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**Correction to "Adaptive Control of Robot  
Manipulators with Flexible Joints"**

B. Brogliato and R. Lozano

I. INTRODUCTION

The above paper<sup>1</sup> contains two flaws that were recently brought to our attention by Hsu [1]. 1) The first mistake concerns the definition of the signal  $q_{2d}$  in (6) and (C.20): At time  $t = 0$ , the term  $q_{2d}(0)$  appears on both sides of the equalities in (6) and (C.20). Hence the initial conditions on the state and on the desired trajectory  $q_{1d}$  and its

first and second derivatives are constrained. 2) The second mistake is that the regressor matrix  $Y_4$  in (C.18) contains integral terms which are not shown to be bounded. Consequently, the stability analysis in Appendix C [(C.42)–(C.47)], based on some signal chasing, is not complete.

- Concerning point 1), it has been shown in [2] that the positive definite function  $V$  in (3) can be modified to

$$V = \frac{1}{2}v_1^T D(q_1)v_1 + \frac{1}{2}v_2^T J_m v_2 + \frac{1}{2}(\tilde{q}_1 - \tilde{q}_2)^T K(\tilde{q}_1 - \tilde{q}_2) \quad (1)$$

so that  $q_{2d}$  is now defined as (see [2, (23)])

$$q_{2d} = q_{1d} + K^{-1}(D(q_1)\dot{q}_{1r} + C(q_1, \dot{q}_1)\dot{q}_{1r} + g(q_1) - B_1 v_1) \quad (2)$$

where  $B_1 = B_1^T > 0$  is a constant feedback gain. This allows us to remove the constraint on the initial conditions and to prove exponential Lyapunov stability of the fixed parameters scheme; see [2]. In the adaptive case, the signal  $q_{2d}$  in (C.20) can be modified as well to avoid the initial conditions' constraints problem. Indeed the term  $K \int_0^t (v_1 - v_2) dt$  in the right-hand side of (C.1) is replaced by  $K(\tilde{q}_1 - \tilde{q}_2)$ . Then  $w_5$  in (C.2), (C.5), and (C.20) is modified to  $w_5 = -q_{1d}$ . This does not affect the definition of the other terms in (C.20) since only  $w_5$  has been modified.

Another possible solution was suggested to us [1], presented here in the nonadaptive case (compare with [3, (3) and (6)])

$$V = \frac{1}{2}v_1^T D(q_1)v_1 + \frac{1}{2}v_2^T J_m v_2 + \frac{1}{2}z^T K z \quad (3)$$

where  $z$  and  $q_{2d}$  are defined as

$$q_{2d} = q_{1d} - \lambda x + K^{-1}(-v_1 + D(q_1)\dot{q}_{1r} + C(q_1, \dot{q}_1)\dot{q}_{1r} + g(q_1)) \quad (4)$$

$$\dot{q}_{1r} := \dot{q}_{1d} - \lambda \tilde{q}_1 \quad (5)$$

$$\dot{x} := \tilde{q}_1 - \tilde{q}_2 \quad (6)$$

$$z = \lambda x + (\tilde{q}_1 - \tilde{q}_2) \quad (\dot{z} = v_1 - v_2) \quad (7)$$

$$u = -v_2 - J_m[-\ddot{q}_{2d} + \lambda \dot{q}_{2d}] - K[q_{1d} - q_{2d} - \lambda x]. \quad (8)$$

*Proof.* Consider  $V(v_1, v_2, z)$  as given by (3). Differentiating (3) we obtain

$$\dot{V} = v_1^T [D\dot{v}_1 + C v_1 + K z] + v_2^T [J_m v_2 - K z] \quad (9)$$

where we have used property P3 of [3] and the fact that [from (6) and (7)]  $\dot{z} = v_1 - v_2$ . Now, closely following [3, Appendix B]

$$D\dot{v}_1 + C v_1 + K z = \quad (10)$$

$$-g - K(q_1 - q_2) + D[-\dot{q}_{1d} + \lambda \dot{q}_1] + C[-\dot{q}_{1d} + \lambda \dot{q}_1] + K z = \quad (11)$$

$$-g - K(q_1 - q_2) + D[-\dot{q}_{1d} + \lambda \dot{q}_1] + C[-\dot{q}_{1d} + \lambda \dot{q}_1] + K[\lambda x + (q_1 - q_2) - (q_{1d} - q_{2d})] = \quad (12)$$

$$K q_{2d} + \{-g - D\dot{q}_{1r} - C\dot{q}_{1r} - K[\lambda x - q_{1d}]\}. \quad (13)$$

The last term within braces is the new  $w_1$ , using the notation of Appendix B of the original paper. From (4), obtained by requiring the expression in (13) to be equal to  $-v_1$ , we then have

$$\dot{V} = -v_1^T v_1 + v_2^T [J_m v_2 - K(\lambda x + (\tilde{q}_1 - \tilde{q}_2))]. \quad (14)$$

The last term within brackets is the new  $w_2$ , following our original notation. After some manipulations using the robot motor equations, one gets

$$w_2 = u + J_m[-\ddot{q}_{2d} + \lambda \dot{q}_{2d}] + K[q_{1d} - q_{2d} - \lambda x]. \quad (15)$$

Then, as in [3], setting  $w_2 = -v_2$ , one obtains (8). Consequently, the proposed control law yields (as in [3])

$$\dot{V} = -v_1^T v_1 - v_2^T v_2 \quad (16)$$

which is semi-definite negative as required.

- Concerning point 2): Fundamentally, the flaw comes from the fact that to be able to define signals  $q_{2d}$ ,  $\dot{q}_{2d}$  which are measurable and  $\ddot{q}_{2d}$  that does not depend on  $\ddot{q}_2$  (so that the control input in (14) is well defined), we need to filter the first dynamical equation (1) to suitably parameterize  $Dv_1$  in (C.18). In the above paper we filtered (1) with an integrator, hence the possible unboundedness of the regressor  $Y_4$  in (C.18). We now propose to use a first-order filter with transfer function  $\frac{1}{s+1}$  instead of the integrator.

Applying the filter to (1) one gets (we drop the arguments for convenience)

$$\frac{1}{s+1} \{D\ddot{q}_1 + C\dot{q}_1 + g + K(q_1 - q_2)\} = 0. \quad (17)$$

Now we have

$$\frac{1}{s+1} \{D\ddot{q}_1\} = D\dot{q}_1 - D(0)\dot{q}_1(0) - \frac{1}{s+1} \times \{D\dot{q}_1 - D(0)\dot{q}_1(0)\} - \frac{1}{s+1} \{D\dot{q}_1\}. \quad (18)$$

Indeed (18) can be obtained by first noting that  $D\ddot{q}_1 = \frac{d}{dt}(D\dot{q}_1) - \dot{D}\dot{q}_1$ . Now

$$\frac{1}{s+1} \{D\ddot{q}_1\} = \int_0^t e^{-(t-\tau)} D\ddot{q}_1 d\tau. \quad (19)$$

Then using integration by parts one gets

$$\begin{aligned} \frac{1}{s+1} \{D\ddot{q}_1\} &= e^{-t} \left[ \int_0^t e^{\tau} (D\dot{q}_1 - D(0)\dot{q}_1(0) - \int_0^{\tau} \dot{D}\dot{q}_1 dy) \right]_0^t \\ &\quad - \int_0^t e^{\tau} (D\dot{q}_1 - D(0)\dot{q}_1(0) - \int_0^{\tau} \dot{D}\dot{q}_1 dy) d\tau \end{aligned} \quad (20)$$

which finally yields

$$\begin{aligned} \frac{1}{s+1} \{D\ddot{q}_1\} &= D\dot{q}_1 - D(0)\dot{q}_1(0) - \int_0^t \dot{D}\dot{q}_1 d\tau - \int_0^t e^{-(t-\tau)} \\ &\quad \times \left( D\dot{q}_1 - D(0)\dot{q}_1(0) - \int_0^{\tau} \dot{D}\dot{q}_1 dy \right) d\tau. \end{aligned} \quad (21)$$

Using the fact that (still integrating by parts)

$$\begin{aligned} \int_0^t e^{-(t-\tau)} \dot{D}\dot{q}_1 d\tau &= \int_0^t \dot{D}\dot{q}_1 d\tau - \int_0^t e^{-(t-\tau)} \left( \int_0^{\tau} \dot{D}\dot{q}_1 dy \right) d\tau \end{aligned} \quad (22)$$

we then get (18) by combining (21) and (22).

Now from (17) and (18) we have

$$\begin{aligned}
 D\dot{q}_1 &= D(0)\dot{q}_1(0) + \frac{1}{s+1}\{D\dot{q}_1 - D(0)\dot{q}_1(0)\} \\
 &+ \frac{1}{s+1}\{\dot{D}\dot{q}_1\} - \frac{1}{s+1}\{C\dot{q}_1 + g + Kq_1\} \\
 &+ \frac{1}{s+1}\{Kq_2\}. \tag{23}
 \end{aligned}$$

The terms between brackets can be written as  $Y_i(q_1, \dot{q}_1)\theta_i$  for some constant vector  $\theta_i$ . Hence  $\frac{1}{s+1}\{Y_i(q_1, \dot{q}_1)\theta_i\} = \frac{1}{s+1}\{Y_i(q_1, \dot{q}_1)\}\theta_i \triangleq Y_{if}\theta_i$  with  $\dot{Y}_{if} + Y_{if} = Y_i(q_1, \dot{q}_1)$ . It follows that (C.18) can be written as

$$Dv_1 = Y_4\theta_4 \tag{24}$$

with  $\dot{Y}_4 + Y_4 = Y_4$  for some  $Y_4(q_1, \dot{q}_1, q_2)$ . Note also that  $q_{2d}, \dot{q}_{2d}$  in (C.20) can be computed from position and velocities measurements only which is crucial for the algorithm to be implementable. From the new parameterization, it follows that if  $q_1, \dot{q}_1$  and  $q_2$  are bounded, so is  $Y_4$ . This was not the case when integrators were used.

The stability analysis is much simplified. Note first that the modification of  $Y_4\theta_4$  leaves the rest of the analysis and algorithm unchanged. Hence from (C.41),  $q_1$  and  $\dot{q}_1$  are bounded. From (23) it follows that  $q_{2f} = \frac{1}{s+1}\{q_2\}$  is bounded. Hence from (C.20)  $q_{2d}$  is bounded, and consequently  $q_2$  and  $\dot{q}_1$  are bounded. Boundedness of  $\dot{q}_2$  follows by differentiating (C.20) which proves that  $\dot{q}_{2d}$  is bounded. Hence from (C.41)  $\dot{q}_2$  is bounded. Boundedness of  $u$  can be inferred by inspecting (C.27) and (C.32). Consequently  $\ddot{q}_2$  is bounded also.

#### REFERENCES

- [1] L. Hsu, private correspondence, COPPE/Federal Univ., Rio de Janeiro, Brazil, Mar. 1995.
- [2] B. Brogliato, R. Ortega, and R. Lozano, "Global tracking controllers for flexible joint manipulators: A comparative study," *Automatica*, vol. 31, no. 7, pp. 941-956, 1995.
- [3] R. Lozano and B. Brogliato, "Adaptive control of robot manipulators with flexible joints," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 174-181, Feb. 1992.