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Data base for the direct kinematics of cable-driven parallel robot (CDPR) with sagging cables

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1 Introduction

CDPR is a specific type of robot where several cables are connecting a moving platform to fixed points A_i. The attachment points of the cables on the platform are denoted B_i. The lengths of these cables can be independently modified by winches and it can be shown that with at least 6 cables the control of the cable lengths allows one to control the 6 degrees of freedom of the platform. The direct kinematics problem (DK), introduced in the next section, is to determine what is the pose of the platform being given the lengths of the cables. Here we assume that the cables have elasticity and their own mass. Such cable type is called *sagging cables* as their shape is not the straight line segment between A_i and B_i. **Solving the DK is difficult and we are interested in determining exactly all solutions** (indeed the DK has usually several solutions). They are methods that are able to solve this problem, namely interval analysis and continuation, but they may miss solution(s) and are extremely time consuming.

We have considered using artificial intelligence, namely Multi-Layer Perceptron (MLP), for simple CDPR with good results in some cases by using a strategy that use the MLP prediction as guess for the Newton method. The databases provided in this document may be used as learning and verification sets for AI and the proposed case is probably one of the most difficult DK problem that can be considered.

Errors on the DK solutions may be acceptable but shall be very low. We will see that a DK solution is constituted of three coordinates x, y, z of a point, 3 angles ϕ_i for defining the orientation of the platform together with 16 forces components. DK may be claimed to be solved in the proposed case if the following constraints are all satisfied:

- an approximation for **all** solutions is found
- the errors on x, y, z is less than 1cm
- the errors on the ϕ_i angles is less than 0.1 degree
- the errors on the force components is less than 5N

Important note: the data bases in this document have been established by solving the DK with interval analysis and continuation for a few sets of cable lengths. We don't claim that all solutions have been determined for these sets. If the solving method you have used is able to find new DK solution(s) (exact or satisfying the above constraints) for the set of lengths provided in the verification set we will be happy to hear from you so that we can check your solution(s) and enrich the data bases. Just send an email to `Jean-Pierre.Merlet@inria.fr` with as subject `DK databases`. You may also ask any relevant question on the data bases with the same process.

The databases are free of use under the Creative Commons Attribution license. It must be referenced in a publication as:

8-cables cable-driven parallel robot direct kinematics databases, J-P. Merlet, INRIA, December 2021

2 Direct kinematics (DK)

The purpose of the DK is to determine the platform pose(s) for a given set of cable lengths L_0. We are considering here that the platform is only submitted to gravity. There are multiple ways to write the direct kinematics equations. Let A_i, B_i be the cable extremity and G the center of mass while O is the origin of the reference frame. A vector $\mathbf{A_iB_i}$ may be written as

$$\mathbf{A_iB_i} = \mathbf{OG} + \mathbf{GB_i} - \mathbf{OA_i}$$

Let $\mathbf{R}$ be the rotation matrix derived from the ϕ_is angles. As we know the components of the vector $\mathbf{GB_i}$ in the mobile frame, denoted as $\mathbf{GB_i}^r$, we have $\mathbf{GB_i} = \mathbf{RGB_i}^r$. Hence the components of $\mathbf{A_iB_i}$ may be expressed as functions of $x, y, z, \phi_1, \phi_2\phi_3$ and if we define ρ_i as $||\mathbf{A_iB_i}||$, then for each cable we have:

$$\begin{aligned} x_b &= \sqrt{A_iB_{i_x}^2 + A_iB_{i_y}^2}/\rho_i \\ z_b &= A_iB_{i_z} \end{aligned}$$

Using the Irvine equations we are therefore able to determine $2n$ constraints for a CDPR with n cables. We have then to consider that the platform is in mechanical equilibrium i.e. that the sum of the forces and torques applied on the platform are equal to 0 except for the vertical force that should equilibrate the weight of the platform as we have assumed that the only external force applied on the platform is gravity which exert a vertical force of amplitude Mg where M is the platform mass. Regarding the forces we must therefore have

$$\begin{aligned} \sum_{j=1}^{j=8} F_{x_j} A_j B_{j_x}/\rho_j &= 0 \\ \sum_{j=1}^{j=8} F_{x_j} A_j B_{j_y}/\rho_j &= 0 \\ \sum_{j=1}^{j=8} F_{z_j} + Mg &= 0 \end{aligned}$$

Let $\tau_j = (-F_{x_j} A_j B_{j_x}/\rho_j, -F_{x_j} A_j B_{j_y}/\rho_j, -F_{z_j})$ be the force vector applied by cable j on the platform at point B_j. The torque applied by this force around G is $\mathbf{T_j} = \mathbf{GB_j} \times \tau_\mathbf{j}$ and the torque equilibrium is written as

$$\sum_{j=1}^{j=8} \mathbf{T_j} = \mathbf{0} \tag{1}$$

Static equilibrium therefore provides 6 constraints so that we have a total of $2n+6$ constraints that are functions of the $2n+6$ unknowns $x, y, z, \phi_1, \phi_2, \phi_3$, the n F_{x_i} and the n F_{z_i}. Therefore the DK problem is always a square system whatever the number of cables is. Note that the DK has usually several solutions and that we cannot determine the number of solutions of the DK for a given set of cable lengths without solving the DK system. The DK solutions we are providing in the data set has been obtained by starting with a limited number of L_0 sets and computing their DK solutions by using interval analysis and a continuation approaches [3, 4] (but we cannot guarantee that all DK solutions have been obtained). Then we have used another continuation approach to derive DK solutions from the initial DK solutions sets.

The robot

We consider a 8-cables CDPR [1] where the cable winch output points A_i have the following x, y, z coordinates in meters in an arbitrary reference frame:

```
-7.175120 -5.243980 5.462460
-7.315910 -5.102960 5.472220
-7.302850 5.235980 5.476150
7.182060 5.347600 5.488300
-7.160980 5.372810 5.485390
7.323310 5.205840 5.499030
7.301560 -5.132550 5.489000
7.161290 -5.269460 5.497070
```

The cable attachment points B_i on the platform are defined in a frame whose origin is the center of mass of the platform. Their coordinates are (in meters):

```
0.503210 -0.492830 0.000000
-0.509740 0.350900 0.997530
-0.503210 -0.269900 0.000000
-0.503210 0.492830 0.000000
0.496070 0.355620 0.999540
0.499640 -0.340280 0.999180
0.502090 0.274900 -0.000620
-0.504540 -0.346290 0.997520
```

3 Pose parameters

The pose parameters are the coordinates of the center of mass of the platform and the RTL angles ϕ_1, ϕ_2, ϕ_3 so that the rotation matrix is

$$\begin{bmatrix} \cos(\phi_2)\cos(\phi_3) & -\cos(\phi_2)\sin(\phi_3) & \sin(\phi_2) \\ \cos(\phi_3)\sin(\phi_1)\sin(\phi_2)+\cos(\phi_1)\sin(\phi_3) & -\sin(\phi_1)\sin(\phi_2)\sin(\phi_3)+\cos(\phi_1)\cos(\phi_3) & -\sin(\phi_1)\cos(\phi_2) \\ -\cos(\phi_1)\cos(\phi_3)\sin(\phi_2)+\sin(\phi_1)\sin(\phi_3) & \cos(\phi_1)\sin(\phi_2)\sin(\phi_3)+\cos(\phi_3)\sin(\phi_1) & \cos(\phi_1)\cos(\phi_2) \end{bmatrix}$$

4 Sagging cables

The cable sagging model we use can be found in the Irvine textbook [2] and is written as

$$\begin{aligned} x_b &= F_x\left(\frac{L_0}{EA_0} + \frac{sinh^{-1}(F_z) - sinh^{-1}(F_z - \frac{\mu g L_0}{F_x})}{\mu g}\right) \\ z_b &= \frac{F_z L_0}{EA_0} - \mu g L_0^2/2 + \frac{\sqrt{F_x^2+F_z^2} - \sqrt{F_x^2+(F_z-\mu g L_0)^2}}{\mu g} \end{aligned}$$

This model is planar and assume that a cable is attached at a point of coordinates (0,0) while its other extremity B is located at $x_b > 0, z_b < 0$. In these equations L_0 is the cable length at rest, F_x, F_z the horizontal and vertical forces applied on the cable at B. A_0 is the surface of a cable cross-section (in m^2) E is the Young modulus of the cable material (in N/m^2) and μ its linear mass (in kg/m). **In the data sets we use $E = 0.1e^{12}$, $\mu = 0.079$ and the diameter of the cable is 4mm**: this correspond to a Dyneema cable.

5 The branches data set

5.1 Definition of the set

We have considered various poses $\mathbf{X_0}$ in the $\mathbf{X}$ space. For a given pose we have calculated the distance between the A_i, B_i for each of the 8 cables. We have then used these distances L_0^r as the L_0 for the DK problem and solved the DK for these L_0^r, obtaining n solutions $S_1, S_2, \ldots, S_n$.

Assume now that we define the L_0 as

$$L_0 = L_0^r + \lambda \mathbf{d}$$

which define a curve C_{L_0} in in the L_0 space. Now consider a specific solution S_j for the DK when $L_0 = L_0^r$. For the L_0 corresponding to $\lambda = 0$ one of the DK solution is S_j while for $\lambda = \epsilon$ with ϵ very small one of the DK solutions will be very close to S_j. Hence the curve C_{L_0} is mapped to n curves in the $\mathbf{X}$ space each one including one of the S_j: these curves are called the *kinematic branches*. The branches data set is a set of files describing a set of kinematic branches that have been obtained for a given $\mathbf{X_0}$ (that allows to obtain L_0^r), a given DK solution S_j obtained for L_0^r and various $\mathbf{d}$. All kinematic branches in such a file include one of the S_j which is the starting point of all branches. In such a file we have several values for L_0 obtained for various λ and the DK solution obtained for these L_0 that belong to the same branch than S_j.

Note: we have imposed several constraints on the branches file:

- the load mass is 1 kg
- the F_x shall be lower than 100
- the z of the platform cannot be larger than 3.5.

As these constraints are not satisfied for some DK solutions S_k obtained for the $L_{0_i}^r$ there is no branch file for these solutions.

5.2 Initial DK solutions for the branches set

A key element for the branches set are the DK solutions for the various $\mathbf{X_0}$ pose that have been used to calculate the $L_{0_i}^r$. These solutions are given in different files in the `Solution_DK` repertory. A typical file name in this repertory is:

`SOL_DK_FINAL_1_1_2_0_120_0.maple`

The 6 numbers in this name are the $x, y, z, \phi_1, \phi_2, \phi_3$ values in meters and degrees of the pose $\mathbf{X_0}$. Each file is a Maple file so that any line starting with # is a comment. In a file you will find:

- `POSE:=[`$a_1, a_2, \ldots a_{14}$`]::` a_1 to a_6 are the $x, y, z, \phi_1, \phi_2, \phi_3$ values in meters and radian of the pose $\mathbf{X_0}$, a_7 to a_{14} are the 8 L_0^r

- `SOL_DK` written as

```
SOL_DK:=[
[u1,u2,... u6,v1,v2,... v16],
[u1,u2,... u6,v1,v2,... v16],
.....
]:
```

  $u1$ to $u6$ are the $x, y, z, \phi_1, \phi_2, \phi_3$ of the DK solution in meters and radian, $v1$ to $v8$ are the 8 cables F_x and $v9$ to $v16$ are the 8 cables F_z.

- the line

```
#sol continuation n
```

  provides in `n` the number of DK solutions

5.3 Structure of the branches data set files

A typical branch file name in the branches data set files is

`learning_SOL_DK_FINAL_4_4_1_10_0_0.maple_SOL_3`

The 6 first numbers in this name are the $x, y, z, \phi_1, \phi_2, \phi_3$ values in meters and degrees of the pose $\mathbf{X_0}$ pose that has been used to calculate the $L_{0_i}^r$. The last number in the file name indicates that the branches have been calculated with the third DK solution for the $L_{0_i}^r$. Hence for the example we have used the pose $\mathbf{X_0}$ defined by

$$x = 4 \quad y = 4 \quad z = 1 \quad \phi_1 = 10^\circ \quad \phi_2 = \phi_3 = 0^\circ$$

A branch file is written in `Maple` format so that any line starting with # is a comment. The number of sample in a branches file is given at the beginning of the file. In a branches file we have

- `POSE` written as $[a_1, a_2, \ldots a_n]$. This is the pose for which the $L_{0_i}^r$ have been calculated. The 6 first a_i are the pose parameters (in meters and radian), followed by the 8 $L_{0_i}^r$

- `SOLUTION_DK` written as $[a_1, a_2, \ldots a_{22}]$. This is the DK solution (among all DK solutions obtained for L_0^r) that is the starting point of the branches. The a_i and a_6 are the pose parameters (in meters and radian), a_7 to a_{14} are the 8 F_x, a_{15} to a_{19} are the 8 F_z

- `BRANCH` written as

```
BRANCH:=[
[
[u1,u2,... u8],
```

```
[v1,v2,... v22]
],
[
[u1,u2,... u8],
[v1,v2,... v22]
],
...
]:
```

where the u_i are the 8 cable lengths and the v_i represent the DK solution for these lengths: $v1$ to $v6$ are the pose parameters (in meters and radian), $v7$ to $v14$ the F_x, $v15$ to $v22$ the F_z

- # `L0`[1..8]: the two numbers following such a line are the minimal and maximal cable lengths for this set

We have 122 branches files and a total of 60861 DK solutions in these files. Each branch file contains around 500 points on branches emanating from a specific DK solution obtained for L_0^r.

5.4 Changing the format

If `Maple` is not available and you are a linux user you may transform a branch file `XX` into another format. with the following structure:

- line beginning by a `L` followed by 8 numbers separated by blanks: these are the L_0 values that will be used for the DK
- line beginning by a `S` followed by 22 numbers separated by blanks: this is a DK solution for the current L_0 giving $x, y, z, \phi_1, \phi_2, \phi_3$ in meters and radian followed by the 8 F_x and then the 8 F_z

To obtain this file you must write a `pawk0` file with

```
BEGIN {x=0 }
{
n=split($0,array," ")
if ( n==8) {
print  "L "  $0}
if ( n==22) {
print  "S "  $0}
}
END {exit}
```

Then you use the command

```
grep "BRANCH:" -A 20000 XX | tr -d "]" |tr "," " " \
 |tr -d "["|grep -v BRAN | awk -f pawk0 >temp
```

The created file will be `temp`.

6 The verification data set

The purpose of this data set is to provide all DK solutions for a large set of L_0s.

Important note: at the opposite of the previous database we don't impose any constraint on the solutions

6.1 Structure of the verification set

The verification set is written in `Maple` format so that any line starting with # is a comment. The verification set is located in the `Verification` repository. It has been established for platform mass of 1kg. An element of the verification set is the *solution set*. A solution set S is described as a Maple list:

```
S:=[
[L01,L02,...,L08],
[
[x1,y1,z1,phi11,phi21,phi31,Fx11,Fx21,..Fx81,Fz11,Fz21,..,Fz81],
[x2,y2,z2,phi12,phi22,phi32,Fx12,Fx22,..Fx82,Fz22,Fz22,..,Fz82],
.....
]
]:
```

The `L0i` are the cable lengths and

```
[xi,yi,zi,phi1i,phi2i,phi3i,Fx1i,Fx2i,..Fx8i,Fz1i,Fz2i,..,Fz8i],
```

is the i-th DK solution for the `L0i`. All phi angles are in radian and the x,y,z are in meters.

The file includes the following information:

- `L0Si:=[L01,L02,...,L08]:`. Such a line provides a set of cable lengths for which all DK solutions have been calculated. All DK solutions in the set have been derived from this basic DK solution
- `#we have XX pairs in this set`: `XX` is the number of solution sets we have in the verification set
- the solution sets `Si` where i is a number from 1 to `X`

6.2 Another file format

If `Maple` is not available for you and are the linux you may obtain another file with the following structure:

- line beginning by a `L` followed by 8 numbers separated by blanks: these are the L_0 values that will be used for the DK
- line beginning by a `S` followed by 22 numbers separated by blanks: this is a DK solution for the current L_0 giving $x, y, z, \phi_1, \phi_2, \phi_3$ in meters and radian followed by the 8 F_x and then the 8 F_z

To obtain this file you must write a `pawk` file that is

```
BEGIN {x=0 }
 { print "L "  $3}
{
n=split($0,array,"[")
for (i=4;i<=n;i++){
if ( length(array[i])>0) {
print  "S " array[i] }
}
}
END {exit}
```

then you run the command:

```
grep ^S[0-9]*: verification_database_DK |tr -d "]"|tr "," " " |awk -F
"[" -f pawk >temp
```

The created file will be `temp`.

References

[1] Gouttefarde M., Nguyen D.Q., and Baradat C. Kinetostatics analysis of cable-driven parallel robots with consideration of sagging and pulleys. In *ARK*, pages 213–221, Ljulbjana, June 29- July 3, 2014.

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[3] Merlet J-P. The forward kinematics of cable-driven parallel robots with sagging cables. In *2nd Int. Conf. on cable-driven parallel robots (CableCon)*, pages 3–16, Duisburg, August, 24-27, 2014.

[4] Merlet J-P. A generic numerical continuation scheme for solving the direct kinematics of cable-driven parallel robot with deformable cables. In *IEEE Int. Conf. on Intelligent Robots and Systems (IROS)*, Daejeon, October, 9-14, 2016.