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Analysis of Ocean-Atmosphere Coupling from the Point of View of Domain Decomposition Methods

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Context
Atmospheric and Oceanic Coupled Models (AOCMs)

- Cyclone
- El-nino
- Global ocean circulation
- Monsoon
- Climate Change
- Paleoclimate

Timeline:
- 0.1 yr
- 1 yr
- 10 yrs
- 100 yrs
- 1000 yrs

Climate Change
Paleoclimate
Context

Atmospheric and Oceanic Coupled Models (AOCMs)

- Message-passing
- Synchronisation in time
- Interpolation/Extrapolation
- Bulk formulae
- Parameterized turbulent fluxes
- Radiative fluxes
- Concurrent vs sequential coupling
- Loosely coupled vs tightly coupled

OCEAN
ATMOSPHERE
COUPLER
AIR-SEA FLUXES
COUPLING ALGORITHM

HPC
Physics

F. Lemarié – OA coupling formulation and algorithms
Content

1. Interface conditions and turbulent boundary fluxes

2. Algorithmic considerations and shortcomings of existing approaches

3. Schwarz algorithms for OA coupling: analysis of the iterative process

4. Application of Schwarz iterations in a climate model
1

Interface conditions and turbulent boundary fluxes
Nature of interface conditions

The ocean and the atmosphere are two open systems exchanging both energy and matter through their common interface.

$n_0$ unit normal to the interface

- **Net mass flux:**

$$J^{\text{mass}} = \mathcal{E} - \mathcal{P}_r - \mathcal{P}_s$$

- **Internal energy (enthalpy) flux:**

$$h^{(sw)} J^{\text{mass}} + J^{\text{oce}}_H \cdot n_0 = J^{\text{atm}}_H \cdot n_0 + h^{(wv)} \mathcal{E} - h^{(liq)} \mathcal{P}_r - h^{(ice)} \mathcal{P}_s$$

**Blue terms** represent energy associated to phase changes.

$$J^{\text{atm}}_H = J^{\text{atm}}_{\text{sen}} + J^{\text{atm}}_{\text{rad}}$$ denotes the sum of sensible and radiative heat fluxes.

- **Mechanical energy flux: continuity of stresses at the free boundary**

  Normal stress:
  $$-p^{\text{oce}} + \sigma^{\text{oce}}_{33} = -p^{\text{atm}} + \sigma^{\text{atm}}_{33}$$

  Tangential stress:
  $$\sigma^{\text{atm}}_{i3} = \sigma^{\text{oce}}_{i3} = \tau^{\text{atm}}_i, \quad (i = 1, 2)$$

**Main concern:** necessity to conserve energy and mass at the air–sea interface
Coupled problem (without phase changes)

\[
\begin{align*}
\partial_t \mathbf{v}^\text{oce} + 2\Omega \times \mathbf{u}^\text{oce} + \nabla \cdot (\mathbf{u}^\text{oce} \otimes \mathbf{v}^\text{oce}) &= -\frac{1}{\rho_0} \nabla p^\text{oce} - \frac{\rho^\text{oce}}{\rho_0} \mathbf{g} + \nabla \cdot \mathbf{\sigma}^\text{oce} \\
\mathbf{u}^\text{oce} &= (u^\text{oce}, v^\text{oce}, w^\text{oce}) \\
\mathbf{v}^\text{oce} &= (u^\text{oce}, v^\text{oce}, 0)
\end{align*}
\]

\[
\nabla \cdot \mathbf{u}^\text{oce} = 0
\]

\[
\begin{align*}
\rho_0 c_p^\text{oce} \frac{D\theta^\text{oce}}{Dt} &= -\nabla \cdot \mathbf{J}^\text{oce} = -\nabla \cdot (\mathbf{J}_T^\text{oce} + \mathbf{J}_\text{rad}^\text{oce}) \\
\rho^\text{oce} &= \rho(\theta^\text{oce}, p_0(z))
\end{align*}
\]

\[
\begin{align*}
\rho^\text{atm} \left( \partial_t \mathbf{u}^\text{atm} + 2\Omega \times \mathbf{u}^\text{atm} + \nabla \cdot (\mathbf{u}^\text{atm} \otimes \mathbf{u}^\text{atm}) \right) &= -\nabla p^\text{atm} - \rho^\text{atm} \mathbf{g} + \rho^\text{atm} \nabla \cdot \mathbf{\sigma}^\text{atm} \\
\partial_t \rho^\text{atm} + \nabla \cdot (\rho^\text{atm} \mathbf{u}^\text{atm}) &= 0 \\
\rho^\text{atm} c_p^\text{atm} \frac{D\theta^\text{atm}}{Dt} &= -\nabla \cdot \mathbf{J}^\text{atm} = (\mathbf{J}_T^\text{atm} + \mathbf{J}_\text{rad}^\text{atm}) \\
p^\text{atm} &= g(\rho^\text{atm}, \theta^\text{atm})
\end{align*}
\]

Interface conditions

\[
\begin{align*}
p^\text{oce} \big|_{z=\eta} &= p^\text{atm} \\
\mathbf{J}^\text{oce}_H \cdot \mathbf{n}_0 &= \mathbf{J}^\text{atm}_H \cdot \mathbf{n}_0 = \mathcal{J}_\text{sen} + \mathcal{J}_\text{LW} + \mathcal{J}_\text{SW} \\
\rho^\text{oce} (\mathbf{\sigma}^\text{oce} \mathbf{n}_0) &= \rho^\text{atm} (\mathbf{\sigma}^\text{atm} \mathbf{n}_0) = (\mathbf{\tau}^a, 0)^T
\end{align*}
\]
Computation of the boundary fluxes $\tau^a$ and $J_{\text{sen}}$

The boundary fluxes are the result of processes in turbulent boundary layers

$\Rightarrow$ they need be parameterized in terms of resolved (Reynolds-averaged) quantities of the model

$\Rightarrow$ Semi-empirical Monin-Obukhov similarity theory is the basis to derive turbulent quantities from the mean variables available from the models

$\rightarrow$ Generalization of the classical law of the wall to stratified conditions

The MO theory provides functional relationships to compute fundamental turbulent parameters $u_*$ and $\theta_*$ from quantities at the first model level

$$\parallel \tau^a \parallel = \rho_{\text{atm}} u_*^2,$$

$$J_{\text{sen}} = -\rho_{\text{atm}} c_p \rho_{\text{atm}} u_* \theta_*$$
MO theory: standard wall law + stability corrections

\[ u^{\text{atm}}(z) = u^{\text{oce}}(z_0) - \frac{u_*}{\kappa} \left[ \ln \left( \frac{z}{z_0} \right) - \psi_m \left( \frac{z}{L_{\text{Ob}}} \right) + \psi_m \left( \frac{z_0}{L_{\text{Ob}}} \right) \right] e^{i \theta_\tau} \]

\[ \theta^{\text{atm}}(z) = \theta^{\text{oce}}(z_0) - \frac{\theta_*}{\kappa} \left[ \ln \left( \frac{z}{z_0} \right) - \psi_s \left( \frac{z}{L_{\text{Ob}}} \right) + \psi_s \left( \frac{z_0}{L_{\text{Ob}}} \right) \right] \]

▷ Usual form:

\[ \tau^a = \rho^{\text{atm}} C_D \| u^{\text{atm}}(z_{\text{atm}}^1) - u^{\text{oce}}(z_{\text{oce}}^1) \| (u^{\text{atm}}(z_{\text{atm}}^1) - u^{\text{oce}}(z_{\text{oce}}^1)) \]

\[ J_{\text{sen}} = \rho^{\text{atm}} c_p^{\text{atm}} C_H \| u^{\text{atm}}(z_{\text{atm}}^1) - u^{\text{oce}}(z_{\text{oce}}^1) \| (\theta^{\text{atm}}(z_{\text{atm}}^1) - \theta^{\text{oce}}(z_{\text{oce}}^1)) \]
General comments

- Even under ideal conditions, the theory has an accuracy of only about 10–20% [Foken, 2006]

- Bill Large (2006) "Surface fluxes for practitioners of global ocean data assimilation"
  - For hourly fluxes on a spatial scale of 10 km, there is at least a factor of 2 uncertainty due to transfer coefficient variability on these scales
  - Long-term averaging is required before the uncertainty in bulk fluxes is minimized

⇒ Internal time-scale $\Delta t_{\text{MO}}$ to keep uncertainty on turbulent flux estimates at a "reasonable level"
Algorithmic considerations and shortcomings of existing approaches
Various alternatives

- Concurrent coupling vs sequential coupling (aka parallel vs multiplicative)
- Global-in-time vs local-in-time
- Intergrid transfer operator in time

Some criteria to choose an appropriate coupling algorithm

- Practical aspects
  - Computational efficiency (time to solution & scalability)
  - Minimal modification to existing codes

- Numerical & physical aspects
  - Numerical stability and consistency
  - Conservation properties
    (in a weak or strong sense ~ local-in-time vs global-in-time)
  - Consistent with underlying assumptions of physical parameterizations
    (e.g. $\Delta t_{MO}$, $\Delta t_{rad}$, PBL scheme, ...)
  - Vertical physics is handled implicitly in time
Standard partitioned time-stepping methods in CMs

4 variants:

- Global-in-time concurrent coupling (e.g. IPSL-CM6, CNRM-CM)
- Global-in-time "atmosphere first" sequential coupling (e.g. ECMWF)
- Global-in-time "atmosphere first" concurrent coupling (e.g. RPN)
- Local-in-time (used exclusively in regional coupled models so far)

Illustration on a simplified 1D coupling problem

Momentum exchange between two vertical columns

\[
\begin{align*}
\partial_t u^{atm} &= -\partial_z \sigma^{atm}_{xz} = \partial_z \left(K^{a}_z \partial_z u^{atm}\right), \quad \text{in } (0, h_{atm}) \times (0, T) \\
\partial_t u^{oce} &= -\partial_z \sigma^{oce}_{xz} = \partial_z \left(K^{o}_z \partial_z u^{oce}\right), \quad \text{in } (-h_{oce}, 0) \times (0, T) \\
\rho^{oce} K^{o}_z \partial_z u^{oce}(0^-, t) &= \rho^{atm} K^a_z \partial_z u^{atm}(0^+, t) = \tau^a_x, \quad t \in (0, T) \\
\tau^a_x &= \rho^{atm} C_D \left\| u^{atm}(z^{1}_{atm}) - u^{oce}(z^{1}_{oce}) \right\| (u^{atm}(z^{1}_{atm}) - u^{oce}(z^{1}_{oce})) 
\end{align*}
\]

+ appropriate initial and external B.C.
Global-in-time concurrent coupling \((0, T) = \bigcup_{N=0}^{K} \mathcal{T}_N, \mathcal{T}_N = (t_N, t_{N-1})\)

- \(\tau_x^a = F_{oa}(u_{atm}^1(z_1^a), u_{oce}^1(z_1^o))\)
- Boundary fluxes are computed within the atmospheric component
  - Flux conservation: \(\left(\sum_{N=1}^K \langle \tau_x^a(t) \rangle_N\right)_{oce} = \left(\langle \tau_x^a(t) \rangle_{0,\ldots,K-1}\right)_{atm}\)
Global-in-time "atmosphere first" sequential coupling

- $\tau_x^a = F_{oa}(u^{atm}(z_1^a), u^{oce}(z_1^o))$
- Boundary fluxes are computed within the atmospheric component
- Flux conservation is re-synchronized in time
Global-in-time "atmosphere first" concurrent coupling

- $\tau^a_x = F_{oa}(u^{atm}(z^1_a), u^{oce}(z^1_o))$
- Boundary fluxes are computed within the atmospheric component
Global-in-time approach (sequential or concurrent)

+ Both models forced by the exact same mean flux on a given time window (except a one-window lag in the concurrent coupling)
+ Models communicate only once per time window
+ Consistent with the underlying assumptions of some physical parameterizations (decouples the dynamical and physical time steps)
  - Shifted retroaction → loosely coupled solution
  - Sequential coupling suppresses a level of parallelism and generally increases time to solution
  - In the oceanic component: discontinuity in forcing fields between two successive time windows

• Existing global-in-time approaches = one single iteration of a global-in-time Schwarz algorithm
Local-in-time approach ($\Delta t_{oce} = \Delta t_{atm}$)

**ATM model**

\[
\begin{align*}
    u_{atm}^{n+1} &= u_{atm}^n + \Delta t_{atm} \partial_z (K_a^z \partial_z u_{atm}^n) \\
    K_a^z \partial_z u_{atm}^{n+1}(z = 0) &= CD \| u_{atm}^n(z_a^0) - u_{oce}^n(z_o^1) \| (u_{atm}^{n+a}(z_a^1) - u_{oce}^n(z_o^1))
\end{align*}
\]

**OCE model**

\[
\begin{align*}
    u_{oce}^{n+1} &= u_{oce}^n + \Delta t_{oce} \partial_z (K_o^z \partial_z u_{oce}^n) \\
    \rho_o K_o^z \partial_z u_{oce}^{n+1}(z = 0) &= \rho_a K_a^z \partial_z u_{atm}^{n+1}(z = 0)
\end{align*}
\]

- $a = 1$: implicit flux coupling (sequential)
- $a = 0$: explicit flux coupling (conditionally stable, e.g. [Zhang et al., 2020])
Local-in-time approach

- lagged coupling $\rightarrow$ loosely coupled partitioned scheme
- Relevance of instantaneous air-sea fluxes ? (large uncertainties)
- Very frequent exchanges/remapping of interface data between models
+ Ability to represent processes related to diurnal cycle

• Existing local-in-time approaches = one single iteration of a local-in-time Schwarz algorithm
How to achieve a tighter coupling: Schwarz algorithms

- [Keyes et al. (2013)]: "Using an approach that ignores strong couplings between components gives a false sense of completion"

If the iterations actually fail to converge, using only one iteration won’t reveal this fact but in this case numerical results would be questionable

\[
\begin{align*}
\partial_t u_1^k - \nabla \cdot (\nu_1 \nabla u_1^k) &= f_1, \quad x \in \Omega_1 \\
\partial_t u_2^k - \nabla \cdot (\nu_2 \nabla u_2^k) &= f_2, \quad x \in \Omega_2 \\
u_1^k &= u_2^{k-1}, \quad x \in \Gamma \\
\nu_2 \nabla u_2^k &= \nu_1 \nabla u_1^k, \quad x \in \Gamma
\end{align*}
\]

Convergence rate: \( R = \sqrt{\nu_2/\nu_1} \)

- **BUT:** tight coupling between components require smoothness

- **Easy way to try it:** Schwarz algorithms (global-in-time for multiphysics problems; only require ”perfect” restartibility of numerical models)
Global-in-time domain decomposition based on Schwarz method

For $t \in (t_{N-1}, t_N)$:

$$\partial_t u_{atm}^k = \partial_z \left( K_z^a \partial_z u_{atm}^k \right)$$

$$\partial_t u_{oce}^k = \partial_z \left( K_z^o \partial_z u_{oce}^k \right)$$

$$K_z^a \partial_z u_{atm}^k(0^+, t) = F_{oa}(u_{atm}^k, \langle u_{oce}^{k-1} \rangle_N)$$

$$\rho_{oce} K_z^o \partial_z u_{oce}^k(0^-, t) = \left\langle \rho_{atm} K_z^a \partial_z u_{atm}^k(0^+, t) \right\rangle_N$$

Repeat steps 1 to 4 iteratively

Does it converge? If yes, what’s the impact on the physics?
Schwarz algorithms for OA coupling: analysis of the iterative process
A convenient framework: Schwarz methods

- Huge gap to fill between the theory and practical applications in the analysis of the iterative process
- The convergence analysis usually done at a continuous level for linear problems with constant coefficients and no averaging of boundary data

In the OA coupling case:
- Non-linear transmission conditions defined in a semi-discrete sense
- Turbulent viscosities and diffusivities vary greatly in time and space
- Boundary data are averaged in time

- The convergence properties depend on the whole viscosity profile $\nu(z)$. Strong influence of the turbulent zone [Thery et al. (2021)]

- Impact of boundary averaging for linear problems: solution has the correct average [Thery (2020)]
Semi-discrete coupled Ekman problem

\[
\partial_t (u_{atm}^m)^k + f e_z \times (u_{atm}^m)^k = \nu^a_m \frac{u_{atm}^m - u_{atm}^{m+1}}{h_m} - \nu^{a} \frac{u_{atm}^m - u_{atm}^{m-1}}{h_m} \\
\partial_t (u_{oce}^m)^k + f e_z \times (u_{oce}^m)^k = \nu^o_m \frac{u_{oce}^m - u_{oce}^{m+1}}{h_m} - \nu^{o} \frac{u_{oce}^m - u_{oce}^{m-1}}{h_m} \\

\left( \nu^{a} \frac{u_{atm}^1 - u_{atm}^0}{h_{1/2}} \right)^k = C_D \| (u_{atm}^{k-1}) - (u_{oce}^{k-1}) \| \left( (u_{atm}^{k-1+\theta}) - (u_{oce}^{k-1}) \right) \\
\left( \nu^{o} \frac{u_{oce}^N+1 - u_{oce}^N}{h_{N+1/2}} \right)^k = \frac{\rho^a}{\rho^{oce}} \left( \nu^{a} \frac{u_{atm}^1 - u_{atm}^0}{h_{1/2}} \right)^k \\
\]
Semi-discrete coupled Ekman problem

Asymptotic convergence rate independent from iterates \( \left( \varepsilon = \frac{\rho_{\text{atm}}}{\rho_{\text{oce}}} \right) \)

\[
\lim_{(\omega+f)\to 0} \xi = \frac{1}{\theta} \left| \frac{3}{2} - \theta + \frac{3}{2} \varepsilon \sqrt{\frac{\nu_a}{\nu_o}} \right|, \quad \lim_{(\omega+f)\to \infty} \xi = 0
\]

- Optimization for low frequencies: \( \theta^{\text{opt}} = 3/2 \)
- Assuming \( C_D \| (u_{\text{atm}}^k)^{k-1} - (u_{\text{oce}}^N)^{k-1} \| = \text{cst} \) would give \( \theta^{\text{opt}} = 1 \)

**Fig. 2** Evolution of the \( L^2 \) norm of the errors. Black lines represent the observed convergence; grey lines are the estimated convergence with slopes \( \xi_0 \) for linear cases and \( \xi_0^q \) for quadratic cases.
Application of Schwarz iterations in a climate model
Application of Schwarz iterations in a realistic CM

Experiments:

- State of the art model: ocean, atmosphere, sea-ice, land (used for CMIP6)
- Atm. component: $3.75^\circ \times 1.875^\circ$ (39 vertical levels)
- Oce. component: $2^\circ \times 2^\circ$ (31 vertical levels)
- 3 numerical coupling methods:
  - Global-in-time concurrent (parallel)
  - Global-in-time ”atmosphere first” sequential
  - Global-in-time Schwarz (50 iterations with $\theta = 1$)
- 5 day experiments
- Time windows: 1 hour or 4 hours
Behavior during iterations
Sea surface temperature evolution with respect to the iterations at 4 different locations

(a) 4 hour time window; (c,d) 1 hour time window
Diagnosing the error $\mathcal{E}$ of lagged coupling

For a given time window $(t_{N-1}, t_N)$, $\mathcal{E} = \frac{\text{SST}^{\text{cvg}}(t_N) - \text{SST}^{\text{par}}(t_N)}{\text{SST}^{\text{cvg}}(t_N) - \text{SST}^{\text{cvg}}(t_{N-1})}$

$\Rightarrow$ large differences after sunrise and before sunset, when the external forcing (insolation at the top of the atmosphere) has the fastest pace of change
Application of Schwarz iterations in a realistic CM

Results from 1 year ensemble simulations (20 ensembles)

Monthly average of the ensemble mean sea surface temperature (SST) difference between Schwarz vs concurrent (parallel) coupling

SST difference, taken at month of maximum difference at each location

°C

-2.0  -1.5  -1.0  -0.5  -0.2  0.2  0.5  1.0  1.5  2.0

F. Lemarié – OA coupling formulation and algorithms
Summary

• Present ocean-atmosphere coupling methods correspond to loosely coupled partitioned scheme

• Schwarz domain decomposition methods due to their inherent simplicity provide a convenient framework for evaluating existing methods

• The convergence/divergence of the iterative Schwarz method can be used as an indicator for the degree of compatibility of models and particularly their physical parameterizations

• As soon as the MO theory remains the standard for air-sea fluxes estimate, some form of averaging in the boundary data is needed

Future work

▷ The iterative process is not viable for production runs. How to get the best possible estimate of the converged solution with only 1 or 2 iterations?

▷ Need for more systematic benchmarking of coupled models under simplified settings (!)
References


