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## Quantum Safe Symmetric Cryptography - Invited talk

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# New Results on Symmetric Quantum Cryptanalysis and Perspectives

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# Outline

- ▶ Introduction  
On Quantum-Safe **Symmetric** Cryptography
- ▶ Quantum Cryptanalysis - Illustrative Case  
Even-Mansour and FX:  
Best Classical and Quantum attacks
- ▶ Open Problems and Conclusion

# Symmetric Cryptography

# Classical Cryptography

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Enable secure communications even in the presence of malicious adversaries.

Asymmetric (e.g. RSA) (*computationally costly*)

Security based on well-known hard mathematical problems (e.g. factorization).

Symmetric (e.g. AES) (*key exchange needed BUT efficient.*)

Ideal security defined by generic attacks ( $2^{|K|}$ ).

Need of continuous security evaluation (cryptanalysis).

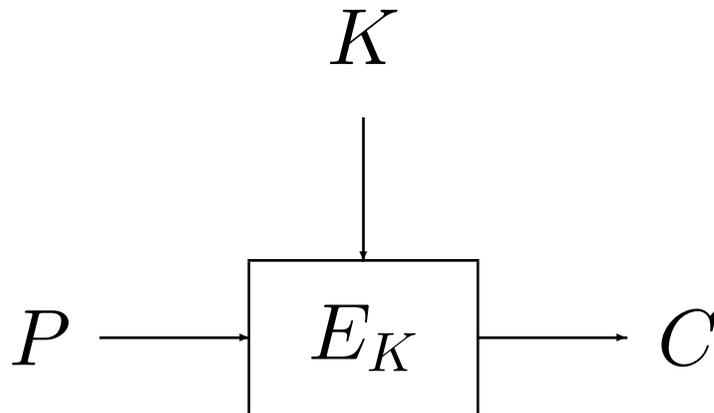
⇒ Hybrid systems! (e.g. in SSH)

# Symmetric primitives

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- ▶ Block ciphers, (stream ciphers, hash functions..)

Message decomposed into blocks, each transformed by the same function  $E_K$ .



$E_K$  is composed of a round transform repeated through several similar rounds.

# Generic Attacks on Ciphers

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- ▶ Security provided by an **ideal block cipher** defined by the best generic attack: **exhaustive search** for the key in  $2^{|K|}$ .

⇒ typical key sizes  $|K| = 128$  to  $256$  bits.

- ▶ Recovering the key from a secure cipher must need at least  $2^{|K|}$  operations.

# Cryptanalysis: Foundation of Confidence

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- ▶ Ideal security defined by generic attacks.  
Does real security meet this ideal one?  
Need of continuous security evaluations

Any attack better than the generic one  
is considered a “break” .

- ▶ We are often left with an empirical measure of the security: Cryptanalysis.

# Current scenario

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▶ Competitions (AES, SHA-3, eSTREAM, CAESAR, LW..). ▶ New needs: Lightweight, FHE-friendly, easy-masking.

⇒ Many good proposals/candidates.

- ▶ How to choose?
- ▶ How to be ahead of possible weaknesses?
- ▶ How to keep on trusting the chosen ones?

# Cryptanalysis: Foundation of Confidence

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When can we consider a primitive as secure?

- A primitive is secure as far as no attack on it is known.
- The more we analyze a primitive without finding any weaknesses, the more reliable it is.

**Design new attacks + improvement of existing ones:**

- ▶ essential to keep on **trusting** the primitives,
- ▶ **or to stop using the insecure ones!**

# Very Important Notion: Security Margin

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If no attack is found on a given cipher, what can we say about its robustness?

The security of a cipher is not a 1-bit information:

- Round-reduced attacks.
- Analysis of components.

⇒ determine and adapt the security margin, which is the highest number of rounds reached by an attack.

# On high complexities

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When considering large keys, sometimes attacks breaking the ciphers might have a very high complexity far from practical e.g..  $2^{120}$  for a key of 128 bits.

Still dangerous because:

- Weak properties not expected by the designers.
  - Experience shows us that **attacks only get better**.
  - Other existing ciphers without the "ugly" properties.
- ▶ When determining the **security margin**:  
Allows to **compare** primitives and to anticipate problems.

# Post-Quantum Symmetric Cryptography

# Post-Quantum Cryptography

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Adversaries have access to **quantum computers**.

Asymmetric (e.g. RSA):

Shor's algorithm: Factorization in polynomial time

⇒ **current systems not secure!**

Solutions: lattice-based, code-based cryptography...

Symmetric (e.g. AES):

Grover's algorithm: Exhaustive search from  $2^{|K|}$  to  $2^{|K|/2}$ .

Double the key length for equivalent ideal security.

**Much to learn about cryptanalysis of current ciphers when having quantum computing available.**

# Post-Quantum Cryptography

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Problem for present existing long-term secrets.  
⇒ start using quantum-safe primitives NOW.

## Important tasks:

- ▶ Conceive the **cryptanalysis algorithms** for evaluating the security of symmetric primitives in the P-Q world.
- ▶ Use them to evaluate and **design** symmetric primitives for the P-Q world.

# On Quantum Attacks

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- ▶ Compare to best generic attack,
- ▶ generic attack is accelerated, so
- ▶ broken classical primitive might be unbroken in a quantum setting:

e.g. a primitive might not have 256-bits security against a classical adversary but might have 128-bit security against a quantum one.

# Scenarios and Models

# Considered Scenarios

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- ▶ **Model  $Q_0$**   
classical attacks with classical computers.
- ▶ **Model  $Q_1$**   
 $Q_0$  + access to a quantum computer.
- ▶ **Model  $Q_2$**   
 $Q_1$  + superposition queries to a quantum cryptographic oracle (QCO).
- ▶ **Model  $Q_3$**   
 $Q_1$  + superposition queries with the differences of a secret key in a QCO.

# Model $Q_0$

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Nothing new here.

# Model $Q_1$

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- ▶ So far, the best we have obtained is a quadratic speed-up, but it can be smaller.

Regarding **exhaustive search**:

- If a primitive is safe<sup>1</sup> in  $Q_0$ , it will also be in  $Q_1$ .

- ▶ Does this mean that (so far) these  $Q_1$  scenario/results are not interesting?

**No!**

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<sup>1</sup>safe = no attack better than generic attack

# Model $Q_1$

---

- ▶ **Until recently**, the best we have obtained was a quadratic speed-up, but it can be smaller.

Regarding **exhaustive search**:

- If a primitive is safe in  $Q_0$ , it will also be in  $Q_1$ .
- ▶ Are  $Q_1$  scenario/results not interesting?

**No!**

# Model $Q_1$

---

1) In a post-quantum future:

- ▶ Classical or quantum surnames will disappear:  
Expected security given by their best generic attack (e.g. Grover).  
And **security margin**? → determined by the highest number of rounds cryptanalyzed with **any** attack more performant than generic.
- ▶  $Q_1$  results: important information needed for determining the unique and future security margin.

# Model $Q_1$

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2) Regarding Collisions this is a bit different, as quantum collision speed-up is **less than quadratic**:

**Hosoyamada and Sasaki 2020**: showed that we could build quantum rebound attacks on hash functions that reach **more rounds** than classical attacks!

# Model $Q_2$

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Very powerful, BUT...

Many good reasons to study security in this scenario:

- ▶ **Simple**: used in security proofs.
- ▶ **Non-trivial**: Many constructions still seem resistant.
- ▶ **Inclusive** of all intermediate scenarios: protocols, obfuscation, hybrid machines, incompetent users...

## Model $Q_2$

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Defined and used in many results:

[Zhandry12], [Boneh-Zhandry13], [Damgård et al13], [Mossayebi-Schack16], [Song-Yun17], [Ito et al19], [Cid et al20], Simon's attacks, FX, AEZ...

An attack in this model  $\Rightarrow$  we need to be extra careful when implementing the primitive in a quantum computer and using it in certain applications.

Ideas have proved useful for improving  $Q_1$  attacks.

# Model $Q_3$

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Super strong model:

Everything is broken [Roetteler-Steinwandt 15]

Too strong model!

# Another scenario classification

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Different types of **memory**:

- ▶ Classical memory,
- ▶ Small amount of qubits,
- ▶ QRAM memories (QRACM or QRAQM).

Less **complex** memory, less **amount**  $\Rightarrow$   
more realistic scenario

Evolution

# First Results

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- ▶ Quantum analysis of CubeHash [Leurent 10]
- ▶ Simon on 3-round Feistel [Kuwakado Morii 10]
- ▶ Simon on Even-Mansour [Kuwakado Morii 12]
- ▶ Quantum MITM iterated ciphers [Kaplan14]
- ▶ Quantum Related-Key [Roetteler-Steinwandt15]
- ▶ Simon on modes+slides [Kaplan-Leurent-Leverrier-NP16b][Santolli-Schaffner17]
- ▶ Difflinear [Kaplan-Leurent-Leverrier-NP16b]

# Quantum Symmetric Cryptanalysis

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Many new results since:

**FX** [Leander-May17], **parallel multi-preim.** [Banegas-Bernstein17],  
**Multicollision** [Hosoyamada-Sasaki-Xagawa17], **Mitm Q1** [Hosoyamada  
Sasaki 18], **DS Mitm Feistel** [Hosoyamada Sasaki 18], **Miss-in-the-  
middle** [Xie, Yang 18], **Feistel key-recovery** [Dong, Wang 18], **CCA  
on Feistel** [Ito et al 19], **Quantum Rebounds** [Hosoyamada Sasaki 20],  
**Simon's evaluations** [Bonnetain, Jacques20] [Shi21] , **Q1 better than  
quadratic**[Bonnetain, Schrottenloher, Sibleyras21] ...

# Main activity from QUASYModo

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- ▶ Efficient Collisions [Chailloux NP Schrottenloher Asiacrypt17],
- ▶ On modular additions [Bonnetain NP Asiacrypt 2018]
- ▶ K-xor [Grassi NP Schrottenloher Asiacrypt2018] [NP Schrot EC20]
- ▶ AES quantum evaluation [Bonnetain NP Schrottenloher ToSC18]
- ▶ On quantum slide attacks [Bonnetain NP Schrottenloher SAC18]
- ▶ Off-line Simon [Bonnetain Hosoyamada NP Schrott. Sasaki AC19]
- ▶ Algos subset-sum [Bonnetain Bricout Schrottenloher Shen AC20]
- ▶ Saturnin [Canteaut Duval Leurent NP Perrin Pornin Schrot. ToSC20]
- ▶ QCB [Bhaumik Bonnet. Chailloux Leurent NP Schrot Seurin AC21]
- ▶ Q linearization attacks [Bonnetain Leurent NP Schrot. AC21]

# Most Useful Quantum Tools

# Some Quantum Tools...

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...that have been useful so far.

- ▶ Amplitude Amplification (AA) / Grover
- ▶ Quantum Collisions
- ▶ Simon
- ▶ Kuperberg

# Amplitude Amplification

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*Exhaustive search:*

Given  $f : \{0, 1\}^n \rightarrow \{0, 1\}$ , find one element  $x \in \{0, 1\}^n$  such that  $f(x) = 1$ .

- ▶ Classical complexity:  $\Omega\left(\frac{2^n}{|\text{supp}(f)|}\right)$ .
- ▶ Quantum complexity [Brassard-Hoyer 97]:  
 $\mathcal{O}\left(\sqrt{\frac{2^n}{|\text{supp}(f)|}}\right)$ .

# Quantum Collision Algorithms

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*Collision problem:* Given a random function  $H : \{0, 1\}^n \rightarrow \{0, 1\}^n$ , find  $x, y \in \{0, 1\}^n$  with  $x \neq y$  such that  $H(x) = H(y)$ .

- ▶ Classical complexity:  $\Omega(2^{n/2})$ .
- ▶ Quantum complexity:  
[Brassard-Hoyer-Tapp 97]  $\mathcal{O}(2^{n/3})$  in queries, in time and in quantum memory

# Simon's algorithm

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*Simon's problem:*

Given  $f : \{0, 1\}^n \rightarrow \{0, 1\}^n$  such that

$\exists s \mid f(x) = f(y) \iff [x = y \text{ or } x \oplus y = s],$

find  $s$ .

- ▶ Classical complexity:  $\Omega(2^{n/2})$ .
- ▶ Quantum complexity [Simon 94]:  
 $T = \mathcal{O}(n^3), D = \mathcal{O}(n)$

# Kuperberg's algorithm

---

*Hidden Shift Problem with modular addition:*

Let  $f, g$  be two injective functions,  $(\mathbb{G}, +)$  a group. Given the promise that there exists  $s \in \mathbb{G}$  such that, for all  $x$ ,  $f(x) = g(x + s)$ , retrieve  $s$ .

- ▶ Classical complexity:  $\Omega(2^{n/2})$ .
- ▶ Quantum complexity: [Kuperberg 05]  $2^{\tilde{O}(\sqrt{n})}$ .

Some new Results  
New useful Quantum Tools

# Some New Tools

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New quantum tools for cryptanalysis:

- ▶ New Quantum Collision Algorithm
- ▶ Quantum K-xor Algorithms
- ▶ Multicollisions
- ▶ Grover-meets-Simon
- ▶ Off-line Simon
- ▶ Simon-meets-Kuperberg
- ▶ Framework for quantizing classical attacks
- ▶ Quantumly efficient DDT equivalent...

# Recent Quantum Cryptanalysis: Illustrative Case

# Outline of the case

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- Even-Mansour Construction
- Best generic classical attacks
- Best quantum attacks in Q1 and Q2
- FX construction - Grover-meets-Simon Q2
- Reducing the queries
- Off-line Simon.
- Better than quadratic in Q1.

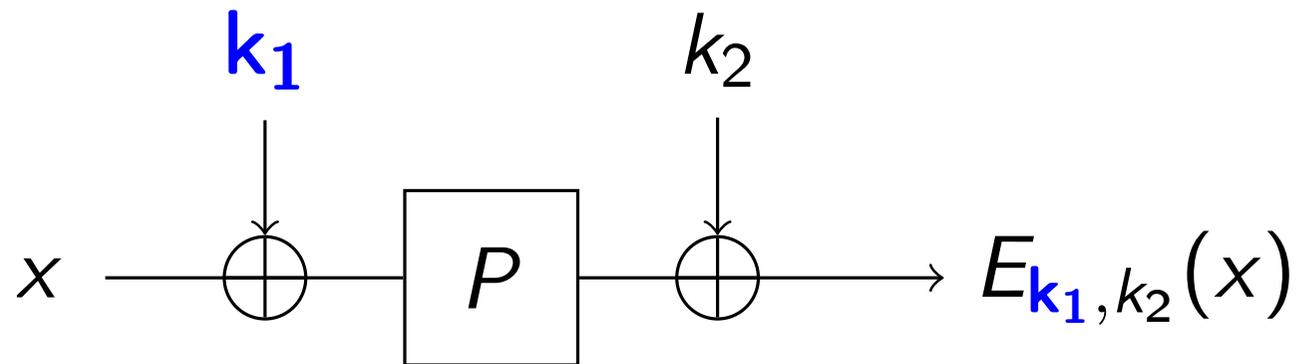
# Even-Mansour [EM97]

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From a public random permutation

$P : \{0, 1\}^n \rightarrow \{0, 1\}^n$  and

$2n$  bits of key,  $k_1$  and  $k_2$ :

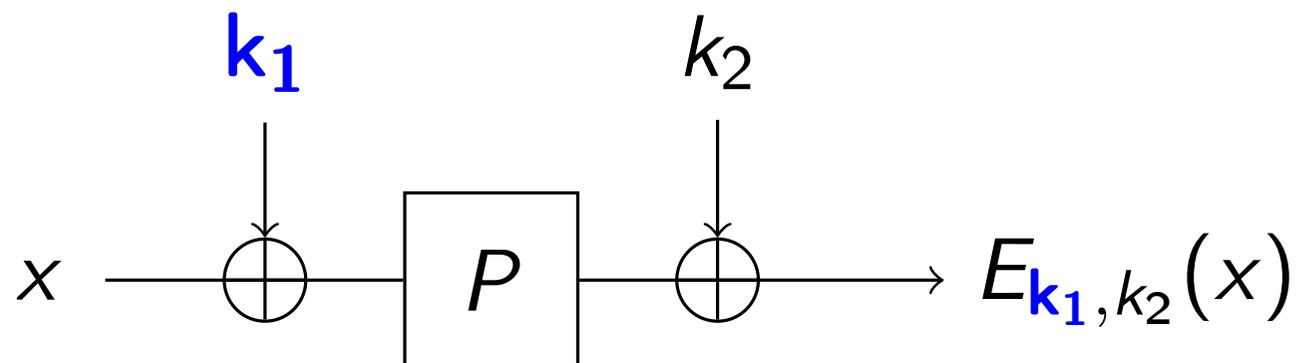


# Even-Mansour [EM97]

---

Any classical attack requires  $T \times D \geq 2^n$ .

Ex1) Guess  $k_1 \Rightarrow k_2$ :  $D = 2$  and  $T = 2^n$ .



Ex2) Collision search on  $f$ :

$$f(x) = E_{k_1, k_2}(x) \oplus P(x) = k_2 \oplus P(x) \oplus P(x \oplus k_1)$$

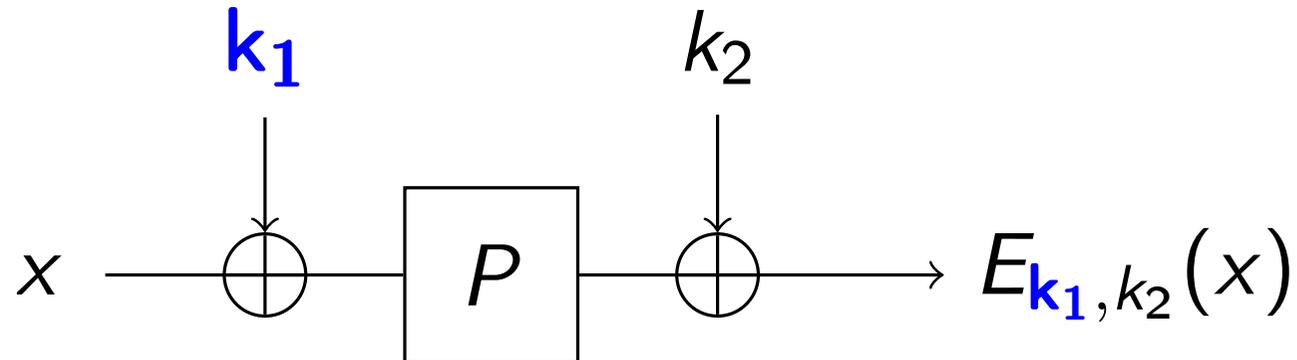
as  $f(x) = f(x \oplus k_1)$ , with  $T = D = 2^{n/2}$ .

All calls to  $f$  are queries  $\rightarrow D \geq 2^{n/2}$

# Even-Mansour [EM97]

---

Any classical attack requires  $T \times D \geq 2^n$ .



Ex3) We can define  $h(x)$  and  $g(x)$  [Daemen 91]:

$$h(x) = E_{k_1, k_2}(x) \oplus E_{k_1, k_2}(x \oplus 1) \text{ and}$$
$$g(x) = P(x) \oplus P(x \oplus 1) \text{ (not queries).}$$

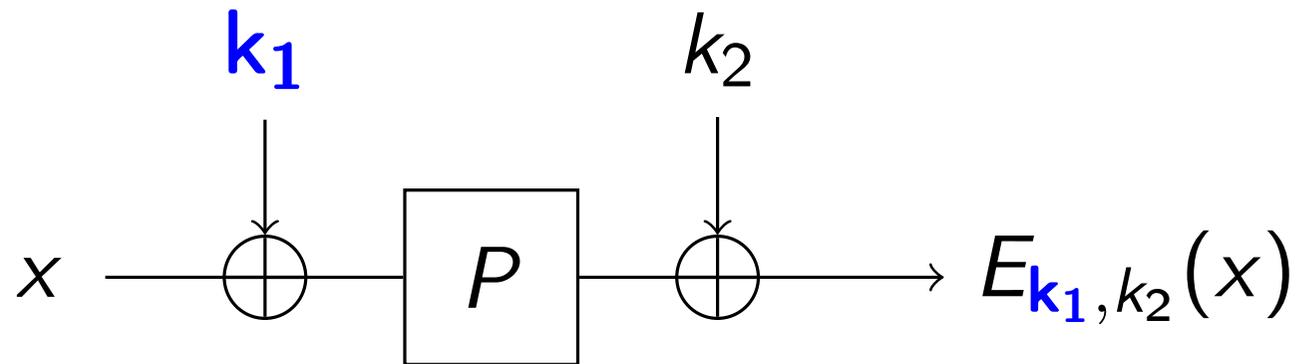
We have  $\forall x, g(x \oplus k_1) = h(x)$ .

**Complexity:**  $D$  and  $T = 2^n / D$

# Even-Mansour [EM97]

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And quantumly?



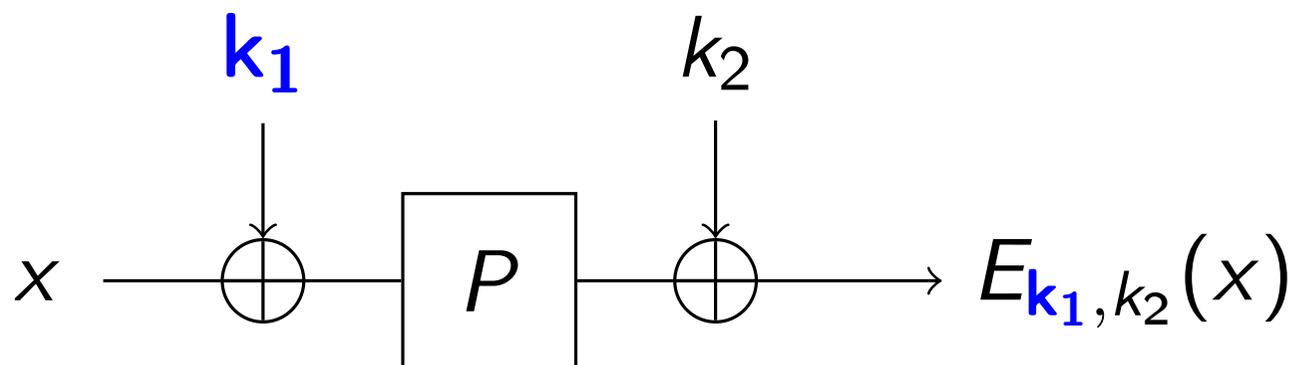
# Even-Mansour Q1 A [KM12]

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And quantumly?

We could use BHT with  $h$  and  $g$ :

$D \times T^2 = 2^n$  needs  $D$  qRAM.



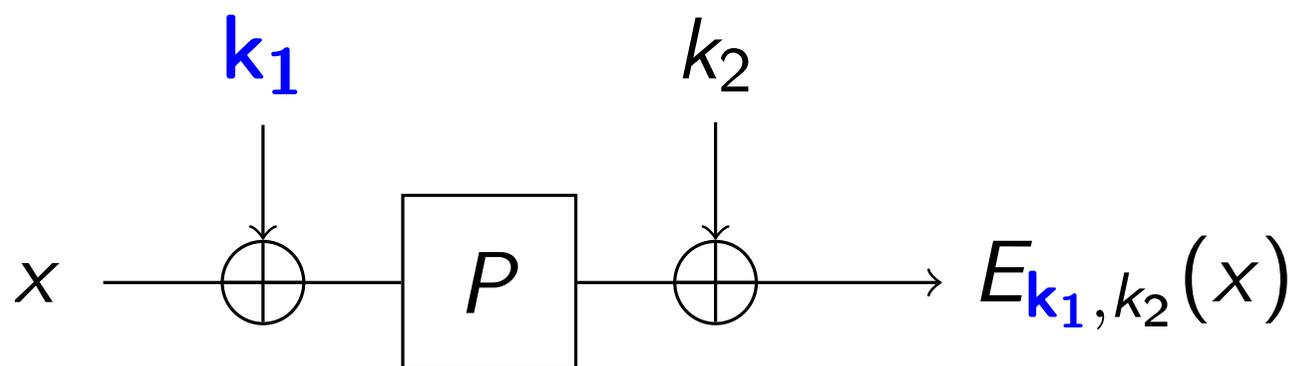
# Even-Mansour Q1 B [HS18]

---

And quantumly?

We could use [CNPS17] and no qRAM (but classical memo =  $2^{n/7}$ ):

$$D = T = 2^{3n/7}$$

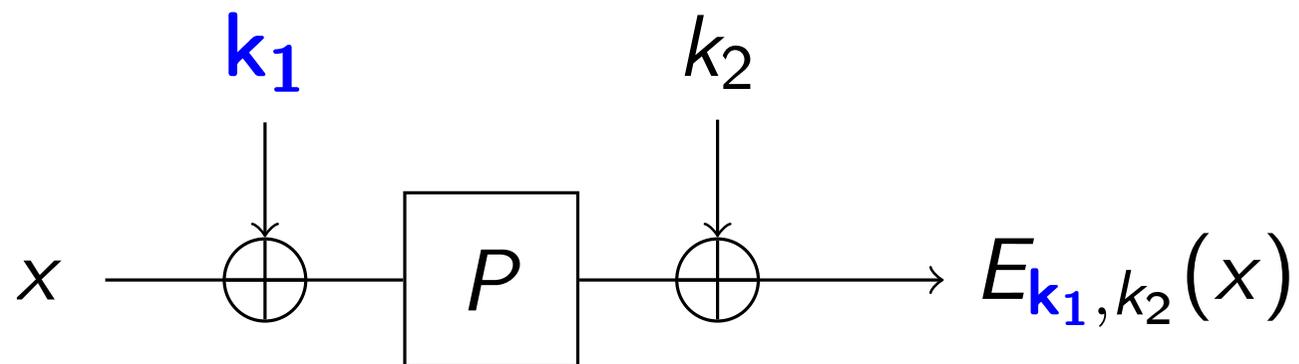


# Even-Mansour Q2 [KM12]

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And quantumly?

In Q2 we can directly use Simon on  $f$ :



$$f(x) = E_{k_1, k_2}(x) \oplus P(x) = k_2 \oplus P(x) \oplus P(x \oplus k_1)$$

that has a secret period  $k_1$ : we can recover it in **polynomial time**.

# Even-Mansour

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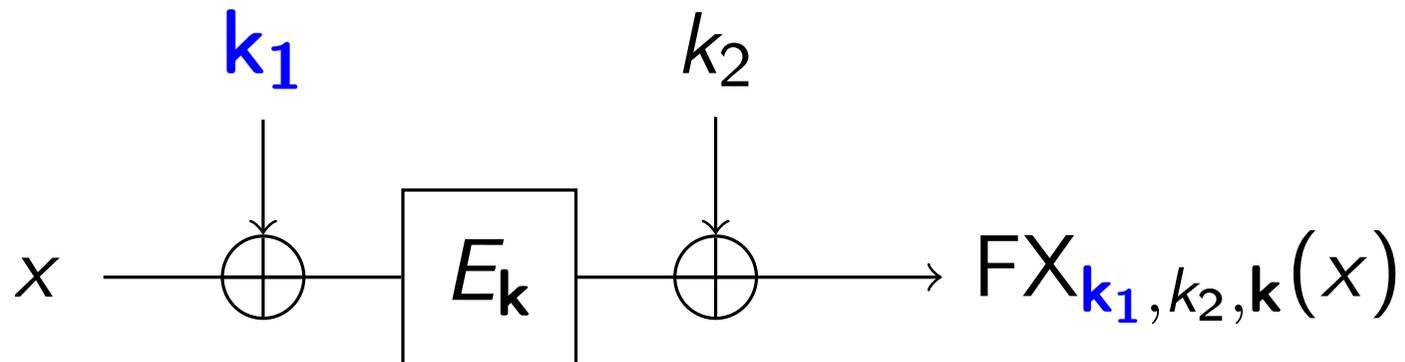
Previously open questions:

- ▶ Could we improve previous attacks?
- ▶ Could we use Simon's algorithm in  $Q_1$ ?

Let us have a look at FX first...

# The FX construction

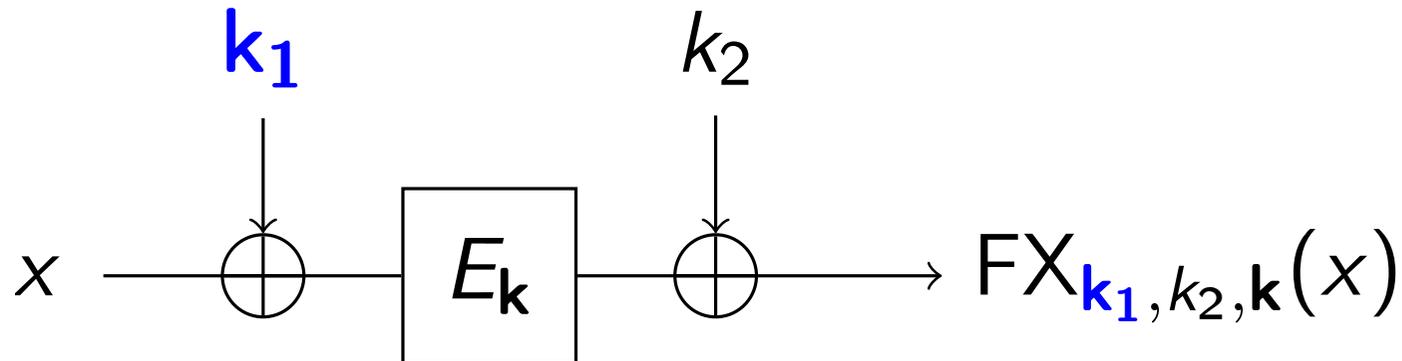
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Classically, verifies  $T \times D \geq 2^{2n}$ .

# Grover meets Simon [LM17]

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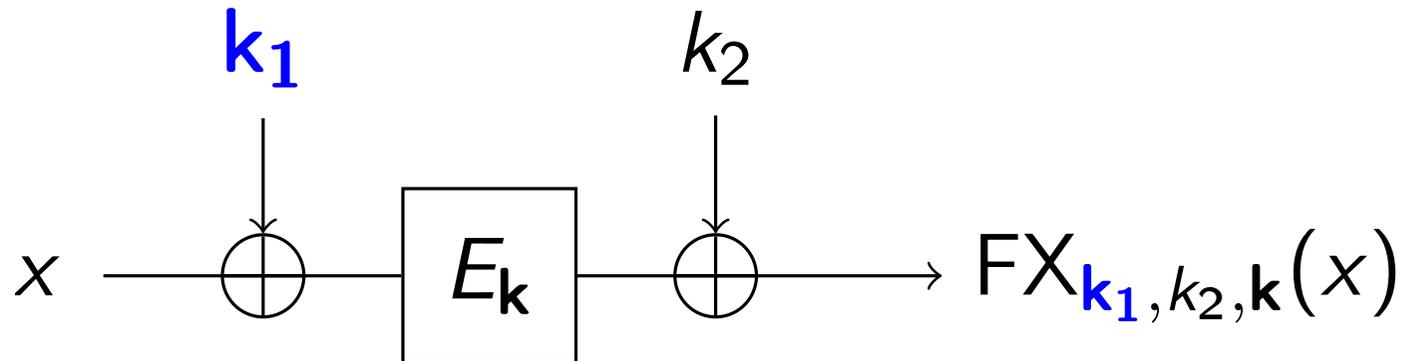
In Q2: Grover search on the cipher key  $k$  with variable  $k'$  and use as a test Simon's attack on Even-Mansour to check if the corresponding function  $f_{k'}$  has a period:

$$f_{k'}(x) = F_{k,k_1,k_2}(x) \oplus E_{k'}(x)$$

$$T = n^3 \times 2^{n/2} \quad D = n \times 2^{n/2}$$

# Reducing D: Idea [BHNPSS19]

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$$f_{k'}(x) = F_{k,k_1,k_2}(x) \oplus E_{k'}(x)$$

Depends on encryption oracle, but not on  $k'$

Depends on guess  $k'$  but not on encryption oracle

In order to reduce  $D$ , makes sense to make only one superposition query to  $F_{k,k_1,k_2}$  and reuse it.

# Reducing D: Idea [BHNPSS19]

---

$$f_{k'}(x) = F X_{k,k_1,k_2}(x) \oplus E_{k'}(x)$$

Precompute  $\mathcal{O}(n)$  states with a superposition query to  $F X_{k,k_1,k_2}(x)$ .

Grover iteration test: compute the superposition of the  $E_{k'}(x)$ , on the precomputed states,

check with Simon if it's periodic,

and next un-do the computation to go back to the original states.

$D = \mathcal{O}(n)$  instead  $D = 2^{n/2}$  with the same  $T = \mathcal{O}(n) 2^{n/2}$ .

# Simulating Q2 queries

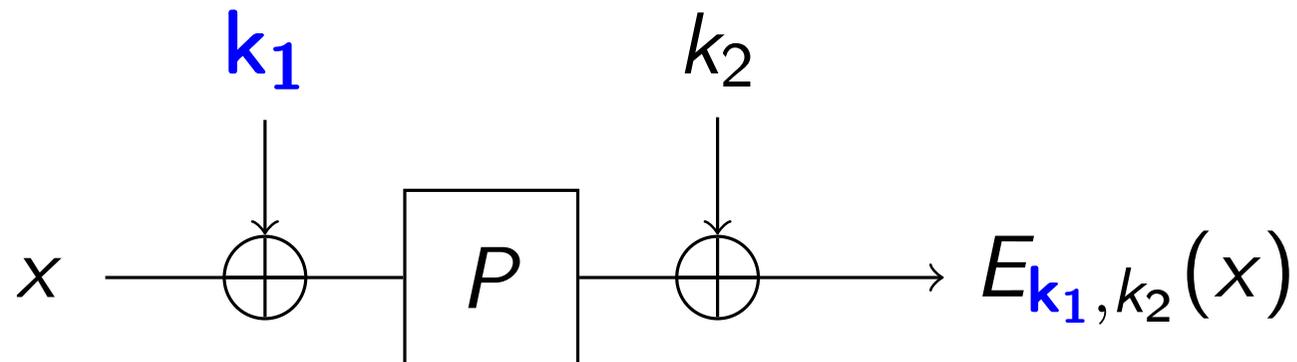
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Can we apply Simon in Q1?

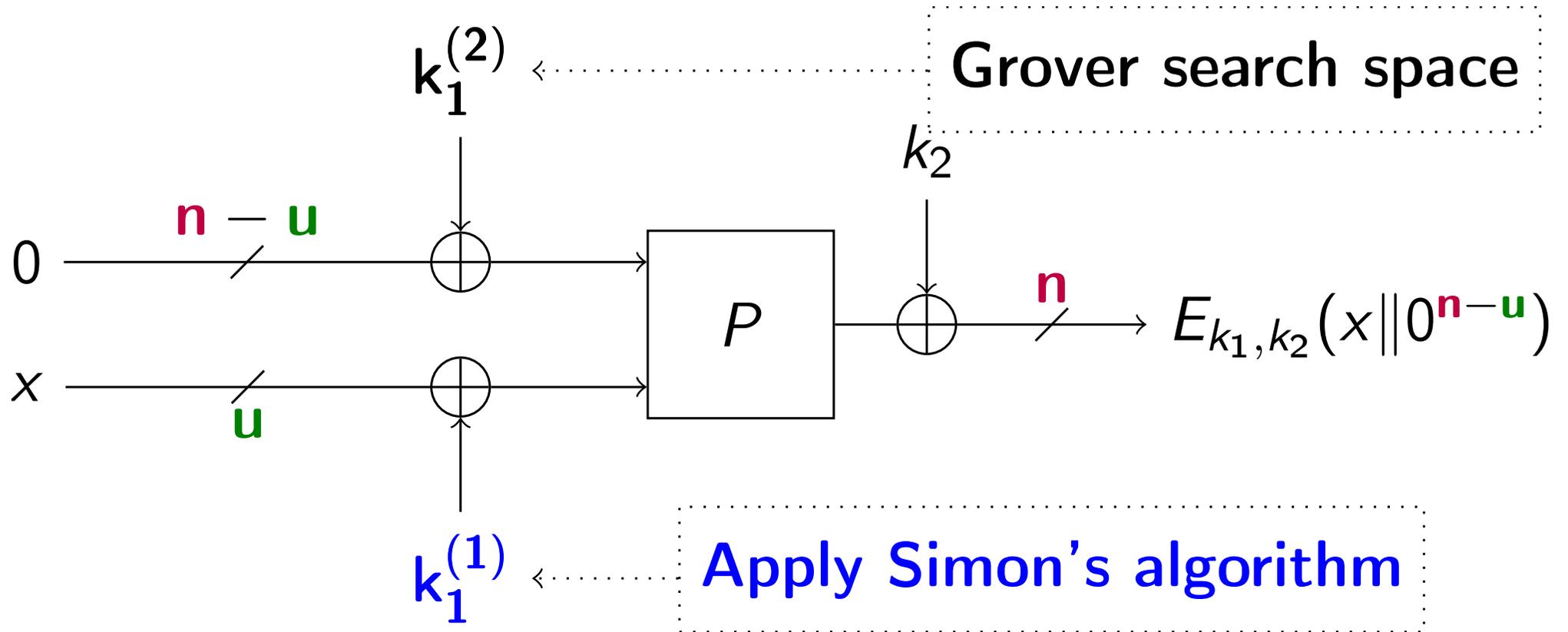
First ideas: We could perform classical queries and superpose them later simulating a Q2 query.

BUT it would need  $2^n$  D and T

$\Rightarrow$  not very interesting.



# Off-line Simon

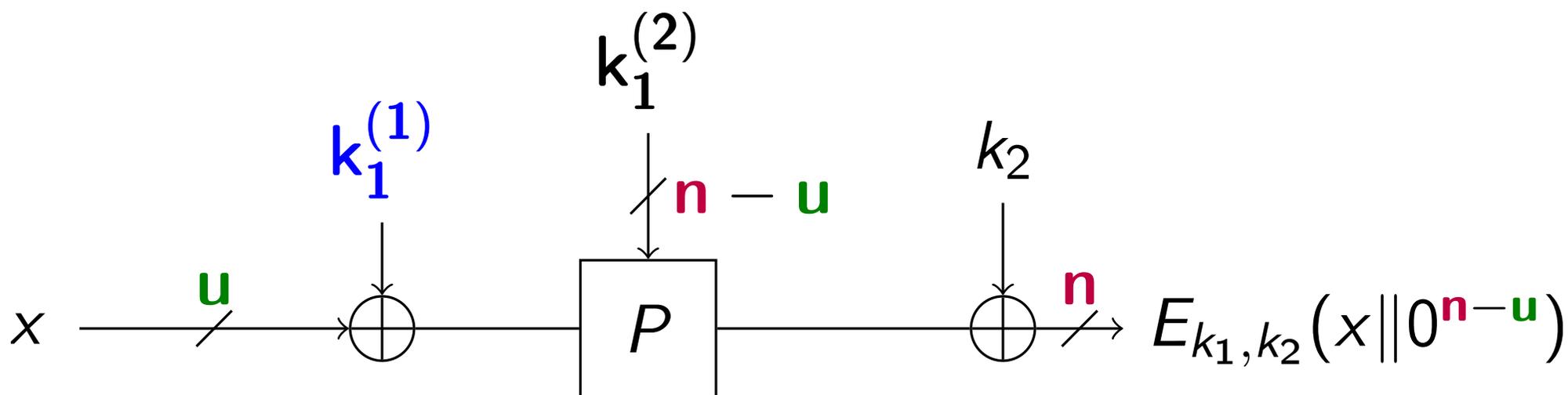


# Off-line Simon [BHNPSS19]

$(n - u)$   $k_1$  bits guessed with Grover.

The remaining  $u$  bits: Simon's secret period.

We can simulate the superposition query in  $2^u$



$$D = 2^u \text{ and } T = 2^u + 2^{(n-u)/2} \text{ which implies } DT^2 = 2^n$$

# Comparison [BHNPS19]

EM:

Model	Queries	Time	Q-memory	C-memory	Reference
Q2	$\mathcal{O}(n)$	$\mathcal{O}(n^3)$	$\mathcal{O}(n)$	$\mathcal{O}(n^2)$	[KM12]
Q1	$\mathcal{O}(2^{n/3})$	$\mathcal{O}(2^{n/3})$	$\mathcal{O}(2^{n/3})$	$\mathcal{O}(2^{n/3})$	[KM12]
Q1	$\mathcal{O}(2^{3n/7})$	$\mathcal{O}(2^{3n/7})$	$\mathcal{O}(n)$	$\mathcal{O}(2^{n/7})$	[HS18]
<b>Q1</b>	$\mathcal{O}(2^{n/3})$	$\mathcal{O}(n^3 2^{n/3})$	<b><math>\mathcal{O}(n^2)</math></b>	$\mathcal{O}(n)$	<b>[BHNPS19]</b>

FX:

Q2	$\mathcal{O}(n 2^{m/2})$	$\mathcal{O}(n^3 2^{m/2})$	$\mathcal{O}(n^2)$	0	[LM17]
Q2	<b><math>\mathcal{O}(n)</math></b>	$\mathcal{O}(n^3 2^{m/2})$	$\mathcal{O}(n^2)$	$\mathcal{O}(n)$	<b>[BHNPS19]</b>
Q1	$\mathcal{O}(2^{3(m+n)/7})$	$\mathcal{O}(2^{3(m+n)/7})$	$\mathcal{O}(n)$	$\mathcal{O}(2^{(m+n)/7})$	[HS18]
Q1	$\mathcal{O}(2^{(m+n)/3})$	$\mathcal{O}(n^3 2^{(m+n)/3})$	$\mathcal{O}(n^2)$	$\mathcal{O}(n)$	<b>[BHNPS19]</b>

## Conclusion [BHNPS19]

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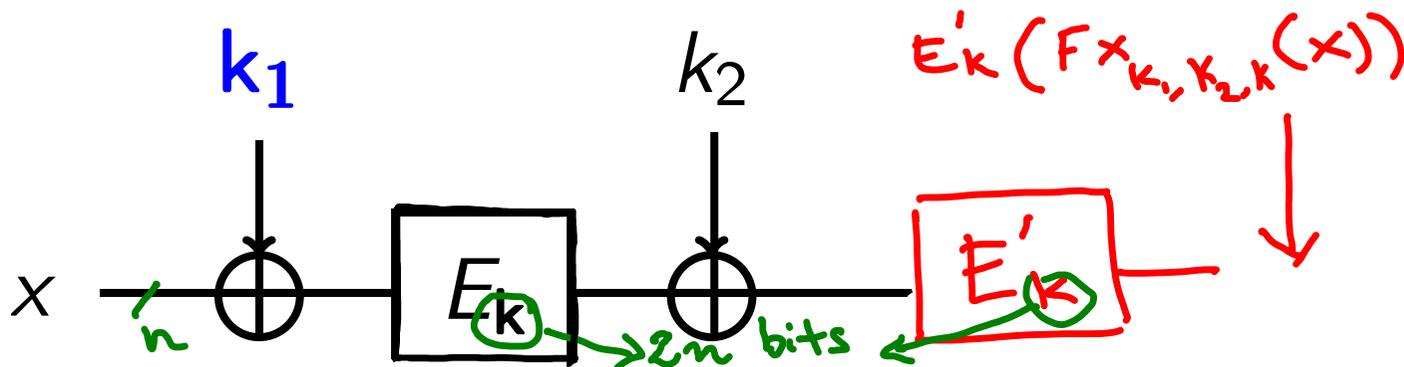
- ▶ Best known attacks on Q1 and Q2 for EM and FX (improves memory and queries).
- ▶ Simon in Q1
- ▶ There are other applications (slides, related-key in Q1)

Is a better than quadratic speed-up possible?

⇒ YES [Bonnetain, Schrottenloher, Sibleyras 21].

# $Q_1$ Better than quadratic [BSS21]

- ▶ 2-xor Cascade [Gazi Tessaro EC 2012]



Classical security:  $2^{2n}2^{n/2} = 2^{2.5n}$

Quantum security [Bonnetain Schrottenloher Sibleyras 21]:

off-line Simon, we have  $2^n$  computations and  $2^n$  classical queries.

# Open Problems

# Many Open problems

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- ▶ New quantum attacks: QFT and linear cryptanalysis ?
- ▶ Quantum security evaluation of primitives(LW)
- ▶ Design of primitives with bigger states
- ▶ Generic key-length/state extensions?
- ▶ Improved quantum attacks on hash functions?
- ▶ Evaluating quantum implementation of algorithms
- ▶ Integral/Saturation cryptanalysis...

# Final Conclusion

# General Conclusion (for now) 1/2

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- ▶ **No reason to panic**, symmetric crypto seems to be holding on well
- ▶ Bigger internal states/keys? How?
- ▶ Ideas from quantum analysis might improve classical analysis
- ▶ **Many things yet to do** to precisely evaluate security, to find best attacks, to adjust parameters...

## General Conclusion (for now) 2/2

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- ▶ **What about Q2?** No consensus:  
Surprising-scary results **vs** useless model?
  - IMHO: Very strong model but **when possible**, better to avoid Q2 attacks: symmetric modulus operandi works well in part because we are never too paranoid: (attacks on  $2^{200}$  declare ciphers broken,...)
- ▶ **Q2 attacks have improved Q1 attacks.**

# Quantum-Safe Symmetric Primitives

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Lots of things to do !

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<sup>1</sup>Many thanks to André Schrottenloher, Xavier Bonnetain, Anne Canteaut, Gaetan Leurent, Anthony Leverrier...

# ERC QUASYModo

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<https://project.inria.fr/quasymodo/>

