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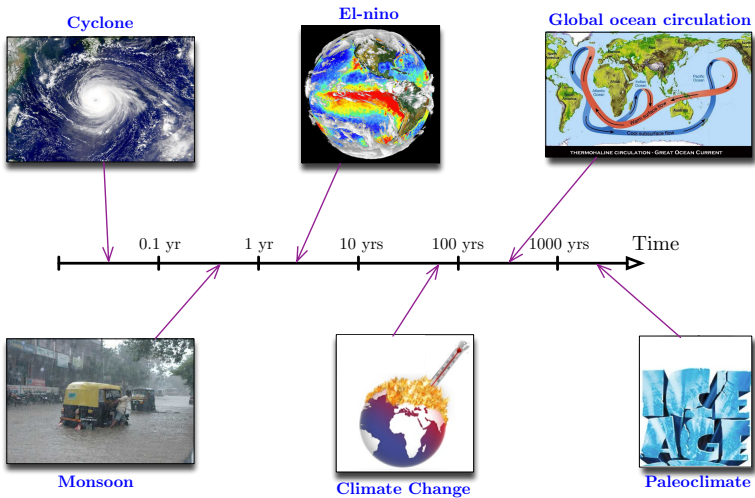
Analysis of Ocean-Atmosphere Coupling from the Point of View of Domain Decomposition Methods

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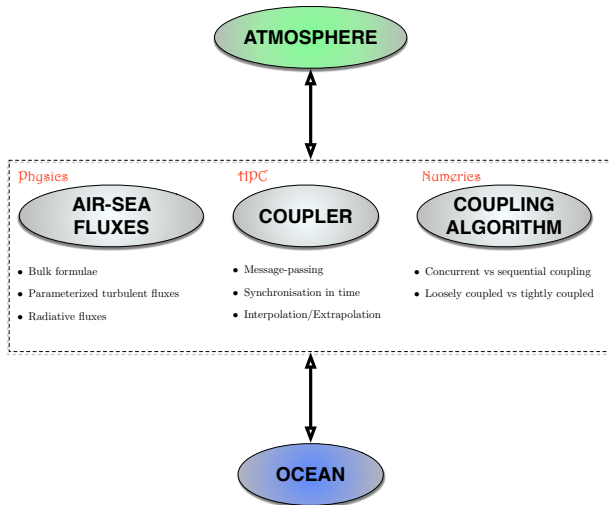
Context

Atmospheric and Oceanic Coupled Models (AOCMs)



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Content

1. Interface conditions and turbulent boundary fluxes
2. Algorithmic considerations and shortcomings of existing approaches
3. Schwarz algorithms for OA coupling: analysis of the iterative process
4. Application of Schwarz iterations in a climate model

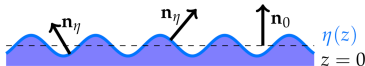
1

Interface conditions and turbulent boundary fluxes

Nature of interface conditions

The ocean and the atmosphere are two open systems exchanging both energy and matter through their common interface.

\mathbf{n}_0 unit normal to the interface



- Net mass flux:

$$\mathcal{J}^{\text{mass}} = \mathcal{E} - \mathcal{P}_r - \mathcal{P}_s$$

- Internal energy (enthalpy) flux:

$$h^{(sw)} \mathcal{J}^{\text{mass}} + \mathbf{J}_H^{\text{oce}} \cdot \mathbf{n}_0 = \mathbf{J}_H^{\text{atm}} \cdot \mathbf{n}_0 + h^{(wv)} \mathcal{E} - h^{(liq)} \mathcal{P}_r - h^{(ice)} \mathcal{P}_s$$

Blue terms represent energy associated to phase changes.

$\mathbf{J}_H^{\text{atm}} = \mathbf{J}_{\text{sen}}^{\text{atm}} + \mathbf{J}_{\text{rad}}^{\text{atm}}$ denotes the sum of sensible and radiative heat fluxes.

- Mechanical energy flux: continuity of stresses at the free boundary

Normal stress: $-p^{\text{oce}} + \sigma_{33}^{\text{oce}} = -p^{\text{atm}} + \sigma_{33}^{\text{atm}}$

Tangential stress: $\sigma_{i3}^{\text{atm}} = \sigma_{i3}^{\text{oce}} = \tau_i^{\text{atm}}, \quad (i = 1, 2)$

Main concern: necessity to conserve energy and mass at the air–sea interface

Coupled problem (without phase changes)

$$\partial_t \mathbf{v}^{\text{oce}} + 2\boldsymbol{\Omega} \times \mathbf{u}^{\text{oce}} + \nabla \cdot (\mathbf{u}^{\text{oce}} \otimes \mathbf{v}^{\text{oce}}) = -\frac{1}{\rho_0} \nabla p^{\text{oce}} - \frac{\rho^{\text{oce}}}{\rho_0} \mathbf{g} + \nabla \cdot \boldsymbol{\sigma}^{\text{oce}}$$

$$\mathbf{u}^{\text{oce}} = (u^{\text{oce}}, v^{\text{oce}}, w^{\text{oce}}) \quad \nabla \cdot \mathbf{u}^{\text{oce}} = 0$$

$$\mathbf{v}^{\text{oce}} = (u^{\text{oce}}, v^{\text{oce}}, 0) \quad \rho_0 c_p^{\text{oce}} \frac{D\theta^{\text{oce}}}{Dt} = -\nabla \cdot \mathbf{J}_H^{\text{oce}} = -\nabla \cdot (\mathbf{J}_T^{\text{oce}} + \mathbf{J}_{\text{rad}}^{\text{oce}})$$

$$\rho^{\text{oce}} = \rho(\theta^{\text{oce}}, p_0(z))$$

$$\rho^{\text{atm}} (\partial_t \mathbf{u}^{\text{atm}} + 2\boldsymbol{\Omega} \times \mathbf{u}^{\text{atm}} + \nabla \cdot (\mathbf{u}^{\text{atm}} \otimes \mathbf{u}^{\text{atm}})) = -\nabla p^{\text{atm}} - \rho^{\text{atm}} \mathbf{g} + \rho^{\text{atm}} \nabla \cdot \boldsymbol{\sigma}^{\text{atm}}$$

$$\partial_t \rho^{\text{atm}} + \nabla \cdot (\rho^{\text{atm}} \mathbf{u}^{\text{atm}}) = 0$$

$$\rho^{\text{atm}} c_p^{\text{atm}} \frac{D\theta^{\text{atm}}}{Dt} = -\nabla \cdot \mathbf{J}_H^{\text{atm}} = (\mathbf{J}_T^{\text{atm}} + \mathbf{J}_{\text{rad}}^{\text{atm}})$$

$$p^{\text{atm}} = g(\rho^{\text{atm}}, \theta^{\text{atm}})$$

Interface conditions

$$p^{\text{oce}}|_{z=\eta} = p^{\text{atm}}$$

$$\mathbf{J}_H^{\text{oce}} \cdot \mathbf{n}_0 = \mathbf{J}_H^{\text{atm}} \cdot \mathbf{n}_0 = \mathcal{J}_{\text{sen}} + \mathcal{J}_{\text{LW}} + \mathcal{J}_{\text{SW}}$$

$$\rho^{\text{oce}} (\boldsymbol{\sigma}^{\text{oce}} \mathbf{n}_0) = \rho^{\text{atm}} (\boldsymbol{\sigma}^{\text{atm}} \mathbf{n}_0) = (\boldsymbol{\tau}^a, 0)^T$$

Computation of the boundary fluxes τ^a and \mathcal{J}_{sen}

The boundary fluxes are the result of processes in turbulent boundary layers

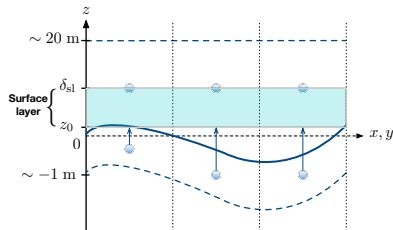
⇒ they need be parameterized in terms of resolved (Reynolds-averaged) quantities of the model

⇒ **Semi-empirical Monin-Obukhov similarity theory** is the basis to derive turbulent quantities from the mean variables available from the models

→ Generalization of the classical law of the wall to stratified conditions

The MO theory provides functional relationships to compute fundamental turbulent parameters u_\star and θ_\star from quantities at the first model level

$$\|\tau^a\| = \rho^{\text{atm}} u_\star^2, \quad \mathcal{J}_{\text{sen}} = -\rho^{\text{atm}} c_p^{\text{atm}} u_\star \theta_\star$$



MO theory: standard wall law + stability corrections

$$\begin{aligned}\mathbf{u}^{\text{atm}}(z) &= \mathbf{u}^{\text{oce}}(z_0) - \frac{u_\star}{\kappa} \left[\ln \left(\frac{z}{z_0} \right) - \psi_m \left(\frac{z}{L_{\text{Ob}}} \right) + \psi_m \left(\frac{z_0}{L_{\text{Ob}}} \right) \right] e^{i\theta_\tau} \\ \theta^{\text{atm}}(z) &= \theta^{\text{oce}}(z_0) - \frac{\theta_\star}{\kappa} \left[\ln \left(\frac{z}{z_0} \right) - \psi_s \left(\frac{z}{L_{\text{Ob}}} \right) + \psi_s \left(\frac{z_0}{L_{\text{Ob}}} \right) \right]\end{aligned}$$

► Usual form :

$$\begin{aligned}\tau^a &= \rho^{\text{atm}} C_D \|\mathbf{u}^{\text{atm}}(z_{\text{atm}}^1) - \mathbf{u}^{\text{oce}}(z_{\text{oce}}^1)\| (\mathbf{u}^{\text{atm}}(z_{\text{atm}}^1) - \mathbf{u}^{\text{oce}}(z_{\text{oce}}^1)) \\ \mathcal{J}_{\text{sen}} &= \rho^{\text{atm}} c_p^{\text{atm}} C_H \|\mathbf{u}^{\text{atm}}(z_{\text{atm}}^1) - \mathbf{u}^{\text{oce}}(z_{\text{oce}}^1)\| (\theta^{\text{atm}}(z_{\text{atm}}^1) - \theta^{\text{oce}}(z_{\text{oce}}^1))\end{aligned}$$

General comments

- Even under ideal conditions, the theory has an accuracy of only about 10–20% [Foken, 2006]
- Bill Large (2006) "Surface fluxes for practitioners of global ocean data assimilation"
 - *For hourly fluxes on a spatial scale of 10 km, there is at least a factor of 2 uncertainty due to transfer coefficient variability on these scales*
 - *Long-term averaging is required before the uncertainty in bulk fluxes is minimized*

⇒ Internal time-scale Δt_{MO} to keep uncertainty on turbulent flux estimates at a "reasonable level"

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Algorithmic considerations and shortcomings of existing approaches

Various alternatives

- ▷ Concurrent coupling vs sequential coupling (aka parallel vs multiplicative)
- ▷ Global-in-time vs local-in-time
- ▷ Intergrid transfer operator in time

Some criteria to choose an appropriate coupling algorithm

- Practical aspects
 - Computational efficiency (time to solution & scalability)
 - Minimal modification to existing codes
- Numerical & physical aspects
 - Numerical stability and consistency
 - Conservation properties
(in a weak or strong sense \sim local-in-time vs global-in-time)
 - Consistent with underlying assumptions of physical parameterizations
(e.g. Δt_{MO} , Δt_{rad} , PBL scheme, ...)
 - Vertical physics is handled implicitly in time

Standard partitioned time-stepping methods in CMs

4 variants:

- Global-in-time concurrent coupling (e.g. IPSL-CM6, CNRM-CM)
- Global-in-time "atmosphere first" sequential coupling (e.g. ECMWF)
- Global-in-time "atmosphere first" concurrent coupling (e.g. RPN)
- Local-in-time (used exclusively in regional coupled models so far)

Illustration on a simplified 1D coupling problem

Momentum exchange between two vertical columns

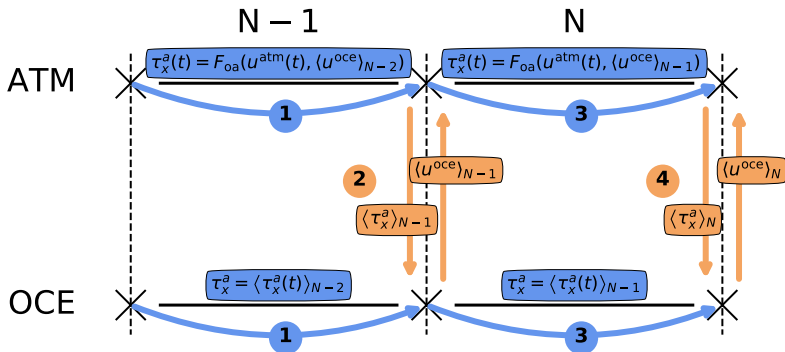
$$\begin{aligned}\partial_t u^{\text{atm}} &= -\partial_z \sigma_{xz}^{\text{atm}} = \partial_z (K_z^a \partial_z u^{\text{atm}}), & \text{in } (0, h_{\text{atm}}) \times (0, T) \\ \partial_t u^{\text{oce}} &= -\partial_z \sigma_{xz}^{\text{oce}} = \partial_z (K_z^o \partial_z u^{\text{oce}}), & \text{in } (-h_{\text{oce}}, 0) \times (0, T)\end{aligned}$$

$$\begin{aligned}\rho^{\text{oce}} K_z^o \partial_z u^{\text{oce}}(0^-, t) &= \rho^{\text{atm}} K_z^a \partial_z u^{\text{atm}}(0^+, t) = \tau_x^a, & t \in (0, T) \\ \tau_x^a &= \rho^{\text{atm}} C_D \|u^{\text{atm}}(z_{\text{atm}}^1) - u^{\text{oce}}(z_{\text{oce}}^1)\| (u^{\text{atm}}(z_{\text{atm}}^1) - u^{\text{oce}}(z_{\text{oce}}^1))\end{aligned}$$

+ appropriate initial and external B.C.

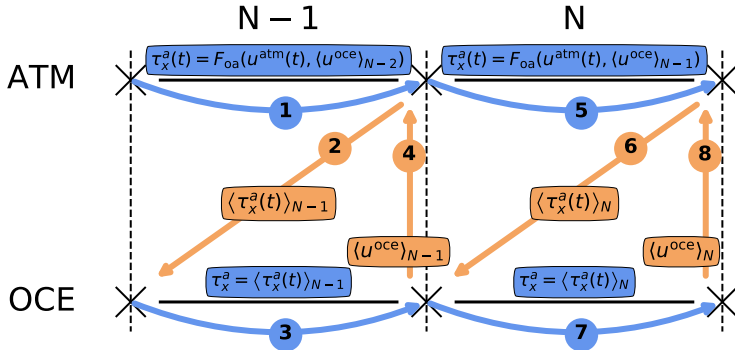
Global-in-time concurrent coupling $(0, T) = \bigcup_{N=0}^K \mathcal{T}_N, \mathcal{T}_N = (t_N, t_{N-1})$

- $\tau_x^a = F_{\text{oa}}(u^{\text{atm}}(z_a^1), u^{\text{oce}}(z_o^1))$
- Boundary fluxes are computed within the atmospheric component
- ▷ Flux conservation: $\left(\sum_{N=1}^K \langle \tau_x^a(t) \rangle_N \right)^{\text{oce}} = \left(\langle \tau_x^a(t) \rangle_{0, \dots, K-1} \right)^{\text{atm}}$



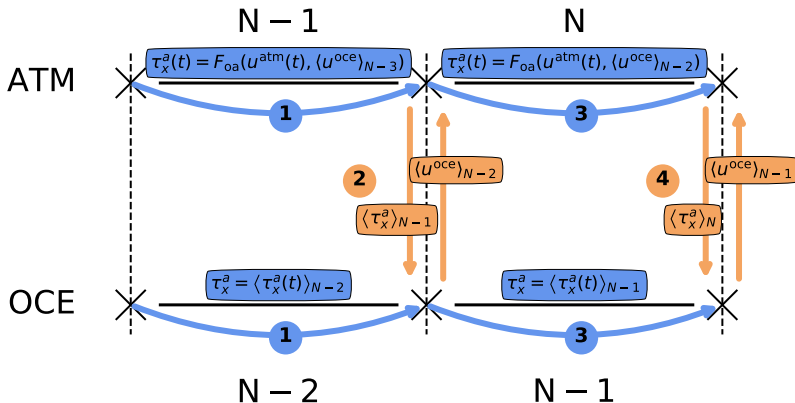
Global-in-time "atmosphere first" sequential coupling

- $\tau_x^a = F_{\text{oa}}(u^{\text{atm}}(z_a^1), u^{\text{oce}}(z_o^1))$
- Boundary fluxes are computed within the atmospheric component
- Flux conservation is re-synchronized in time



Global-in-time "atmosphere first" concurrent coupling

- $\tau_x^a = F_{\text{oa}}(u^{\text{atm}}(z_a^1), u^{\text{oce}}(z_o^1))$
- Boundary fluxes are computed within the atmospheric component



Global-in-time approach (sequential or concurrent)

- + Both models forced by the exact same mean flux on a given time window (except a one-window lag in the concurrent coupling)
 - + Models communicate only once per time window
 - + Consistent with the underlying assumptions of some physical parameterizations (decouples the dynamical and physical time steps)
 - Shifted retroaction → **loosely coupled solution**
 - Sequential coupling suppresses a level of parallelism and generally increases time to solution
 - In the oceanic component: discontinuity in forcing fields between two successive time windows
- **Existing global-in-time approaches** = one single iteration of a **global-in-time Schwarz algorithm**

Local-in-time approach ($\Delta t_{\text{oce}} = \Delta t_{\text{atm}}$)

ATM model

$$\begin{cases} u_{\text{atm}}^{n+1} &= u_{\text{atm}}^n + \Delta t_{\text{atm}} \partial_z (K_z^a \partial_z u_{\text{atm}}^{n+1}) \\ K_z^a \partial_z u_{\text{atm}}^{n+1}(z=0) &= C_D \|u_{\text{atm}}^n(z_a^1) - u_{\text{oce}}^n(z_o^1)\| (u_{\text{atm}}^{n+a}(z_a^1) - u_{\text{oce}}^n(z_o^1)) \end{cases}$$

OCE model

$$\begin{cases} u_{\text{oce}}^{n+1} &= u_{\text{oce}}^n + \Delta t_{\text{oce}} \partial_z (K_z^o \partial_z u_{\text{oce}}^{n+1}) \\ \rho_o K_z^o \partial_z u_{\text{oce}}^{n+1}(z=0) &= \rho_a K_z^a \partial_z u_{\text{atm}}^{n+1}(z=0) \end{cases}$$

- $a = 1$: implicit flux coupling (sequential)
- $a = 0$: explicit flux coupling (conditionally stable, e.g. [\[Zhang et al., 2020\]](#))

Local-in-time approach

- lagged coupling → loosely coupled partitioned scheme
- Relevance of instantaneous air-sea fluxes ? (large uncertainties)
- Very frequent exchanges/remapping of interface data between models
- + Ability to represent processes related to diurnal cycle

• Existing local-in-time approaches = one single iteration of a local-in-time Schwarz algorithm

How to achieve a tighter coupling: Schwarz algorithms

- [Keyes et al. (2013)]: *"Using an approach that ignores strong couplings between components gives a false sense of completion"*

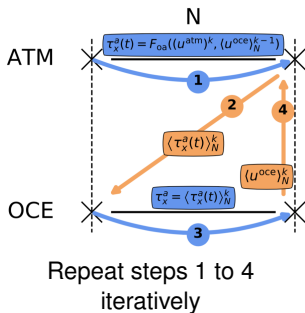
If the iterations actually fail to converge, using only one iteration won't reveal this fact but in this case numerical results would be questionable

$$\left\{ \begin{array}{ll} \partial_t u_1^k - \nabla \cdot (\nu_1 \nabla u_1^k) &= f_1, & x \in \Omega_1 \\ \partial_t u_2^k - \nabla \cdot (\nu_2 \nabla u_2^k) &= f_2, & x \in \Omega_2 \\ u_1^k &= u_2^{k-1}, & x \in \Gamma \\ \nu_2 \nabla u_2^k &= \nu_1 \nabla u_1^k, & x \in \Gamma \end{array} \right.$$

$$\text{Convergence rate : } R = \sqrt{\nu_2/\nu_1}$$

- **BUT:** tight coupling between components require smoothness
- *Easy way to try it :* Schwarz algorithms (global-in-time for multiphysics problems; only require "perfect" restartability of numerical models)

Global-in-time domain decomposition based on Schwarz method



For $t \in (t_{N-1}, t_N)$:

$$\partial_t u_{atm}^k = \partial_z \left(K_z^a \partial_z u_{atm}^k \right)$$

$$\partial_t u_{oce}^k = \partial_z \left(K_z^o \partial_z u_{oce}^k \right)$$

$$K_z^a \partial_z u_{atm}^k(0^+, t) = F_{oa}(u_{atm}^k, \langle u_{oce}^{k-1} \rangle_N)$$

$$\rho_{oce} K_z^o \partial_z u_{oce}^k(0^-, t) = \langle \rho_{atm} K_z^a \partial_z u_{atm}^k(0^+, t) \rangle_N$$

Does it converge ? If yes, what's the impact on the physics ?

3

Schwarz algorithms for OA coupling: analysis of the iterative process

A convenient framework: Schwarz methods

- Huge gap to fill between the theory and practical applications in the analysis of the iterative process
- The convergence analysis usually done at a continuous level for linear problems with constant coefficients and no averaging of boundary data

In the OA coupling case:

- Non-linear transmission conditions defined in a semi-discrete sense
- Turbulent viscosities and diffusivities vary greatly in time and space
- Boundary data are averaged in time

- ▷ The convergence properties depend on the whole viscosity profile $\nu(z)$. Strong influence of the turbulent zone [Thery et al. (2021)]
- ▷ Impact of boundary averaging for linear problems: solution has the correct average [Thery (2020)]

Semi-discrete coupled Ekman problem

$$\partial_t(\mathbf{u}_m^{\text{atm}})^k + f\mathbf{e}_z \times (\mathbf{u}_m^{\text{atm}})^k = \frac{\nu_{m+1/2}^a}{h_m} \left(\frac{\mathbf{u}_{m+1}^{\text{atm}} - \mathbf{u}_m^{\text{atm}}}{h_{m+1/2}} \right)^k - \frac{\nu_{m-1/2}^a}{h_m} \left(\frac{\mathbf{u}_m^{\text{atm}} - \mathbf{u}_{m-1}^{\text{atm}}}{h_{m-1/2}} \right)^k$$

$$\partial_t(\mathbf{u}_m^{\text{oce}})^k + f\mathbf{e}_z \times (\mathbf{u}_m^{\text{oce}})^k = \frac{\nu_{m+1/2}^o}{h_m} \left(\frac{\mathbf{u}_{m+1}^{\text{oce}} - \mathbf{u}_m^{\text{oce}}}{h_{m+1/2}} \right)^k - \frac{\nu_{m-1/2}^o}{h_m} \left(\frac{\mathbf{u}_m^{\text{oce}} - \mathbf{u}_{m-1}^{\text{oce}}}{h_{m-1/2}} \right)^k$$

$$\left(\nu_{1/2}^a \frac{\mathbf{u}_1^{\text{atm}} - \mathbf{u}_0^{\text{atm}}}{h_{1/2}} \right)^k = C_D \|(\mathbf{u}_1^{\text{atm}})^{k-1} - (\mathbf{u}_N^{\text{oce}})^{k-1}\| \left((\mathbf{u}_1^{\text{atm}})^{k-1+\theta} - (\mathbf{u}_N^{\text{oce}})^{k-1} \right)$$

$$\left(\nu_{N+1/2}^o \frac{\mathbf{u}_{N+1}^{\text{oce}} - \mathbf{u}_N^{\text{oce}}}{h_{N+1/2}} \right)^k = \frac{\rho^{\text{atm}}}{\rho^{\text{oce}}} \left(\nu_{1/2}^a \frac{\mathbf{u}_1^{\text{atm}} - \mathbf{u}_0^{\text{atm}}}{h_{1/2}} \right)^k$$

$$\text{with } (\mathbf{u}_1^{\text{atm}})^{k-1+\theta} = \theta(\mathbf{u}_1^{\text{atm}})^k + (1-\theta)(\mathbf{u}_1^{\text{atm}})^{k-1}.$$

Asymptotic convergence rate independent from iterates $\left(\varepsilon = \frac{\rho^{\text{atm}}}{\rho^{\text{oce}}} \right)$

$$\lim_{(\omega+f) \rightarrow 0} \xi = \frac{1}{\theta} \left| \frac{3}{2} - \theta + \frac{3}{2} \varepsilon \sqrt{\frac{\nu_a}{\nu_o}} \right|, \quad \lim_{(\omega+f) \rightarrow \infty} \xi = 0$$

- Optimization for low frequencies: $\theta^{\text{opt}} = 3/2$

Semi-discrete coupled Ekman problem

Asymptotic convergence rate independent from iterates $\left(\varepsilon = \frac{\rho^{\text{atm}}}{\rho^{\text{oce}}}\right)$

$$\lim_{(\omega+f) \rightarrow 0} \xi = \frac{1}{\theta} \left| \frac{3}{2} - \theta + \frac{3}{2} \varepsilon \sqrt{\frac{\nu_a}{\nu_o}} \right|, \quad \lim_{(\omega+f) \rightarrow \infty} \xi = 0$$

- Optimization for low frequencies: $\theta^{\text{opt}} = 3/2$
- Assuming $C_D \|(\mathbf{u}_1^{\text{atm}})^{k-1} - (\mathbf{u}_N^{\text{oce}})^{k-1}\| = \text{cst}$ would give $\theta^{\text{opt}} = 1$

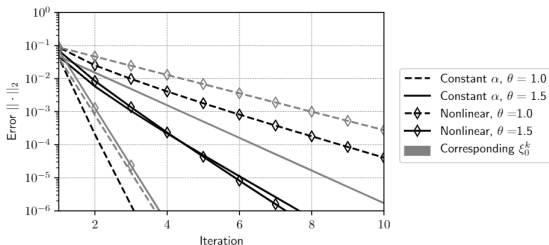


Fig. 2 Evolution of the L^2 norm of the errors. Black lines represent the observed convergence; grey lines are the estimated convergence with slopes ξ_0 for linear cases and ξ_0^q for quadratic cases.

[Clément et al.
(2021), DD26
proceedings]

4

Application of Schwarz iterations in a climate model

Application of Schwarz iterations in a realistic CM

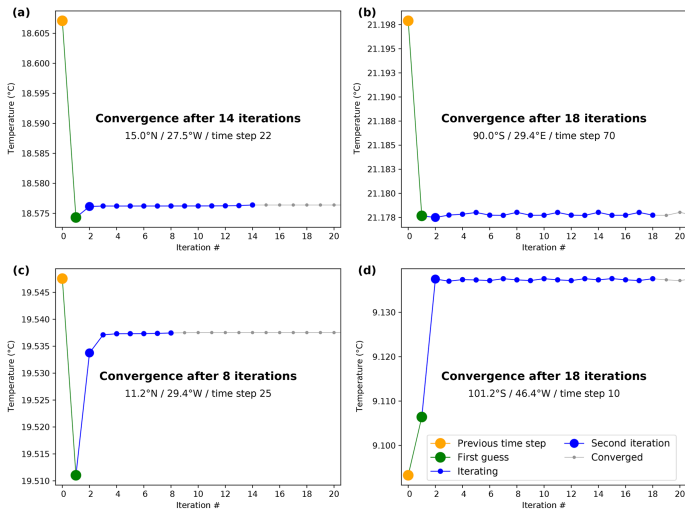
Marti O., S. Nguyen, P. Braconnot, S. Valcke, F. Lemarié, and E. Blayo, 2021: *A Schwarz iterative method to evaluate ocean-atmosphere coupling schemes. Implementation and diagnostics in IPSL-CM6-SW-VLR*, Geosci. Model Dev.

Experiments:

- State of the art model: ocean, atmosphere, sea-ice, land (used for CMIP6)
- Atm. component: $3.75^\circ \times 1.875^\circ$ (39 vertical levels)
- Oce. component: $2^\circ \times 2^\circ$ (31 vertical levels)
- 3 numerical coupling methods:
 - Global-in-time concurrent (parallel)
 - Global-in-time "atmosphere first" sequential
 - Global-in-time Schwarz (50 iterations with $\theta = 1$)
- 5 day experiments
- Time windows: 1 hour or 4 hours

Behavior during iterations

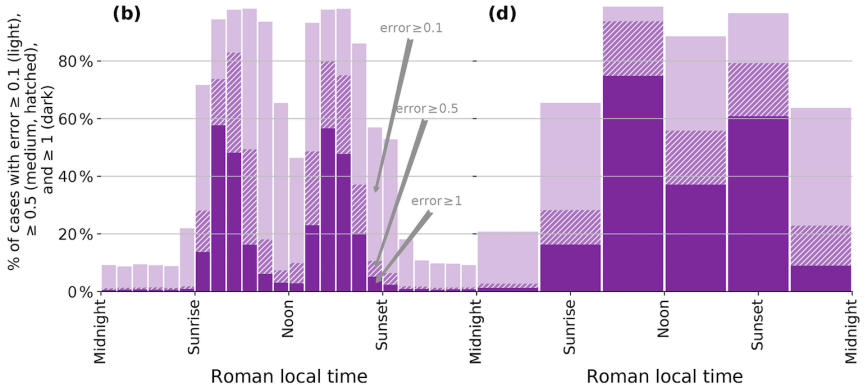
Sea surface temperature evolution with respect to the iterations at 4 different locations



(a,b) 4 hour time window; (c,d) 1 hour time window

Diagnosing the error \mathcal{E} of lagged coupling

For a given time window (t_{N-1}, t_N) , $\mathcal{E} = \frac{SST^{cvg}(t_N) - SST^{par}(t_N)}{SST^{cvg}(t_N) - SST^{cvg}(t_{N-1})}$

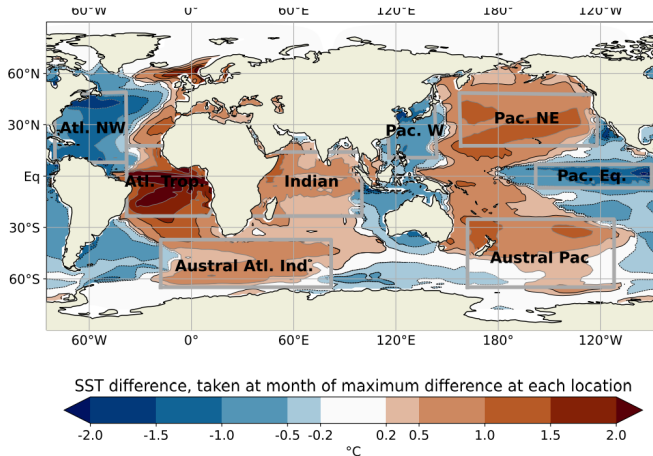


⇒ large differences after sunrise and before sunset, when the external forcing (insolation at the top of the atmosphere) has the fastest pace of change

Application of Schwarz iterations in a realistic CM

Results from 1 year ensemble simulations (20 ensembles)

Monthly average of the ensemble mean sea surface temperature (SST) difference between Schwarz vs concurrent (parallel) coupling



Summary

- Present ocean-atmosphere coupling methods correspond to loosely coupled partitioned scheme
- Schwarz domain decomposition methods due to their inherent simplicity provide a convenient framework for evaluating existing methods
- The convergence/divergence of the iterative Schwarz method can be used as an indicator for the degree of *compatibility* of models and particularly their physical parameterizations
- As soon as the MO theory remains the standard for air-sea fluxes estimate, some form of averaging in the boundary data is needed

Future work

- ▷ The iterative process is not viable for production runs. How to get the best possible estimate of the converged solution with only 1 or 2 iterations ?
- ▷ Need for more systematic benchmarking of coupled models under simplified settings (!)

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