]	OK (Res 0)	–	OK (Res +)	–
Discrete Shell (+ LibShell) [Grinspun et al. 2003]	OK (Res +)	–	KO	–
ArcSim [Narain et al. 2012]	KO	–	×	–
Discrete Shell + ArcSim (**tentative fix** of ArcSim)	KO	–	×	–
Projective Dynamics [Bouaziz et al. 2014] (**fit**)	KO	–	×	–
Contact & friction				
Viscous Friction (+ Super-Helix 2D)	–	–		
So-Bogus [Daviet et al. 2011] (+ Super-Helix 2D)	–	–		
Argus (≈ ArcSim + So-Bogus) [Li et al. 2018]	–	–		
Argus Non Adaptive (**fix** of Argus)	–	–		
Bridson-Harmon [Bridson et al. 2002; Harmon et al. 2008] (+ ArcSim)	–	–		
Projective Friction [Ly et al. 2020]	–	–		
Reference codes in Mechanical Engineering				
FenicsShell [Hale et al. 2018]	OK (Res 0, $P_{2,3}$ elts)	–	OK (Res 0, $P_{2,3}$ elts)	–
©Abaqus	OK (200 P_2 elts)	KO	KO	OK (dt=9 µs)

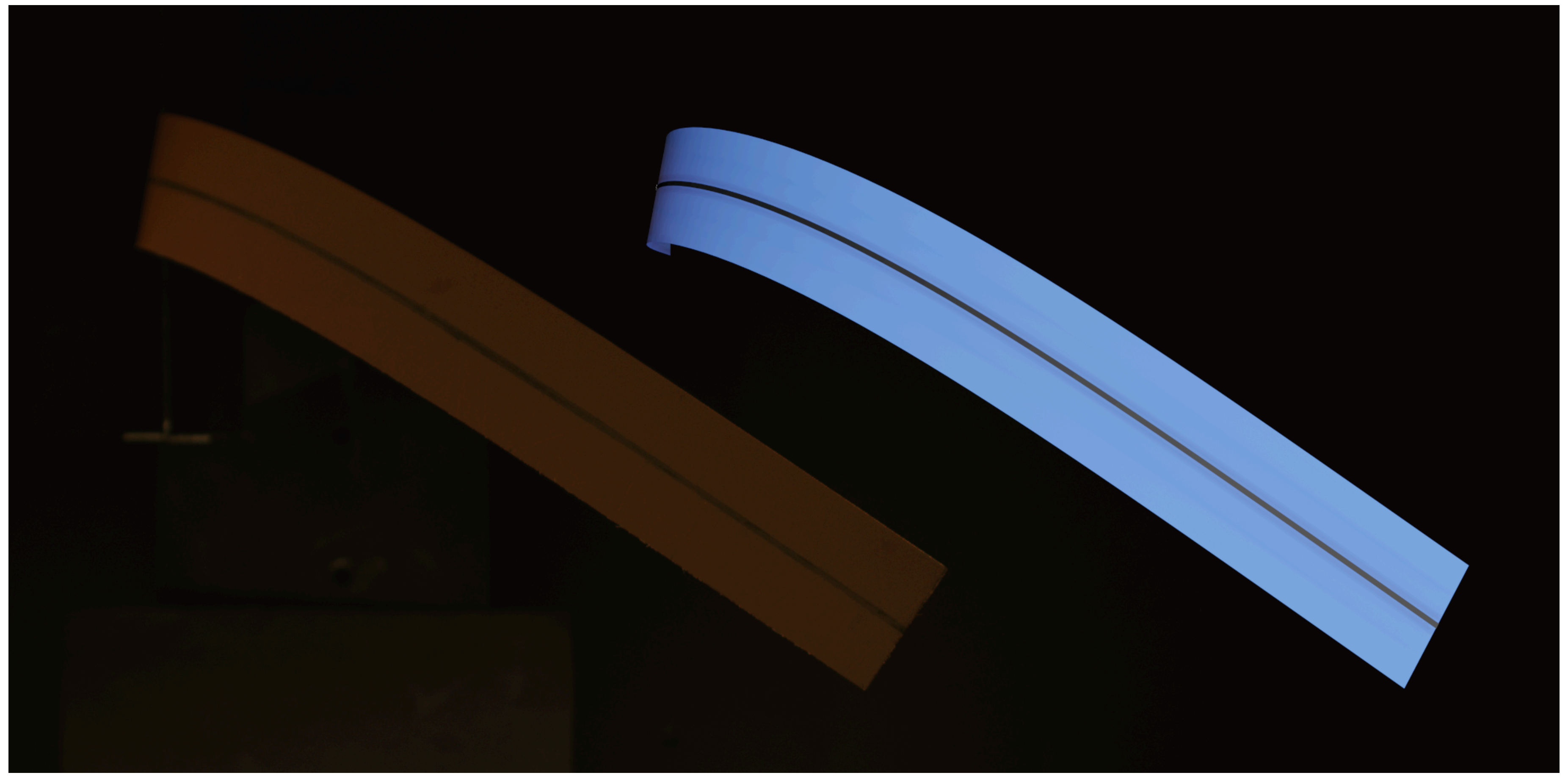
The 4 tests



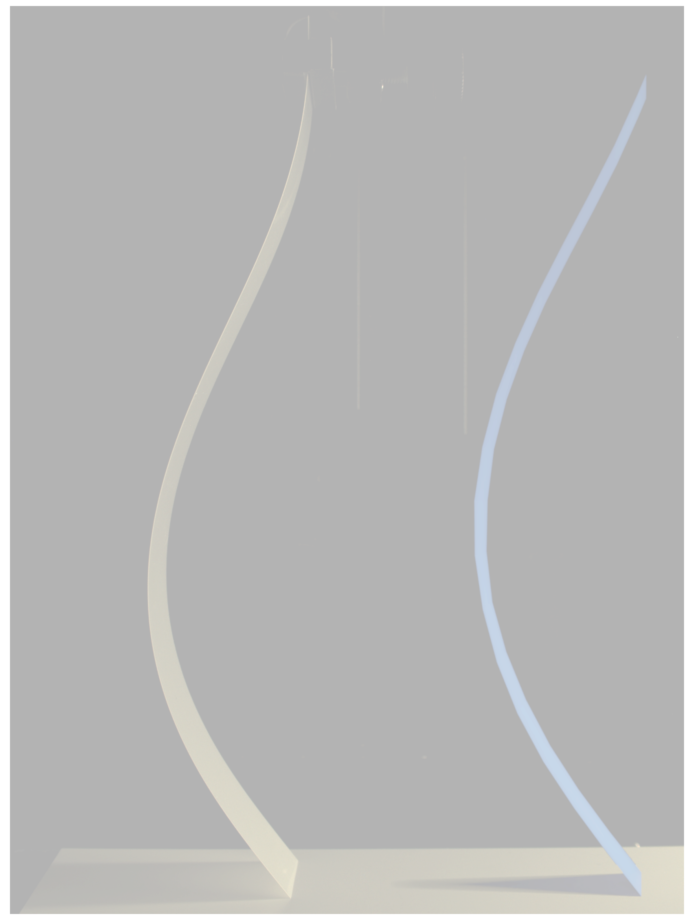
Cantilever



Bend-Twist

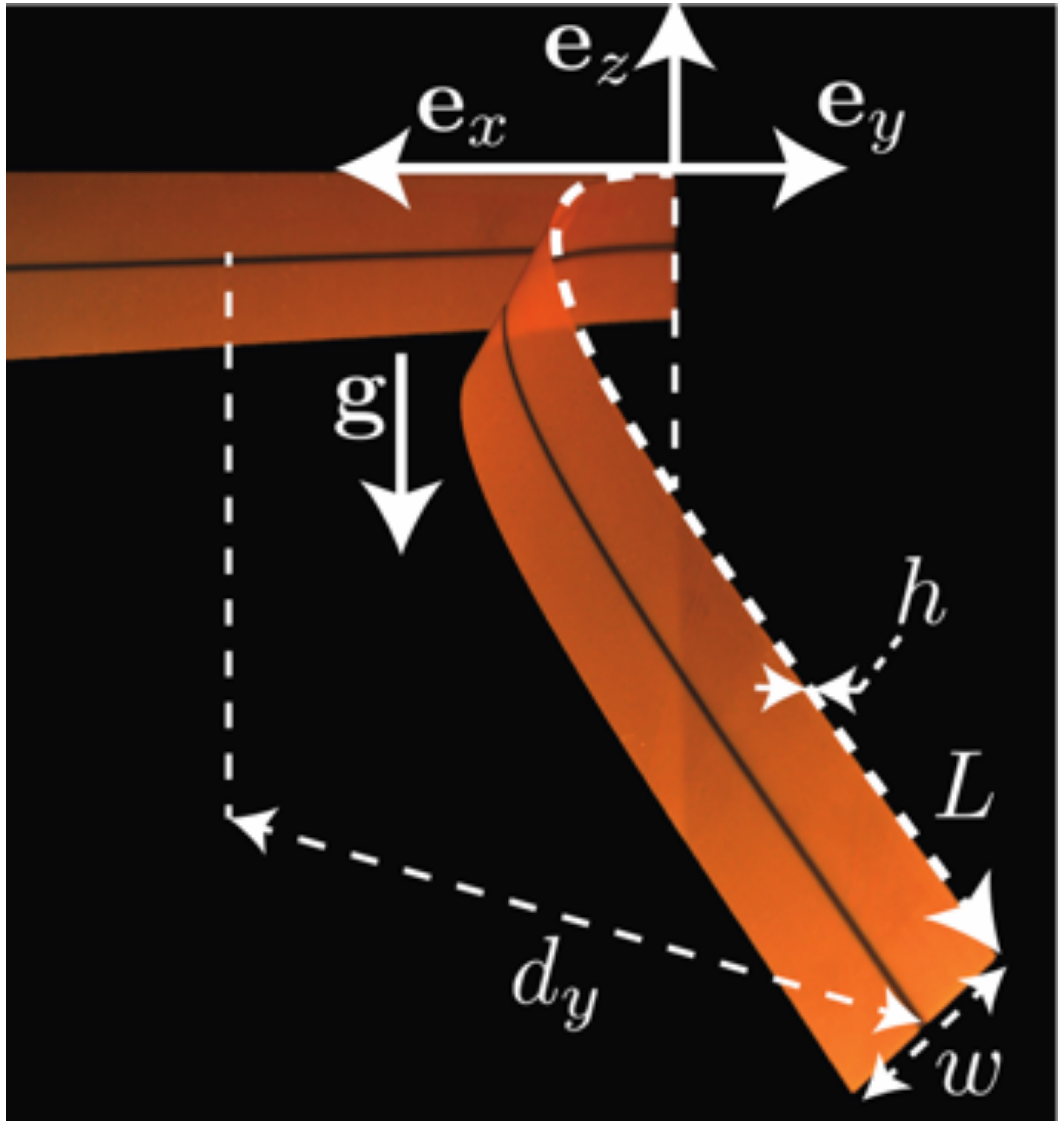


Lateral Buckling



Stick-Slip

Lateral-Buckling test



Stretching rigidity $A = E\,h = 12\,(1-\nu^2)\,D\,h^2$
Poisson's ratio $\nu = 0.35$

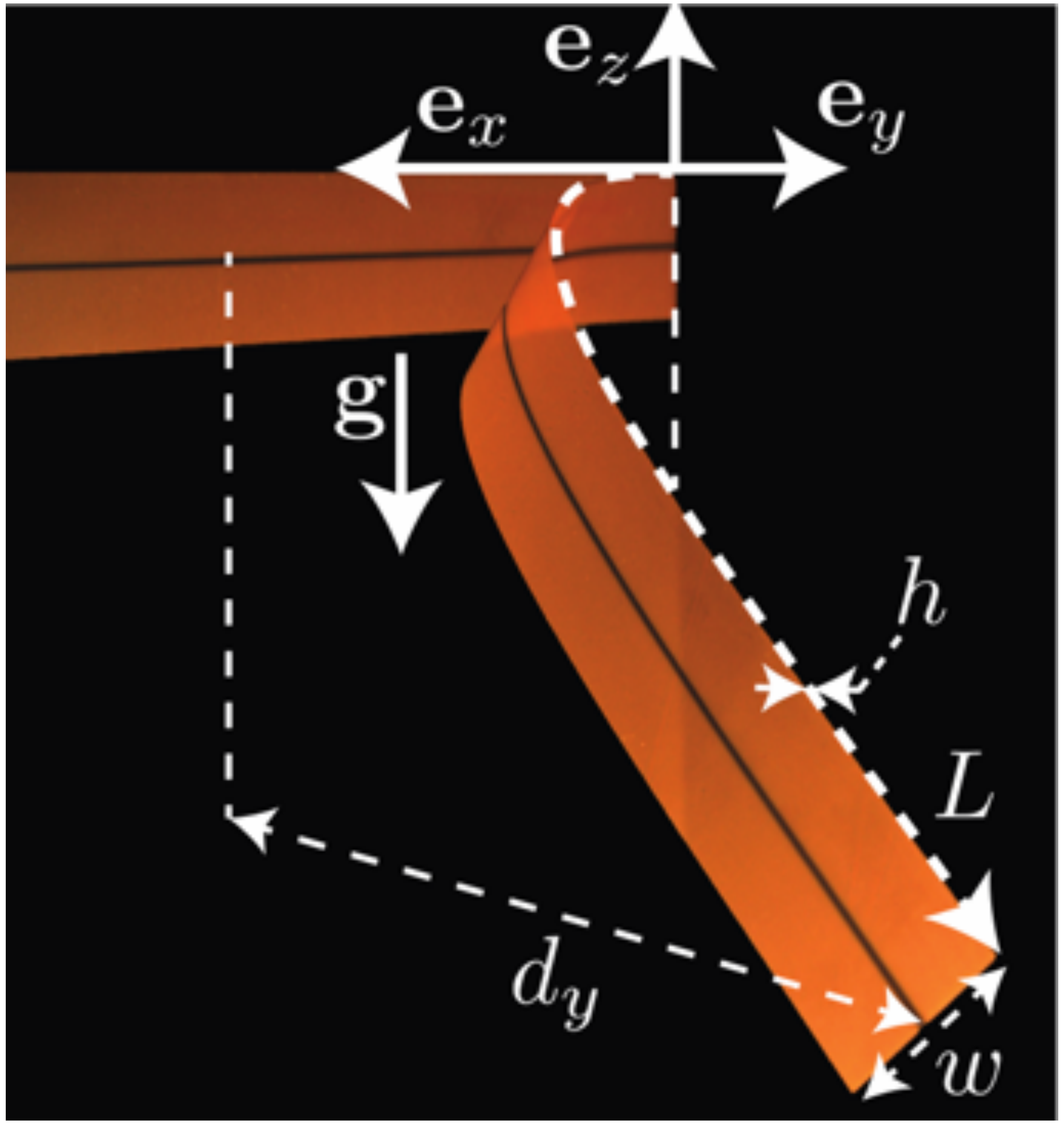
Classical failure mode in Civil Engineering

Bending rigidity	D
Length	L
Weight	$M g$
Thickness	h
Width	w

$$\downarrow$$

$$\Gamma^* = \frac{M g}{D w / L^2} \text{ and } \frac{w}{L} \text{ and } \frac{h}{L}$$

Lateral-Buckling test



Stretching rigidity $A = E\,h = 12\,(1-\nu^2)\,D\,h^2$
Poisson's ratio $\nu = 0.35$

Classical failure mode in Civil Engineering

Bending rigidity	D
Length	L
Weight	Mg
Thickness	h
Width	w

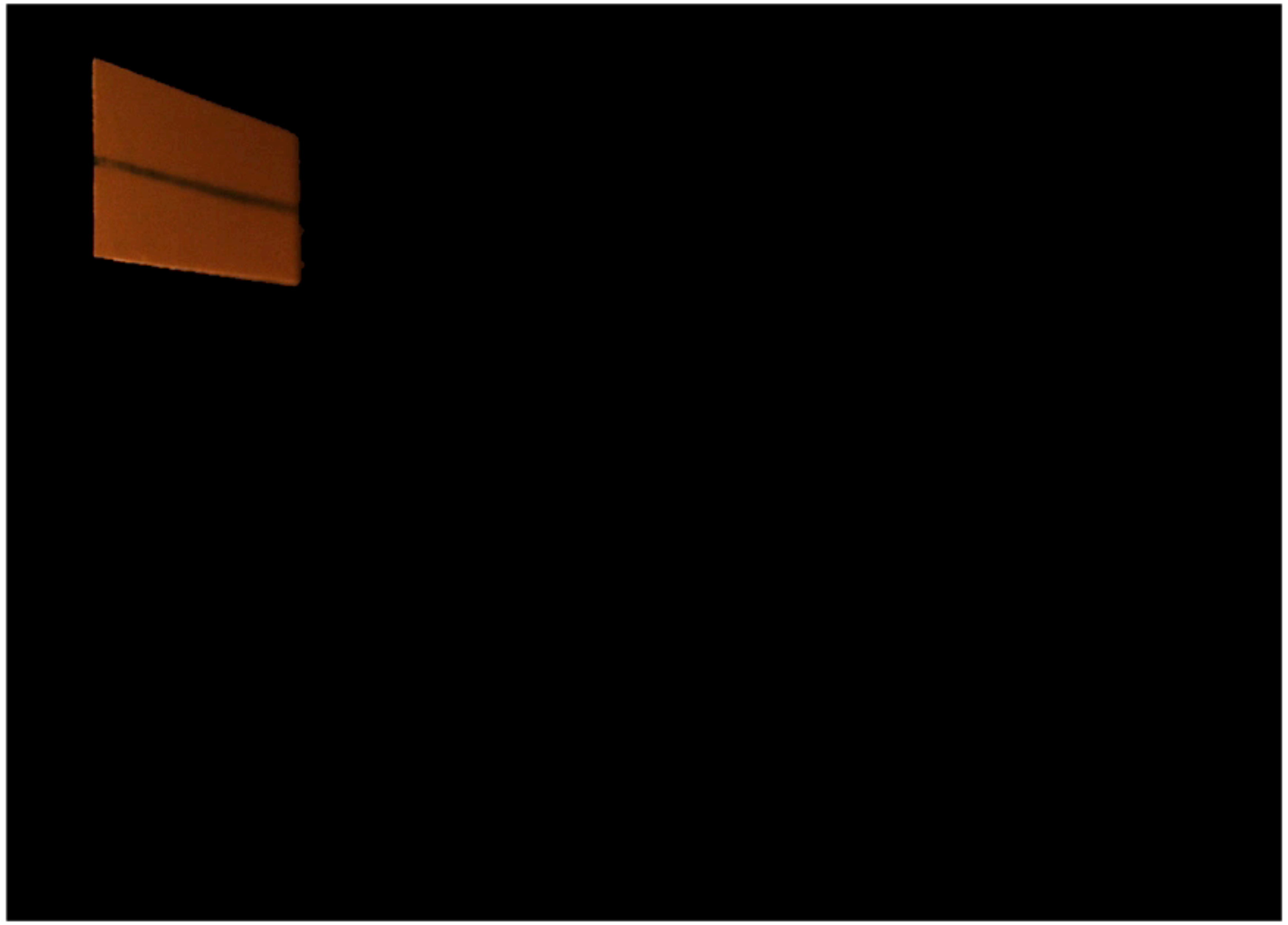
$\downarrow$

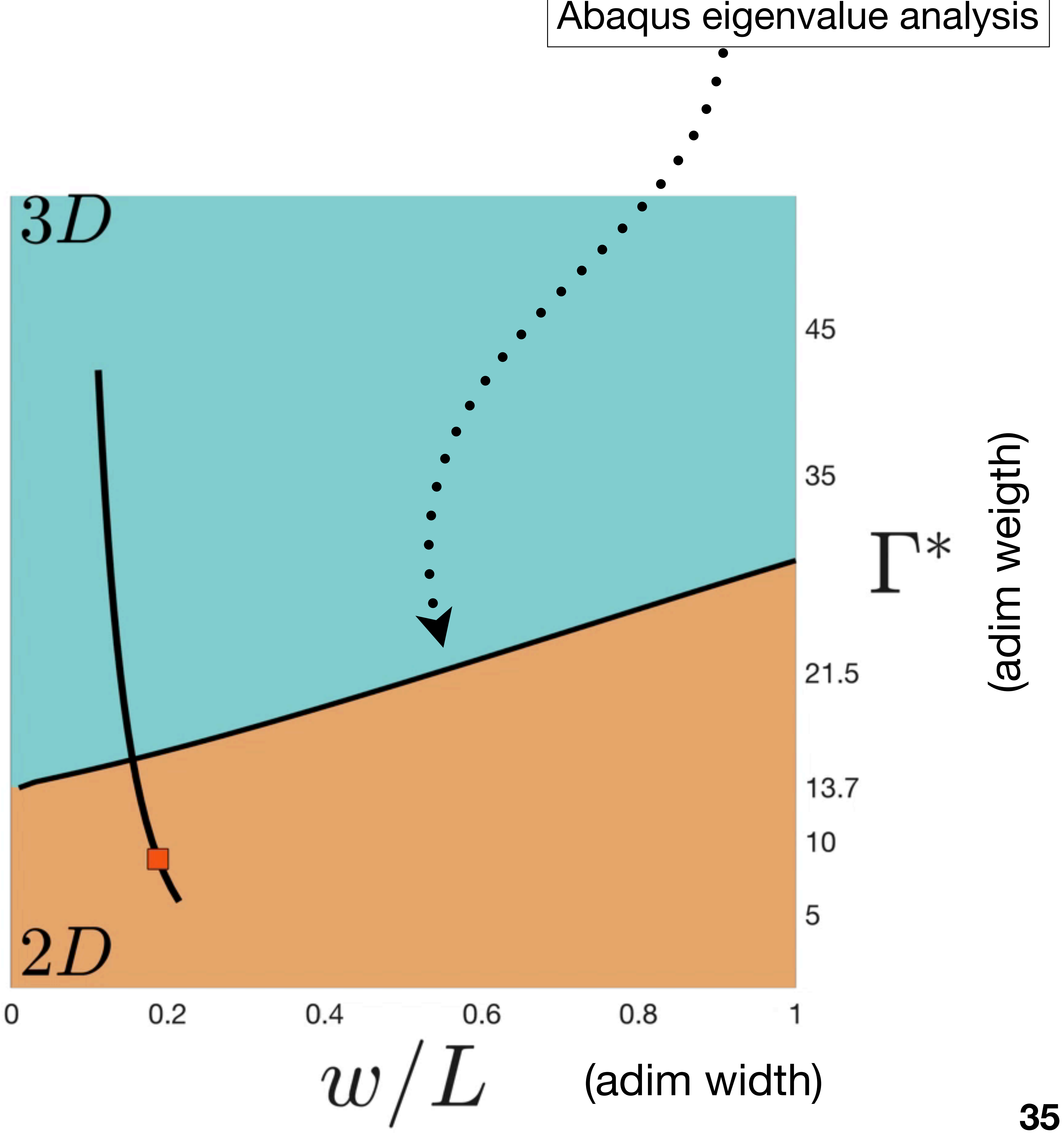
limit $h \to 0$

$$\Gamma^* = \frac{Mg}{Dw/L^2} \text{ and } \frac{w}{L} \text{ and } \cancel{\frac{h}{L}}$$

Lateral-Buckling test experimental validation

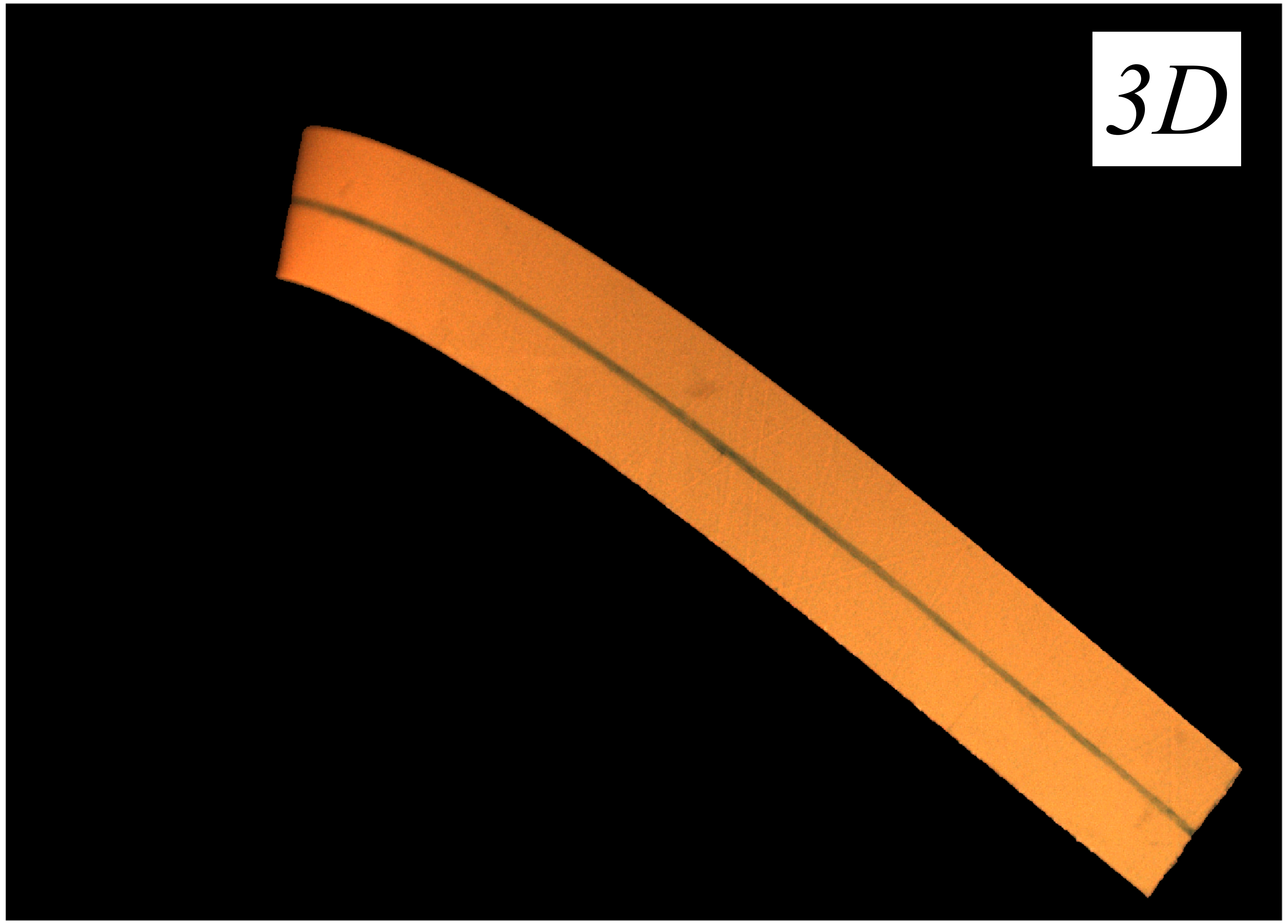
experiment: increase L

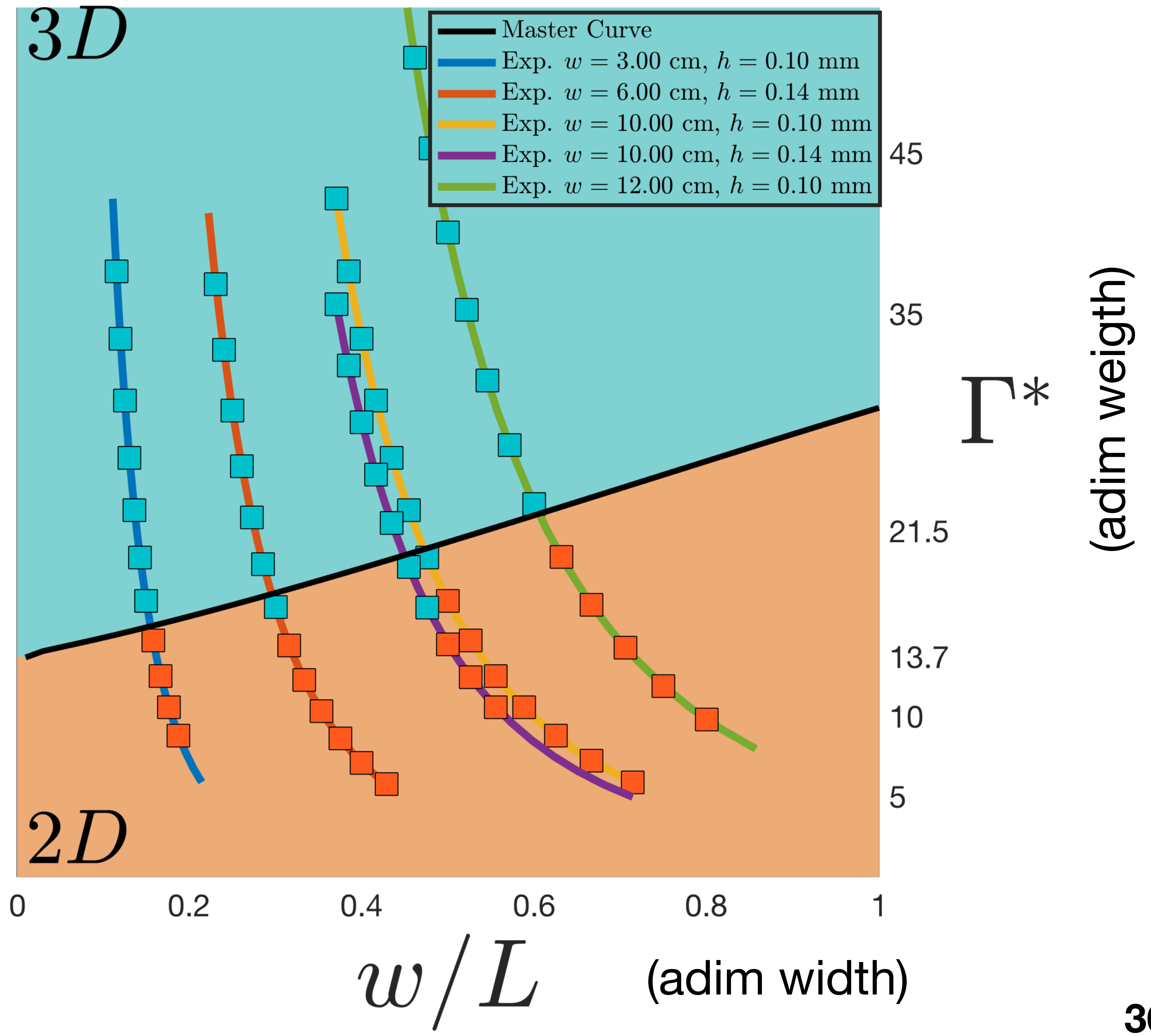




Lateral-Buckling test experimental validation

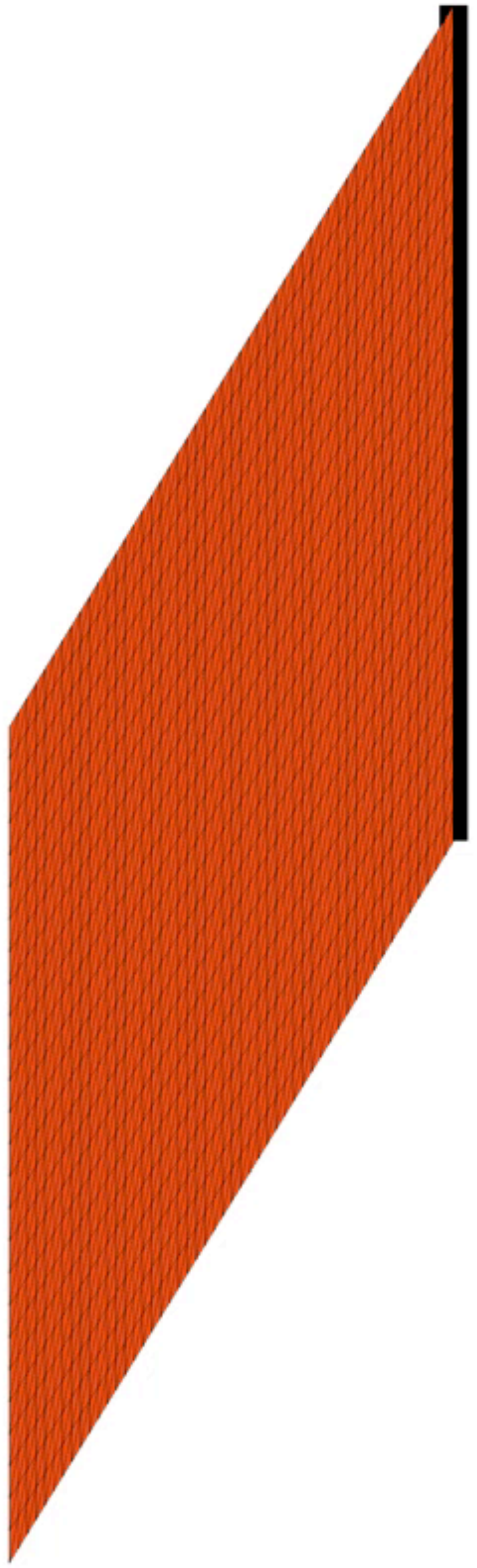
experiment: increase L

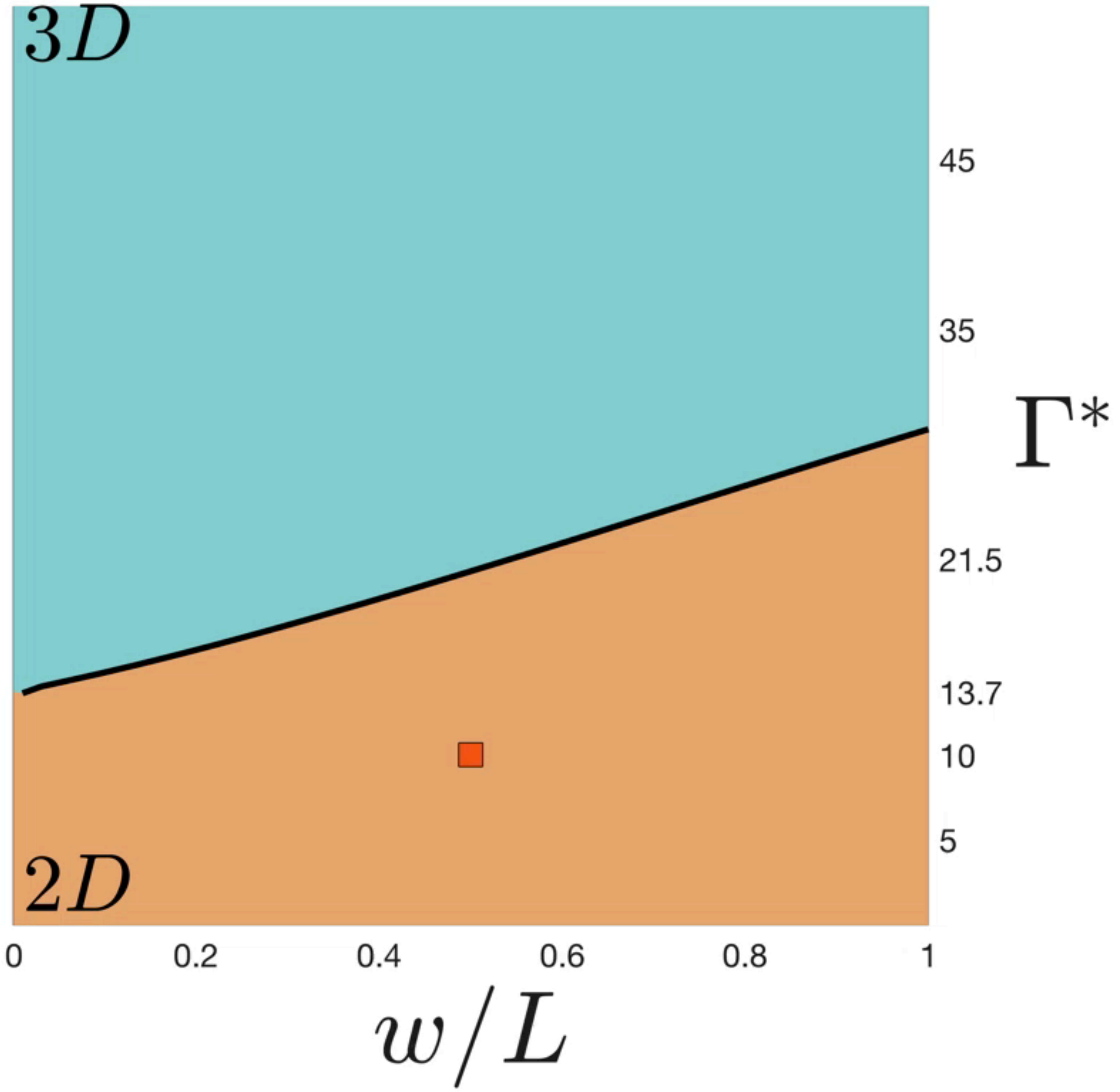




Lateral Buckling test: numerics

FEniCS Shell [Hale et al, Comptuers & Structures 2018]

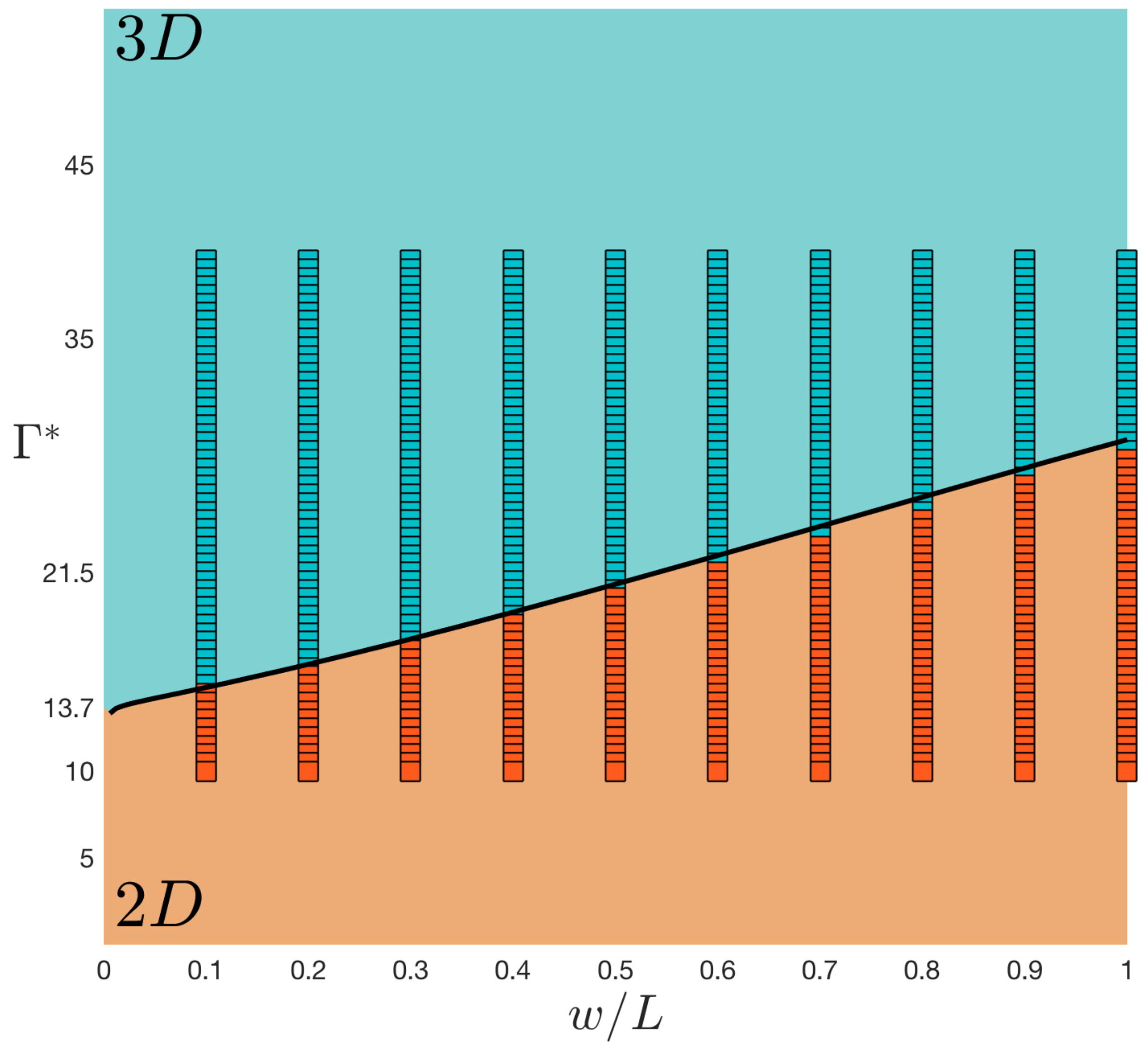




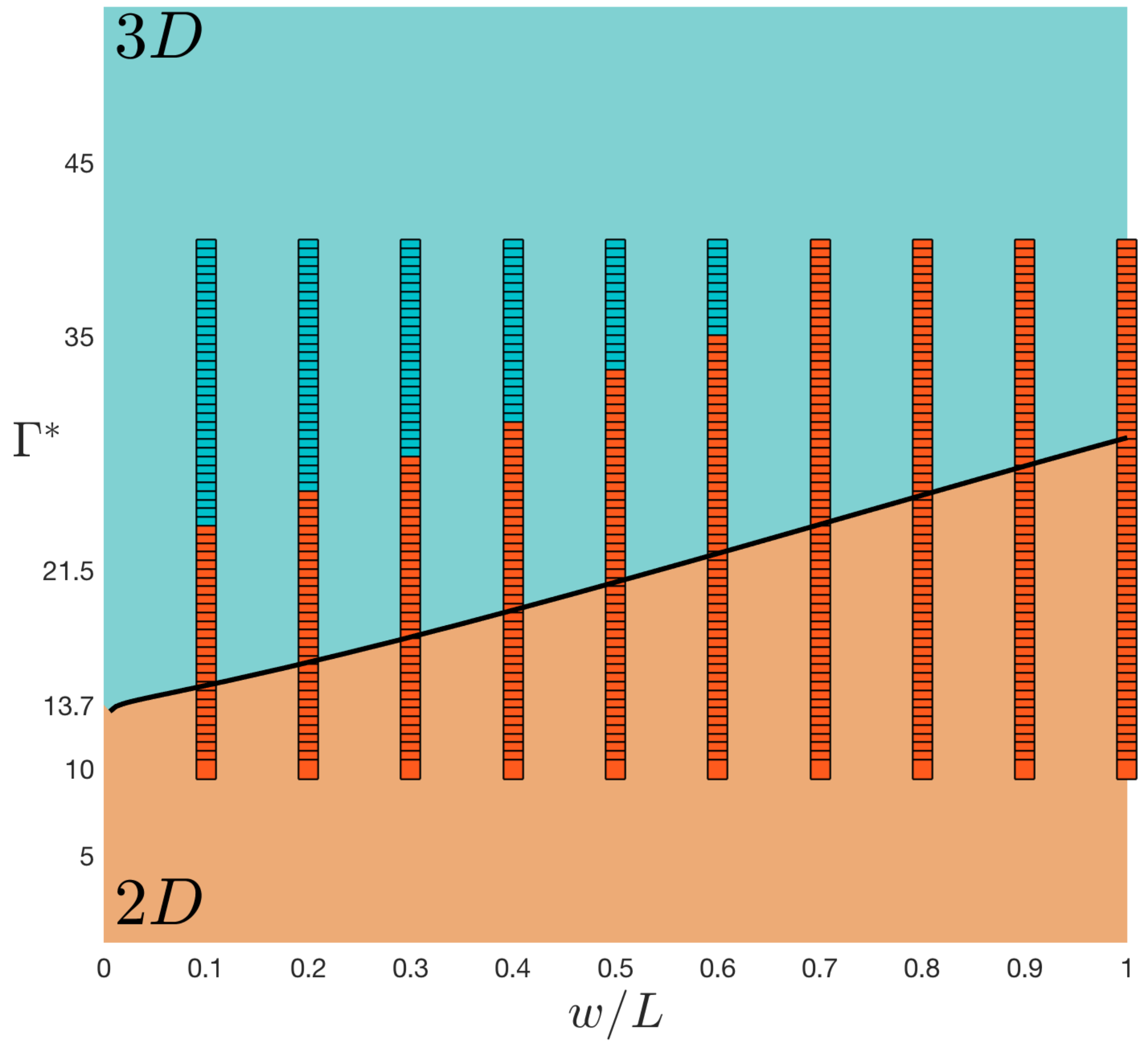
Lateral Buckling test

Computer Graphics codes

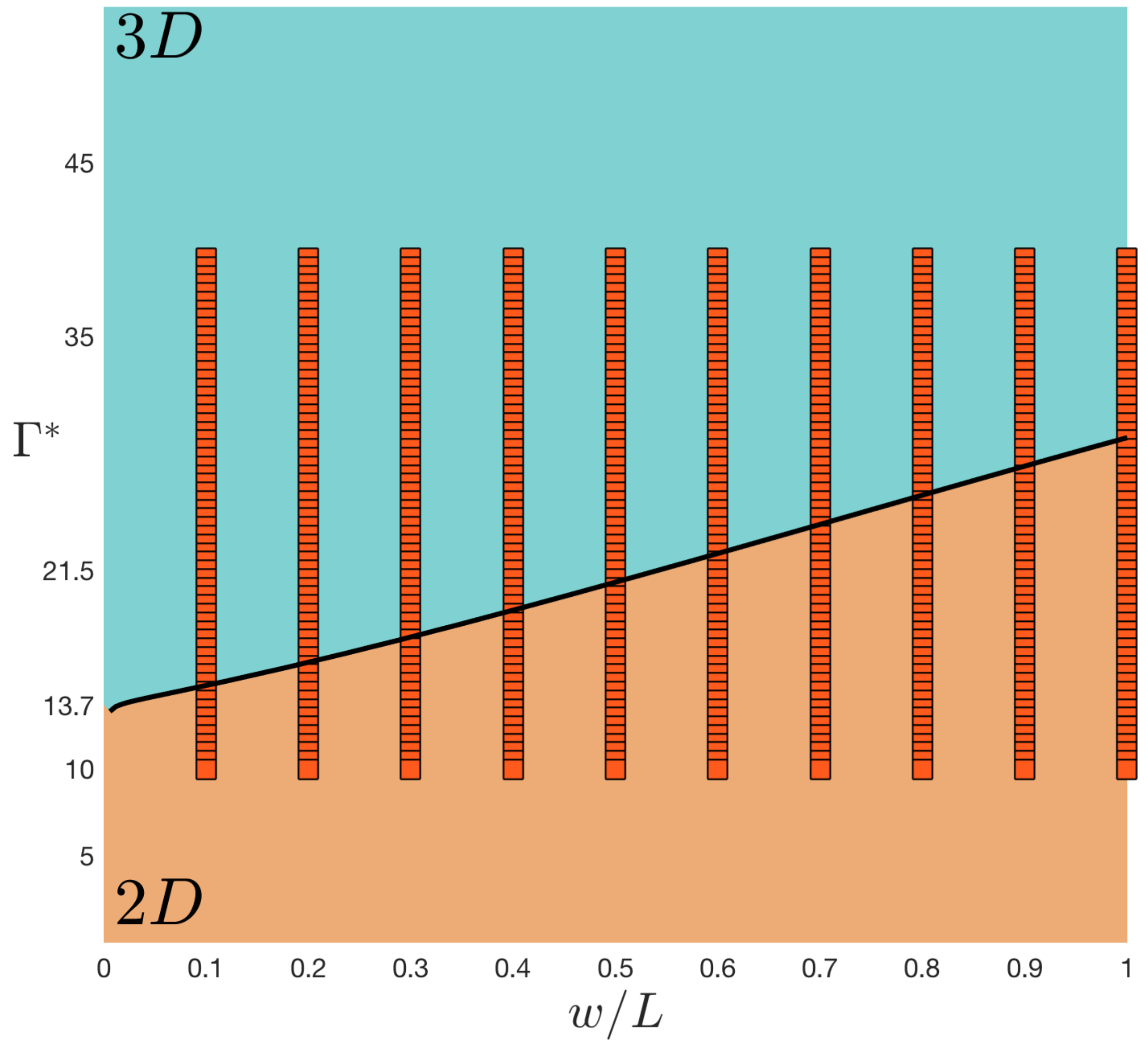
LibShell **OK**



Super-Ribbon **KO**



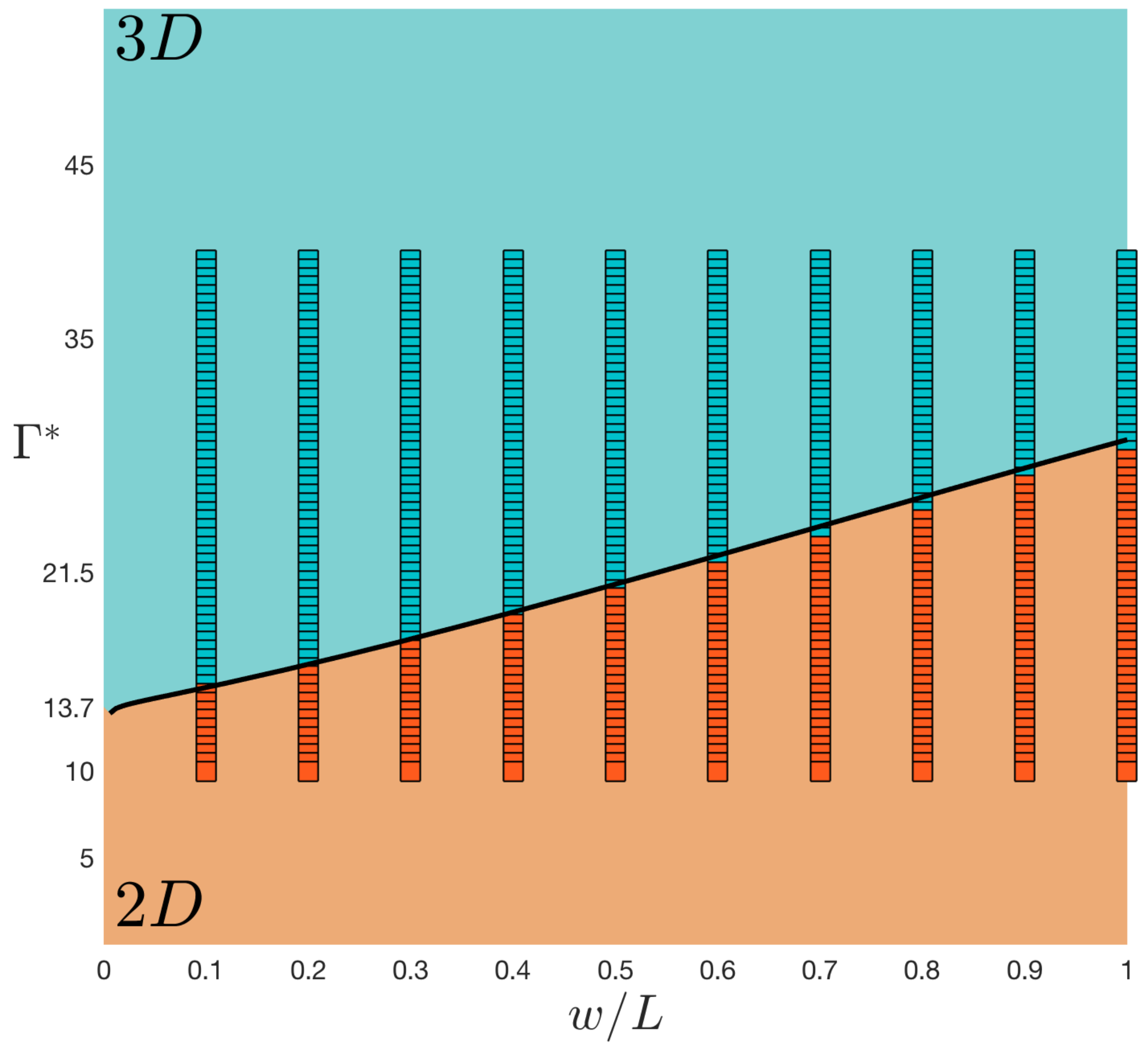
Discrete Shell **KO**



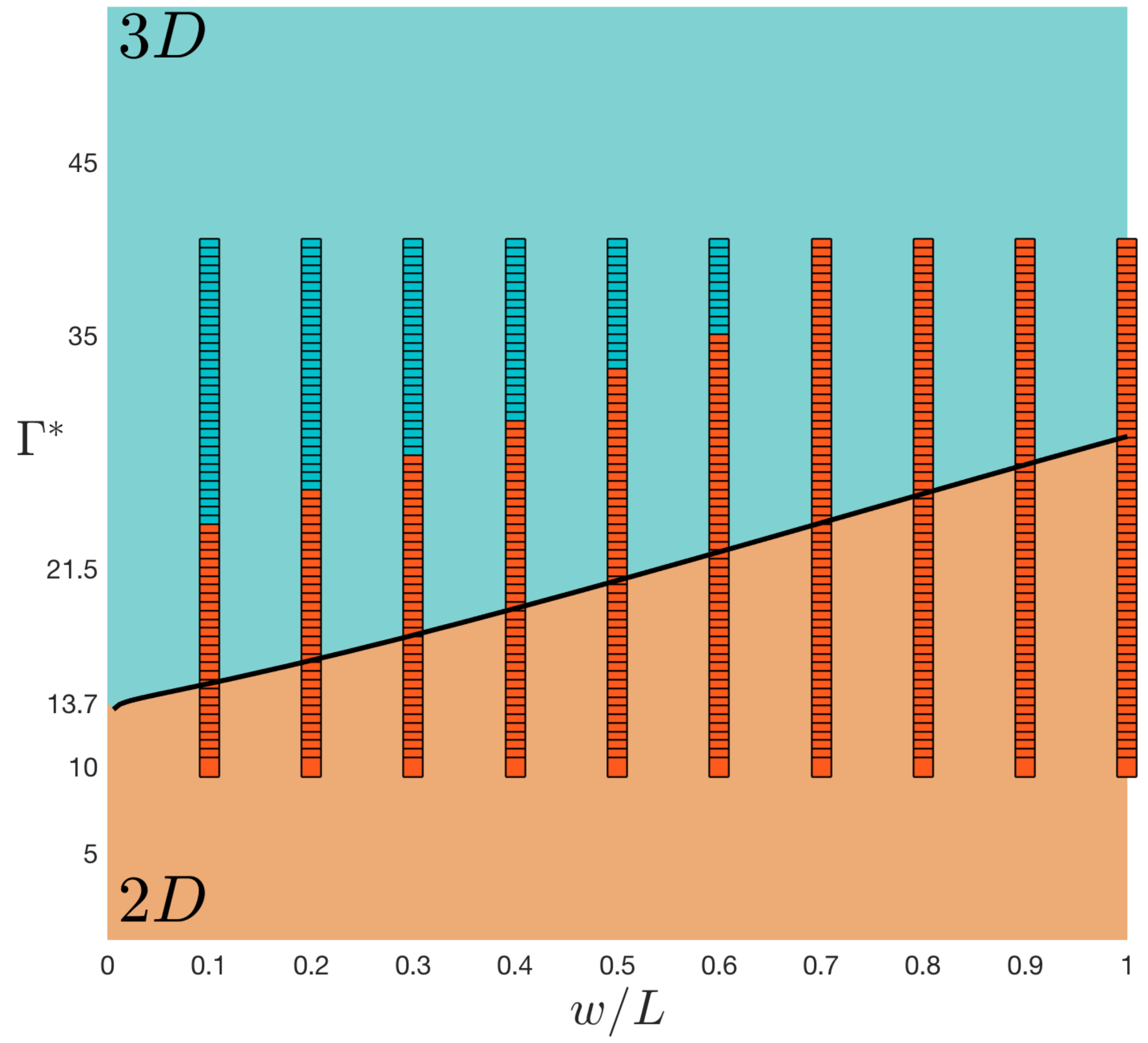
Lateral Buckling test

Computer Graphics codes

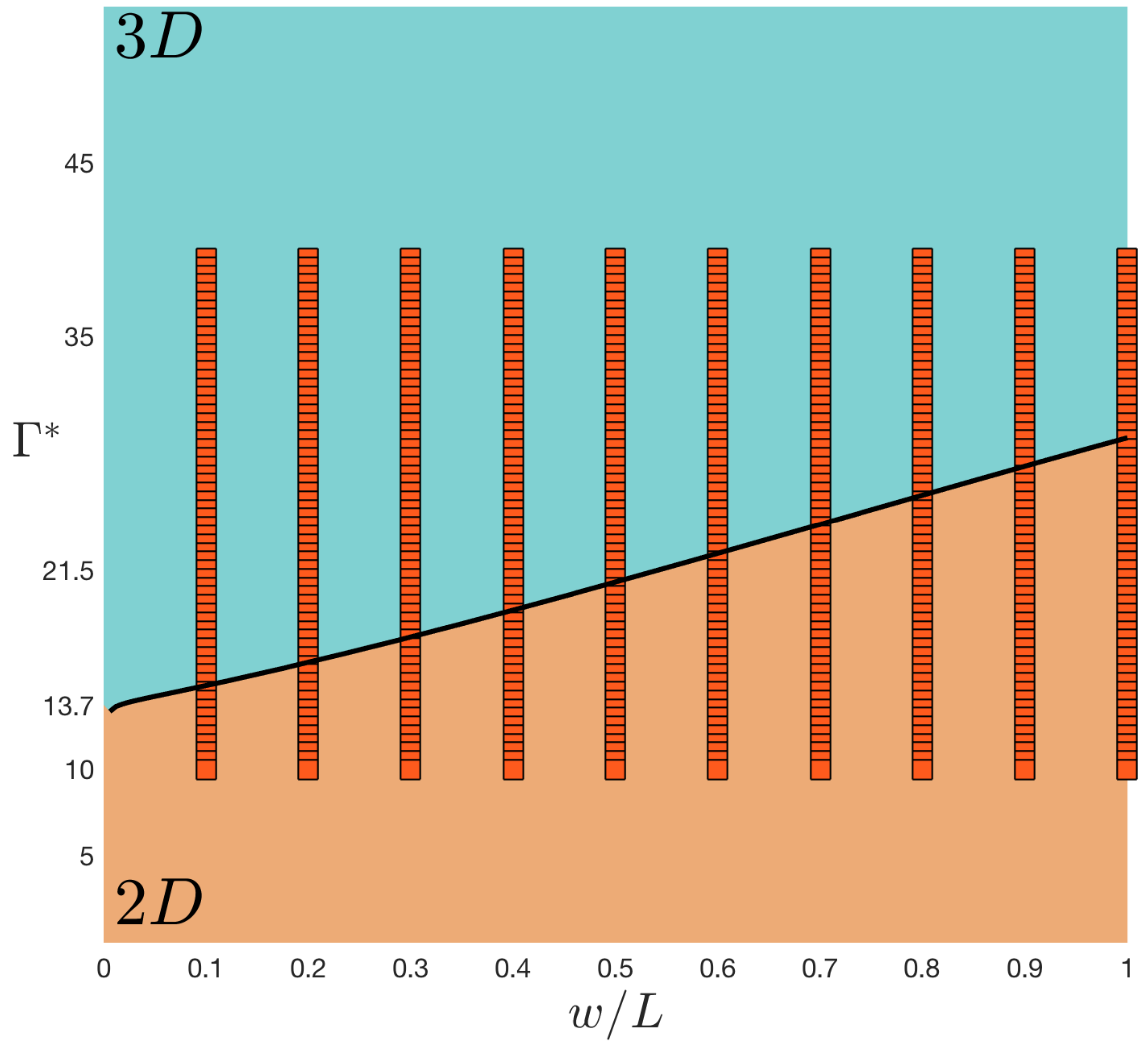
LibShell **OK**



Super-Ribbon **KO**

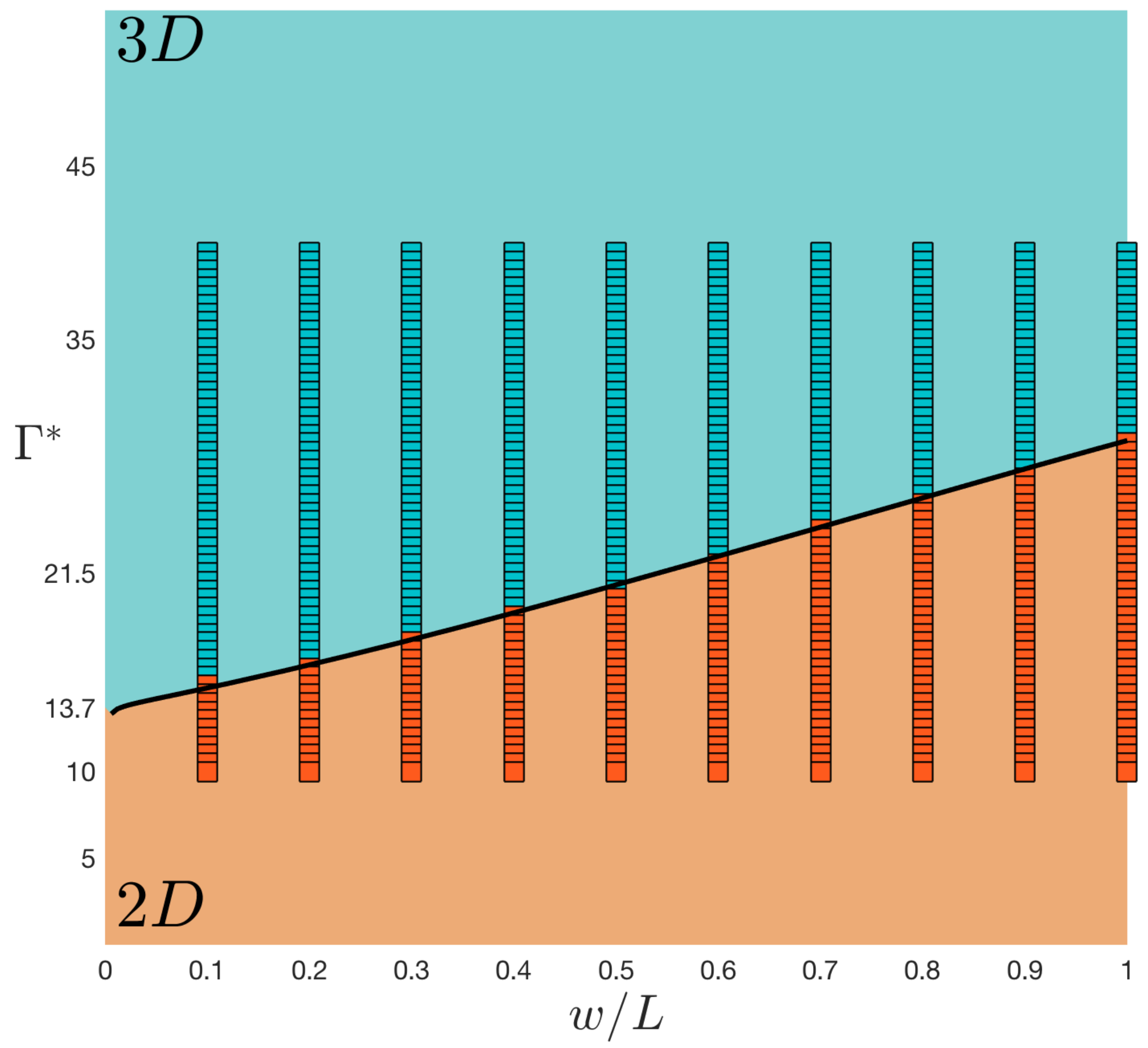


Discrete Shell **KO**

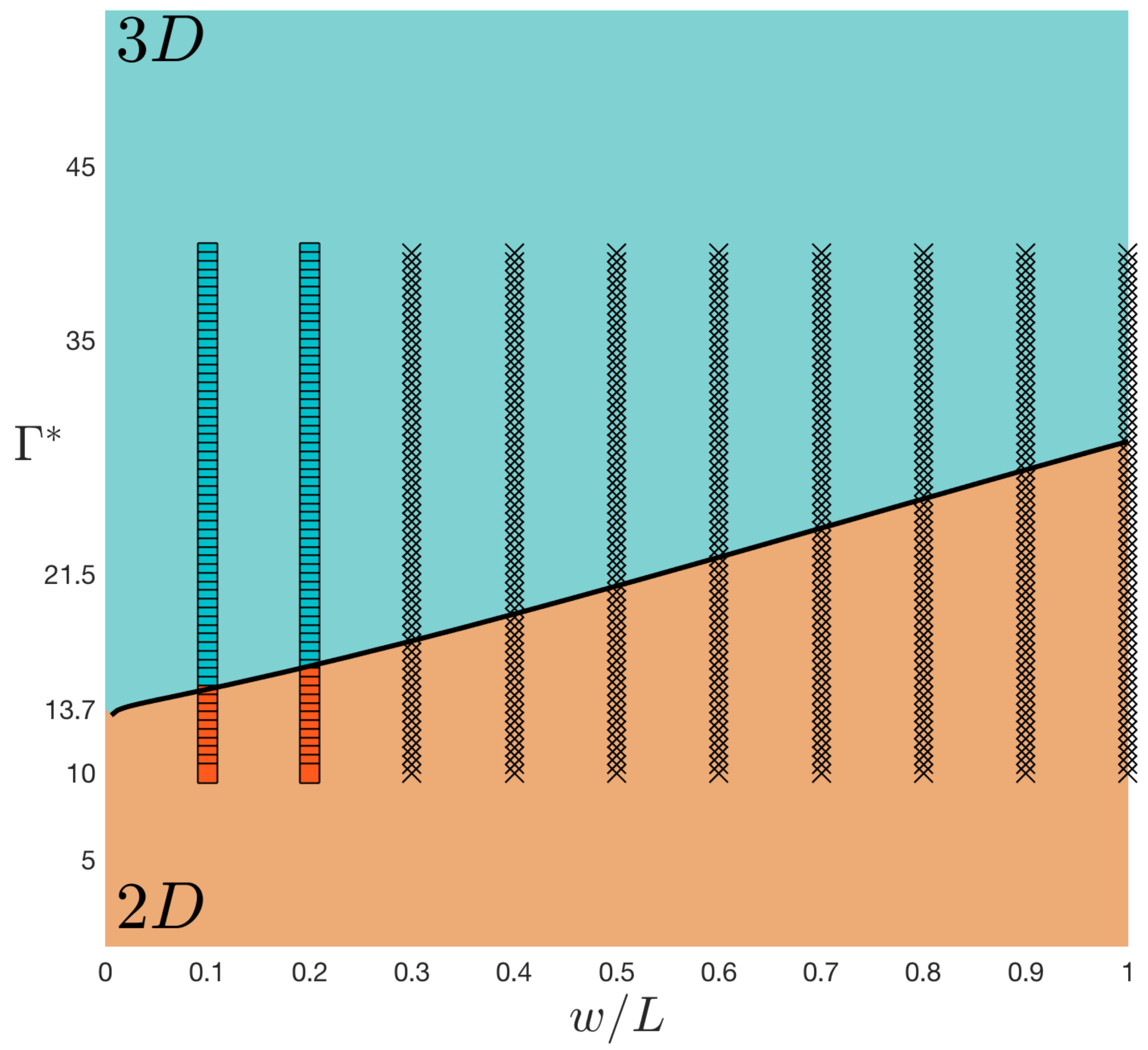


Solid Mechanics codes

FeniCS Shell **OK**



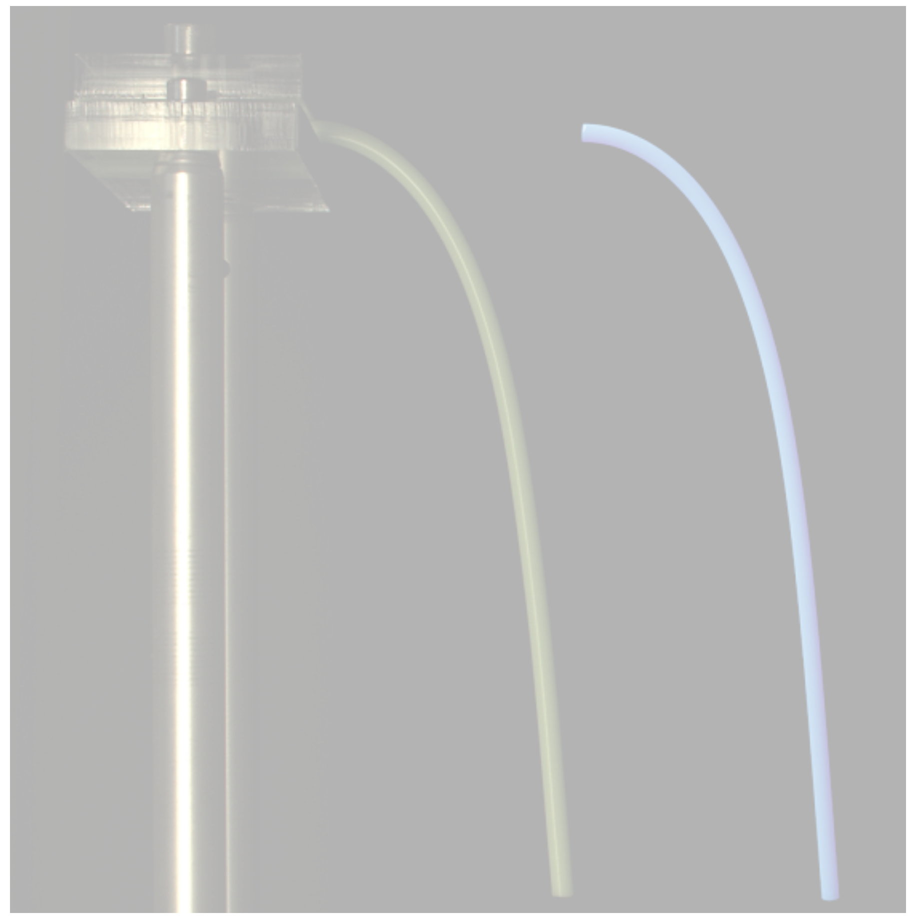
©Abaqus **KO**



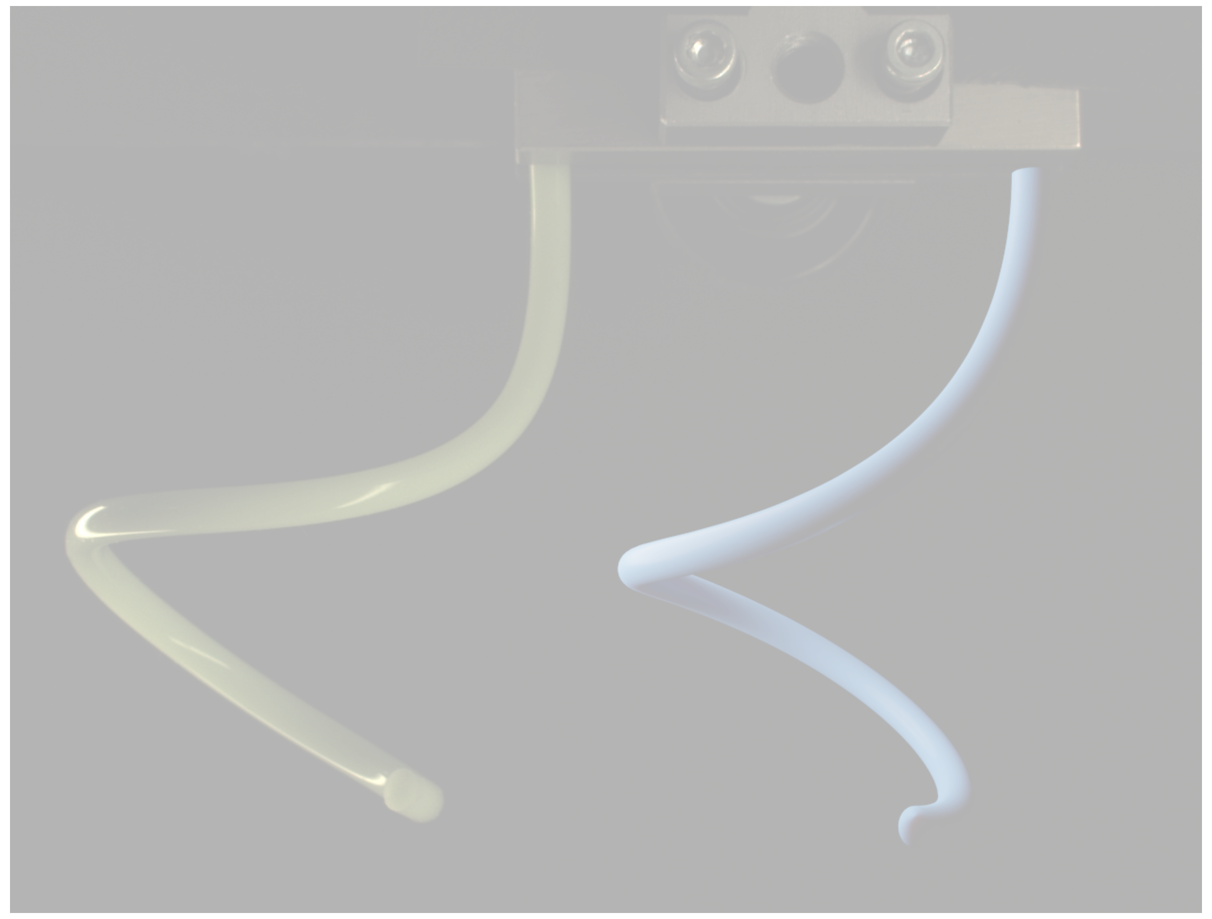
Lateral Buckling test

Tested Code	Cantilever	Bend-Twist	Lateral Buckling	Stick-Slip
Rod				
Discrete Elastic Rod [Bergou et al. 2010]	OK (300 elts)	OK (200 elts)	–	–
Super-Helix [Bertails et al. 2006]	OK (50 elts)	OK (30 elts)	–	–
Super-Clothoid [Casati and Bertails-Descoubes 2013]	OK (20 elts)	OK (25 elts)	–	–
Ribbon				
Super-Ribbon [Charrondière et al. 2020]	OK (20 elts)	–	KO	–
Plate				
LibShell [Chen et al. 2018]	OK (Res 0)	–	OK (Res +)	–
Discrete Shell (+ LibShell) [Grinspun et al. 2003]	OK (Res +)	–	KO	–
ArcSim [Narain et al. 2012]	KO	–	×	–
Discrete Shell + ArcSim (**tentative fix** of ArcSim)	KO	–	×	–
Projective Dynamics [Bouaziz et al. 2014] (**fit**)	KO	–	×	–
Contact & friction				
Viscous Friction (+ Super-Helix 2D)	–	–	–	KO
So-Bogus [Daviet et al. 2011] (+Super-Helix 2D)	–	–	–	OK (dt=0.5 ms, tol $= 10^{-13}$ N)
Argus (≈ ArcSim + So-Bogus) [Li et al. 2018]	–	–	–	KO
Argus Non Adaptive (**fix** of Argus)	–	–	–	OK (dt=0.5 ms, tol $= 10^{-13}$ N)
Bridson-Harmon [Bridson et al. 2002; Harmon et al. 2008] (+ ArcSim)	–	–	–	KO
Projective Friction [Ly et al. 2020]	–	–	–	KO
Reference codes in Mechanical Engineering				
FenicsShell [Hale et al. 2018]	OK (Res 0, $P_{2,3}$ elts)	–	OK (Res 0, $P_{2,3}$ elts)	–
©Abaqus	OK (200 P_2 elts)	KO	KO	OK (dt=9 µs)

Measurement protocols

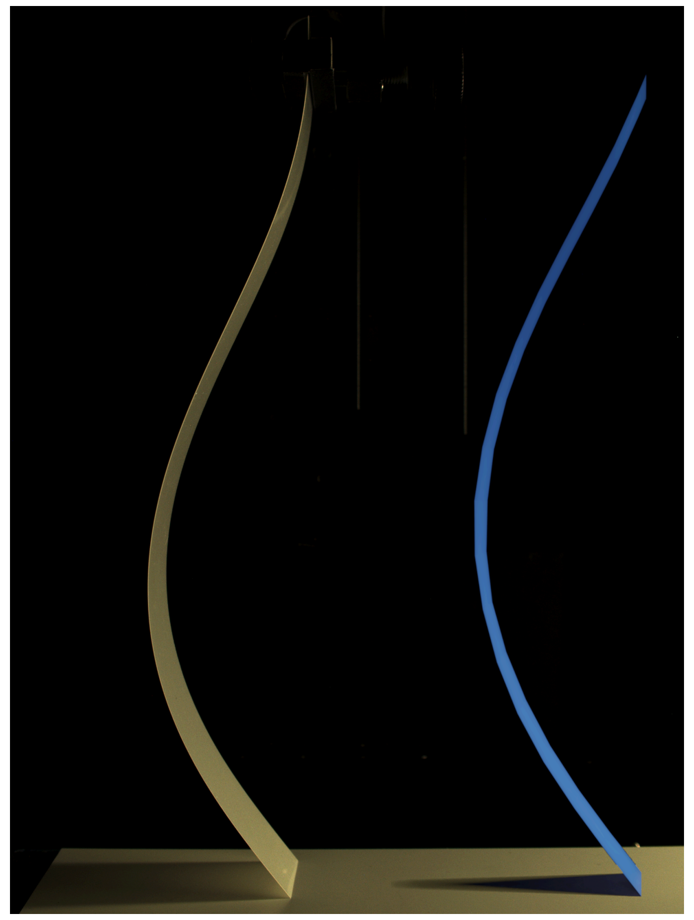


Cantilever



Bend-Twist

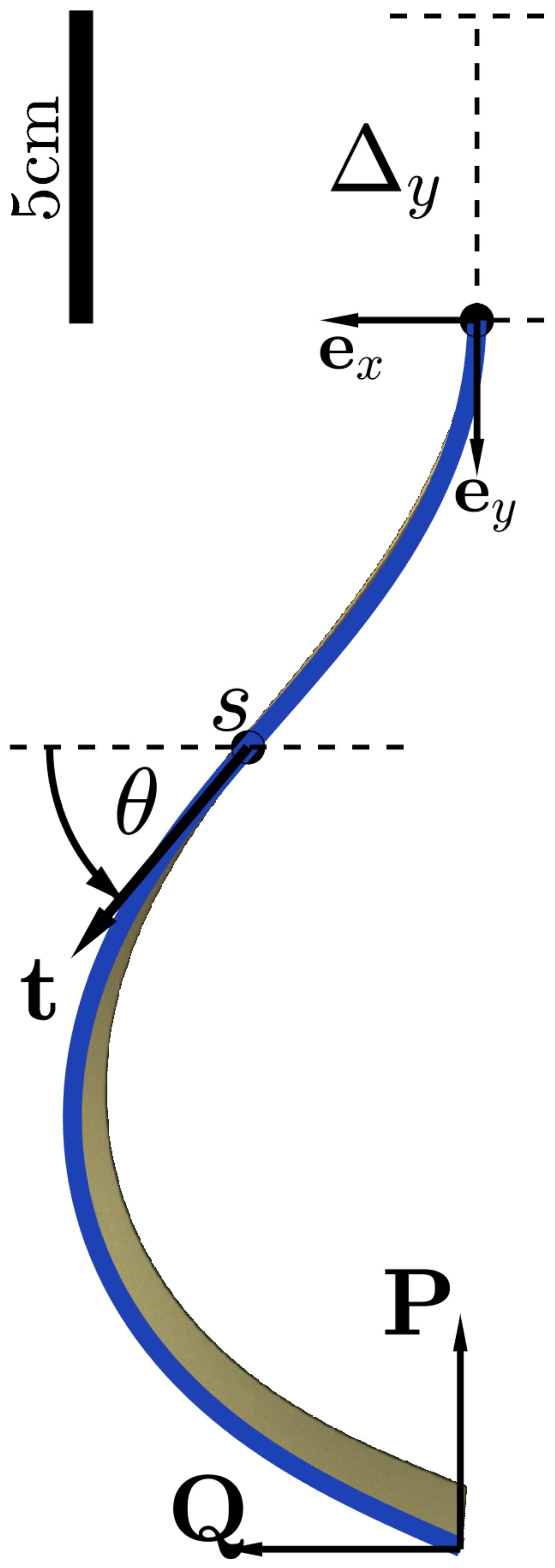
Lateral Buckling



Stick-Slip

Measurement protocols

Stick-Slip test

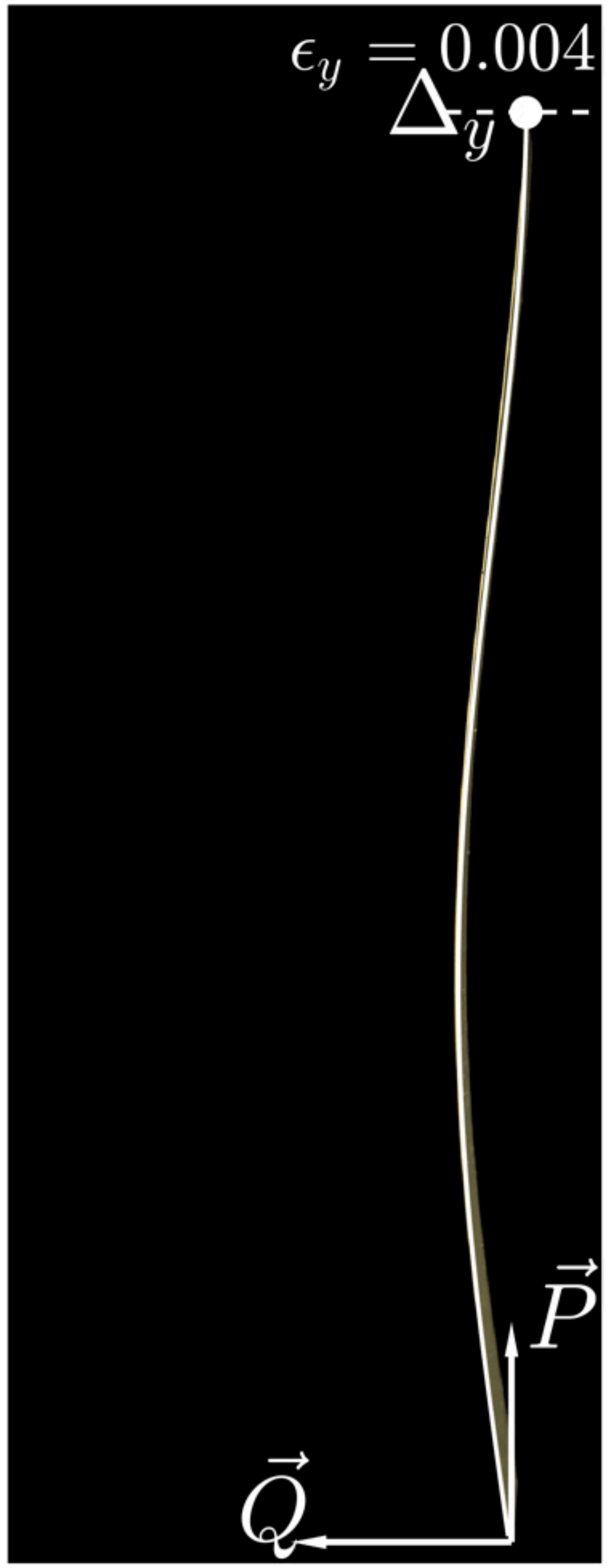


Experiment Inspired by Sano et al..

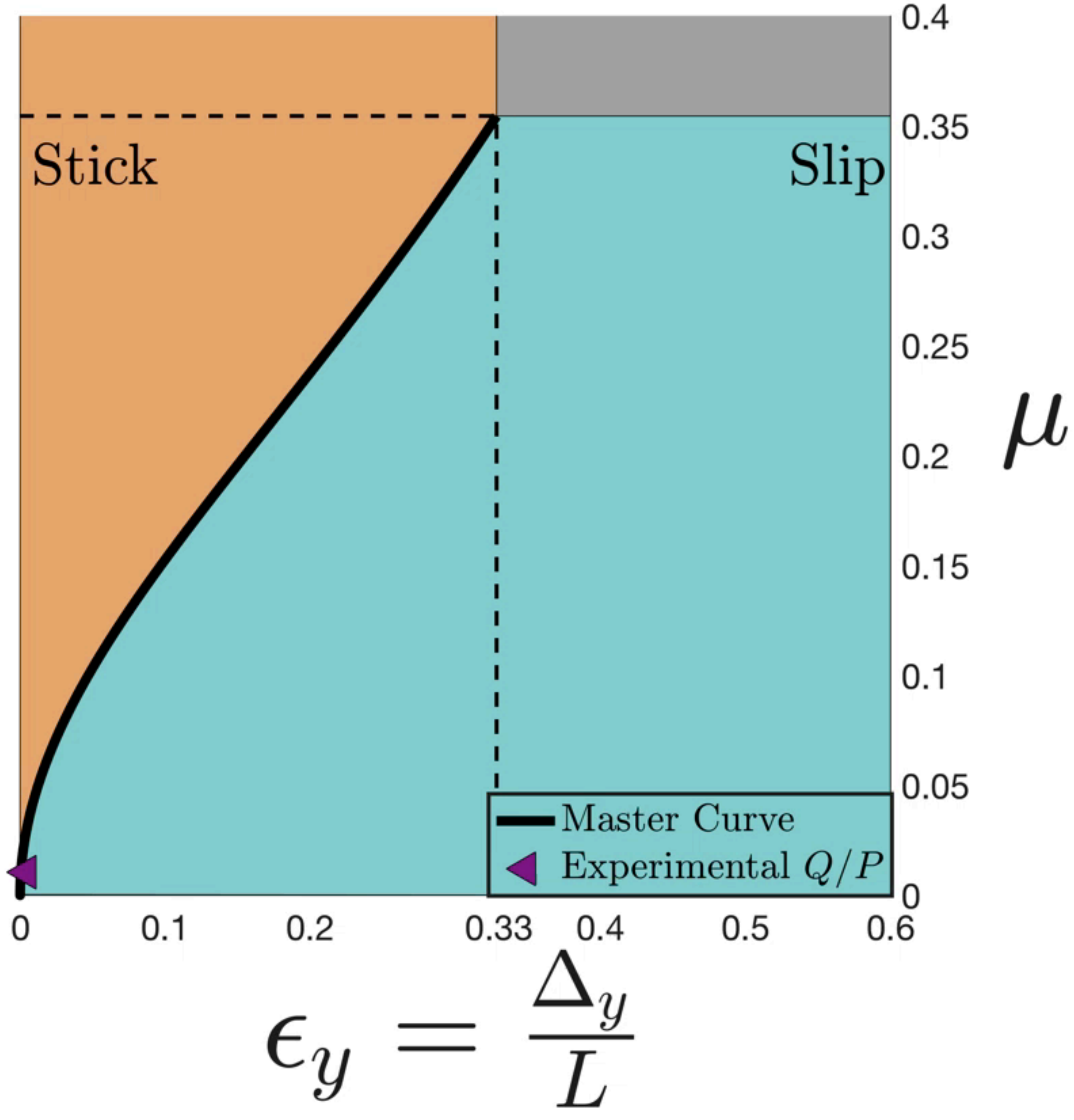
Gravity negligible	
Normal Force:	P
Tangential Force:	Q
Lenght:	L
Vertical displacement:	Δ_y
Friction coefficient:	μ

$$\text{Force ratio} = \frac{Q}{P} \text{ and Vertical strain } \epsilon_y = \frac{\Delta_y}{L}$$

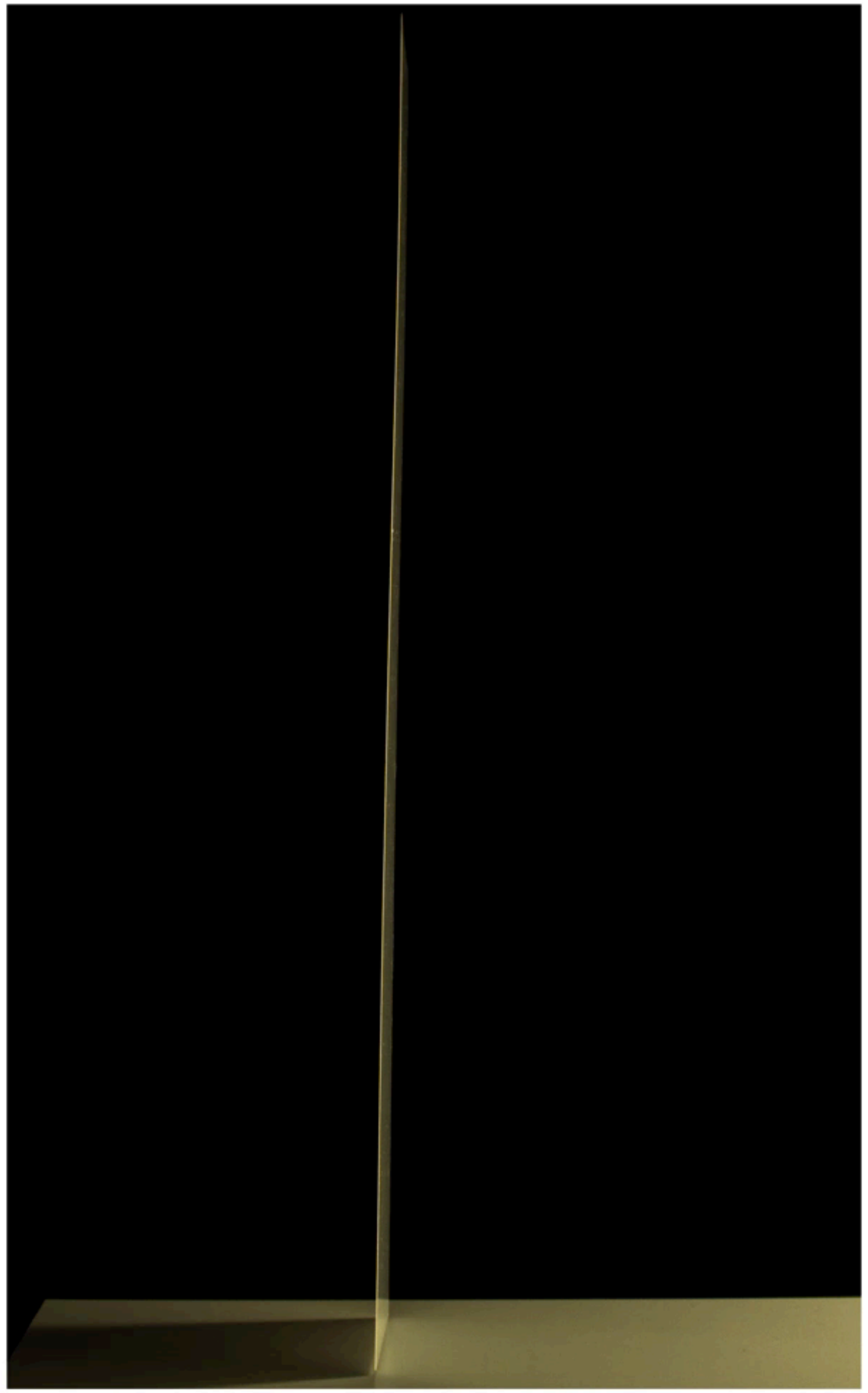
Measurement protocols
Stick-Slip test – experimental validation

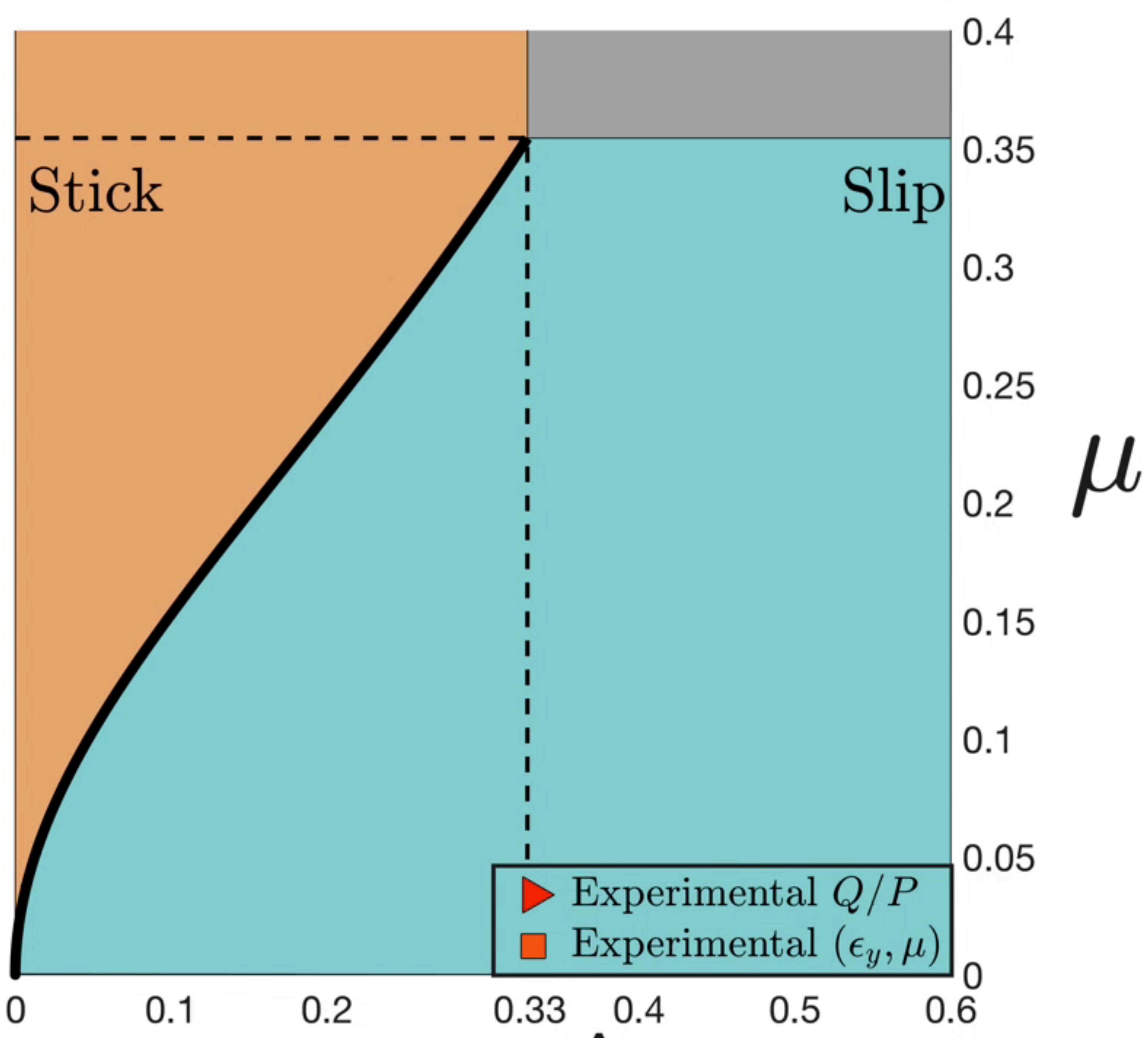


Extended from Sano et al. 2017



Measurement protocols
Stick-Slip test – experimental validation





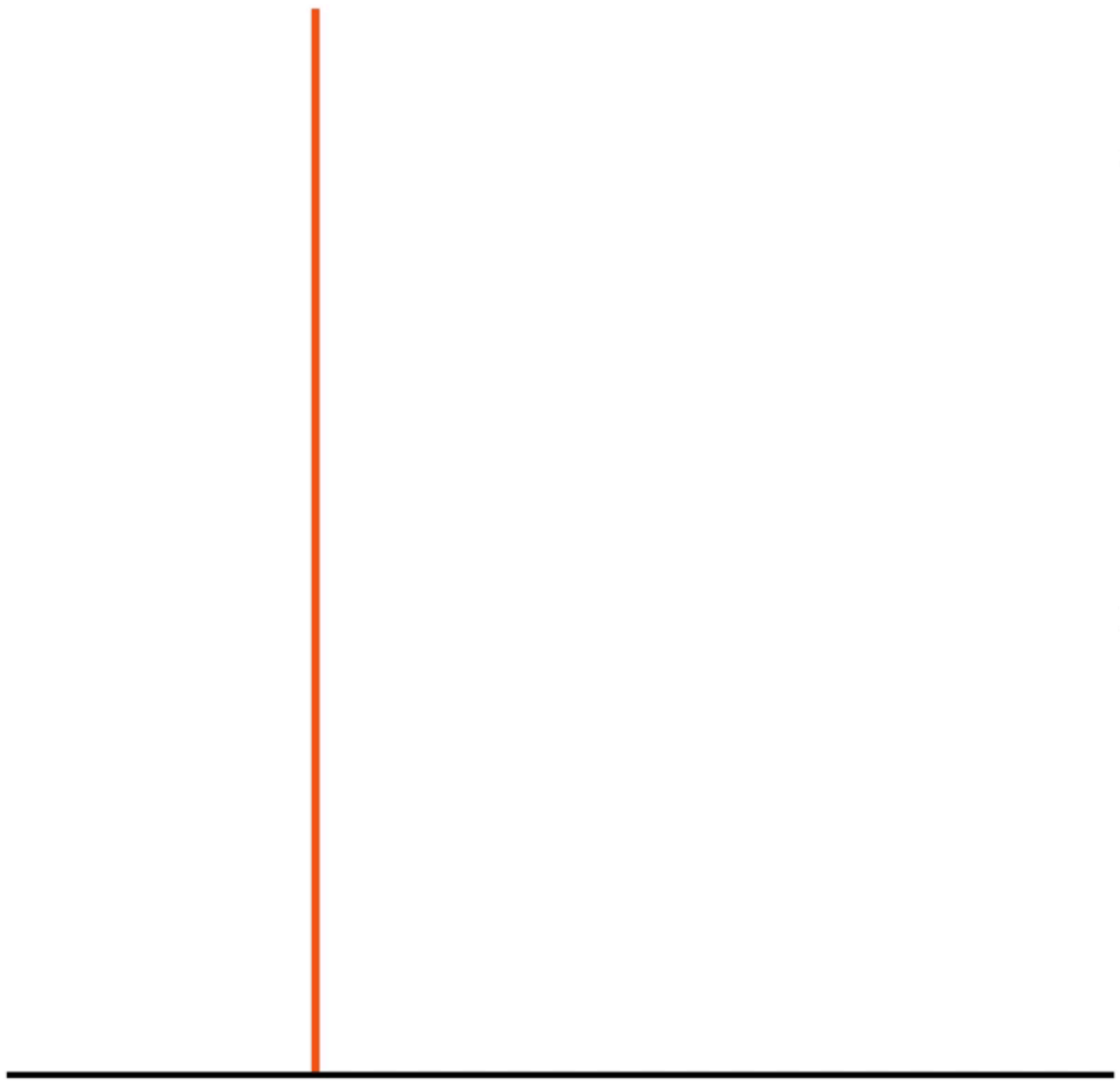
$$\epsilon_y = \frac{\Delta_y}{L}$$

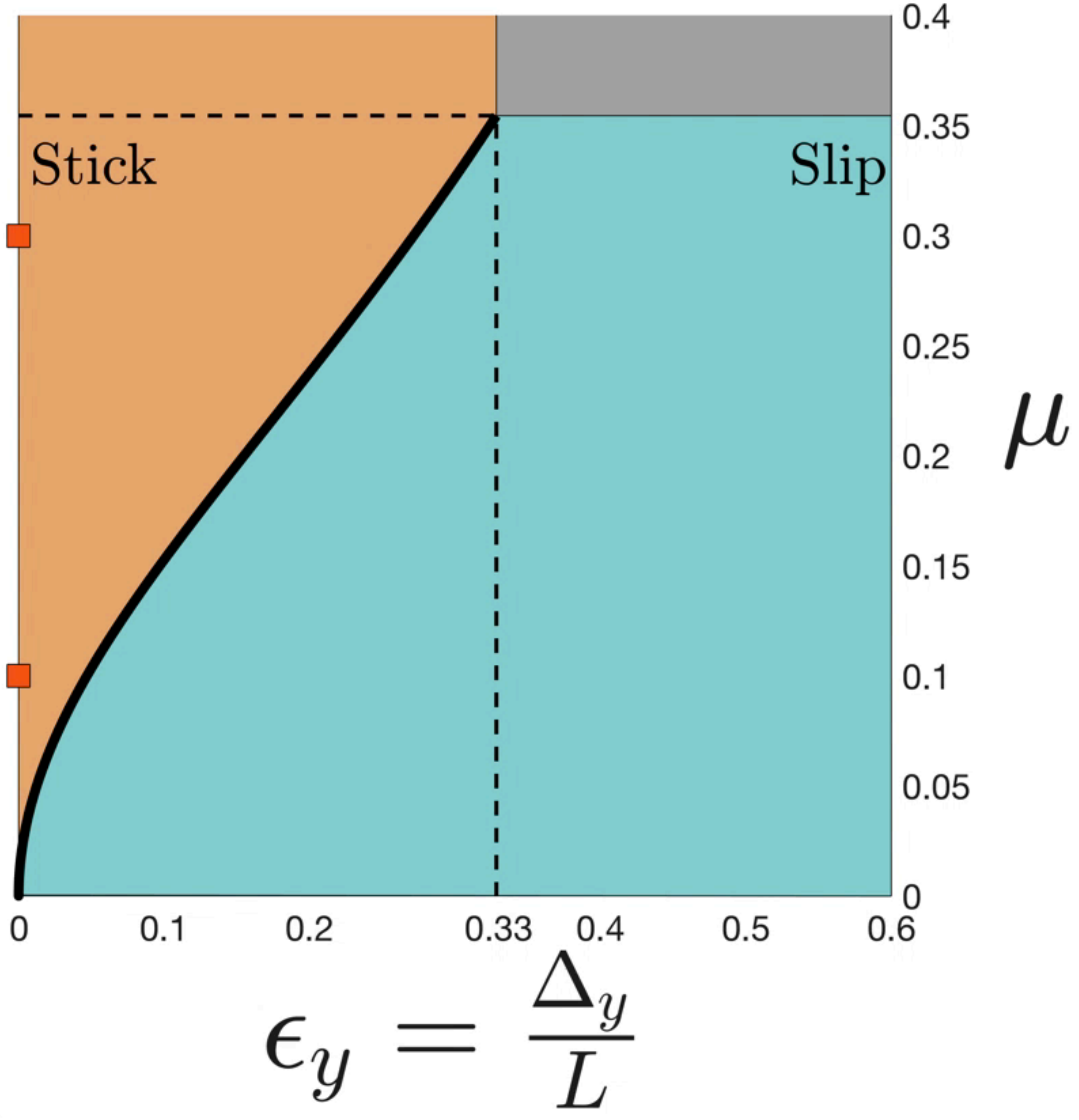
Results
Stick-slip test

Tested Code	Cantilever	Bend-Twist	Lateral Buckling	Stick-Slip
Rod				
Discrete Elastic Rod [Bergou et al. 2010]	OK (300 elts)	OK (200 elts)	–	–
Super-Helix [Bertails et al. 2006]	OK (50 elts)	OK (30 elts)	–	–
Super-Clothoid [Casati and Bertails-Descoubes 2013]	OK (20 elts)	OK (25 elts)	–	–
Ribbon				
Super-Ribbon [Charrondière et al. 2020]	OK (20 elts)	–	KO	–
Plate				
LibShell [Chen et al. 2018]	OK (Res 0)	–	OK (Res +)	–
Discrete Shell (+ LibShell) [Grinspun et al. 2003]	OK (Res +)	–	KO	–
ArcSim [Narain et al. 2012]	KO	–	×	–
Discrete Shell + ArcSim (**tentative fix** of ArcSim)	KO	–	×	–
Projective Dynamics [Bouaziz et al. 2014] (**fit**)	KO	–	×	–
Contact & friction				
Viscous Friction (+ Super-Helix 2D)	–	–	–	KO
So-Bogus [Daviet et al. 2011] (+Super-Helix 2D)	–	–	–	OK (dt=0.5 ms, tol $=10^{-13}$ N)
Argus (≈ ArcSim + So-Bogus) [Li et al. 2018]	–	–	–	KO
Argus Non Adaptive (**fix** of Argus)	–	–	–	OK (dt=0.5 ms, tol $=10^{-13}$ N)
Bridson-Harmon [Bridson et al. 2002; Harmon et al. 2008] (+ ArcSim)	–	–	–	KO
Projective Friction [Ly et al. 2020]	–	–	–	KO
Reference codes in Mechanical Engineering				
FenicsShell [Hale et al. 2018]	OK (Res 0, $P_{2,3}$ elts)	–	OK (Res 0, $P_{2,3}$ elts)	–
©Abaqus	OK (200 P_2 elts)	KO	KO	OK (dt=9 μs)

Results

Stick-slip test



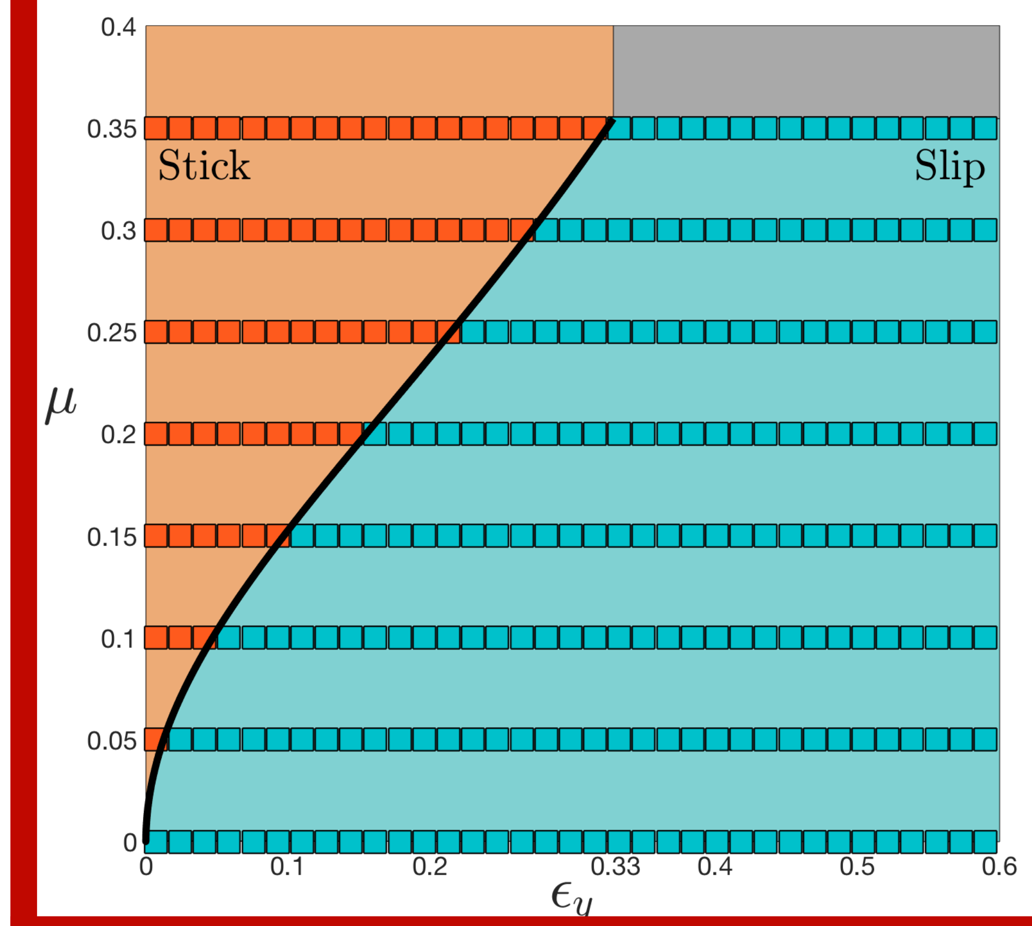


Results

Stick-slip test

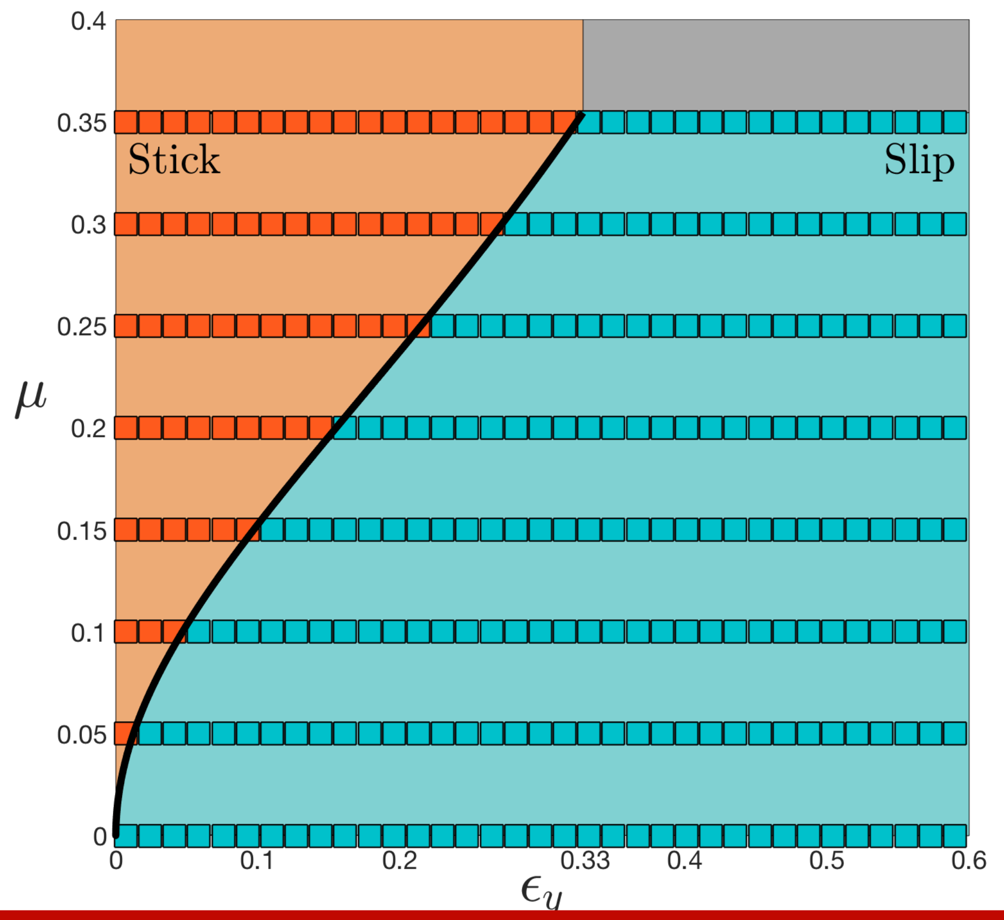
©Abaqus

OK



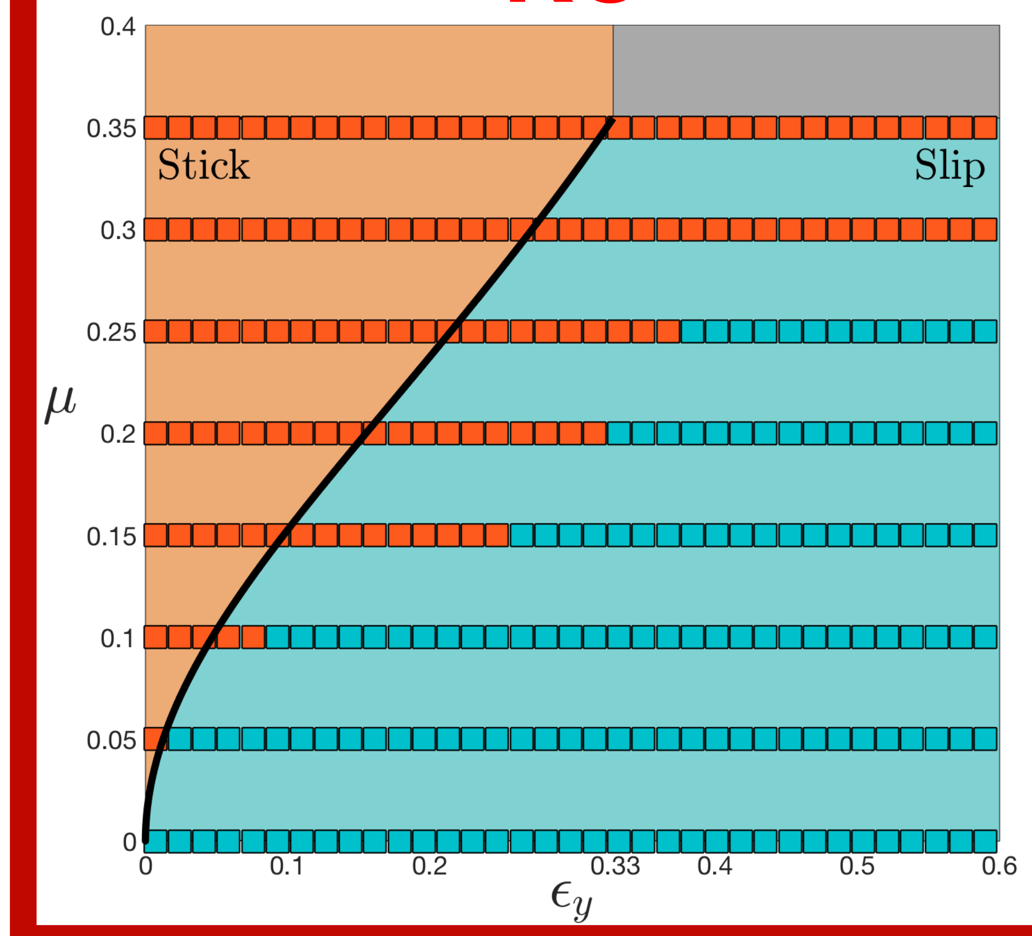
So-Bogus

OK



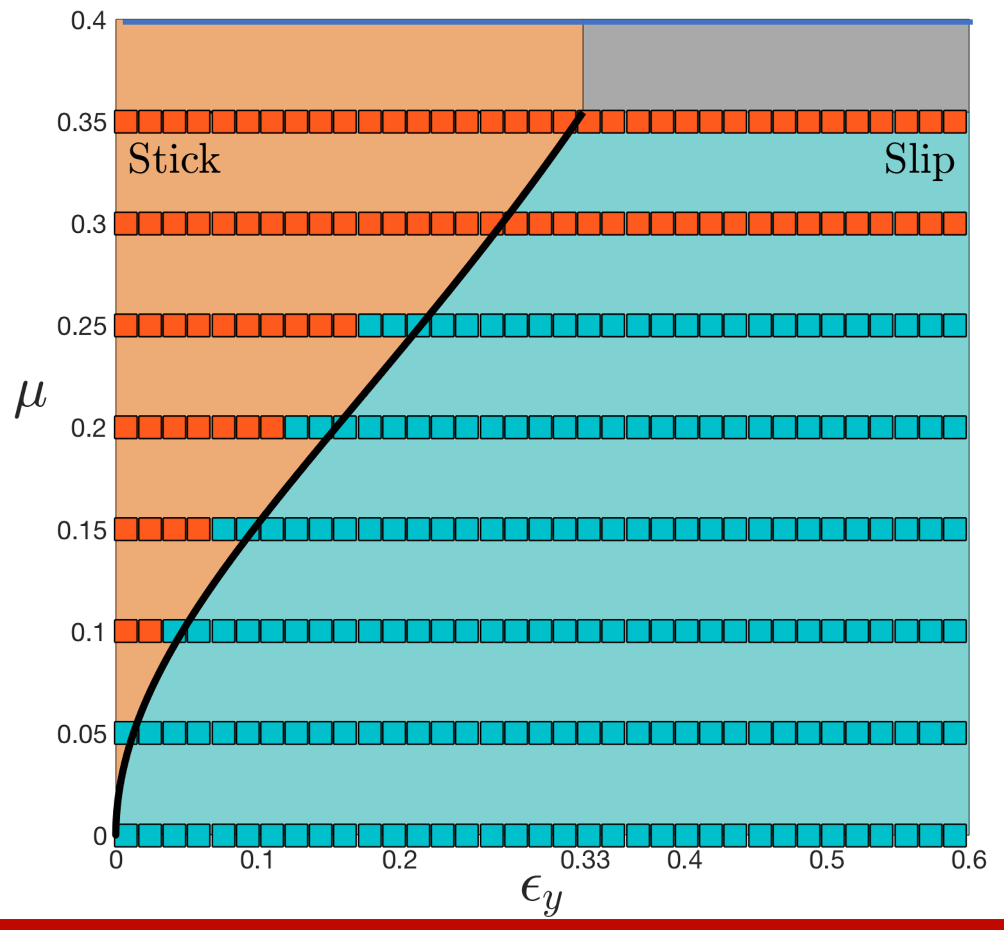
Bridson-Harmon

KO



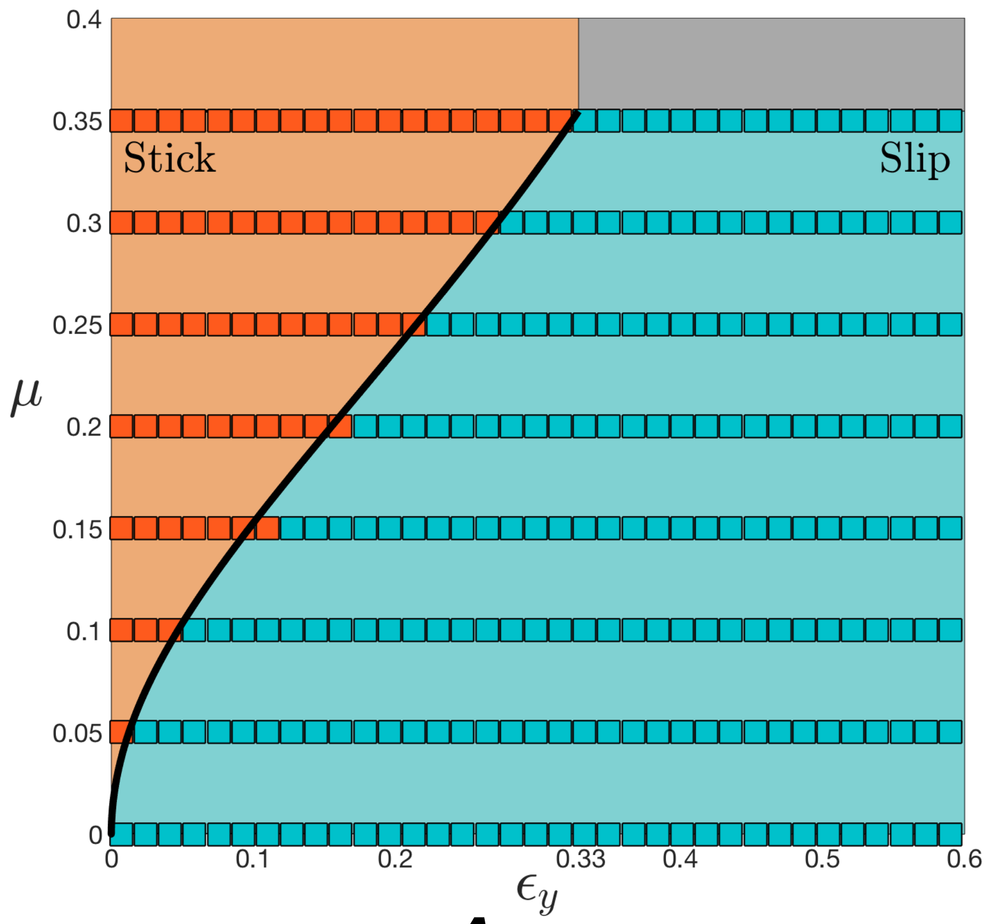
Projective Friction

KO



Argus no remeshing

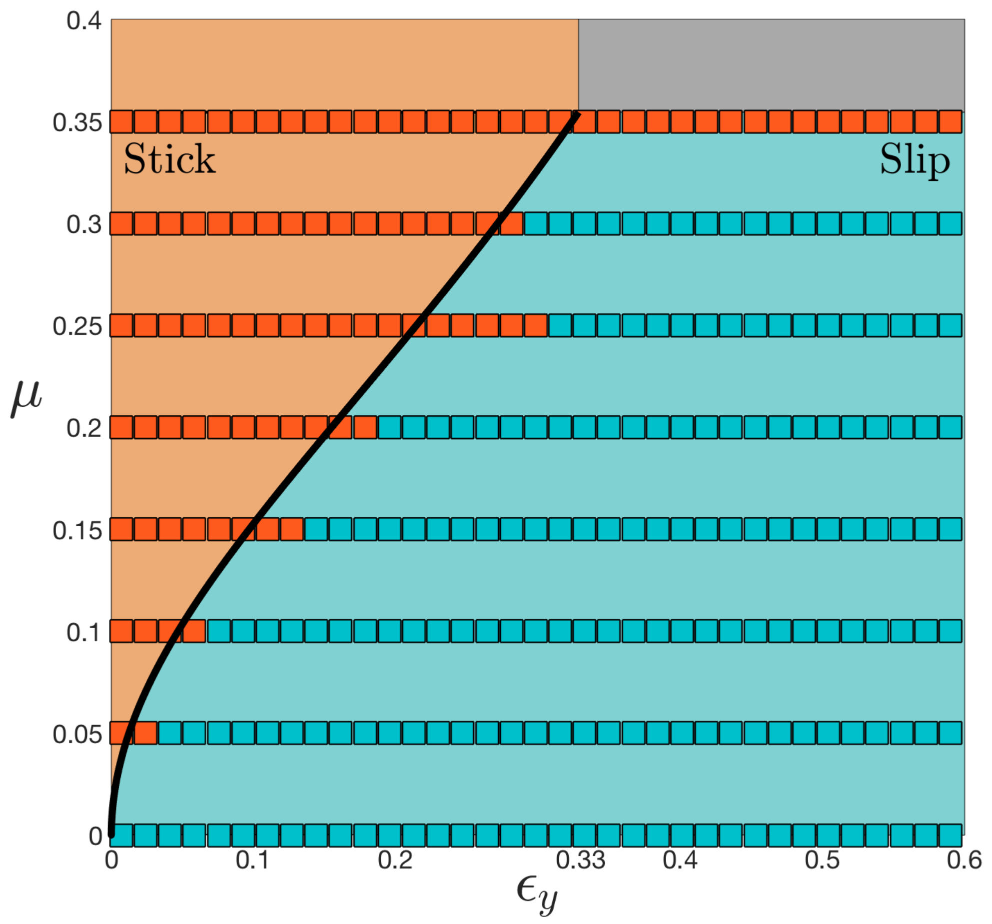
OK



Argus

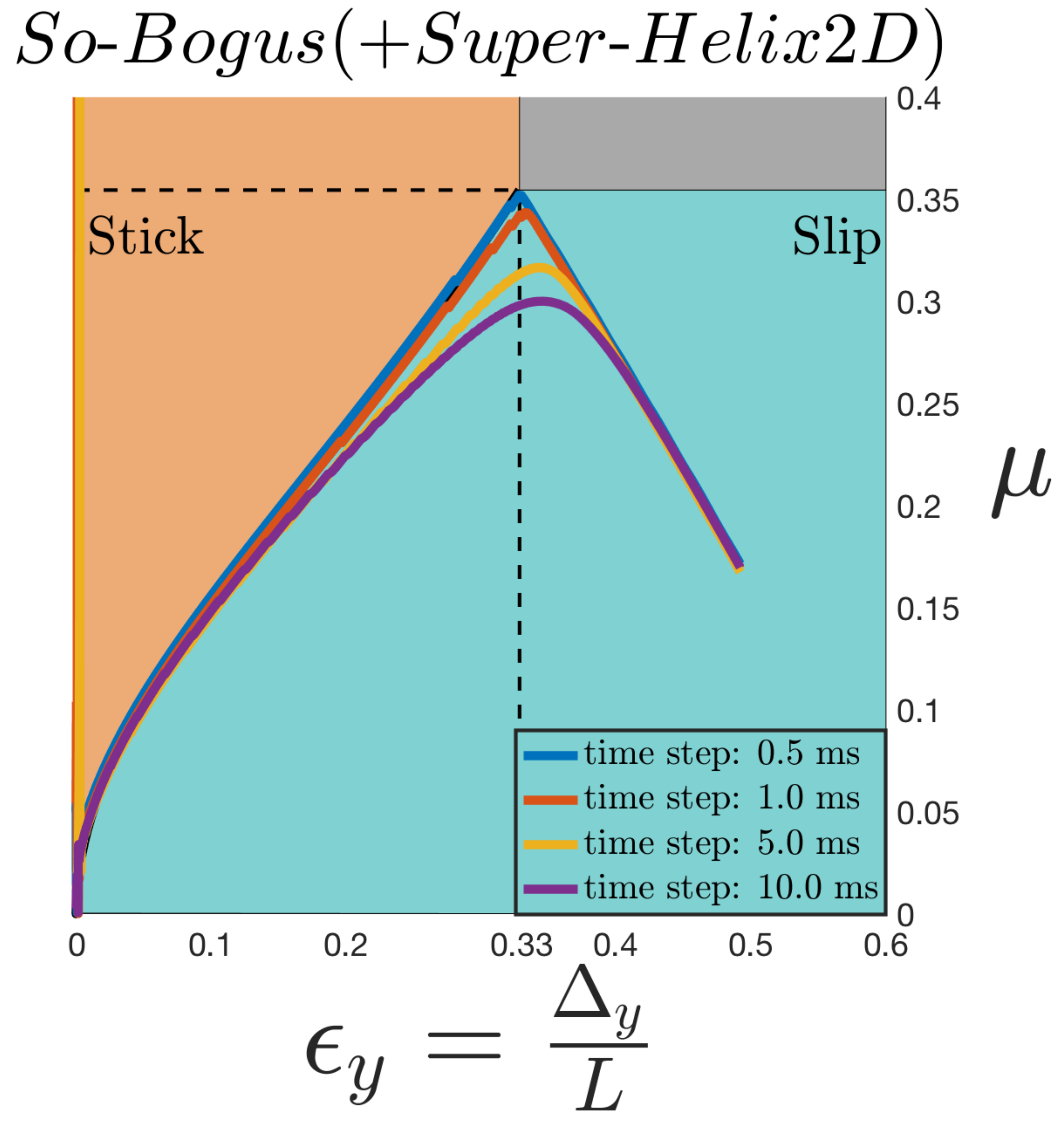
adaptive remeshing

KO



Results

Stick-slip test – sensitivity to the time-step



Results

Tested Code	Cantilever	Bend-Twist	Lateral Buckling	Stick-Slip
Rod				
Discrete Elastic Rod [Bergou et al. 2010]	OK (300 elts)	OK (200 elts)	–	–
Super-Helix [Bertails et al. 2006]	OK (50 elts)	OK (30 elts)	–	–
Super-Clothoid [Casati and Bertails-Descoubes 2013]	OK (20 elts)	OK (25 elts)	–	–
Ribbon				
Super-Ribbon [Charrondière et al. 2020]	OK (20 elts)	–	KO	–
Plate				
LibShell [Chen et al. 2018]	OK (Res 0)	–	OK (Res +)	–
Discrete Shell (+ LibShell) [Grinspun et al. 2003]	OK (Res +)	–	KO	–
ArcSim [Narain et al. 2012]	KO	–	×	–
Discrete Shell + ArcSim (**tentative fix** of ArcSim)	KO	–	×	–
Projective Dynamics [Bouaziz et al. 2014] (**fit**)	KO	–	×	–
Contact & friction				
Viscous Friction (+ Super-Helix 2D)	–	–	–	KO
So-Bogus [Daviet et al. 2011] (+Super-Helix 2D)	–	–	–	OK (dt=0.5 ms, tol $= 10^{-13}$ N)
Argus (≈ ArcSim + So-Bogus) [Li et al. 2018]	–	–	–	KO
Argus Non Adaptive (**fix** of Argus)	–	–	–	OK (dt=0.5 ms, tol $= 10^{-13}$ N)
Bridson-Harmon [Bridson et al. 2002; Harmon et al. 2008] (+ ArcSim)	–	–	–	KO
Projective Friction [Ly et al. 2020]	–	–	–	KO
Reference codes in Mechanical Engineering				
FenicsShell [Hale et al. 2018]	OK (Res 0, $P_{2,3}$ elts)	–	OK (Res 0, $P_{2,3}$ elts)	–
©Abaqus	OK (200 P_2 elts)	KO	KO	OK (dt=9 µs)

And the Oscar goes to ...

2020 Technical Academy Award (Oscar)

Weta FX special effects (NZ)

“for the Synapse Hair Simulation System”

Alita: Battle Angel (2019)

Gilles Daviet

Gilles Daviet

PhD (2016) INRIA Grenoble
(dir. F. Bertails-Descoubes)

So'Bogus

SIGGRAPH Asia 2011
2000 fibers (25 Hours CPU)

SIGGRAPH 2020

100 000 fibers (with contacts)
4×10^6 DoF

P. Alart and A. Curnier. 1991. A Mixed Formulation for Frictional Contact Problems Prone to Newton like Solution Methods. *Comput. Methods Appl. Mech. Eng.* 92, 3 (1991).

M. Jean and J.J. Moreau. 1988. Dynamics in the presence of unilateral contacts and dry friction : a numerical approach. In *Second Meeting on Unilateral Problems in Structural Analysis (Unilateral Problems in Structural Analysis, 2)*. Springer.

Conclusion

- Four protocols based on solid mechanics
- dimensionless parameters
- well-established master (bifurcation) curves
- limitations: only statics, no self-contact, purely elastic
- Results on 15 codes from Computer Graphics and 2 from Mechanical Engineering
- Encouraging results for Computer Graphics Models

Download the master curves:
https://elan.inrialpes.fr/people/vromerog/Validation134_sggph2021.html

Thank you for your attention