



HAL
open science

Correction to "Adaptive control of flexible joint manipulators"

Bernard Brogliato, Rogelio Lozano

► **To cite this version:**

Bernard Brogliato, Rogelio Lozano. Correction to "Adaptive control of flexible joint manipulators". IEEE Transactions on Automatic Control, 1996, 41 (6), pp.920-922. 10.1109/9.506251 . hal-03592564

HAL Id: hal-03592564

<https://inria.hal.science/hal-03592564>

Submitted on 8 Mar 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

**Correction to "Adaptive Control of Robot
Manipulators with Flexible Joints"**

B. Brogliato and R. Lozano

I. INTRODUCTION

The above paper¹ contains two flaws that were recently brought to our attention by Hsu [1]. 1) The first mistake concerns the definition of the signal q_{2d} in (6) and (C.20): At time $t = 0$, the term $q_{2d}(0)$ appears on both sides of the equalities in (6) and (C.20). Hence the initial conditions on the state and on the desired trajectory q_{1d} and its

first and second derivatives are constrained. 2) The second mistake is that the regressor matrix Y_4 in (C.18) contains integral terms which are not shown to be bounded. Consequently, the stability analysis in Appendix C [(C.42)–(C.47)], based on some signal chasing, is not complete.

- Concerning point 1), it has been shown in [2] that the positive definite function V in (3) can be modified to

$$V = \frac{1}{2}v_1^T D(q_1)v_1 + \frac{1}{2}v_2^T J_m v_2 + \frac{1}{2}(\tilde{q}_1 - \tilde{q}_2)^T K(\tilde{q}_1 - \tilde{q}_2) \quad (1)$$

so that q_{2d} is now defined as (see [2, (23)])

$$q_{2d} = q_{1d} + K^{-1}(D(q_1)\dot{q}_{1r} + C(q_1, \dot{q}_1)\dot{q}_{1r} + g(q_1) - B_1 v_1) \quad (2)$$

where $B_1 = B_1^T > 0$ is a constant feedback gain. This allows us to remove the constraint on the initial conditions and to prove exponential Lyapunov stability of the fixed parameters scheme; see [2]. In the adaptive case, the signal q_{2d} in (C.20) can be modified as well to avoid the initial conditions' constraints problem. Indeed the term $K \int_0^t (v_1 - v_2) dt$ in the right-hand side of (C.1) is replaced by $K(\tilde{q}_1 - \tilde{q}_2)$. Then w_5 in (C.2), (C.5), and (C.20) is modified to $w_5 = -q_{1d}$. This does not affect the definition of the other terms in (C.20) since only w_5 has been modified.

Another possible solution was suggested to us [1], presented here in the nonadaptive case (compare with [3, (3) and (6)])

$$V = \frac{1}{2}v_1^T D(q_1)v_1 + \frac{1}{2}v_2^T J_m v_2 + \frac{1}{2}z^T K z \quad (3)$$

where z and q_{2d} are defined as

$$q_{2d} = q_{1d} - \lambda x + K^{-1}(-v_1 + D(q_1)\dot{q}_{1r} + C(q_1, \dot{q}_1)\dot{q}_{1r} + g(q_1)) \quad (4)$$

$$\dot{q}_{1r} := \dot{q}_{1d} - \lambda \tilde{q}_1 \quad (5)$$

$$\dot{x} := \tilde{q}_1 - \tilde{q}_2 \quad (6)$$

$$z = \lambda x + (\tilde{q}_1 - \tilde{q}_2) \quad (\dot{z} = v_1 - v_2) \quad (7)$$

$$u = -v_2 - J_m[-\ddot{q}_{2d} + \lambda \dot{q}_{2d}] - K[q_{1d} - q_{2d} - \lambda x]. \quad (8)$$

Proof. Consider $V(v_1, v_2, z)$ as given by (3). Differentiating (3) we obtain

$$\dot{V} = v_1^T [D\dot{v}_1 + C v_1 + K z] + v_2^T [J_m v_2 - K z] \quad (9)$$

where we have used property P3 of [3] and the fact that [from (6) and (7)] $\dot{z} = v_1 - v_2$. Now, closely following [3, Appendix B]

$$D\dot{v}_1 + C v_1 + K z = \quad (10)$$

$$-g - K(q_1 - q_2) + D[-\dot{q}_{1d} + \lambda \dot{q}_1] + C[-\dot{q}_{1d} + \lambda \dot{q}_1] + K z = \quad (11)$$

$$-g - K(q_1 - q_2) + D[-\dot{q}_{1d} + \lambda \dot{q}_1] + C[-\dot{q}_{1d} + \lambda \dot{q}_1] + K[\lambda x + (q_1 - q_2) - (q_{1d} - q_{2d})] = \quad (12)$$

$$K q_{2d} + \{-g - D\dot{q}_{1r} - C\dot{q}_{1r} - K[\lambda x - q_{1d}]\}. \quad (13)$$

The last term within braces is the new w_1 , using the notation of Appendix B of the original paper. From (4), obtained by requiring the expression in (13) to be equal to $-v_1$, we then have

$$\dot{V} = -v_1^T v_1 + v_2^T [J_m v_2 - K(\lambda x + (\tilde{q}_1 - \tilde{q}_2))]. \quad (14)$$

The last term within brackets is the new w_2 , following our original notation. After some manipulations using the robot motor equations, one gets

$$w_2 = u + J_m[-\ddot{q}_{2d} + \lambda \dot{q}_{2d}] + K[q_{1d} - q_{2d} - \lambda x]. \quad (15)$$

Then, as in [3], setting $w_2 = -v_2$, one obtains (8). Consequently, the proposed control law yields (as in [3])

$$\dot{V} = -v_1^T v_1 - v_2^T v_2 \quad (16)$$

which is semi-definite negative as required.

- Concerning point 2): Fundamentally, the flaw comes from the fact that to be able to define signals q_{2d} , \dot{q}_{2d} which are measurable and \ddot{q}_{2d} that does not depend on \ddot{q}_2 (so that the control input in (14) is well defined), we need to filter the first dynamical equation (1) to suitably parameterize Dv_1 in (C.18). In the above paper we filtered (1) with an integrator, hence the possible unboundedness of the regressor Y_4 in (C.18). We now propose to use a first-order filter with transfer function $\frac{1}{s+1}$ instead of the integrator.

Applying the filter to (1) one gets (we drop the arguments for convenience)

$$\frac{1}{s+1} \{D\ddot{q}_1 + C\dot{q}_1 + g + K(q_1 - q_2)\} = 0. \quad (17)$$

Now we have

$$\frac{1}{s+1} \{D\ddot{q}_1\} = D\dot{q}_1 - D(0)\dot{q}_1(0) - \frac{1}{s+1} \times \{D\dot{q}_1 - D(0)\dot{q}_1(0)\} - \frac{1}{s+1} \{D\dot{q}_1\}. \quad (18)$$

Indeed (18) can be obtained by first noting that $D\ddot{q}_1 = \frac{d}{dt}(D\dot{q}_1) - \dot{D}\dot{q}_1$. Now

$$\frac{1}{s+1} \{D\ddot{q}_1\} = \int_0^t e^{-(t-\tau)} D\ddot{q}_1 d\tau. \quad (19)$$

Then using integration by parts one gets

$$\begin{aligned} \frac{1}{s+1} \{D\ddot{q}_1\} &= e^{-t} \left[\int_0^t e^{\tau} \left(D\dot{q}_1 - D(0)\dot{q}_1(0) - \int_0^{\tau} \dot{D}\dot{q}_1 dy \right) \right]_0^t \\ &\quad - \int_0^t e^{\tau} \left(D\dot{q}_1 - D(0)\dot{q}_1(0) - \int_0^{\tau} \dot{D}\dot{q}_1 dy \right) d\tau \end{aligned} \quad (20)$$

which finally yields

$$\begin{aligned} \frac{1}{s+1} \{D\ddot{q}_1\} &= D\dot{q}_1 - D(0)\dot{q}_1(0) - \int_0^t \dot{D}\dot{q}_1 d\tau - \int_0^t e^{-(t-\tau)} \\ &\quad \times \left(D\dot{q}_1 - D(0)\dot{q}_1(0) - \int_0^{\tau} \dot{D}\dot{q}_1 dy \right) d\tau. \end{aligned} \quad (21)$$

Using the fact that (still integrating by parts)

$$\begin{aligned} \int_0^t e^{-(t-\tau)} \dot{D}\dot{q}_1 d\tau &= \int_0^t \dot{D}\dot{q}_1 d\tau - \int_0^t e^{-(t-\tau)} \left(\int_0^{\tau} \dot{D}\dot{q}_1 dy \right) d\tau \end{aligned} \quad (22)$$

we then get (18) by combining (21) and (22).

Now from (17) and (18) we have

$$\begin{aligned}
 D\dot{q}_1 &= D(0)\dot{q}_1(0) + \frac{1}{s+1}\{D\dot{q}_1 - D(0)\dot{q}_1(0)\} \\
 &+ \frac{1}{s+1}\{\dot{D}\dot{q}_1\} - \frac{1}{s+1}\{C\dot{q}_1 + g + Kq_1\} \\
 &+ \frac{1}{s+1}\{Kq_2\}.
 \end{aligned} \tag{23}$$

The terms between brackets can be written as $Y_i(q_1, \dot{q}_1)\theta_i$ for some constant vector θ_i . Hence $\frac{1}{s+1}\{Y_i(q_1, \dot{q}_1)\theta_i\} = \frac{1}{s+1}\{Y_i(q_1, \dot{q}_1)\}\theta_i \triangleq Y_{if}\theta_i$ with $\dot{Y}_{if} + Y_{if} = Y_i(q_1, \dot{q}_1)$. It follows that (C.18) can be written as

$$Dv_1 = Y_4\theta_4 \tag{24}$$

with $\dot{Y}_4 + Y_4 = Y_4$ for some $Y_4(q_1, \dot{q}_1, q_2)$. Note also that q_{2d}, \dot{q}_{2d} in (C.20) can be computed from position and velocities measurements only which is crucial for the algorithm to be implementable. From the new parameterization, it follows that if q_1, \dot{q}_1 and q_2 are bounded, so is Y_4 . This was not the case when integrators were used.

The stability analysis is much simplified. Note first that the modification of $Y_4\theta_4$ leaves the rest of the analysis and algorithm unchanged. Hence from (C.41), q_1 and \dot{q}_1 are bounded. From (23) it follows that $q_{2f} = \frac{1}{s+1}\{q_2\}$ is bounded. Hence from (C.20) q_{2d} is bounded, and consequently q_2 and \dot{q}_1 are bounded. Boundedness of \dot{q}_2 follows by differentiating (C.20) which proves that \dot{q}_{2d} is bounded. Hence from (C.41) \dot{q}_2 is bounded. Boundedness of u can be inferred by inspecting (C.27) and (C.32). Consequently \ddot{q}_2 is bounded also.

REFERENCES

- [1] L. Hsu, private correspondence, COPPE/Federal Univ., Rio de Janeiro, Brazil, Mar. 1995.
- [2] B. Brogliato, R. Ortega, and R. Lozano, "Global tracking controllers for flexible joint manipulators: A comparative study," *Automatica*, vol. 31, no. 7, pp. 941-956, 1995.
- [3] R. Lozano and B. Brogliato, "Adaptive control of robot manipulators with flexible joints," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 174-181, Feb. 1992.