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Bounding and simulating contextual correlations in quantum theory

Armin Tavakoli, Emmanuel Zambrini Cruzeiro, Roope Uola, Alastair A. Abbott

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Hierarchy 1

“Bounding and Simulating Contextual Correlations in Quantum Theory”

Armin Tavakoli, Emmanuel Zambrini Cruzeiro, Roope Uola, Alastair A. Abbott

PRX Quantum 2, 020334 (2021); arXiv:2010.04751

Alastair A. Abbott

Inria Grenoble



**UNIVERSITÉ
DE GENÈVE**



Inria

SDP Hierarchies for Quantum Correlations

- Outer approximations of sets of quantum correlations using semidefinite programming (SDP)

- **Bell scenarios:** $p(a, b|x, y) = \langle \psi | E_a^x E_b^y | \psi \rangle$

- Navascues-Pironio-Acin (NPA) hierarchy [NJP 2008]

- **Prepare-and-measure scenarios:**

- Dimension bounded correlations [Navascues, Vertesi, PRL 2015]
 - Bounded overlap between states [Y. Wang *et al.*, npjQI 2019]
- } $p(b|x, y) = \langle \psi_x | E_b^y | \psi_x \rangle = \text{Tr}(E_b^y |\psi_x\rangle\langle\psi_x|)$

SDP Hierarchies for Quantum Correlations

- Outer approximations of sets of quantum correlations using semidefinite programming (SDP)
- **Bell scenarios:** $p(a, b|x, y) = \langle \psi | E_a^x E_b^y | \psi \rangle$
 - Navascues-Pironio-Acin (NPA) hierarchy [NJP 2008]
- **Prepare-and-measure scenarios:**
 - Dimension bounded correlations [Navascues, Vertesi, PRL 2015]
 - Bounded overlap between states [Y. Wang *et al.*, npjQI 2019]
 - Contextuality?
- **Difficulties:**
 - How to enforce operational equivalences?
 - Mixed states and POVMs? (Standard purification arguments don't go through)

$$\left. \begin{array}{l} \text{Dimension bounded correlations [Navascues, Vertesi, PRL 2015]} \\ \text{Bounded overlap between states [Y. Wang et al., npjQI 2019]} \end{array} \right\} p(b|x, y) = \langle \psi_x | E_b^y | \psi_x \rangle = \text{Tr}(E_b^y |\psi_x\rangle\langle\psi_x|)$$

Preparation Noncontextuality & Zero-Information Games

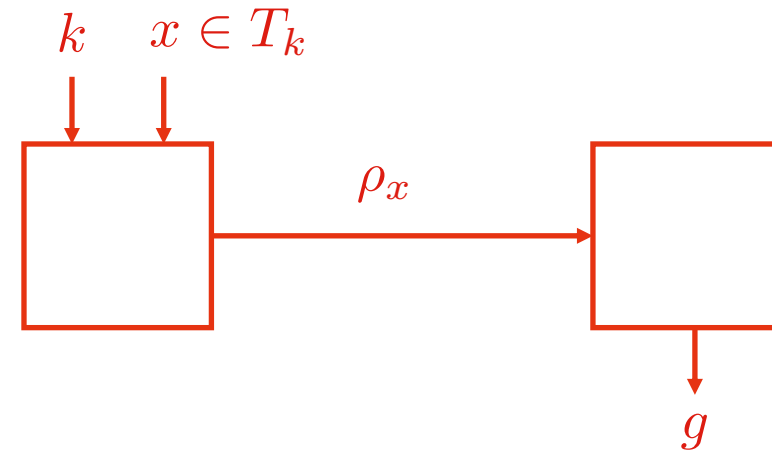
- Focus on preparation noncontextuality for simplicity

$$\frac{1}{2}(\rho_0 + \rho_1) = \frac{1}{2}(\rho_2 + \rho_3) = \frac{1}{2}(\rho_4 + \rho_5)$$

$$T_1 = \{\rho_0, \rho_1\}$$

$$T_2 = \{\rho_2, \rho_3\}$$

$$T_3 = \{\rho_4, \rho_5\}$$



$$\begin{aligned} \text{operational equivalences satisfied} &\iff \max p_{\text{guess}} = \max p(g = k) = \frac{1}{3} \\ &\iff I_{\text{accessible}} = \log_2 3 + \log_2 \max p_{\text{guess}} = 0 \end{aligned}$$

Preparation Noncontextuality & Zero-Information Games

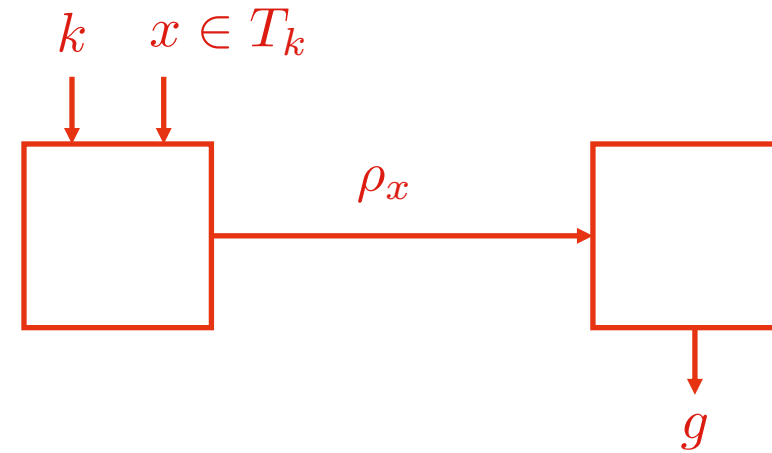
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- Adapt results from “informationally-restricted correlations” to handle OEs
 - [A. Tavakoli *et al.*, Quantum (2020)]
 - SDP hierarchy: [A. Tavakoli, E. Zambrini Cruzeiro, E. Woodhead, S. Pironio, arXiv:2007.16145]

SDP Hierarchy for Preparation Noncontextuality

Given a distribution $p(b|x, y)$ are there quantum states $\{\rho_x\}$ and POVMs $\{E_b^y\}$ reproducing the statistics $p(b|x, y) = \text{Tr}(E_b^y \rho_x)$ and satisfying the operational equivalences?

Mixed states
Unbounded dimension

Projective measurements
(preparation NC)

SDP Hierarchy for Preparation Noncontextuality

Given a distribution $p(b|x, y)$ are there quantum states $\{\rho_x\}$ and POVMs $\{E_b^y\}$ reproducing the statistics $p(b|x, y) = \text{Tr}(E_b^y \rho_x)$ and satisfying the operational equivalences?

- Moment matrix $\Gamma : \Gamma_{ij} = \text{Tr}(O_i^\dagger O_j)$ $O_i, O_j \in \{\mathbb{1}, \rho_x, E_b^y, \sigma, \rho_x \rho_{x'}, \rho_x E_b^y, \dots\}$

	$\mathbb{1}$	ρ_0	ρ_1	E_0^0	E_0^1	σ	$\rho_0 E_0^0$	\dots
$\mathbb{1}$								
ρ_0								
ρ_1								
E_0^0								
E_0^1								

PSD operators used to enforce zero-information constraints (OEs)

SDP Hierarchy for Preparation Noncontextuality

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	$\mathbb{1}$	ρ_0	ρ_1	E_0^0	E_0^1	σ	$\rho_0 E_0^0$	\dots
$\mathbb{1}$	$\text{Tr}(\mathbb{1})$	$\text{Tr}(\rho_0)$	$\text{Tr}(\rho_1)$					
ρ_0								
ρ_1								
E_0^0								
E_0^1								

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	$\mathbb{1}$	ρ_0	ρ_1	E_0^0	E_0^1	σ	$\rho_0 E_0^0$	\dots
$\mathbb{1}$	$\text{Tr}(\mathbb{1})$	1	1					
ρ_0								
ρ_1								
E_0^0								
E_0^1								

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	$\mathbb{1}$	ρ_0	ρ_1	E_0^0	E_0^1	σ	$\rho_0 E_0^0$	\dots
$\mathbb{1}$	$\text{Tr}(\mathbb{1})$	1	1				$\text{Tr}(\rho_0 E_0^0)$	
ρ_0				$\text{Tr}(\rho_0 E_0^0)$	$\text{Tr}(\rho_0 E_0^1)$			
ρ_1				$\text{Tr}(\rho_1 E_0^0)$	$\text{Tr}(\rho_1 E_0^1)$			
E_0^0							$\text{Tr}(E_0^0 \rho_0 E_0^0)$	
E_0^1								

PSD operators used to enforce zero-information constraints (OEs)

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	$\mathbb{1}$	ρ_0	ρ_1	E_0^0	E_0^1	σ	$\rho_0 E_0^0$	\dots
$\mathbb{1}$	$\text{Tr}(\mathbb{1})$	1	1				$p(0 0, 0)$	
ρ_0				$p(0 0, 0)$	$p(0 0, 1)$			
ρ_1				$p(0 1, 0)$	$p(0 1, 1)$			
E_0^0							$p(0 0, 0)$	
E_0^1								

PSD operators used to enforce zero-information constraints (OEs)

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	$\mathbb{1}$	ρ_0	ρ_1	E_0^0	E_0^1	σ	$\rho_0 E_0^0$	\dots
$\mathbb{1}$	$\text{Tr}(\mathbb{1})$	1	1	$\text{Tr}(E_0^0)$	$\text{Tr}(E_0^1)$	$\text{Tr}(\sigma)$	$p(0 0, 0)$	
ρ_0		$\text{Tr}(\rho_0^2)$		$p(0 0, 0)$	$p(0 0, 1)$			
ρ_1			$\text{Tr}(\rho_1^2)$	$p(0 1, 0)$	$p(0 1, 1)$			
E_0^0				$\text{Tr}(E_0^0)$			$p(0 0, 0)$	
E_0^1					$\text{Tr}(E_0^1)$			

PSD operators used to enforce zero-information constraints (OEs)

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	$\mathbb{1}$	ρ_0	ρ_1	E_0^0	E_0^1	σ	$\rho_0 E_0^0$	\dots
$\mathbb{1}$	$\text{Tr}(\mathbb{1})$	1	1	$\text{Tr}(E_0^0)$	$\text{Tr}(E_0^1)$	$\text{Tr}(\sigma)$	$p(0 0, 0)$	
ρ_0		$\text{Tr}(\rho_0^2)$		$p(0 0, 0)$	$p(0 0, 1)$			
ρ_1			$\text{Tr}(\rho_1^2)$	$p(0 1, 0)$	$p(0 1, 1)$			
E_0^0				$\text{Tr}(E_0^0)$			$p(0 0, 0)$	
E_0^1					$\text{Tr}(E_0^1)$			

PSD operators used to enforce zero-information constraints (OEs)

Necessary Conditions

$$\begin{aligned} \Gamma &\succeq 0 \\ \Gamma_{\mathbb{1}, \rho_x} &= 1 \\ \Gamma_{\rho_x, E_b^y} &= p(b|x, y) \\ &\vdots \end{aligned}$$

- How to enforce the operational equivalences, positivity ($\rho_x \succeq 0$)?

Localising Matrices for Noncontextuality

- State positivity: $\Upsilon^x : (\Upsilon^x)_{ij} = \text{Tr}(L_i^\dagger \rho_x L_j)$

$$L_i, L_j \in \{\mathbb{1}, E_b^y, \rho_x, \dots\}$$

	$\mathbb{1}$	E_0^0	E_0^1	\dots
$\mathbb{1}$	1	$p(0 x, 0)$	$p(0 x, 1)$	
E_0^0		$p(0 x, 0)$	$\text{Tr}(\rho_x E_0^1 E_0^0)$	
E_0^1			$p(0 x, 1)$	
\vdots				

$$\succeq 0$$

	$\mathbb{1}$	ρ_0	ρ_1	E_0^0	E_0^1	σ	$\rho_0 E_0^1$	\dots
$\mathbb{1}$	$\text{Tr}(\mathbb{1})$	1	1	$\text{Tr}(E_0^0)$	$\text{Tr}(E_0^1)$	$\text{Tr}(\sigma)$	$p(0 0, 1)$	
ρ_0		$\text{Tr}(\rho_0^2)$		$p(0 0, 0)$	$p(0 0, 1)$			
ρ_1			$\text{Tr}(\rho_1^2)$	$p(0 1, 0)$	$p(0 1, 1)$			
E_0^0				$\text{Tr}(E_0^0)$			$\text{Tr}(\rho_0 E_0^1 E_0^0)$	
E_0^1					$\text{Tr}(E_0^1)$			

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E_0^0		$p(0 x, 0)$	$\text{Tr}(\rho_x E_0^1 E_0^0)$	
E_0^1			$p(0 x, 1)$	
\vdots				

$$\succeq 0$$

	$\mathbb{1}$	ρ_0	ρ_1	E_0^0	E_0^1	σ	$\rho_0 E_0^1$	\dots
$\mathbb{1}$	$\text{Tr}(\mathbb{1})$	1	1	$\text{Tr}(E_0^0)$	$\text{Tr}(E_0^1)$	$\text{Tr}(\sigma)$	$p(0 0, 1)$	
ρ_0		$\text{Tr}(\rho_0^2)$		$p(0 0, 0)$	$p(0 0, 1)$			
ρ_1			$\text{Tr}(\rho_1^2)$	$p(0 1, 0)$	$p(0 1, 1)$			
E_0^0				$\text{Tr}(E_0^0)$			$\text{Tr}(\rho_0 E_0^1 E_0^0)$	
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Localising Matrices for Noncontextuality

- State positivity: $\Upsilon^x : (\Upsilon^x)_{ij} = \text{Tr}(L_i^\dagger \rho_x L_j)$

$$L_i, L_j \in \{\mathbb{1}, E_b^y, \rho_x, \dots\}$$

$$\begin{array}{c|cccc}
 & \mathbb{1} & E_0^0 & E_0^1 & \dots \\
 \hline
 \mathbb{1} & 1 & p(0|x, 0) & p(0|x, 1) & \\
 E_0^0 & & p(0|x, 0) & \text{Tr}(\rho_x E_0^1 E_0^0) & \\
 E_0^1 & & & p(0|x, 1) & \\
 \vdots & & & &
 \end{array} \succeq 0$$

$$\begin{array}{c|cccccccc}
 & \mathbb{1} & \rho_0 & \rho_1 & E_0^0 & E_0^1 & \sigma & \rho_0 E_0^1 & \dots \\
 \hline
 \mathbb{1} & \text{Tr}(\mathbb{1}) & 1 & 1 & \text{Tr}(E_0^0) & \text{Tr}(E_0^1) & \frac{1}{3} & p(0|0, 1) & \\
 \rho_0 & & \text{Tr}(\rho_0^2) & & p(0|0, 0) & p(0|0, 1) & & & \\
 \rho_1 & & & \text{Tr}(\rho_1^2) & p(0|1, 0) & p(0|1, 1) & & & \\
 E_0^0 & & & & \text{Tr}(E_0^0) & & & \text{Tr}(\rho_0 E_0^1 E_0^0) & \\
 E_0^1 & & & & & \text{Tr}(E_0^1) & & &
 \end{array}$$

- Operational equivalences:

$$\frac{1}{2}(\rho_0 + \rho_1) = \frac{1}{2}(\rho_2 + \rho_3) = \frac{1}{2}(\rho_4 + \rho_5)$$

$$T_1 = \{\rho_0, \rho_1\}, T_2 = \{\rho_2, \rho_3\}, T_3 = \{\rho_4, \rho_5\}$$

$$\exists \sigma : \begin{cases} \text{Tr}(\sigma) = \frac{1}{3} \quad (= \max p_{\text{guess}}) \\ \sigma = \frac{1}{3} \left[\frac{1}{2}(\rho_0 + \rho_1) \right] \\ \sigma = \frac{1}{3} \left[\frac{1}{2}(\rho_2 + \rho_3) \right] \\ \sigma = \frac{1}{3} \left[\frac{1}{2}(\rho_4 + \rho_5) \right] \end{cases}$$

$$\Lambda^k : (\Lambda^k)_{ij} = \text{Tr} \left(L_i^\dagger \left(\sigma - \frac{1}{3} \sum_{x \in T_k} \frac{1}{2} \rho_x \right) L_j \right)$$

$$\Lambda^k = 0$$

Localising Matrices for Noncontextuality

- State positivity: $\Upsilon^x : (\Upsilon^x)_{ij} = \text{Tr}(L_i^\dagger \rho_x L_j)$

$$L_i, L_j \in \{\mathbb{1}, E_b^y, \rho_x, \dots\}$$

$$\begin{array}{c|cccc} & \mathbb{1} & E_0^0 & E_0^1 & \dots \\ \hline \mathbb{1} & 1 & p(0|x,0) & p(0|x,1) & \\ E_0^0 & & p(0|x,0) & \text{Tr}(\rho_x E_0^1 E_0^0) & \\ E_0^1 & & & p(0|x,1) & \\ \vdots & & & & \end{array} \succeq 0$$

$$\begin{array}{c|cccccccc} & \mathbb{1} & \rho_0 & \rho_1 & E_0^0 & E_0^1 & \sigma & \rho_0 E_0^1 & \dots \\ \hline \mathbb{1} & \text{Tr}(\mathbb{1}) & 1 & 1 & \text{Tr}(E_0^0) & \text{Tr}(E_0^1) & \frac{1}{3} & p(0|0,1) & \\ \rho_0 & & \text{Tr}(\rho_0^2) & & p(0|0,0) & p(0|0,1) & & & \\ \rho_1 & & & \text{Tr}(\rho_1^2) & p(0|1,0) & p(0|1,1) & & & \\ E_0^0 & & & & \text{Tr}(E_0^0) & & & \text{Tr}(\rho_0 E_0^1 E_0^0) & \\ E_0^1 & & & & & \text{Tr}(E_0^1) & & & \end{array}$$

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$$\frac{1}{2}(\rho_0 + \rho_1) = \frac{1}{2}(\rho_2 + \rho_3) = \frac{1}{2}(\rho_4 + \rho_5)$$

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$$\exists \sigma : \begin{cases} \text{Tr}(\sigma) = \frac{1}{3} \quad (= \max p_{\text{guess}}) \\ \sigma \succeq \frac{1}{3} \left[\frac{1}{2}(\rho_0 + \rho_1) \right] \\ \sigma \succeq \frac{1}{3} \left[\frac{1}{2}(\rho_2 + \rho_3) \right] \\ \sigma \succeq \frac{1}{3} \left[\frac{1}{2}(\rho_4 + \rho_5) \right] \end{cases}$$

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$$\Lambda^k \succeq 0$$

Necessary Conditions

$$\Gamma \succeq 0$$

$$\Upsilon^x \succeq 0$$

$$\Lambda^k \succeq 0$$

$$\Gamma_{\mathbb{1}, \rho_x} = 1$$

$$\Gamma_{\rho_x, E_b^y} = p(b|x, y)$$

$$\Gamma_{\mathbb{1}, \sigma} = \frac{1}{K}$$

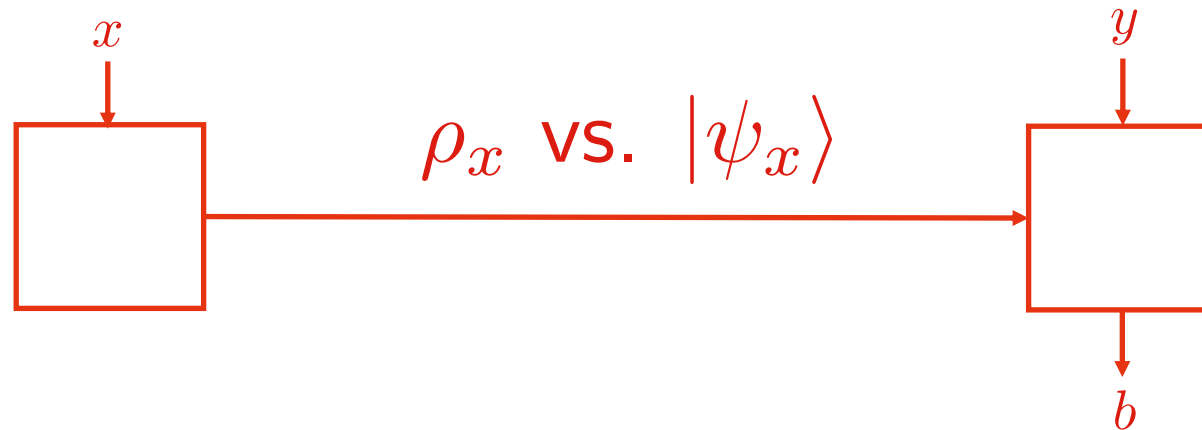
Applications

- Tight upper bounds for several noncontextuality inequalities
- Implementation freely available on github

See paper!

Are pure states as useful as mixed states in contextuality scenarios?

- Purifications may not satisfy preparation equivalences

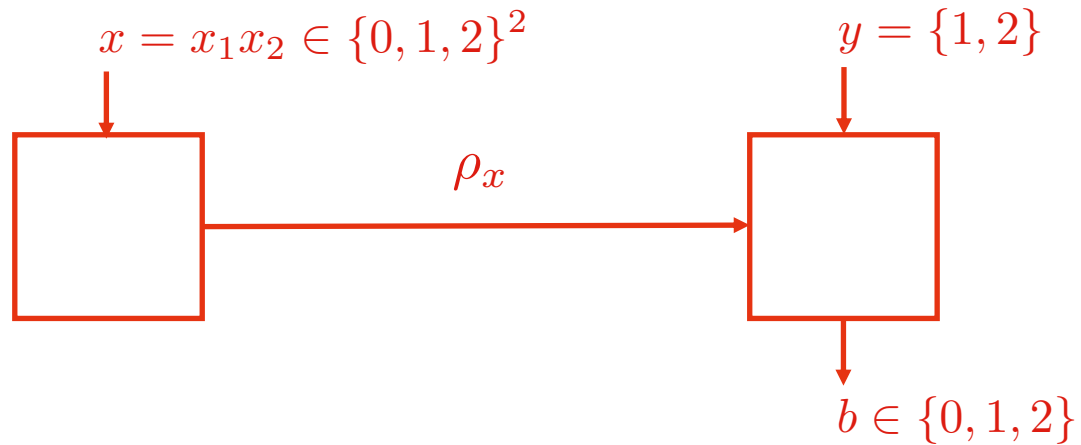


Pure hierarchy:

- Simplify: $(\rho_x)^2 = \rho_x$
- No need for positivity localising matrices (Υ^x) : $(\rho_x)^2 = \rho_x \implies \rho_x \succeq 0$

Contextual Advantage from Mixed States

- Hameedi-Tavakoli-Marques-Bourennane inequality [PRL 2017]



RAC-based communication game:

$$p_{\text{win}} = \frac{1}{18} \sum_{x,y} p(b = x_y | x, y)$$

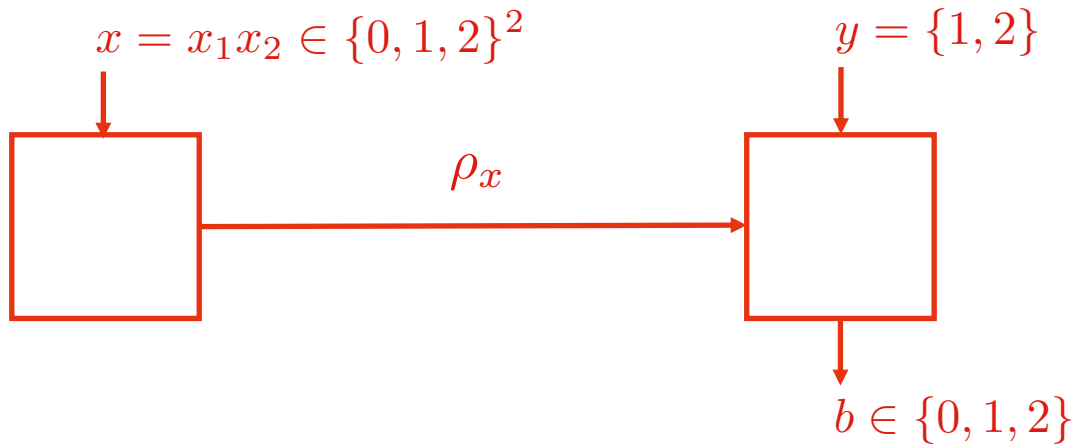
Operational equivalences:

$$\frac{1}{3}(\rho_{00} + \rho_{12} + \rho_{21}) = \frac{1}{3}(\rho_{01} + \rho_{10} + \rho_{22}) = \frac{1}{3}(\rho_{02} + \rho_{20} + \rho_{11})$$
$$\frac{1}{3}(\rho_{00} + \rho_{11} + \rho_{22}) = \frac{1}{3}(\rho_{02} + \rho_{10} + \rho_{21}) = \frac{1}{3}(\rho_{01} + \rho_{12} + \rho_{20})$$

Noncontextuality inequality: $p_{\text{win}}^{\text{NC}} \leq 2/3$

Contextual Advantage from Mixed States

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RAC-based communication game:

$$p_{\text{win}} = \frac{1}{18} \sum_{x,y} p(b = x_y | x, y)$$

Operational equivalences:

$$\frac{1}{3}(\rho_{00} + \rho_{12} + \rho_{21}) = \frac{1}{3}(\rho_{01} + \rho_{10} + \rho_{22}) = \frac{1}{3}(\rho_{02} + \rho_{20} + \rho_{11})$$
$$\frac{1}{3}(\rho_{00} + \rho_{11} + \rho_{22}) = \frac{1}{3}(\rho_{02} + \rho_{10} + \rho_{21}) = \frac{1}{3}(\rho_{01} + \rho_{12} + \rho_{20})$$

Noncontextuality inequality: $p_{\text{win}} \stackrel{\text{NC}}{\leq} 2/3$

➤ SDP hierarchy: No contextual advantage from pure states! $p_{\text{win}} \stackrel{\Psi}{\lesssim} 2/3$

➤ Mixed states:

○ Achievable (d=6): $p_{\text{win}} \stackrel{\mathcal{Q}}{\approx} 0.6979$

○ SDP hierarchy: $p_{\text{win}} \stackrel{\mathcal{Q}}{\lesssim} 0.704$

Simulating Contextuality

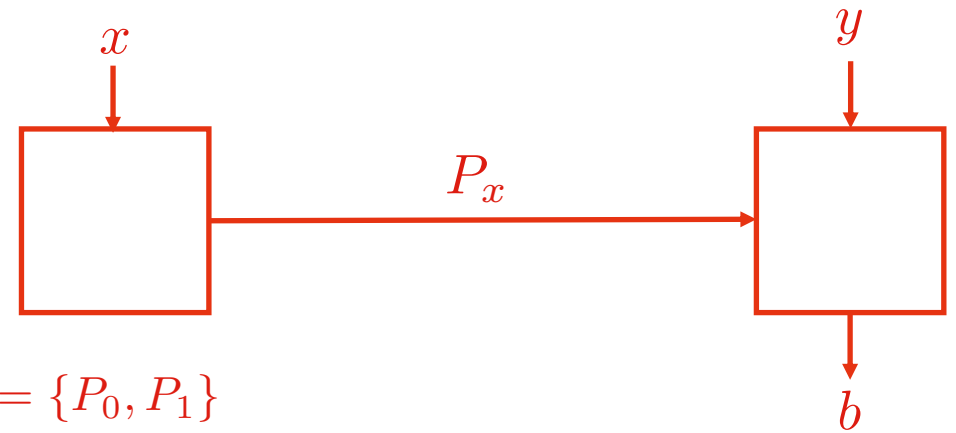
What resources are needed to simulate contextual correlations (maybe post-quantum ones)

1. with a noncontextual, classical model?
2. with a quantum model?

Bell Nonlocality



Contextuality



$$T_1 = \{P_0, P_1\}$$

$$T_2 = \{P_2, P_3\}$$

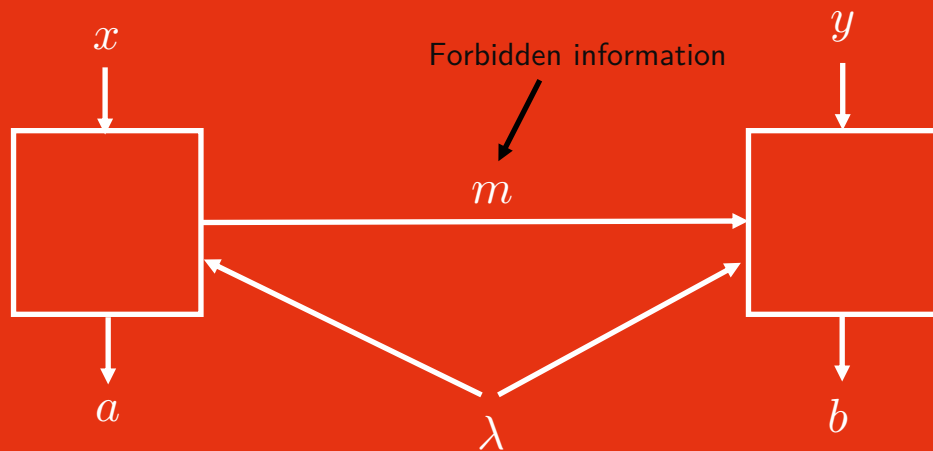
$$P_0 + P_1 \simeq P_2 + P_3$$

Simulating Contextuality

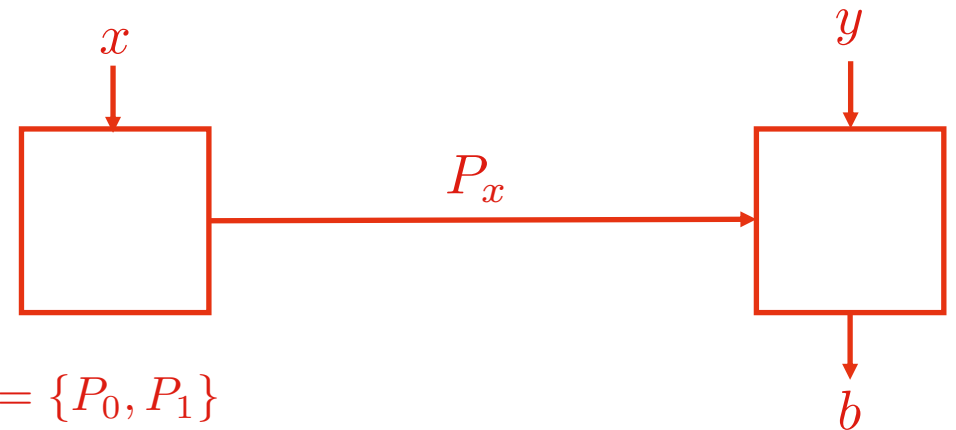
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Contextuality



$$T_1 = \{P_0, P_1\}$$

$$T_2 = \{P_2, P_3\}$$

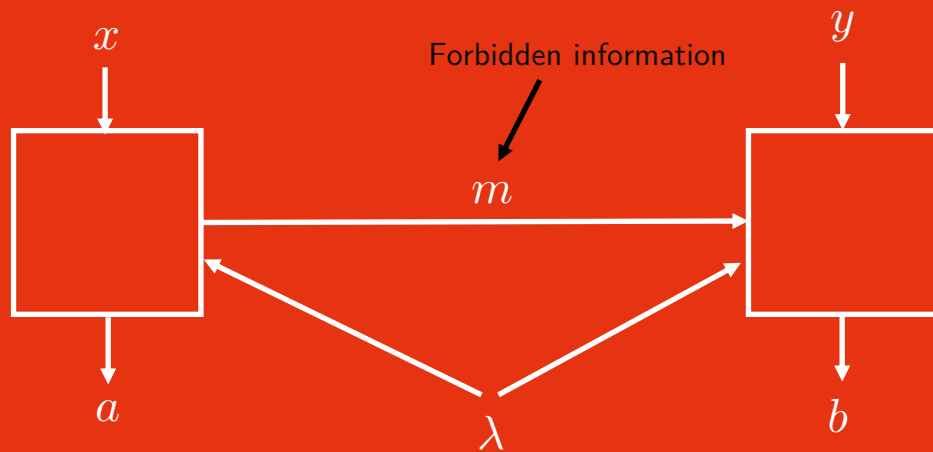
$$P_0 + P_1 \simeq P_2 + P_3$$

Simulating Contextuality

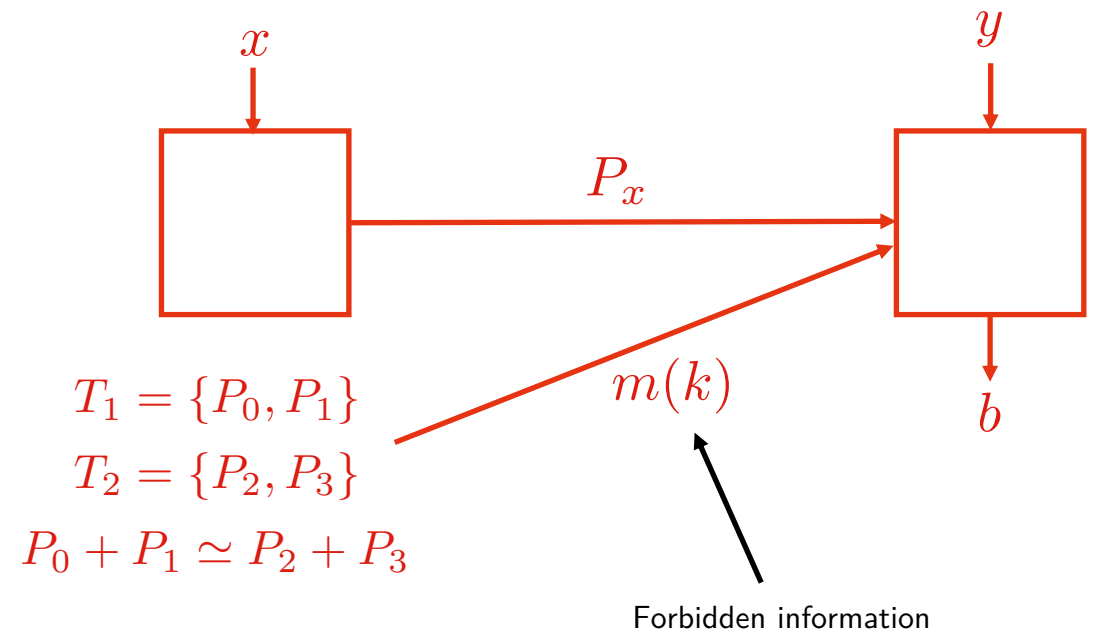
What resources are needed to simulate contextual correlations (maybe post-quantum ones)

1. with a noncontextual, classical model?
2. with a quantum model?

Bell Nonlocality



Contextuality



Simulation Cost of Preparation Contextuality

Simulation cost: quantifies information about preparation equivalences available to Bob

$$C(p) = \min \mathcal{I}_{\text{accessible}} = \log_2 K + \log_2(\max p_{\text{guess}})$$

min over all models (classical or quantum)
reproducing $p(b|x,y)$

Can lower bound $C(p)$ using SDP hierarchy:

- Rather than imposing $\Gamma_{\mathbb{1},\sigma} = \frac{1}{K}$ we instead minimise $\Gamma_{\mathbb{1},\sigma} = \text{Tr}(\sigma)$ and use

$$p_{\text{guess}} \leq \text{Tr}(\sigma)$$

- Classical models \approx commuting operators

Parity Oblivious Multiplexing

