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Improved coupling formulations and time integration strategies for ocean-atmosphere interactions



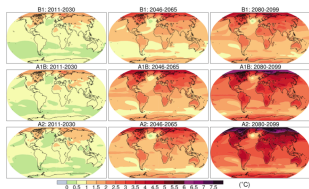
E. Blayo, F. Lemarié, C. Pelletier, S. Thery and S. Clément

Laboratoire Jean Kuntzmann

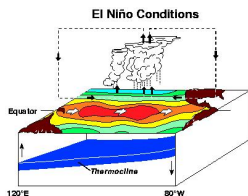
Univ. Grenoble Alpes and Inria, France



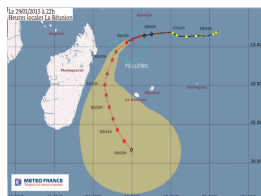
Context: ocean - atmosphere coupling



climate modeling



seasonal forecasts

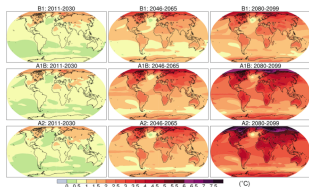


short term predictions

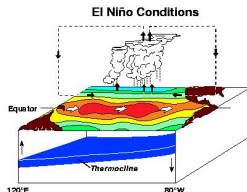
Representing their complex interactions mostly means

- ▶ estimating OA fluxes
 - ▶ imposing these fluxes through the OA interface
- Numerous physical / mathematical / numerical / algorithmic issues

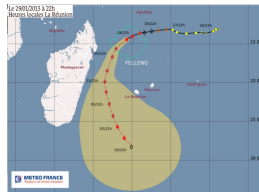
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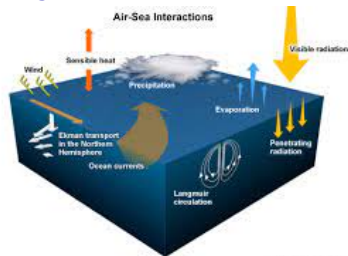
Some mathematical and numerical inputs

- ▶ **Modeling aspects:** provide a general framework for the formulation of the coupled OA problem
- ▶ **Algorithmic aspects:** revisit the coupling strategies presently used in such systems

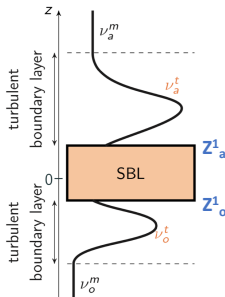
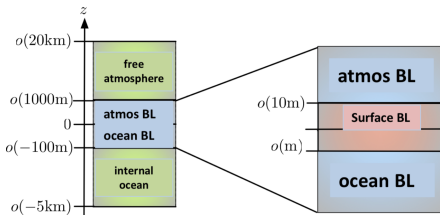
Air-sea interactions: 3 different domains

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\ \mathcal{L}_{\text{oce}} \mathbf{U}^o = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \end{cases}$$

$$\text{with } \mathbf{U}^a = \begin{pmatrix} \mathbf{u}_h^a \\ T^a \end{pmatrix} \quad \mathbf{U}^o = \begin{pmatrix} \mathbf{u}_h^o \\ T^o \end{pmatrix}$$



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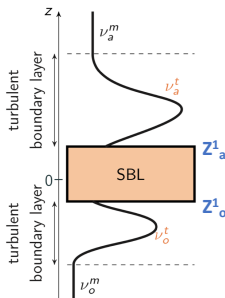
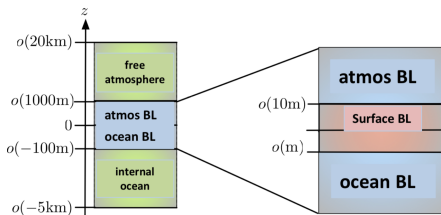
Air-sea interactions: 3 different domains

Exchange conditions:

momentum $\rho^a K_m^a \frac{\partial \mathbf{u}_h^a}{\partial z}(z_a^1, t) = \rho^o K_m^o \frac{\partial \mathbf{u}_h^o}{\partial z}(z_o^1, t) = \boldsymbol{\tau}(t)$

heat flux $\rho^a c_p^a K_T^a \frac{\partial T^a}{\partial z}(z_a^1, t) = \rho^o c_p^o K_T^o \frac{\partial T^o}{\partial z}(z_o^1, t) = Q_S(t) + \mathcal{R}(t)$

where $\boldsymbol{\tau}(t)$ and $Q_S(t)$ are obtained via a *bulk closure*.



Bulk closure: a model of the Surface Boundary Layer

Monin-Obukhov similarity theory (1954): a generalized law of the wall for stratified fluids

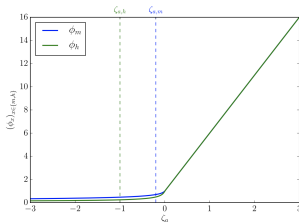
- ▶ Key parameters: **turbulent scales** (u_a^* , θ_a^*)
- ▶ Ingredients: constant flux hypothesis, stability functions, roughness lengths...

$$\begin{cases} \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z} \Phi_m \left(\frac{z}{L_O^a} \right) & z \in (z_u^{a,r}; z_a^1) \\ \frac{\partial \theta}{\partial z} = \frac{\theta_a^*}{\kappa z} \Phi_h \left(\frac{z}{L_O^a} \right) & z \in (z_\theta^{a,r}; z_a^1) \end{cases}$$

with

- ▶ $L_O^a = \frac{(u_a^*)^2}{g\kappa} \frac{\theta(z_a^1)}{\theta_a^*}$ *Obukhov length*

- ▶ Φ_m, Φ_h *empirical stability functions*

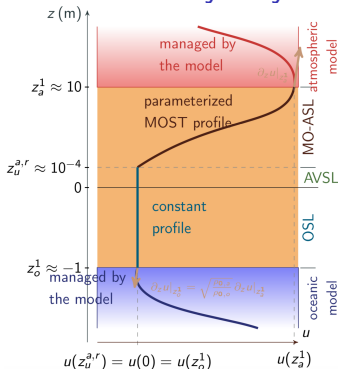


Stability functions
(Large et al., 1994)

Bulk closure: a model of the Surface Boundary Layer

Usual practice in coupled OA models:
constant profiles in AVSL and OSL

$$\frac{\partial u}{\partial z} = \frac{\partial \theta}{\partial z} = 0 \quad z \in (z_o^1, z_x^{a,r})$$



By integration: nonlinear system of equations for (u_a^*, θ_a^*)

$$\begin{cases} \llbracket u \rrbracket_{z_o^1}^{z_a^1} = \frac{u_a^*}{\kappa} \left[\ln \left(\frac{z_a^1}{z_u^{a,r}(u_a^*, \theta_a^*)} \right) - \psi_m \left(\frac{z_a^1}{L_O^a(u_a^*, \theta_a^*)} \right) \right] \\ \llbracket \theta \rrbracket_{z_o^1}^{z_a^1} = \frac{\theta_a^*}{\kappa} \left[\ln \left(\frac{z_a^1}{z_\theta^{a,r}(u_a^*, \theta_a^*)} \right) - \psi_h \left(\frac{z_a^1}{L_O^a(u_a^*, \theta_a^*)} \right) \right] \end{cases}$$

Bulk closure: a model of the Surface Boundary Layer

- ▶ These so-called “**bulk formulas**” are solved by fixed-point iterations.
- ▶ Hence the fluxes: $\tau = \rho_a (u_a^*)^2$ $Q_S = \rho_a c_p^a u_a^* \theta_a^*$
- ▶ Several such formulas have been developed (e.g. Fairall et al., 2002; Large, 2006; ECMWF, 2016), corresponding to different parameterizations for $z^{a,r}$, (ψ_m, ψ_h) , and the addition of other physical effects (gustiness, warm layers...). Calibration is done w.r.t. field experiment data.

Bulk closure: a model of the Surface Boundary Layer

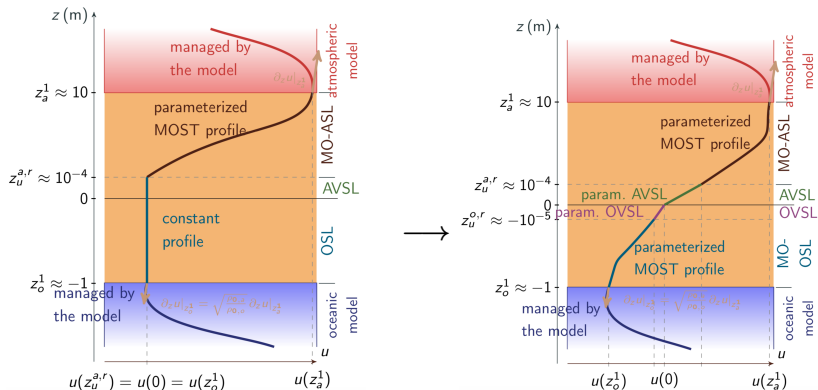
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Classical bulk closures have been developed in the framework of atmosphere-only simulations:

- ▶ No consideration of the ocean surface layer
- ▶ The flux depends on the ocean model vertical discretization

A two-sided surface layer model

Goal: building a framework more adapted to the context of coupled air–sea simulations (symmetry of the problem, regularity of the solution profile...)



A two-sided surface layer model

- ▶ Introduction of **oceanic turbulent scales** (u_o^*, θ_o^*)
- ▶ Symmetric point of view between ocean and atmosphere
- ▶ Mathematical regularity of the vertical profiles

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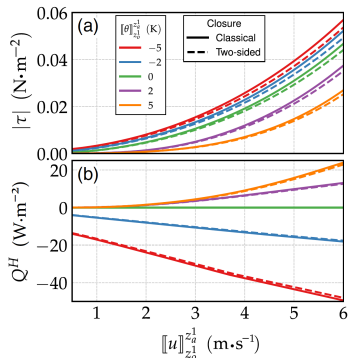
$$\llbracket u \rrbracket_{z_o^1}^{z_a^1} = \frac{u_a^*}{\kappa} \left[\ln \left(\frac{z_a^1}{z_u^{a,r}(u_a^*, \theta_a^*)} \right) - \psi_m^a \left(\frac{z_a^1}{L_O^a(u_a^*, \theta_a^*)} \right) \right. \\ \left. + \lambda \ln \left(\frac{z_o^1}{z_u^{o,r}(u_o^*, \theta_o^*)} \right) - \lambda \psi_m^o \left(\frac{-z_o^1}{L_O^o(u_o^*, \theta_o^*)} \right) \right]$$

$$\lambda = \sqrt{\rho_o^a / \rho_o^o}$$

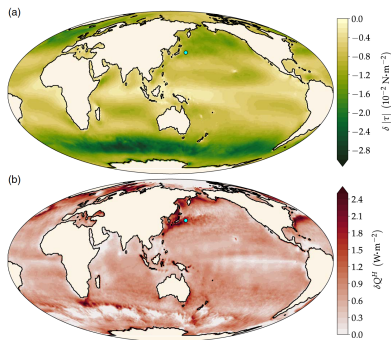
$$\llbracket \theta \rrbracket_{z_o^1}^{z_a^1} = \frac{\theta_a^*}{\kappa} \left[\ln \left(\frac{z_a^1}{z_\theta^{a,r}(u_a^*, \theta_a^*)} \right) - \psi_h^a \left(\frac{z_a^1}{L_O^a(u_a^*, \theta_a^*)} \right) \right. \\ \left. + \lambda \ln \left(\frac{-z_o^1}{z_u^{o,r}(u_o^*, \theta_o^*)} \right) - \lambda \psi_h^o \left(\frac{-z_o^1}{L_O^o(u_o^*, \theta_o^*)} \right) \right]$$

Impact on turbulent fluxes

- ▶ Systematic decrease of the wind stress, slight increase (on average) of the heat flux
- ▶ Indicates that the ocean part is significant. But must not be understood as an error: need for recalibration in the context of two-sided closures



$$|\tau|, Q_H = f\left(\|u\|_{z_0^1}^2, [\theta]_{z_0^1}^1\right)$$



$\delta|\tau|, \delta Q_H$ - 2006 yearly mean

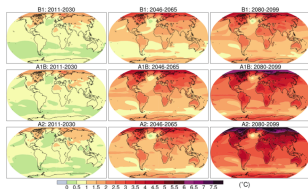
A two-sided surface layer model

- ▶ A generic modeling framework, with a clear view of the hypotheses, and avoiding non physical properties of the vertical profiles
- ▶ Allows for an easy introduction of new parameterizations and/or additional effects: waves, radiative flux...

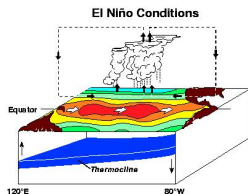
More details in:

Pelletier C., F. Lemarié, E. Blayo, M.-N. Bouin and J.-L. Redelsperger, 2021: Two-sided turbulent surface layer parameterizations for computing air-sea fluxes. *Quart. J. Roy. Meteorol. Soc.* <http://dx.doi.org/10.1002/qj.3991>

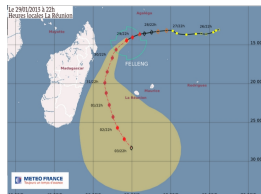
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Some mathematical and numerical inputs

- ▶ **Modeling aspects:** provide a general framework for the representation of the coupled surface layers
- ▶ **Algorithmic aspects:** revisit the coupling strategies presently used in such systems

Algorithmic issues

One has to solve:

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\ \mathcal{L}_{\text{oce}} \mathbf{U}^o = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^a = \mathcal{F}_{\text{oce}} \mathbf{U}^o = F_{\text{OA}}(\mathbf{U}^a, \mathbf{U}^o, \mathcal{R}) & \text{on } \Gamma \times [0, T] \end{cases}$$

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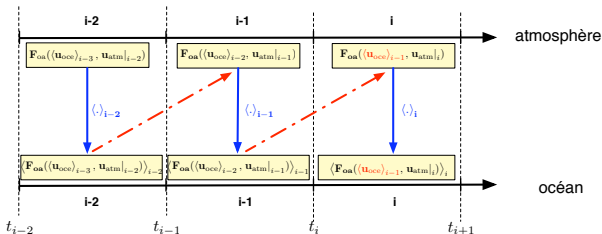
Two usual approaches:

- ▶ Asynchronous coupling by time windows (averaged fluxes)
- ▶ Synchronous coupling at the time step (instantaneous fluxes)

Asynchronous coupling

- ▶ Split the simulation into time windows $[0, T] = \cup_{i=1}^M [t_i, t_{i+1}]$

Example: atmosphere-first sequential algorithm



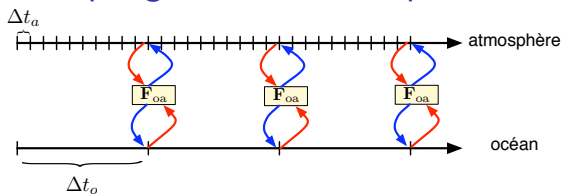
$$\begin{cases} \mathcal{L}_{atm} \mathbf{U}^a &= f_{atm} & \text{in } \Omega_{atm} \times [t_i, t_{i+1}] \\ \mathcal{F}_{atm} \mathbf{U}^a &= F_{OA}(\langle \mathbf{U}^o \rangle_{i-1}, \mathbf{U}^a, \mathcal{R}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

then

$$\begin{cases} \mathcal{L}_{oce} \mathbf{U}^o &= f_{oce} & \text{in } \Omega_{oce} \times [t_i, t_{i+1}] \\ \mathcal{F}_{oce} \mathbf{U}^o &= \langle \mathcal{F}_{atm} \mathbf{U}^a \rangle_i & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

- ▶ same mean fluxes over each time window $[t_i, t_{i+1}]$
- ▶ synchrony issue: time lag between model(s) and interface cond.

Synchronous coupling at the time step



$$\text{and } \begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_i + N \Delta t_a] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^a = F_{\text{OA}}(\mathbf{U}^o(t_i), \mathbf{U}^a(t), \mathcal{R}(t)) & \text{on } \Gamma \times [t_i, t_i + N \Delta t_a] \end{cases}$$

$$\begin{cases} \mathcal{L}_{\text{oce}} \mathbf{U}^o = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_i + \Delta t_o] \\ \mathcal{F}_{\text{oce}} \mathbf{U}^o = F_{\text{OA}}(\mathbf{U}^o(t_i), \mathbf{U}^a(t_i), \mathcal{R}(t_i)) & \text{on } \Gamma \times [t_i, t_i + \Delta t_o] \end{cases}$$

- ▶ much smaller synchrony issue
- ▶ difficult to implement efficiently
- ▶ physical validity issue (w.r.t. Monin-Obukhov hypotheses)
- ▶ stability issues with the coupling algorithm (Lemarié et al., 2015; Beljaars et al., 2017; Zhang et al., 2020)

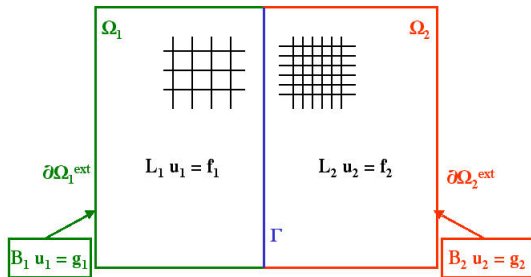
Algorithmic issues

- ▶ Current coupling methods are ad-hoc algorithms which ensure that fluxes are balanced and are computationally cheap.
- ▶ These methods raise a number of issues.

Questions

- ▶ Can the coupling algorithm be improved ?
- ▶ Does it improve the physics of the coupled solution ?
- ▶ Can this be done at a reasonable CPU cost ?

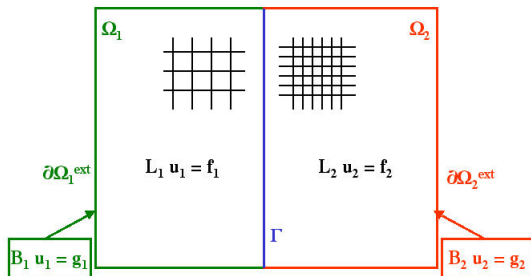
A convenient framework: Schwarz methods



$$\left\{ \begin{array}{l} L_1 u_1 = f_1 \quad \Omega_1 \times [0, T] \\ u_1 \text{ given} \quad \text{at } t = 0 \\ B_1 u_1 = g_1 \quad \partial\Omega_1^{\text{ext}} \times [0, T] \end{array} \right. \quad \left\{ \begin{array}{l} L_2 u_2 = f_2 \quad \Omega_2 \times [0, T] \\ u_2 \text{ given} \quad \text{at } t = 0 \\ B_2 u_2 = g_2 \quad \partial\Omega_2^{\text{ext}} \times [0, T] \end{array} \right.$$

+ physical constraints at the interface : $\mathcal{F}(u_1, u_2) = 0$

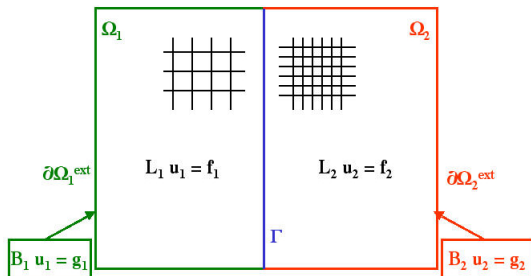
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$$\text{with } (C u_1 = C' u_2 \text{ and } D u_2 = D' u_1) \iff \mathcal{F}(u_1, u_2) = 0$$

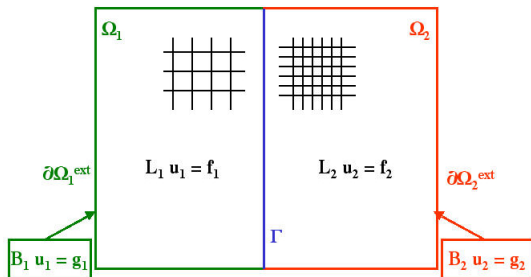
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- Present ocean-atmosphere coupling algorithms correspond to **one** single iteration of a Schwarz-like coupling method

Toward iterative ocean-atmosphere coupling

- ▶ Present ocean-atmosphere coupling algorithms correspond to **one single iteration of a Schwarz-like coupling method**

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^a = F_{\text{OA}}(\mathbf{U}^o, \mathbf{U}^a, \mathcal{R}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

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Toward iterative ocean-atmosphere coupling

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Iterate until convergence

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}_{k+1}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \mathbf{U}_{k+1}^{\text{a}} = F_{\text{OA}}(\mathbf{U}_k^{\text{o}}, \mathbf{U}_{k+1}^{\text{a}}, \mathcal{R}_{k+1}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

then

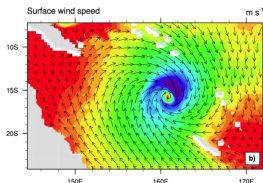
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- ▶ Should lead (if convergence) to the exact solution of the coupled problem

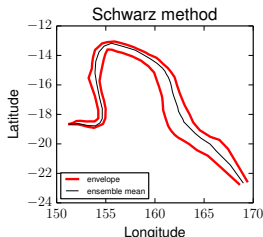
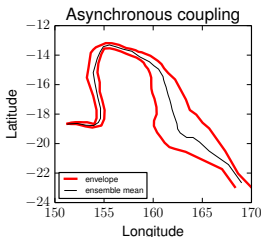
Does it improve the physics of the coupled solution ?

A major difficulty: no idealized coupled ocean-atmosphere testcase with a known reference solution.

Lemarié et al., 2014: ensemble simulations of a tropical cyclone with a coupled ROMS-WRF system.



Trajectory
of the
cyclone

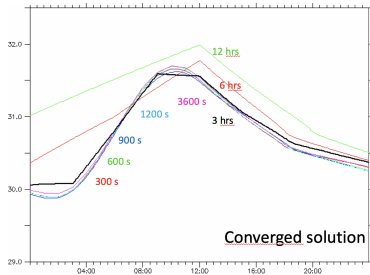
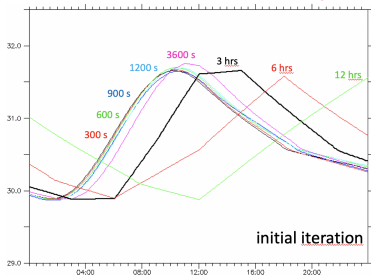


- ▶ 30%-40% decrease of the uncertainty on trajectory and intensity

Does it improve the physics of the coupled solution ?

24h simulations with the 1-D version of the climate model of
Météo-France (S. Valcke and A. Voldoire)

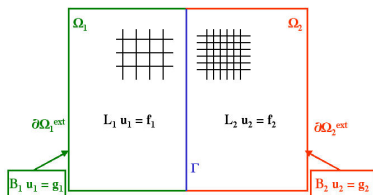
Ocean temperature at $z = -0.5$ m



► Resynchronization of the diurnal cycle

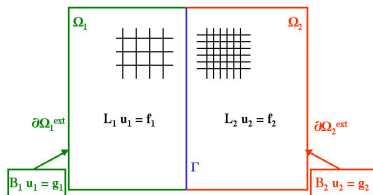
See also next talk by Olivier Marti with the IPSL Climate Model.

Analysis of the iterative process



$$\left\{ \begin{array}{l} L_1 u_1^{k+1} = f_1 \\ u_1^{k+1} \text{ given} \\ B_1 u_1^{k+1} = g_1 \\ C u_1^{k+1} = C' u_2^k \end{array} \right. \quad \begin{array}{l} \Omega_1 \times [0, T] \\ \text{at } t = 0 \\ \partial\Omega_1^{\text{ext}} \times [0, T] \\ \Gamma \times [0, T] \end{array} \quad \left\{ \begin{array}{l} L_2 u_2^{k+1} = f_2 \\ u_2^{k+1} \text{ given} \\ B_2 u_2^{k+1} = g_2 \\ D u_2^{k+1} = D' u_1^k \end{array} \right. \quad \begin{array}{l} \Omega_2 \times [0, T] \\ \text{at } t = 0 \\ \partial\Omega_2^{\text{ext}} \times [0, T] \\ \Gamma \times [0, T] \end{array}$$

Analysis of the iterative process



Systems satisfied by the errors $e_i^k = u_i^k - u_i$:

$$\left\{ \begin{array}{ll} L_1 e_1^{k+1} = 0 & \Omega_1 \times [0, T] \\ e_1^{k+1} = 0 & \text{at } t = 0 \\ B_1 e_1^{k+1} = 0 & \partial\Omega_1^{\text{ext}} \times [0, T] \\ C e_1^{k+1} = C' e_2^k & \Gamma \times [0, T] \end{array} \right. \quad \left\{ \begin{array}{ll} L_2 e_2^{k+1} = 0 & \Omega_2 \times [0, T] \\ e_2^{k+1} = 0 & \text{at } t = 0 \\ B_2 e_2^{k+1} = 0 & \partial\Omega_2^{\text{ext}} \times [0, T] \\ D e_2^{k+1} = D' e_1^k & \Gamma \times [0, T] \end{array} \right.$$

convergence factor

$$\rho_k = \frac{\|e_j^{k+1}\|}{\|e_j^k\|}$$

- ▶ analytical expression ?
- ▶ convergence: $\rho < 1$?
- ▶ minimization of ρ ?

Model problem: coupling of two Ekman layers

$$\begin{cases} \partial_t u_a - f v_a - \partial_z (\nu_a(z) \partial_z u_a) & = F_a^x \\ \partial_t v_a + f u_a - \partial_z (\nu_a(z) \partial_z v_a) & = F_a^y \end{cases}$$

$$\begin{cases} \partial_t u_o - f v_o - \partial_z (\nu_o(z) \partial_z u_o) & = F_o^x \\ \partial_t v_o + f u_o - \partial_z (\nu_o(z) \partial_z v_o) & = F_o^y \end{cases}$$

+ initial conditions

+ boundary conditions

- ▶ influence of the Coriolis term
- ▶ ν depends on z and is discontinuous at $z = 0$
 $\nu(z) \in P_0$ (viscous layer), P_1 (log layer), P_2 , P_3 (KPP)
- ▶ Interface conditions:

Dirichlet-Neumann

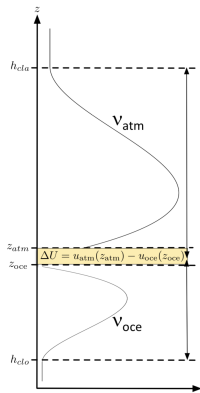
$$\begin{cases} \mathbf{U}_a(0) = \mathbf{U}_o(0) \\ \nu_o(0) \partial_z \mathbf{U}_o(0) = \nu_a(0) \partial_z \mathbf{U}_a(0) \end{cases}$$

Robin-Robin

$$\begin{cases} \nu_a(0) \partial_z \mathbf{U}_a(0) + \lambda \mathbf{U}_a(0) = \nu_o(0) \partial_z \mathbf{U}_o(0) + \lambda \mathbf{U}_o(0) \\ \nu_o(0) \partial_z \mathbf{U}_o(0) + \mu \mathbf{U}_o(0) = \nu_a(0) \partial_z \mathbf{U}_a(0) + \mu \mathbf{U}_a(0) \end{cases}$$

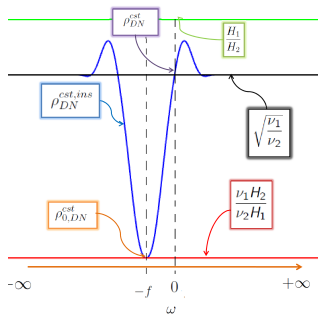
Bulk (linearized or not) $\nu_o(0) \partial_z \mathbf{U}_o(0) = \nu_a(0) \partial_z \mathbf{U}_a(0) = \alpha \| \mathbf{U}_a(0) - \mathbf{U}_o(0) \| (\mathbf{U}_a(0) - \mathbf{U}_o(0))$

With or without an averaging of the fluxes $\nu_o(0) \partial_z \mathbf{U}_o(0, t) = \langle \nu_a(0) \partial_z \mathbf{U}_a(0) \rangle$

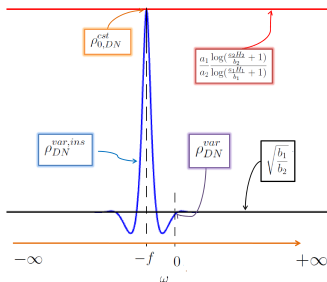


Model problem: coupling of two Ekman layers

- Analytical expressions of ρ for the different viscosity profiles and interface conditions



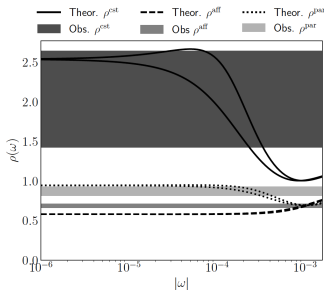
Dir-Neu, cst coeff



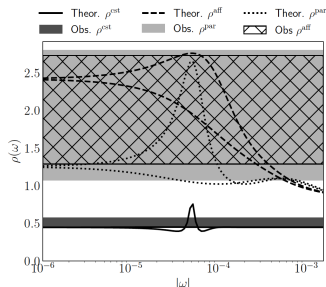
Dir-Neu, P_1 coeff

Model problem: coupling of two Ekman layers

- ▶ Analytical expressions of ρ for the different viscosity profiles and interface conditions
- ▶ The convergence properties depend on the whole vertical profile $\nu(z)$. Strong influence of the turbulent zone.



example 1: $\rho^{cst} > 1$, $\rho^{P1,P2} < 1$



example 2: $\rho^{cst} < 1$, $\rho^{P1,P2} > 1$

Model problem: coupling of two Ekman layers

- ▶ Analytical expressions of ρ for the different viscosity profiles and interface conditions
- ▶ The convergence properties depend on the whole vertical profile $\nu(z)$. Strong influence of the turbulent zone.
- ▶ Optimization results
- ▶ Averaging operator: solution has the correct average. Impact on ρ
- ▶ Impact of the nonlinearity of the bulk formula

References

- Thery S., C. Pelletier, F. Lemarié and E. Blayo, 2021: Analysis of Schwarz waveform relaxation for the coupled Ekman boundary layer problem with continuously variable coefficient. *Numerical Algorithms*, doi 10.1007/s11075-021-01149-y.
- Thery S., 2021: Étude numérique des algorithmes de couplage océan-atmosphère avec prise en compte des paramétrisations physiques de couches limites. *PhD thesis*, Univ. Grenoble Alpes.
- Clement S., F. Lemarié, and E. Blayo, 2021: Discrete analysis of Schwarz Waveform Relaxation for a simplified air-sea coupling problem with nonlinear transmission conditions. *Proceedings of the 26th conference on Domain Decomposition Methods*.

Summary and future work

Modeling aspects

- ▶ A general framework for the formulation of the coupled OA problem: hypotheses are clearer, two-sided point of view, possible additional parameterizations...
- ▶ Still a lot of (difficult) theoretical work to do:
 - ▶ Well-posedness of the coupled surface layers problem (a unique fixed-point ?)
 - ▶ Coherence of the different parameterizations (O and A turbulent closures and bulk formula)

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Modeling aspects

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 - ▶ Coherence of the different parameterizations (O and A turbulent closures and bulk formula)

Algorithmic aspects

- ▶ Present OA coupling algorithms raise consistency and stability issues. Schwarz algorithms are a convenient framework for developing improved methods.
- ▶ Iterative coupling can lead to improved solutions.
- ▶ The CPU cost must be decreased: optimizing the interface conditions, imagining preconditioners...