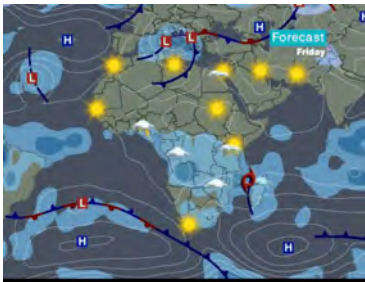




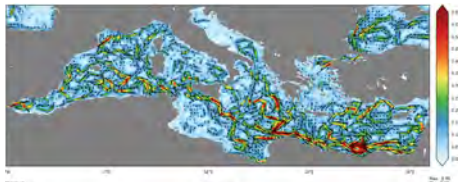
Numerical forecast systems for geophysical fluids and climate

Numerical forecast systems

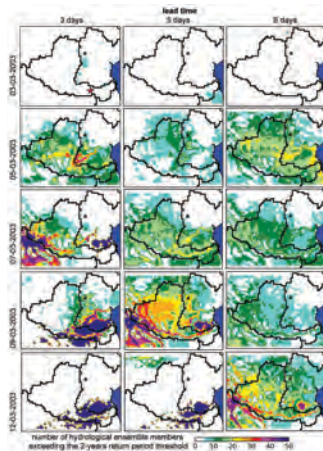
Weather forecast



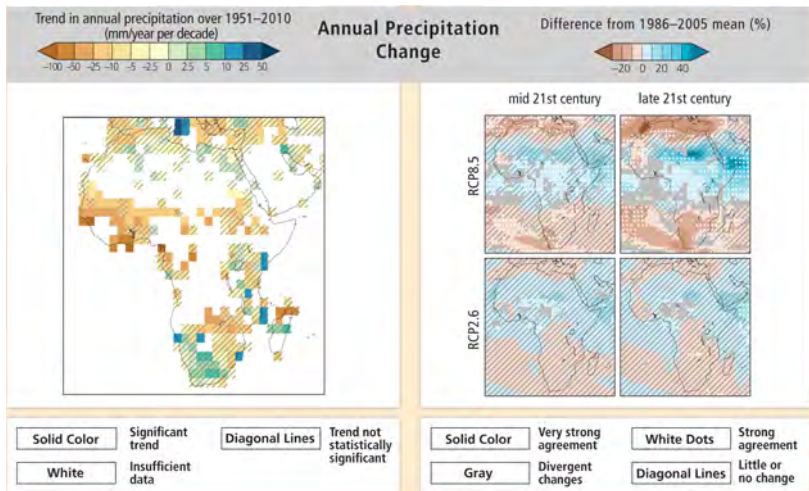
Mercator Ocean



African flood forecasting system



Numerical forecast systems



IPCC AR5 report

Numerical forecast systems



Coastal engineering



Risk management

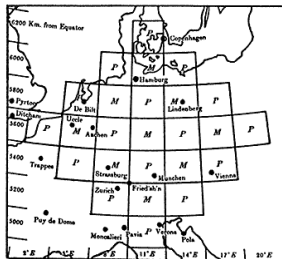
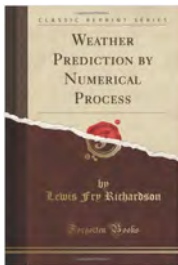


Floods

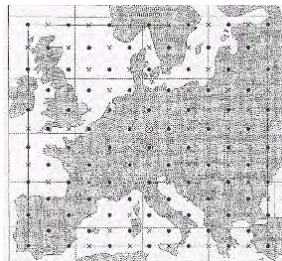
The beginnings of weather forecasting



Lewis F. Richardson (1881-1953)



Meteorological stations



Computation grid

Richardson's forecast factory (1922)

A myriad computers are at work upon the weather of the part of the map where each sits, but each computer attends only to one equation or part of an equation. The work of each region is coordinated by an official of higher rank. Numerous little "night signs" display the instantaneous values so that neighbouring computers can read them. Each number is thus displayed in three adjacent zones so as to maintain communication to the North and South on the map.



The forecast factory

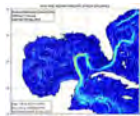
From the floor of the pit a tall pillar rises to half the height of the hall. It carries a large pulpit on its top. In this sits the man in charge of the whole theatre; he is surrounded by several assistants and messengers. One of his duties is to maintain a uniform speed of progress in all parts of the globe. In this respect he is like the conductor of an orchestra in which the instruments are slide-rules and calculating machines.

A short history of numerical forecast systems

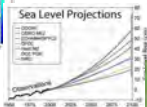
Numerical forecast systems have been developed progressively for all disciplines related to geophysical fluids.



Weather forecast



Ocean simulation and forecast



Climate change forecast



*Flood, air quality,
agriculture...*

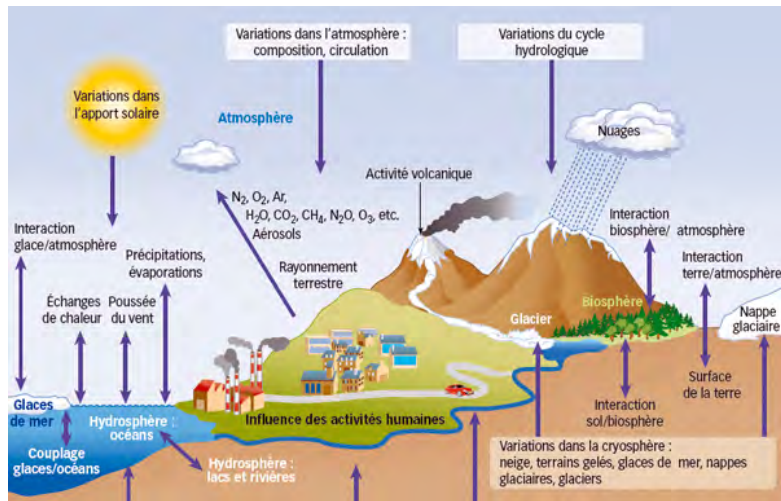
1940

1960

1980

2000

Step 1 : translate reality into equations



Step 1 - Modeling : translate reality into equations

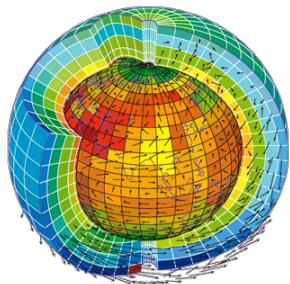
- ▶ **Main physical laws** : conservation of mass, energy, water...
- ▶ **Boundary conditions** : exchange fluxes (heat, mass...)
- ▶ **Some specific aspects** : boundary layers, micro-physics...

Step 1 - Modeling : translate reality into equations

- ▶ **Main physical laws** : conservation of mass, energy, water...
- ▶ **Boundary conditions** : exchange fluxes (heat, mass...)
- ▶ **Some specific aspects** : boundary layers, micro-physics...

$$\begin{aligned} \frac{\partial u}{\partial t} + \mathbf{U} \cdot \nabla u - fv + \frac{1}{\rho_0} \frac{\partial p}{\partial x} - \mathcal{F}(u) &= 0 & \frac{\partial T}{\partial t} + \mathbf{U} \cdot \nabla T - \mathcal{D}_T(T) &= 0 \\ \frac{\partial v}{\partial t} + \mathbf{U} \cdot \nabla v + fu + \frac{1}{\rho_0} \frac{\partial p}{\partial y} - \mathcal{F}(v) &= 0 & \frac{\partial S}{\partial t} + \mathbf{U} \cdot \nabla S - \mathcal{D}_S(S) &= 0 \\ \frac{\partial p}{\partial z} &= -\rho g & \rho &= \rho(T, S, p) \\ \operatorname{div} \mathbf{U} &= 0 & &+ \text{conditions aux limites} \end{aligned}$$

From infinite to finite dimension

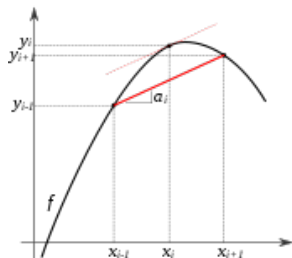


Approximate representation of :

- ▶ the domain : **mesh**
Météo-France : 1,3 km, a few seconds
Mercator-Ocean : $1/36^\circ$ (2-3 km), a few mns
- ▶ the equations : **numerical schemes**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x-h)}{2h}$$

$$\rightarrow f'(x_i) \simeq \frac{1}{2\Delta x} f(x_{i+1}) - \frac{1}{2\Delta x} f(x_{i-1})$$



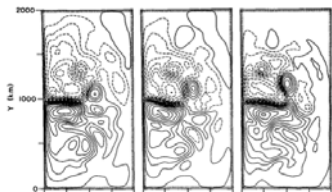
And a lot of calculation

This results in a (large) system of equations, the solution of which allows to compute the system state at the next time step, starting from the present one. ($\approx 10^6 - 10^9$ unknowns)



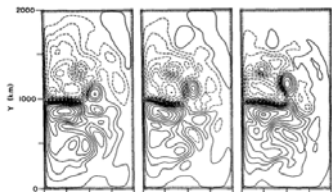
Supercomputer Jean Zay — IDRIS, Orsay — 90 000 cores, 28 Pflops peak

At this stage : a numerical model that is useful to understand the behavior of geophysical systems...

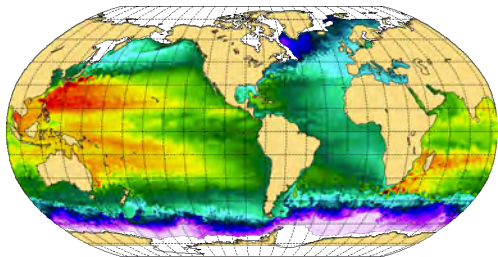


Holland (1978) : evidence for intense meso-scale eddy activity in mid-latitude ocean

At this stage : a numerical model that is useful to understand the behavior of geophysical systems...

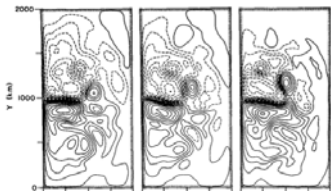


Holland (1978) : evidence for intense meso-scale eddy activity in mid-latitude ocean

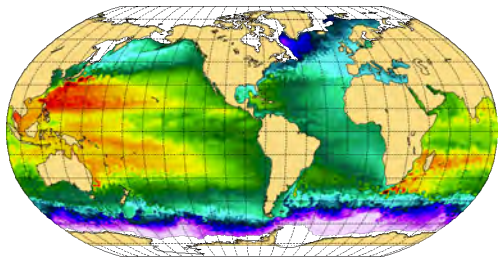


1/4° global ocean simulation (sea surface elevation)

At this stage : a numerical model that is useful to understand the behavior of geophysical systems...



Holland (1978) : evidence for intense meso-scale eddy activity in mid-latitude ocean



1/4° global ocean simulation (sea surface elevation)

...but is unable to produce forecasts. **To provide a forecast, one needs to know the present state of the system.**

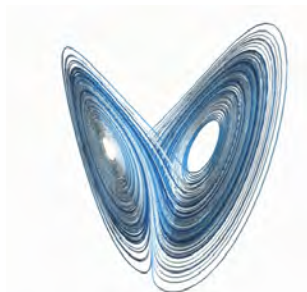
Chaos, butterfly effect...

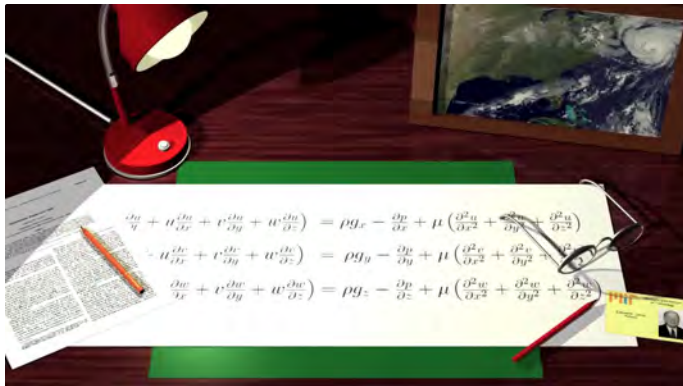
Atmosphere and ocean are chaotic systems : small perturbations may lead to large deviations.



Edward Lorenz
(1917-2008)

$$\begin{cases} \frac{dx}{dt} = \alpha(y - x) \\ \frac{dy}{dt} = \beta x - y - xz \\ \frac{dz}{dt} = -\gamma z + xy \end{cases}$$





<https://www.chaos-math.org>

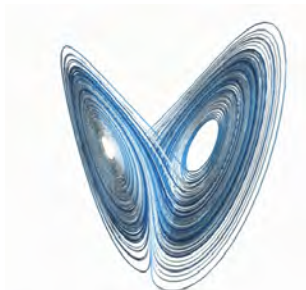
Chaos, butterfly effect...

Atmosphere and ocean are chaotic systems : small perturbations may lead to large deviations.



Edward Lorenz
(1917-2008)

$$\begin{cases} \frac{dx}{dt} = \alpha(y - x) \\ \frac{dy}{dt} = \beta x - y - xz \\ \frac{dz}{dt} = -\gamma z + xy \end{cases}$$



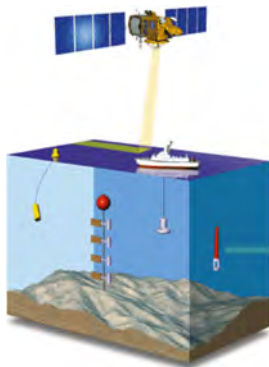
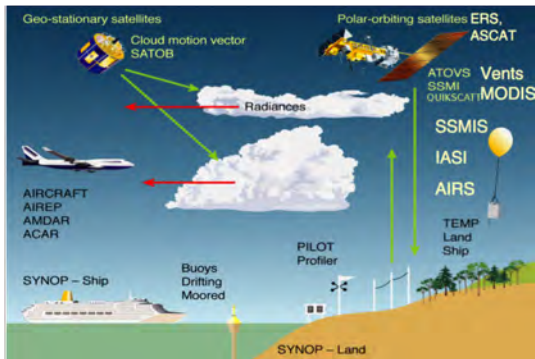
Does the flap of a butterfly's wings in Brazil set off a tornado in Texas? (139th meeting of the American Association for the Advancement of Science, 1972)

Apparent chaos, but an underlying order...

www.chaos-math.org

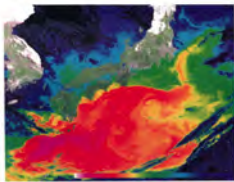
To foresee the future, you have to know the present

- ▶ A wide variety of observations : in situ / satellite, quantitative / qualitative. . .



To foresee the future, you have to know the present

- ▶ A wide variety of observations : in situ / satellite, quantitative / qualitative. . .
- ▶ Earth observation from space has initiated a revolution in geophysical numerical forecast systems



sea surface temperature



satellite observations



of satellite obs in Arpege

To foresee the future, you have to know the present

- ▶ Use all available information to reconstruct the current state of the system



past and present observations

$$\left\{ \begin{array}{l} \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial y} + \sigma \frac{\partial u}{\partial \sigma} - f(v - v_g) + \frac{\partial \phi}{\partial x} \Big|_{\sigma} + F_x = 0 \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + w \frac{\partial v}{\partial y} + \sigma \frac{\partial v}{\partial \sigma} + f(u - u_g) + \frac{\partial \phi}{\partial y} \Big|_{\sigma} + F_y = 0 \\ \frac{\partial \phi}{\partial(T/\theta)} = -C_p g (1 + 0.85q_v) \\ \frac{\partial p_s}{\partial t} = - \int_{\sigma}^{\sigma_0} \left[\frac{\partial(p_s u)}{\partial x} + \frac{\partial(p_s v)}{\partial y} \right] d\sigma \\ \frac{\partial \sigma}{\partial t} = \frac{1}{p_s} \left\{ \sigma \int_{\sigma}^{\sigma_0} \left[\frac{\partial(p_s u)}{\partial x} + \frac{\partial(p_s v)}{\partial y} \right] d\sigma - \int_{\sigma}^{\sigma_0} \left[\frac{\partial(p_s u)}{\partial x} + \frac{\partial(p_s v)}{\partial y} \right] d\sigma \right\} \\ \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} + \sigma \frac{\partial \theta}{\partial \sigma} + \frac{\partial R_{net}}{\partial \sigma} + F_{\theta} + p_{\theta} = 0 \\ \frac{\partial q_v}{\partial t} + u \frac{\partial q_v}{\partial x} + v \frac{\partial q_v}{\partial y} + \sigma \frac{\partial q_v}{\partial \sigma} + F_{q_v} + p_{q_v} = 0 \end{array} \right.$$

mathematical models

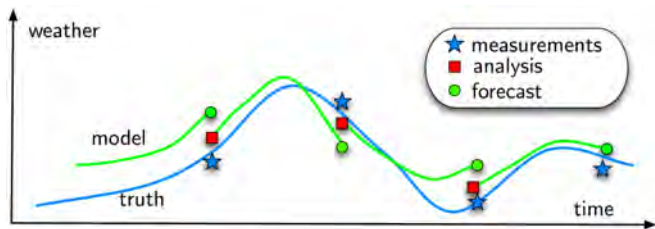


statistics

→ Data assimilation

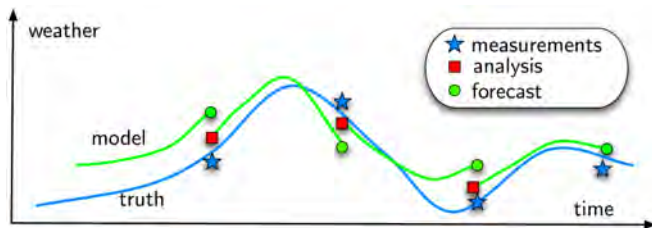
Step 2 : data assimilation

- ▶ You all already experienced data assimilation



Step 2 : data assimilation

- ▶ You all already experienced data assimilation



- ▶ Mathematical techniques that combine different sources of information in an optimal way : optimal control, stochastic filtering...

$$\min_u \int_0^T \|\mathbf{y}^{obs}(t) - \mathcal{H}(\mathbf{x}(u, t))\| dt \quad ; \quad \max p(\mathbf{x}(t) | \mathbf{x}(t-1), \mathbf{y}^{obs}(t))$$

From space rockets to weather forecast...



Lev Pontryagin
(1908-1988)



Rudolf Kalman
(1930-2016)



Jacques-Louis Lions
(1928-2001)

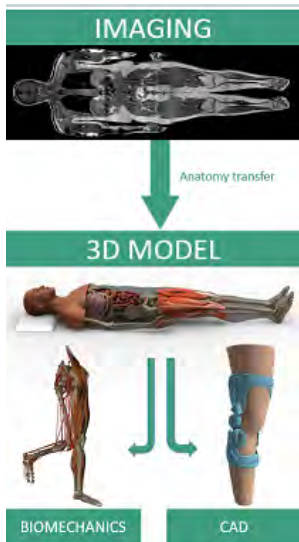


François-Xavier Le Dimet
(1945-)



NASA, 1961

... and from weather forecast to multiple applications



Where are we ?

At this stage, we have :

- ▶ a numerical model
- ▶ a method to initialize it and/or tune its parameters

→ We can therefore make forecasts

Where are we ?

At this stage, we have :

- ▶ a numerical model
- ▶ a method to initialize it and/or tune its parameters

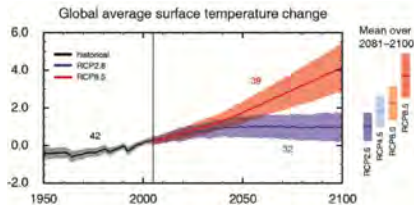
→ We can therefore make forecasts

→ Are they reliable ?

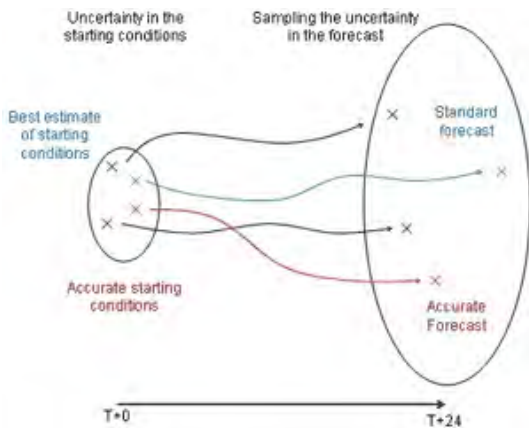
Step 3 : quantifying uncertainty

- ▶ There are still uncertainties all over the place
- ▶ Nothing can be done against chaos

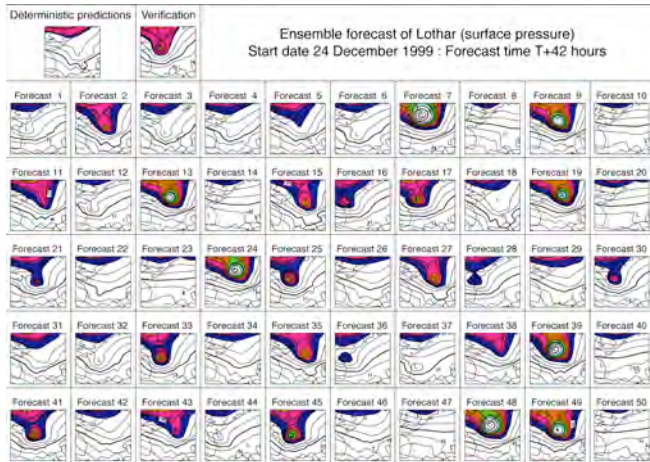
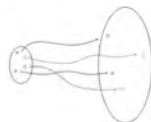
→ provide probabilistic forecasts



A simple concept : ensemble methods



A simple concept : ensemble methods



Source : Palmer, 2006

Building a numerical forecast system

- ▶ A very multidisciplinary work. Progress is being made at all levels, by all actors.
- ▶ Many playgrounds for applied mathematicians and computer scientists

mathematical modeling
mathematical analysis
dynamical systems
...

numerical analysis
numerical methods
optimization
...

image processing
uncertainty quantification
HPC
...

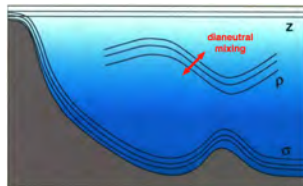
→ a few examples

Modeling diffusion of oceanic tracers

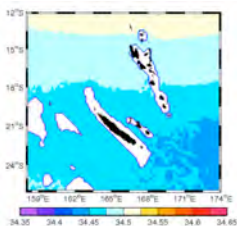
Goal improve the long-term representation of water masses (T,S)

- ▶ **Real ocean**
isopycnal mixing \gg diapycnal mixing
- ▶ **Numerical models**
 Δ or Δ^2 in z or σ coordinates

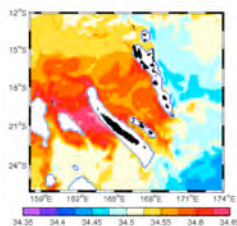
→ **Development of an adaptive diffusion tensor**



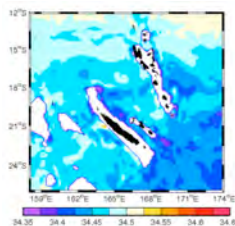
Climatology



Third order Upwind scheme



Third order Upwind scheme
with rotated diffusion



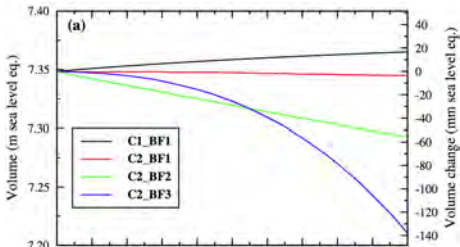
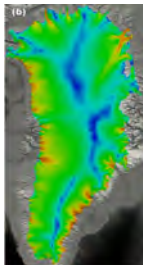
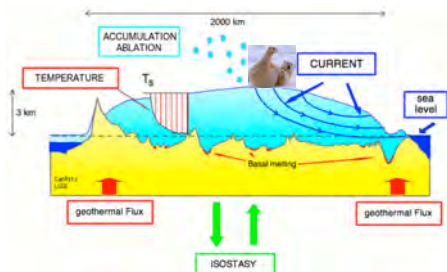
Salinity
at 1000 m

Marchesiello et al., 2009 ; Lemarié et al., 2012.

Parameter estimation for ice caps modeling

Motivation large uncertainties on the topography of the bedrock and the basal friction

→ estimation via assimilation of observations of the surface velocity : EnKF, 4DVar, approximate 4DVar

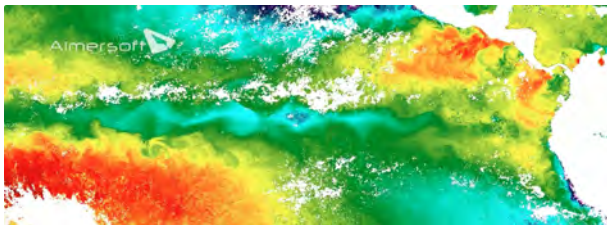


Estimated surface velocity and melting scenarii

Gillet-Chaulet et al., 2012; Bonan et al., 2014

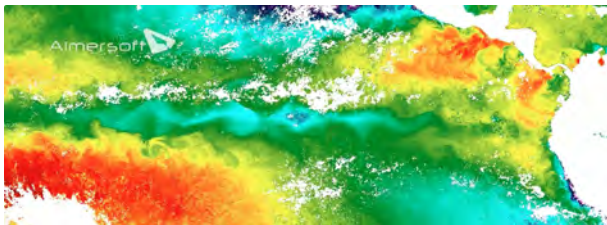
Assimilation of (sequences of) images

- ▶ Huge amount of images, containing significant information



Assimilation of (sequences of) images

- ▶ Huge amount of images, containing significant information



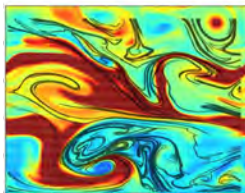
- ▶ Which relation between a photograph and a numerical model?

image analysis



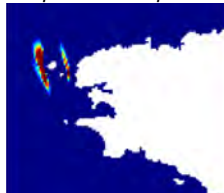
curvelets decomposition

dynamical systems



chlorophyll and Lyapunov exponents

optimal transport

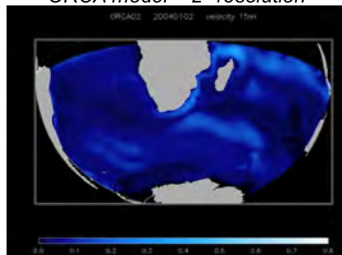


optimal field with obstacles

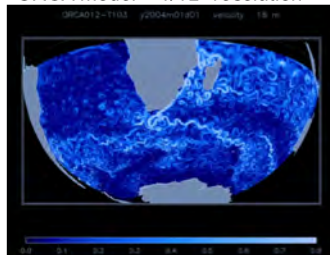
A massive use of supercomputers

- Increasing the resolution is often one of the key points for improving the quality of the model results (small scale activity + scale interactions).

ORCA model – 2° resolution



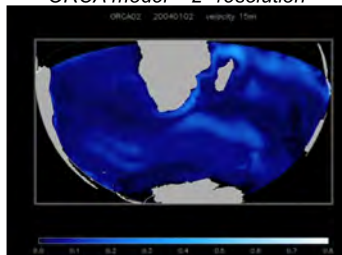
ORCA model – 1/12° resolution



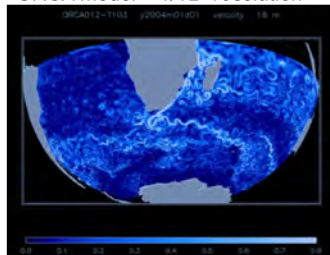
A massive use of supercomputers

- ▶ Increasing the resolution is often one of the key points for improving the quality of the model results (small scale activity + scale interactions).

ORCA model – 2° resolution



ORCA model – 1/12° resolution



- ▶ Data assimilation : \times one or two orders of magnitude
- ▶ Quantification of uncertainty : \times several orders of magnitude

Numerical systems for ocean/atmosphere/climate simulations generally require huge computational resources \rightarrow HPC issues