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A Benchmark of Deterministic Automata for Regular XPath Queries

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Abstract. We provide a benchmark collection of deterministic automata for regular XPATH queries. For this, we select the subcollection of forward navigational XPATH queries from a corpus that Lick and Schmitz extracted from real-world XSLT and XQuery programs, compile them to symbolic stepwise hedge automata, and determinize them. Large blowups by automata determinization are avoided by using schema-based determinization. Therefore, we show how to lift a recent algorithm for schema-based determinization from SHAS to symbolic SHAS. Our collection also provides deterministic symbolic nested word automata, which we obtain by compilation from deterministic symbolic SHAS.

Keywords: Automata, regular path queries, trees, nested words, XPATH.

1 Introduction

Algorithms for validating and querying data trees or other kinds of hierarchically structured documents are often based on deterministic automata [14,2,5]. Determinism is often essential to keep the computational complexity tractable, as well known from the universality and inclusion problems of deterministic automata for trees, unranked trees, hedges, or nested words [4,20]. This observation does also apply to the problem of answering regular path queries on hierarchical documents in streaming mode [7,11], and to the problem of enumerating query answers of document spanners in in-memory mode [8,19].

Collections of deterministic automata for regular queries nested words [12,15,9] are needed for benchmarking algorithms for these problems. But only a few deterministic automata are available so far. Niehren and Sakho [16] provided deterministic nested word automata (NWAs) of decent size for the 10 forward navigational XPATH queries for the XPathMark benchmark [10]. The problem is that determinization remains problematic. As already noticed in [6], the usual determinization algorithm for nested word automata (NWAs) [3,1,18] quickly leads to a size explosion even for simple regular path queries such as $/a/b$. This difficulty can be circumvented by avoiding the top-down determinism of NWAs. This can be done by either restricting NWAs so that they satisfy the weak single entry property, or more directly, by using stepwise hedge automata (SHAs) [16]. Both approaches have the same expressiveness up to quadratic time transformations. But even the determinization of SHAs may lead to unreasonably large automata in practice. It was argued recently by the authors [17] that schema-based determinization might be the key to solving this problem. We confirm this conjecture here based on a practical benchmark.

We start with the collection of 21000 XPATH queries that Lick and Schmitz [13] extracted from real-world XQUERY and XSLT programs available on the Web. The purpose of this corpus is to reflect the form and distribution of XPATH queries in practical applications. The much smaller XPathMark benchmark [10], in contrast, focuses on functionality testing. We then filter the subclass of 4500 forward navigational XPATH queries of Lick’s and Schmitz’s corpus. The other queries contain comparisons of data values, arithmetics, and functions, including higher-order functions to iterate over sequences, which may be nonregular. We also removed boolean queries and kept only node selection queries. We then selected the 180 largest queries of this subcorpus, for instance:

03257	./equation[title or info/title]
07113	following-sibling::*[self::dbk:appendix self::dbk:article self::dbk:book self::dbk:chapter self::dbk:part self::dbk:preface self::dbk:section self::dbk:sect1 self::dbk:sect2 self::dbk:sect3 self::dbk:sect4 self::dbk:sect5] following-sibling::dbk:para[@rnd:style = 'bibliography' or @rnd:style = 'bibliography-title' or @rnd:style = 'glossary' or @rnd:style = 'glossary-title' or @rnd:style = 'qandaset' or @rnd:style = 'qandaset-title']

Query number 03257 is considered large due to the descendant axis. Query number 07113 is large since it contains the following-sibling axis, and also since its parse tree has more than 15 nodes. We then removed duplicates of queries up to renaming of *XML* names and namespaces and syntactical details, such as *./author* or *descendant::author* or *descendant::corpauthor*. This leads us to the collection of 79 queries in the appendix.

We then improved the compilation chain to symbolic SHAs from [16]. First, we increased the coverage of our compilers of forward navigational XPATH queries to FXP formulas, and second the compiler from FXP formulas to nested regular expressions. The latter are then compiled to symbolic SHAs in a third step as before. The remaining algorithmic question is how to lift schema-based determinization for SHAs to symbolic SHAs. We contribute the needed extensions for the symbolic aspects. The treatment of else rules is presented at the core of the paper. The extensions to apply else and typed else rules can be found in the appendix (even though nontrivial).

The application of our algorithm for schema-based determinization of symbolic SHAs leads us to a corpus of deterministic automata for regular XPATH queries that we compiled further to deterministic symbolic NWA_s. All automata are published at <https://gitlab.inria.fr/aalserha/xpath-benchmark>.

Outline. Section 2 starts with preliminaries on nested words and *XML* documents. In Section 3, we recall schema-based determinization for NFAs that we extend with else rules in Section 4 and to symbolic SHAs in Section 5. In Section 6, we formalize the *XML* data model by a symbolic dSHAs schema and show how to define regular queries by symbolic SHAs. The benchmark collection of symbolic dSHAs derived from Lick’s and Schmitz’s XPath queries is discussed in Section 7. Complete information can be found in the appendix.

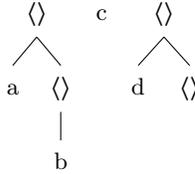
2 Words, Nested Words, and XML Documents

An alphabet Σ is a set. A word over with alphabet Σ is a sequence of letters in $\Sigma^* = \cup_{n=0}^{\infty} \Sigma^n$. The empty word is the unique element $\varepsilon \in \Sigma^0$. The concatenation of two words $w, w' \in \Sigma^*$ by $w \cdot w' \in \Sigma^*$.

Nested words generalize on words by adding parenthesis that must be well-nested. Nested words also generalize on unranked trees and over sequences thereof that are often called hedges. We restrict ourselves to nested words with a single pair of opening and closing parenthesis \langle and \rangle , since named parenthesis can be encoded easily. Nested words in \mathcal{N}_{Σ} have the following abstract syntax.

$$w, w' \in \mathcal{N}_{\Sigma} ::= \varepsilon \mid a \mid \langle w \rangle \mid w \cdot w' \quad \text{where } a \in \Sigma.$$

We assume that concatenation \cdot is associative, and that the empty word ε is a neutral element, that is $w \cdot (w' \cdot w'') = (w \cdot w') \cdot w''$ and $\varepsilon \cdot w = w = w \cdot \varepsilon$. Nested words can be identified with hedges, i.e., words of unranked trees and letters from Σ . Seen as a graph, the inner nodes are labeled by the tree constructor $\langle \rangle$ and the leafs by symbols in Σ or the tree constructor. For instance $\langle a \cdot \langle b \rangle \cdot \varepsilon \rangle \cdot c \cdot \langle d \cdot \langle \varepsilon \rangle \rangle$ corresponds to the hedge on the right.



XML documents are labeled unranked trees that can be serialized into a text, such as for instance: $\langle s:a \text{ name}=\text{"uff"} \rangle \langle s:b \text{ gaga} \langle s:d \rangle \langle /s:b \rangle \langle s:c \rangle \langle /s:a \rangle$. Labeled unranked trees satisfying the *XML* data model can be represented as nested words over a signature that contains the *XML* node-types (*elem*, *attr*, *text*, ...), the *XML* namespaces of the document(s), the *XML* names of the document (a, \dots, d, nam), and the characters of the data values, say UTF8. For the above example, we get the nested word:

$$\langle \text{elem} \cdot s \cdot a \cdot \langle \text{attr} \cdot \text{nam} \cdot u \cdot f \cdot s \cdot f \rangle \rangle \langle \text{elem} \cdot s \cdot b \cdot \langle \text{text} \cdot g \cdot a \cdot g \cdot a \rangle \rangle \langle \text{elem} \cdot s \cdot d \rangle \langle \text{elem} \cdot s \cdot c \rangle$$

We also notice that the alphabet Σ_{XML} of *XML* document has 4 different types of letters: node types, namespaces, names, and characters.

We are interested in finite state automata that are able to read any nested word encoding some *XML* document. Given that the sets of names and namespaces are infinite, such automata must admit infinite alphabets. The infiniteness calls for finite representation by symbolic automata. We will start with finite alphabets Σ but all our algorithms for symbolic SHAs will be able to deal with infinite alphabets without change.

3 Finite State Automata

We recall algorithms for schema-based cleaning [16] and schema-based determinization [17] in the case of finite state automata on words (NFAs).

Definition 1. A NFA is a tuple $A = (\Sigma, \mathcal{Q}, \Delta, I, F)$ such that the alphabet Σ is a finite set of letters, the state set \mathcal{Q} is a finite set, $I, F \subseteq \mathcal{Q}$ are subsets of initial and final states, and the set of transition rules is a subset $\Delta \subseteq (\mathcal{Q} \times \Sigma) \times \mathcal{Q}$. A NFA is called deterministic or equivalently a DFA if Δ is a partial function and I contains at most one initial state.

$$\begin{array}{c}
\frac{I^A \neq \emptyset}{I^A \in I^{\det(A)} \quad I^A \in \mathcal{Q}^{\det(A)}} \quad \frac{Q \in \mathcal{Q}^{\det(A)} \quad Q \cap F^A \neq \emptyset}{Q \in F^{\det(A)}} \\
\frac{Q \in \mathcal{Q}^{\det(A)} \quad Q' = \{q' \in \mathcal{Q}^A \mid q \xrightarrow{a} q' \in \Delta^A, q \in Q\} \neq \emptyset}{Q \xrightarrow{a} Q' \in \Delta^{\det(A)} \quad Q' \in \mathcal{Q}^{\det(A)}}
\end{array}$$

Fig. 1: Accessible determinization $\det(A)$ of an NFA A .

As usual, we draw NFAs as graphs whose nodes are the states in \mathcal{Q} (see e.g. Fig. 6). A state $q \in \mathcal{Q}$ is drawn with a circle \textcircled{q} , an initial state $q \in I$ with an incoming arrow $\rightarrow \textcircled{q}$, and final states with a double circle $\textcircled{\textcircled{q}}$. A transition rule $(q_1, a, q_2) \in \Delta$ is drawn as a black edge $\textcircled{q_1} \xrightarrow{a} \textcircled{q_2}$ that is labeled by a letter.

Transitions for NFAs on words have the form $q \xrightarrow{w} q'$ wrt Δ where $w \in \mathcal{N}_\Sigma$ and $q, q' \in \mathcal{Q}$. They are defined by the following inference rules:

$$\frac{q \in \mathcal{Q}}{q \xrightarrow{\varepsilon} q \text{ wrt } \Delta} \quad \frac{q \xrightarrow{a} q' \in \Delta}{q \xrightarrow{a} q' \text{ wrt } \Delta} \quad \frac{q_0 \xrightarrow{w_1} q_1 \text{ wrt } \Delta \quad q_1 \xrightarrow{w_2} q_2 \text{ wrt } \Delta}{q_0 \xrightarrow{w_1 \cdot w_2} q_2 \text{ wrt } \Delta}$$

The language $\mathcal{L}(A)$ accepted by a NFA A is $\mathcal{L}(A) = \{w \in \Sigma^* \mid q \xrightarrow{w} q' \text{ wrt } \Delta, q \in I, q' \in F\}$. A sink state of a NFA is a state from which no final state can be accessed. We denote the set of all sink states of A by $\text{sink}(A)$. The accessible determinization $\det(A)$ of a NFA can be computed by the rule system in Fig. 1, which describes the usual subset construction, but adapted in such a way that only accessible states are added.

The schema-based cleaning $\text{scl}_S(A)$ of a NFA A with respect to a DFA S keeps only those transitions of A that are needed to recognize some word in the language of the schema $\mathcal{L}(S)$. The NFA $\text{scl}_S(A)$ can be obtained by projecting the accessible product $A \times S$ to A . A one-shot rule system doing so is presented in Fig. 2. An alignment of $q \in \mathcal{Q}^A$ with a schema state $s \in \mathcal{Q}^S$ is denoted as $q \sim s$, meaning that (Q, s) is a state of the accessible product $A \times S$. Sinks of S are treated as if they were removed.

The schema-based determinization $\text{det}_S(A)$ runs the accessible determinization $\det(A)$ in parallel with its schema-based cleaning for schema S . So only those states $Q \in \mathcal{Q}^{\det(A)}$ are kept that can be aligned to some non-sink state $s \in \mathcal{Q}^S \setminus \text{sink}(S)$ in the accessible product $\det(A) \times S$. The rules for schema-based determinization in Fig. 3 are analogous to those of schema-based cleaning in Fig. 2. An alignment of a subset $Q \subseteq \mathcal{Q}^A$ with a schema state s , that is denoted as $Q \sim s$, means that (q, s) is a state in the accessible product $\det(A) \times S$. Furthermore, $Q \in \mathcal{Q}^{\text{det}_S(A)}$ iff $Q \in \mathcal{Q}^{\det(A)}$ and $Q \sim s$ for some $s \notin \text{sink}(S)$. The correctness statement proven in [17] shows that $\text{det}_S(A) = \text{scl}_S(\det(A))$. Moreover, the worst-case complexity of schema-based determinization is lower than of the accessible determinization followed by schema-based cleaning.

$$\begin{array}{c}
\frac{q \in I^A \quad s \in I^S}{q \in I^{scl_S(A)} \quad q \sim s} \quad \frac{q \in F^A \quad s \in F^S \quad q \sim s}{q \in F^{scl_S(A)}} \\
\\
\frac{q_1 \xrightarrow{a} q \in \Delta^A \quad s_1 \xrightarrow{a} s \in \Delta^{exp(S)} \quad s \notin sink(S) \quad q_1 \sim s_1}{q_1 \xrightarrow{a} q \in \Delta^{scl_S(A)} \quad q \sim s} \quad \frac{q \sim s}{q \in \mathcal{Q}^{scl_S(A)}}
\end{array}$$

Fig. 2: Schema-based cleaning $scl_S(A)$ of a NFA A with respect to a DFA S .

$$\begin{array}{c}
\frac{Q \in I^{det(A)} \quad I^S = \{s\}}{Q \in I^{det_S(A)} \quad Q \sim s} \quad \frac{Q \in F^{det(A)} \quad s \in F^S \quad Q \sim s}{Q \in F^{det_S(A)}} \\
\\
\frac{Q \xrightarrow{a} Q' \in \Delta^{det(A)} \quad s \xrightarrow{a} s' \in \Delta^S \quad s' \notin sink(S) \quad Q \sim s}{Q \xrightarrow{a} Q' \in \Delta^{det_S(A)} \quad Q' \sim s'} \quad \frac{Q \sim s}{Q \in \mathcal{Q}^{det_S(A)}}
\end{array}$$

Fig. 3: The schema-based determinization $det_S(A)$ for an NFA A and a DFA S .

4 Symbolic Finite State Automata

We next lift schema-based cleaning and determinization to symbolic NFAs extending NFAs by else rules.

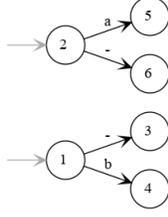
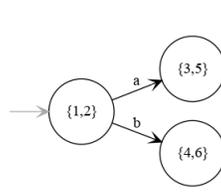
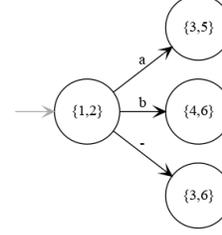
Definition 2. An NFA $[-]$ is a tuple $A = (\Sigma, \mathcal{Q}, \Delta, I, F)$ such that Σ is a finite alphabet, $\Delta = (\Delta', _{}^\Delta)$ and $A' = (\Sigma, \mathcal{Q}, \mathcal{P}, \Delta', I, F)$ is a NFA and $_{}^\Delta \subseteq \mathcal{Q}^2$ a set of else rules. We call A deterministic or equivalently an DFA $[-]$ if A' is a DFA and $_{}^\Delta$ a partial function.

For any else rule $(q, q') \in _{}^\Delta$ we write $q \vec{\rightarrow} q' \in \Delta$ and draw it as $\textcircled{q} \vec{\rightarrow} \textcircled{q'}$ in pictures. It means that the automaton in state q can go to state q' when reading any letter $a \in \Sigma$ such that there exists no q'' with $q \xrightarrow{a} q'' \in \Delta$. Since Σ is finite, any else rule can be expanded to a set of internal rules, so any NFA $[-]$ A can be expanded to some NFA $exp(A)$ as follows:

$$\frac{q \vec{\rightarrow} q' \in \Delta^A \quad a \in \Sigma \quad \neg \exists q'' \in \mathcal{Q}^A. q \xrightarrow{a} q'' \in \Delta^A}{q \xrightarrow{a} q' \in \Delta^{exp(A)}} \quad \frac{q \xrightarrow{a} q' \in \Delta^A}{q \xrightarrow{a} q' \in \Delta^{exp(A)}}$$

In Fig. 4, we lift the accessible determinization from NFAs to NFAs $[-]$. For this, some rule expansion is needed, but we limit it as much as possible. This is illustrated at the example of the NFA $[-]$ E in Fig. 5. The else rule $2 \vec{\rightarrow} 6$ means that E can go from state 2 to state 6 when reading any letter from $\Sigma \setminus \{a\}$. In a similar manner, the rule $1 \vec{\rightarrow} 3$ permits to go from state 1 to state 3 when reading any letter from $\Sigma \setminus \{b\}$. Note that the interpretation of the else rules depends on the alphabet Σ . The same holds for the result of accessible determinization. For $\Sigma = \{a, b\}$, the else rules has two instances for the letters a and b respectively. Since $det(E)$ in Fig. 6 already has an internal transition for both letters, no else rule is produced any more. Whereas for $det(E)$ in Fig. 7 with alphabet

$$\begin{array}{c}
\frac{Q_1 \xrightarrow{a} Q_2 \in \Delta^{\det(\text{exp}(A))} \quad \exists q_1 \in Q_1 \exists q \in \mathcal{Q}^A. q_1 \xrightarrow{a} q \in \Delta^A}{Q_1 \xrightarrow{a} Q_2 \in \Delta^{\det(A)}} \\
\frac{Q_1 \in \mathcal{Q}^{\det(\text{exp}(A))} \quad Q_2 = \{q \in \mathcal{Q}^A \mid q_1 \xrightarrow{a} q \in \Delta^A, q_1 \in Q_1\} \neq \emptyset}{Q_1 \xrightarrow{a} Q_2 \in \Delta^{\det(A)}} \\
\frac{\exists a \in \Sigma \forall q_1 \in Q_1 \neg \exists q' \in \mathcal{Q}^A. q_1 \xrightarrow{a} q' \in \Delta^A}{Q_1 \xrightarrow{a} Q_2 \in \Delta^{\det(A)}}
\end{array}$$

Fig. 4: Accessible determinization for NFAs $[-]$ lifted from Fig. 1 for NFAs.Fig. 5: The NFA $[-]$ E .Fig. 6: $\det(E)$ for $\Sigma = \{a, b\}$.Fig. 7: $\det(E)$ for $\Sigma = \{a, b, c\}$.

$\Sigma = \{a, b, c\}$ an else rule is needed to capture letter c of the alphabet, which does not appear in the graph of E .

Schema-based cleaning for NFAs $[-]$ is shown in Fig. 8 and schema-based determinization in Fig. 9. As in the non-symbolic case, the two rule systems are analogous. The first rule on the left says that an a -transition of A (resp. $\det(A)$) is preserved in scl_A (resp. $\det_S(A)$), if it is aligned to an a -transition of the expansion of S into a non-sink. The second rule on the left says that an else transition of A (resp. $\det(A)$) is preserved in scl_A (resp. $\det_S(A)$), if it can be expanded to an a -transition that is aligned to an a -transition of the expansion of S into a non-sink. The rule on the right states that a -transitions of A (resp. $\det(A)$) must be preserved in scl_A (resp. $\det_S(A)$), if the meaning of some else rule of scl_A (resp. $\det_S(A)$) would be changed otherwise.

5 Symbolic Stepwise Hedge Automata

We next lift schema-based cleaning and determinization from symbolic NFAs to symbolic stepwise hedge automata nested words. Note that the algorithms for nonsymbolic SHAs were presented earlier [16].

Definition 3. A dSHA $[-]$ is a tuple $A = (\Sigma, \mathcal{Q}, \mathcal{P}, \Delta, I, F)$ where $\Delta = (\Delta', \Delta'')$ so that $A' = (\Sigma, \mathcal{Q}, \Delta', I, F)$ is an NFA $[-]$. Furthermore, \mathcal{P} is a finite set of tree states and $\Delta'' = (\diamond^\Delta, @^\Delta, \dashrightarrow^\Delta)$, such that $\diamond^\Delta \subseteq \mathcal{Q}$ is a subset of tree initial states, $@^\Delta \subseteq \mathcal{Q} \times \mathcal{P} \times \mathcal{Q}$ a set of apply rules, and $\dashrightarrow^\Delta \subseteq \mathcal{Q} \times \mathcal{P}$ a set of tree final rules. We call A deterministic or equivalently a dSHA $[-]$, if A' is a DFA $[-]$, there is at most one tree initial state in \diamond^Δ , and $@^\Delta$ and \dashrightarrow^Δ are partial functions.

The notion of determinism for dSHAs $[-]$ extends on the notion of left-to-right determinism of NFAs $[-]$ and on the notion of bottom-up determinism of tree

$$\begin{array}{c}
\frac{q_1 \xrightarrow{a} q \in \Delta^A \quad s_1 \xrightarrow{a} s \in \Delta^{exp(S)} \quad s \notin \text{sink}(S) \quad q_1 \sim s_1}{q_1 \xrightarrow{a} q \in \Delta^{scl_S(A)} \quad q \sim s} \quad \frac{q_1 \xrightarrow{a} q_2 \in \Delta^A \quad q_1 \rightarrow q \in \Delta^{scl_S(A)}}{q_1 \xrightarrow{a} q_2 \in \Delta^{scl_S(A)}} \\
\frac{q_1 \rightarrow q \in \Delta^A \quad \neg \exists q_2. q_1 \xrightarrow{a} q_2 \in \Delta^A \quad s_1 \xrightarrow{a} s \in \Delta^{exp(S)} \quad s \notin \text{sink}(S) \quad q_1 \sim s_1}{q_1 \rightarrow q \in \Delta^{scl_S(A)} \quad q \sim s}
\end{array}$$

Fig. 8: Schema-based cleaning for NFAS_[·] lifted from Fig. 2 for NFAS.

$$\begin{array}{c}
\frac{Q_1 \xrightarrow{a} Q \in \Delta^{\det(A)} \quad s_1 \xrightarrow{a} s \in \Delta^{exp(S)} \quad s \notin \text{sink}(S) \quad Q_1 \sim s_1}{Q_1 \xrightarrow{a} Q \in \Delta^{\det_S(A)} \quad Q \sim s} \quad \frac{Q_1 \xrightarrow{a} Q \in \Delta^{\det(A)} \quad \exists Q_2. Q_1 \rightarrow Q_2 \in \Delta^{\det_S(A)}}{Q_1 \xrightarrow{a} Q \in \Delta^{\det_S(A)}} \\
\frac{Q_1 \rightarrow Q \in \Delta^{\det(A)} \quad \neg \exists Q_2. Q_1 \xrightarrow{a} Q_2 \in \Delta^{\det(A)} \quad s_1 \xrightarrow{a} s \in \Delta^{exp(S)} \quad s \notin \text{sink}(S) \quad Q_1 \sim s_1}{Q_1 \rightarrow Q \in \Delta^{\det_S(A)} \quad Q \sim s}
\end{array}$$

Fig. 9: Schema-based determinization of NFAS_[·] lifted from Fig. 3 for NFAS.

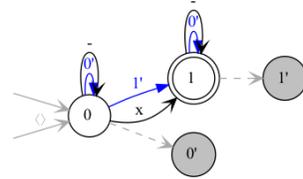
automata. The states in \mathcal{Q} are also called hedge states. The graphs of dSHAS_[·] extend on those for NFAS_[·]. A tree state $p \in \mathcal{P}$ is drawn in gray \textcircled{p} . A tree initial state $q \in \langle \Delta \rangle$ is drawn as a node $\langle \Delta \rangle \textcircled{q}$ with an incoming tree arrow. An apply rule $(q_1, p, q_2) \in @^\Delta$ is written as $q_1 @ p \rightarrow q_2 \in \Delta$ and drawn with a blue edge $\textcircled{q_1} \xrightarrow{p} \textcircled{q_2}$ labeled by a state $p \in \mathcal{P}$ rather than by a letter $a \in \Sigma$. It states that a nested word in state $q_1 \in \mathcal{Q}$ can be extended by a tree in state $p \in \mathcal{P}$ and go into state $q_2 \in \mathcal{Q}$. A tree final rule $(q, p) \in \dashrightarrow^\Delta$ is written as $q \dashrightarrow p \in \Delta$.

We show on the right the graph of the dSHA_[·] one^x that accepts all nested words over $\Sigma \uplus \{x, \neg x\}$ with exactly one occurrence of letter x . Beside of the DFA recognizing all words over $\Sigma \cup \{x, \neg x\}$ with a single occurrence of x , it has three additional apply rules $0@0' \rightarrow 0$, $0@1' \rightarrow 1$,

$1@0' \rightarrow 1 \in \Delta^{one^x}$ for reading the tree states assigned to subtrees. The hedge state 0 is the single tree initial state.

Transitions of dSHAS_[·] have the form $q \xrightarrow{w} q'$ wrt Δ where $w \in \mathcal{N}_\Sigma$ and $q, q' \in \mathcal{Q}$. They are defined by the inference rule for NFAS and the following rule:

$$\frac{q' \in \langle \Delta \rangle \quad q' \xrightarrow{w} q \text{ wrt } \Delta \quad q \dashrightarrow p \in \Delta \quad q_1 @ p \rightarrow q_2 \in @^\Delta}{q_1 \xrightarrow{\langle w \rangle} q_2 \text{ wrt } \Delta}$$



$$\begin{array}{c}
\frac{\diamond^{\Delta^A} \neq \emptyset}{\diamond^{\Delta^A} \in \mathcal{Q}^{det(A)}} \quad \frac{Q \in \mathcal{Q}^{det(A)} \quad P = \{p \in \mathcal{P}^A \mid \exists q \in Q, q \twoheadrightarrow p \in \Delta^A\} \neq \emptyset}{Q \twoheadrightarrow P \in \Delta^{det(A)} \quad P \in \mathcal{P}^{det(A)}} \\
\frac{Q \in \mathcal{Q}^{det(A)} \quad P \in \mathcal{P}^{det(A)} \quad Q' = \{q' \in \mathcal{Q}^A \mid q @ p \rightarrow q', q \in Q, p \in P\} \neq \emptyset}{Q @ P \rightarrow Q' \in \Delta^{det(A)} \quad Q' \in \mathcal{Q}^{det(A)}}
\end{array}$$

Fig. 10: Accessible determinization for dSHAS[-] extending on NFAS[-] in Fig. 4.

$$\begin{array}{c}
\frac{\diamond^{\Delta^S} = \{s\}}{\diamond^{\Delta^A} \in \diamond^{\Delta^{det_S(A)}} \quad \diamond^{\Delta^A} \sim s} \quad \frac{Q \twoheadrightarrow P \in \Delta^{det(A)} \quad s \twoheadrightarrow r \in \Delta^S \quad r \notin \text{sink}(S) \quad Q \sim s}{Q \twoheadrightarrow P \in \Delta^{det_S(A)} \quad P \sim r} \\
\frac{Q_1 @ Q_2 \rightarrow Q' \in \Delta^{det(A)} \quad s_1 @ s_2 \rightarrow s' \in \Delta^S \quad s' \notin \text{sink}(S) \quad Q_1 \sim s_1 \quad Q_2 \sim s_2}{Q_1 @ Q_2 \rightarrow Q' \in \Delta^{det_S(A)} \quad Q' \sim s'}
\end{array}$$

Fig. 11: Schema-based determinization for dSHAS[-] extending on Fig. 9.

It says that a tree $\langle w \rangle$ can transit from a state q_1 to a state q_2 if w can transit from some tree initial state q' to q , so that there is some tree final rule $(q, p) \in \twoheadrightarrow^\Delta$ and some apply rule $(q_1, p, q_2) \in @^\Delta$. The language $\mathcal{L}(A)$ of a dSHA[-] is defined literally as for NFAS[-] except that now nested words may be recognized too.

The accessible determinization is extended from NFAS[-] to dSHAS[-] by adding the inference rules in Fig. 10. It should be noticed that the three additional rules for dSHAS[-] are fully analogous to the three rules for NFAS. Schema-based cleaning and determinization can be extended from NFAS[-] to dSHAS[-] in the obvious manner. We only present the additional inference rules for the schema-based determinization in Fig. 11. The previous inference rules for the schema-based determinization of NFAS[-] from Fig. 9 remain all valid. The same is done for schema-based cleaning of SHAS[-] in Fig. 23 in the appendix.

Extensions. In our application, we will permit extended symbolic SHAS: An apply else rule has the form $q @ _ \rightarrow q'$ and is drawn as with a blue arrow $\textcircled{q} \rightarrow \textcircled{q'}$ in contrast to a standard else rule. The underscore can be expanded to any else state $p \in \mathcal{P}_{else}$ such that there does not exist q'' with $q @ p \rightarrow q'' \in \Delta$. The set of else states $\mathcal{P}_{else} \subseteq \mathcal{P}$ is distinguished by the automaton. Its elements $p \in \mathcal{P}_{else}$ are marked as \textcircled{p} in the pictures. See Fig. 13 for an example. The alphabet will be typed, such that any letter can be given some types in some set T . Typed else rules then have the form $q \xrightarrow{:\tau} q' \in \Delta$ where $\tau \in T$ and are drawn as $\textcircled{q} \xrightarrow{:\tau} \textcircled{q'}$. They can be expanded with all letters $a \in \Sigma$ that can be given the type τ . Untyped else rules as before can be covered by allowing for a universal type $all \in T$. The class of symbolic SHAS with apply and typed else rules is denoted by $\text{SHA}[-; T, @_]$. An example for an $\text{SHA}[-; T, @_]$ is given in Fig. 12.

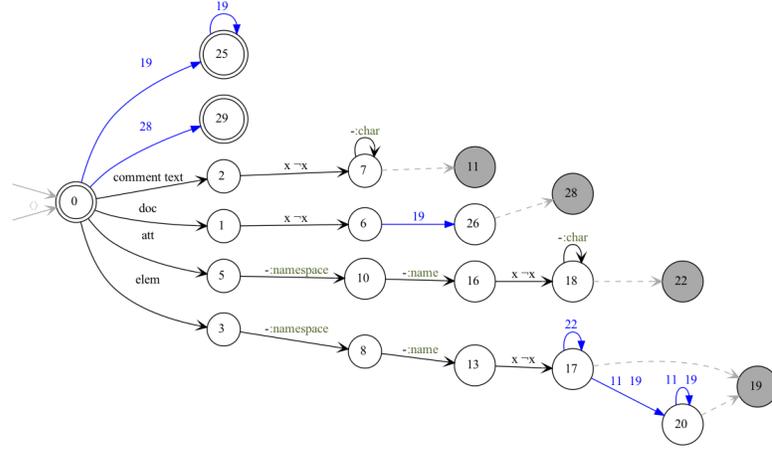


Fig. 12: The symbolic dSHAs x -doc: a schema for x -annotated XML documents.

6 Regular Schemas and Queries for XML Documents

We now discuss how to define regular schemas and queries on nested words by symbolic SHAs in the class $\text{SHA}[_:\text{T}, @_]$.

Since we are interested in monadic node selection queries, we fix a single variable x that is to be annotated to the selected node. In Fig. 12, we present the schema x -doc as a symbolic dSHA that captures the XML data model. Automaton x -doc accepts all valid XML documents, whose nodes are annotated by either x or $\neg x$. The alphabet $\Sigma_{XML} \uplus \{x, \neg x\}$ has the 6 types in $T = \{\text{node-type}, \text{namespace}, \text{name}, \text{char}, \text{var}, \text{all}\}$. Note that x -doc does also accept all subdocuments of XML documents, i.e., sequence of XML elements, texts, and comments.

Monadic queries on nested words select a subset of the nodes of its graph. For this we assume that the nodes of such graphs named by natural numbers in an arbitrary but unique manner. We denote the set of nodes of the graph of a nested word w by $\text{nod}(w)$, generalizing over the set of positions of words. A (total) monadic query \mathbf{Q} is a function that maps any nested word $w \in \mathcal{N}_\Sigma$ to a subset of its nodes $\mathbf{Q}(w) \subseteq \text{nod}(w)$.

We define monadic queries on nested words by query automata relying on the notion of V -structures [21] where $V = \{x\}$. Query automata are symbolic SHAs with variable x in the alphabet. For any node π of a nested word $w \in \mathcal{N}_\Sigma$, let $w * [\pi/x]$ be the nested word obtained by inserting x after the position of node π in w . The word $w * [\pi/x]$ is a V -structure since x occurs exactly once in it. The annotation by x marks a node that is to be selected. Formally, we identify any total monadic query \mathbf{Q} with the language of V -structures such that $L(\mathbf{Q}) = \{w * [\pi/x] \mid w \in \mathcal{N}_\Sigma, \pi \in \mathbf{Q}(w)\}$. Such queries for nested words can be defined by symbolic SHAs A with alphabet $\Sigma \uplus \{x, \neg x\}$, such that $L_{\mathbf{Q}} = \mathcal{L}(A) \cap \mathcal{L}(\text{one}^x)$. If so, a node π of a nested word $w \in \mathcal{N}_\Sigma$ is selected by \mathbf{Q} on w iff $\pi \in \mathbf{Q}(w) \Leftrightarrow w * [\pi/x] \in \mathcal{L}(A)$. In Fig. 13, the minimal symbolic dSHA for the query \mathbf{Q}_{03257} of our XPATH benchmark is given. We note that it has apply

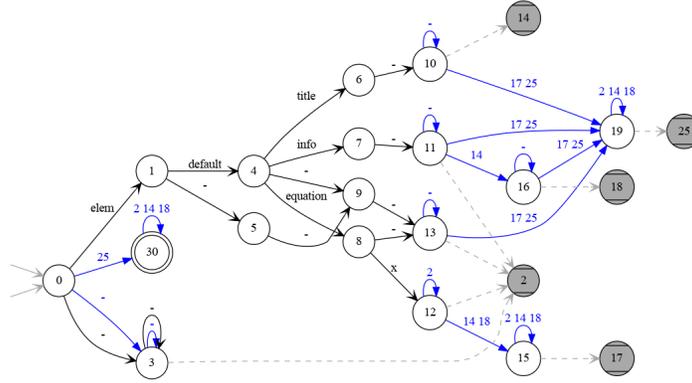


Fig. 13: A symbolic dSHAS for \mathbf{Q}_{03257} : `//equation[title or info/title]`.

Query Id	A = $sha(\mathbf{Q}_{Id})$	$det(A)$	B = $det_{one^x}(A)$	$scl_{x-doc}(B)$	C = $det_{one^x \times x-doc}(A)$	mini(C)
18330	89 (37)	400 (38)	127 (22)	73 (22)	73 (22)	60 (18)
09138	349 (152)		410 (59)	198 (59)	198 (59)	62 (15)
05460	214 (88)	38163 (610)	1039 (173)	628 (173)	628 (173)	69 (18)
06512	120 (49)	407 (42)	202 (34)	114 (34)	114 (34)	74 (22)
06176	1191 (550)		5271 (386)		1202 (386)	
12539	159 (68)	1965 (112)	311 (40)	137 (40)	137 (40)	100 (28)
11780	166 (70)	2286 (128)	317 (41)	142 (41)	142 (41)	98 (27)
05684	1276 (580)		4044 (226)		718 (226)	123 (16)
06947	704 (322)		1575 (129)	443 (129)	443 (129)	70 (14)
06794	256 (114)		399 (51)	177 (51)	177 (51)	63 (15)
04358	966 (424)		3354 (433)		2020 (433)	

Fig. 14: Few results on the automata for the queries in size(number-of-states).

else rules and else states. It also uses the symbol `default` which stands for the default namespace of *XML* elements and attributes.

7 Benchmark of Query Automata

We compiled all forward navigational XPATH queries \mathbf{Q} in our collection to a symbolic SHAS $sha(\mathbf{Q})$ in the class `SHA[:T,@_]` by applying our extension of the compiler from [16], followed by schema-based cleaning with the schemas one^x and $x-doc$. The sizes of all $sha(\mathbf{Q})$ for our subcorpus of 79 XPATH queries are given in the first column of Fig. 14. 40% of the symbolic SHAS have more than 100 states. The biggest is $sha(\mathbf{Q}_{05684})$ with size 1276 and 580 states. This query is selecting a union of 36 attributes of the current node. For each of these attributes a symbolic SHAS with 16 states is constructed. So the overall number of states is $16 * 36 + 4 = 580$.

The second column contains the sizes and number of states of the accessible determinization $\det(A)$ where $A = sha(\mathbf{Q})$. No schema is used. We use a timeout of 5 minutes. Whenever this is not enough, the cell in the table is left blank. Indeed, the accessible determinization fails with this timeout for nearly 40% of the queries of our corpus. Roughly, the determinization fails for all symbolic SHAs with more than 100 states. For instance, $A = sha(\mathbf{Q}_{05460})$ has size 214 and 88 states, while $\det(A)$ has size 38361 and 592 states.

The third column reports on $B = \det_{one^x}(A)$ obtained by schema-based determinization with schema one^x . Indeed, the computation succeeds in all cases in less than 5 minutes, which is a tremendous improvement compared to schema-less determinization. Furthermore, only 88% of the symbolic dSHAs obtained in this way have less than 100 states. In other words, schema-based determinization decreased the automata size in at least 28% of the cases.

The fourth column applies schema-based cleaning schema $x-doc$ to B . It turns out that $scl_{x-doc}(B)$ often has considerably lower size than B , but always the same number of states. Furthermore, the computation with timeout of 5 minutes fails precisely for the 12% of the automata that have more than 100 states.

In the fifth column, we apply schema-based determinization with both schemas one^x and $x-doc$, so we compute $C = \det_{one^x \times x-doc}(A)$. The computation succeeds in all cases in less than 5 minutes. The correctness statement $scl_{x-doc}(B) = \det_{x-doc}(B) = C$ suggested by Theorem 2 in [17] is confirmed in all cases. Note that the theorem cannot be applied directly, since we deal with symbolic SHAs rather than with SHAs.

In the last column, we applied minimization to compute $mini(C)$. This worked out in 97% of the queries within a timeout of 5 minutes, but failed for the 2 biggest automata. One of them is for the query \mathbf{Q}_{04358} .

All minimal dSHAs $mini(C)$ that we could compute in less than 5 minutes have at most 50 states. The biggest minimal automata for \mathbf{Q}_{07113} below has 49 states and size 365. Our experiments show that we can obtain deterministic automata of decent size based on schema-based determinization in practice. Furthermore, based on minimization we can even obtain quite small deterministic automata in 97% of the cases. The remaining 3% could possibly be improved by not using the naïve minimization algorithm.

8 Conclusion

We provide a benchmark of deterministic automata for regular XPath queries obtained with an algorithm for schema-based determinization of symbolic SHAs that we presented. Our benchmark is compiled from forward navigational XPath (FXP) queries: the 79 largest queries modulo renaming of the 4500 FXP queries of the corpus of Lick and Schmitz, and 10 FXP queries from the XPathMark benchmark. While 40% of symbolic SHAs obtained from these FXP queries cannot be determinized in less than 5 minutes by standard determinization; schema-based determinization succeeds for 100% of them. Furthermore, all but 2 of the symbolic dSHAs obtained are sufficiently small so that they can be minimized with the naïve quadratic algorithm, leading us to small symbolic dSHAs with at most 50 states in 97% of the cases.

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Table 2: The 79 largest forward navigational queries of the XPATH corpus of Lick makand Schmitz, without equivalent queries up to renaming.

Id	XPATH Query
18330	/ descendant-or-self::node()/child::parts-of-speech
17914	/ descendant-or-self::node()/child::tei:back/descendant-or-self::node()/child::tei:interpGrp
10745	*//tei:imprint/tei:date[@type='access']
02091	* ../refentry
00744	../@id ../@xml:id
12060	../attDef
02762	../authorgroup/author ../author
06027	../authorinitials ../author
02909	../bibliomisc[@role='serie']
06415	../email address/otheraddr/ulink
03257	../equation[title or info/title]
05122	../procedure[title]
09138	../rng:ref ../tei:elementRef ../tei:classRef ../tei:macroRef ../tei:dataRef
05460	../table//footnote ../informaltable//footnote
12404	../tei:dataRef[@name]
10337	../tei:note[@place='end']
06639	../tgroup//footnote
14340	//*
13804	//GAP/@DISP
13896	//HEADER//IDNO[@TYPE='evans citation']
02194	//annotation
06726	//doc:table //doc:informaltable
13640	//equiv[@filter]
05735	//glossary[@role='auto']
15766	//h:body/h:section[@data-type='titlepage']
15524	//h:section[@data-type='titlepage']
06512	//refentry//text()
06176	//set //book //part //reference //preface //chapter //appendix //article //colophon //refentry //section //sect1 //sect2 //sect3 //sect4 //sect5 //indexterm //glossary //bibliography //*[@id]
12539	//tei:elementSpec //tei:classSpec[@type='atts']
11780	//tei:ref[@type='cite'] //tei:ptr[@type='cite']
11478	//xhtml:p[@class]

11227	/tei:TEI/tei:text//tei:note[@type='action']
05684	@abbr @align @axis @bgcolor @border @cellpadding @cellspacing @char @charoff @class @dir @frame @headers @height @id @lang @nowrap @onclick @ondblclick @onkeydown @onkeypress @onkeyup @onmousedown @onmousemove @onmouseout @onmouseover @onmouseup @rules @scope @style @summary @title @valign @width @xml:id @xml:lang
06947	anchor areaset audiodata audioobject beginpage constraint indexterm itemset keywordset msg doc:anchor doc:areaset doc:audiodata doc:audioobject doc:beginpage doc:constraint doc:indexterm doc:itemset doc:keywordset doc:msg
06794	articleinfo chapterinfo bookinfo doc:info doc:articleinfo doc:chapterinfo doc:bookinfo
06169	article preface chapter appendix refentry section sect1 glossary bibliography
06924	authorblurb formalpara legalnotice note caution warning important tip doc:authorblurb doc:formalpara doc:legalnotice doc:note doc:caution doc:warning doc:important doc:tip
11958	biblStruct//note
01705	book article part reference preface chapter bibliography appendix glossary section sect1 sect2 sect3 sect4 sect5 refentry colophon bibliodiv[title] setindex index
02086	book article topic part reference preface chapter bibliography appendix glossary section sect1 sect2 sect3 sect4 sect5 refentry colophon bibliodiv[title] setindex index
02000	chapter appendix epigraph warning preface index colophon glossary biblioentry bibliography dedication sidebar footnote glossterm glossdef bridgehead part
02697	chapter appendix preface reference refentry article topic index glossary bibliography
14183	content//rng:ref
07106	dbk:appendix dbk:article dbk:book dbk:chapter dbk:part dbk:preface dbk:section dbk:sect1 dbk:sect2 dbk:sect3 dbk:sect4 dbk:sect5
05824	descendant-or-self::*
11368	descendant-or-self::tei:TEI/tei:text/tei:back
15848	descendant::*[@class='refname']
15462	descendant::h:span[@data-type='footnote']
04267	descendant::label

07113	following-sibling::*[self::dbk:appendix self::dbk:article self::dbk:book self::dbk:chapter self::dbk:part self::dbk:preface self::dbk:section self::dbk:sect1 self::dbk:sect2 self::dbk:sect3 self::dbk:sect4 self::dbk:sect5] following-sibling::dbk:para[@rnd:style = 'bibliography' or @rnd:style = 'bibliography-title' or @rnd:style = 'glossary' or @rnd:style = 'glossary-title' or @rnd:style = 'qandaset' or @rnd:style = 'qandaset-title']
03864	guibutton guiicon guilabel guimenu guimenuitem guisubmenu interface
15484	h:pre[@data-type='programlisting']//text()
15461	h:table[descendant::h:span[@data-type='footnote']]
11160	html:table html:tr html:thead html:tbody html:td html:th html:caption html:li
06856	imageobject imageobjectco audioobject videoobject doc:imageobject doc:imageobjectco doc:audioobject doc:videoobject
06458	info refentryinfo referenceinfo refsynopsisdivinfo refsectioninfo refsect1info refsect2info refsect3info setinfo bookinfo articleinfo chapterinfo sectioninfo sect1info sect2info sect3info sect4info sect5info partinfo prefaceinfo appendixinfo docinfo
13710	persName orgName addName nameLink roleName forename surname genName country placeName geogName
06808	personname surname firstname honorific lineage othername contrib doc:personname doc:surname doc:firstname doc:honorific doc:lineage doc:othername doc:contrib
04338	refsynopsisdiv/title refsection/title refsect1/title refsect2/title refsect3/title refsynopsisdiv/info/title refsection/info/title refsect1/info/title refsect2/info/title refsect3/info/title
04358	section/title simplesect/title sect1/title sect2/title sect3/title sect4/title sect5/title section/info/title simplesect/info/title sect1/info/title sect2/info/title sect3/info/title sect4/info/title sect5/info/title section/sectioninfo/title sect1/sect1info/title sect2/sect2info/title sect3/sect3info/title sect4/sect4info/title sect5/sect5info/title
13632	self::placeName self::persName self::district self::settlement self::region self::country self::bloc
01847	set book part preface chapter appendix article reference refentry book/glossary article/glossary part/glossary bibliography colophon
05219	set book part preface chapter appendix article topic reference refentry book/glossary article/glossary part/glossary book/bibliography article/bibliography part/bibliography colophon

05226	set book part preface chapter appendix article topic reference refentry sect1 sect2 sect3 sect4 sect5 section book/glossary article/glossary part/glossary book/bibliography article/bibliography part/bibliography colophon
03325	set book part reference preface chapter appendix article topic glossary bibliography index setindex refentry sect1 sect2 sect3 sect4 sect5 section
03410	set book part reference preface chapter appendix article topic glossary bibliography index setindex refentry refsynopsisdiv refsect1 refsect2 refsect3 refsection sect1 sect2 sect3 sect4 sect5 section
03407	set book part reference preface chapter appendix article glossary bibliography index setindex refentry sect1 sect2 sect3 sect4 sect5 section
04245	set book part reference preface chapter appendix article glossary bibliography index setindex refentry refsynopsisdiv refsect1 refsect2 refsect3 refsection sect1 sect2 sect3 sect4 sect5 section
04953	set book part reference preface chapter appendix article glossary bibliography index setindex topic refentry refsynopsisdiv refsect1 refsect2 refsect3 refsection sect1 sect2 sect3 sect4 sect5 section
07095	sf:stylesheet sf:stylesheet-ref sf:container-hint sf:page-start sf:br sf:selection-start sf:selection-end sf:insertion-point sf:ghost-text sf:attachments
05463	table//footnote informaltable//footnote
12960	tei:classSpec/tei:attList//tei:attDef/tei:datatype/rng:ref
12961	tei:classSpec/tei:attList//tei:attDef/tei:datatype/tei:dataRef
09123	tei:content//rng:ref[@name = 'macro.anyXML']
12514	tei:content/tei:classRef tei:content//tei:sequence/tei:classRef
12964	tei:dataSpec/tei:content//tei:dataRef
08632	tei:front//tei:titlePart/tei:title
10595	tei:label tei:figure tei:table tei:item tei:p tei:title tei:bibl tei:anchor tei:cell tei:lg tei:list tei:sp
12962	tei:macroSpec/tei:content//rng:ref

Table 3: Experiment results on the XPATH subcorpus from Lick and Schmitz in Table 2. For each automaton we present: size(number-of-states).

Query Id	A = $sha(\mathbf{Q}_{Id})$	$det(A)$	B = $det_{one^x}(A)$	$scl_{x-doc}(B)$	C = $det_{one^x \times x-doc}(A)$	mini(C)
18330	89 (37)	400 (38)	127 (22)	73 (22)	73 (22)	60 (18)

17914	161 (67)	4071 (151)	308 (49)	178 (49)	178 (49)	87 (24)
10745	155 (62)	586 (49)	256 (34)	131 (34)	131 (34)	125 (32)
02091	114 (48)	1088 (66)	128 (23)	79 (23)	79 (23)	65 (18)
00744	147 (62)	2038 (109)	183 (30)	97 (30)	97 (30)	53 (15)
12060	80 (32)	333 (34)	97 (18)	62 (18)	62 (18)	50 (14)
02762	153 (64)	3143 (132)	190 (34)	118 (34)	118 (34)	52 (14)
06027	147 (62)	11244 (263)	161 (28)	95 (28)	95 (28)	52 (14)
02909	112 (45)	594 (52)	202 (29)	106 (29)	106 (29)	94 (25)
06415	153 (64)	5736 (169)	247 (40)	149 (40)	149 (40)	109 (27)
03257	143 (58)	3961 (164)	380 (51)	256 (51)	256 (51)	98 (22)
05122	99 (40)	715 (55)	172 (27)	109 (27)	109 (27)	71 (18)
09138	349 (152)		410 (59)	198 (59)	198 (59)	62 (15)
05460	214 (88)	38163 (610)	1039 (173)	628 (173)	628 (173)	69 (18)
12404	100 (40)	521 (47)	154 (24)	83 (24)	83 (24)	71 (20)
10337	108 (43)	566 (50)	184 (27)	98 (27)	98 (27)	86 (23)
06639	134 (54)	1199 (79)	243 (42)	149 (42)	149 (42)	67 (18)
14340	69 (29)	186 (24)	94 (18)	57 (18)	57 (18)	44 (14)
13804	90 (37)	314 (35)	171 (28)	92 (28)	92 (28)	72 (22)
13896	116 (46)	146 (23)	124 (23)	92 (23)	92 (23)	80 (18)
02194	71 (29)	208 (26)	112 (20)	65 (20)	65 (20)	52 (16)
06726	129 (56)	1400 (87)	182 (30)	96 (30)	96 (30)	54 (16)
13640	90 (37)	314 (35)	170 (26)	85 (26)	85 (26)	75 (23)
05735	101 (41)	362 (39)	210 (30)	105 (30)	105 (30)	95 (27)
15766	134 (54)	609 (55)	318 (41)	154 (41)	154 (41)	134 (35)
15524	115 (46)	421 (44)	259 (35)	129 (35)	129 (35)	119 (32)
06512	120 (49)	407 (42)	202 (34)	114 (34)	114 (34)	74 (22)
06176	1191 (550)		5271 (386)		1202 (386)	
12539	159 (68)	1965 (112)	311 (40)	137 (40)	137 (40)	100 (28)
11780	166 (70)	2286 (128)	317 (41)	142 (41)	142 (41)	98 (27)
11478	91 (37)	315 (35)	171 (26)	86 (26)	86 (26)	76 (23)
11227	148 (59)	583 (53)	337 (42)	163 (42)	163 (42)	144 (37)
05684	1276 (580)		4044 (226)		718 (226)	123 (16)
06947	704 (322)		1575 (129)	443 (129)	443 (129)	70 (14)
06794	256 (114)		399 (51)	177 (51)	177 (51)	63 (15)
06169	328 (146)		516 (62)	218 (62)	218 (62)	66 (14)
06924	566 (258)		1141 (105)		361 (105)	66 (14)
11958	102 (40)	347 (35)	143 (24)	89 (24)	89 (24)	75 (20)
01705	732 (330)		2424 (172)	745 (172)	745 (172)	112 (19)

02086	767 (346)		2605 (180)	780 (180)	780 (180)	114 (19)
02000	608 (274)		1212 (110)		386 (110)	82 (14)
02697	363 (162)		589 (68)	239 (68)	239 (68)	68 (14)
14183	103 (40)	361 (36)	154 (25)	93 (25)	93 (25)	79 (21)
07106	433 (194)		747 (80)	281 (80)	281 (80)	72 (14)
05824	78 (32)	305 (32)	79 (16)	54 (16)	54 (16)	42 (12)
11368	118 (48)	1233 (76)	187 (32)	118 (32)	118 (32)	86 (22)
15848	120 (47)	293 (34)	175 (26)	100 (26)	100 (26)	97 (25)
15462	114 (44)	332 (36)	302 (36)	113 (28)	113 (28)	108 (27)
04267	83 (32)	145 (20)	80 (15)	53 (15)	53 (15)	50 (14)
07113	695 (296)		7574 (302)		7570 (302)	365 (49)
03864	258 (114)		382 (50)	176 (50)	176 (50)	62 (14)
15484	173 (66)	656 (62)	411 (50)	188 (47)	188 (47)	173 (42)
15461	134 (52)	685 (56)	423 (48)	155 (37)	155 (37)	141 (34)
11160	293 (130)		447 (56)	197 (56)	197 (56)	64 (14)
06856	290 (130)		465 (57)	197 (57)	197 (57)	58 (14)
06458	783 (354)		1777 (140)		491 (140)	92 (14)
13710	398 (178)		666 (74)	260 (74)	260 (74)	70 (14)
06808	497 (226)		948 (93)	320 (93)	320 (93)	64 (14)
04338	450 (196)		936 (135)	562 (135)	562 (135)	94 (22)
04358	966 (424)		3354 (433)		2020 (433)	
13632	132 (58)	339 (33)	216 (33)	113 (33)	113 (33)	47 (9)
01847	531 (238)		1142 (137)	542 (137)	542 (137)	91 (19)
05219	664 (298)		1422 (164)	650 (164)	650 (164)	93 (19)
05226	874 (394)		2184 (218)	866 (218)	866 (218)	105 (19)
03325	713 (322)		1539 (128)	449 (128)	449 (128)	88 (14)
03410	888 (402)		2164 (158)	554 (158)	554 (158)	98 (14)
03407	678 (306)		1426 (122)	428 (122)	428 (122)	86 (14)
04245	853 (386)		2031 (152)		533 (152)	96 (14)
04953	888 (402)		2164 (158)		554 (158)	98 (14)
07095	363 (162)		589 (68)	239 (68)	239 (68)	68 (14)
05463	150 (60)	1179 (70)	256 (38)	151 (38)	151 (38)	77 (20)
12960	160 (64)	1339 (81)	316 (46)	189 (46)	189 (46)	145 (33)
12961	159 (64)	1317 (80)	302 (45)	185 (45)	185 (45)	141 (32)
09123	157 (60)	704 (59)	367 (43)	174 (43)	174 (43)	163 (40)
12514	161 (66)	2733 (112)	238 (38)	145 (38)	145 (38)	113 (28)
12964	121 (48)	559 (48)	192 (31)	119 (31)	119 (31)	95 (24)
08632	121 (48)	628 (51)	192 (31)	119 (31)	119 (31)	95 (24)

$$\begin{array}{c}
Q_1 @ P \rightarrow Q \in \Delta^{\det(\text{exp}(A))} \quad \exists q_1 \in Q_1 \exists p \in P \exists q' \in Q^A. q_1 @ p \rightarrow q' \in \Delta^A \\
\hline
Q_1 @ P \rightarrow Q \in \Delta^{\det(A)} \\
Q = \{q \in Q^A \mid q_1 @ _ \rightarrow q \in \Delta^{\epsilon} \Delta^A, q_1 \in Q_1\} \neq \emptyset \\
\hline
Q_1 @ _ \rightarrow Q \in \Delta^{\det(A)}
\end{array}$$

Fig. 15: Accessible determinization for $\text{SHA}[-, @]$.

$$\begin{array}{c}
q @ _ \rightarrow q' \in \Delta^A \quad \exists p. p \sim r \wedge q @ p \rightarrow q' \in \Delta^{\text{exp}(A)} \quad q \sim s \\
s @ r \rightarrow s' \in \Delta^{\text{exp}(S)} \quad s' \notin \text{sink}(S) \\
\hline
q @ _ \rightarrow q' \in \Delta^{\text{scl}_S(A)} \quad q' \sim s'
\end{array}$$

Fig. 16: Schema-based cleaning for $\text{SHA}[-, @]$.

10595	433 (194)		747 (80)	281 (80)	281 (80)	72 (14)
12962	122 (48)	575 (49)	204 (32)	123 (32)	123 (32)	99 (25)

9 Adding Apply Else Rules

We extend symbolic stepwise hedge automata with a second kind of symbolic representations called apply else rules.

Definition 4. An $\text{SHA}[-, @]$ is a tuple $A = (\Sigma, \mathcal{Q}, \mathcal{P}, \mathcal{P}_{\text{else}}, \Delta, I, F)$ such that $\Delta = (\Delta', @_{-}^{\Delta})$ and $A' = (\Sigma, \mathcal{Q}, \mathcal{P}, \Delta', I, F)$ is a $d\text{SHA}[-]$. Furthermore, $@_{-}^{\Delta} \subseteq \mathcal{Q}^2$ is a set of apply else transition rules and $\mathcal{P}_{\text{else}} \subseteq \mathcal{P}$ is a subset of else states. We call A deterministic if A' is a $d\text{SHA}[-]$ and $@_{-}^{\Delta}$ is a partial function.

The else states $p \in \mathcal{P}_{\text{else}}$ are distinguished graphically by dark gray filling color with parallel bars \overline{p} . An apply else rule $(q, q') \in @_{-}^{\Delta}$ is drawn with a blue arrow as $\overrightarrow{q} \rightarrow \overrightarrow{q'}$ and written as $q @ _ \rightarrow q' \in \Delta$. It says that q can go to q' when applied an else state $p \in \mathcal{P}_{\text{else}}$, under the condition that there does not exist any state q'' with $q @ p \rightarrow q'' \in \Delta$. Any $\text{SHA}[-, @]$ can be expanded to some SHA by lifting the expansion for $\text{NFAS}[-]$ as follows:

$$\frac{q @ _ \rightarrow q' \in \Delta^A \quad p \in \mathcal{P}_{\text{else}}^A \quad \neg \exists q'' \in \mathcal{Q}. q @ p \rightarrow q'' \in \Delta^A}{q @ p \rightarrow q' \in \Delta^{\text{exp}(A)}} \quad \frac{q @ p \rightarrow q' \in \Delta^A}{q @ p \rightarrow q' \in \Delta^{\text{exp}(A)}}$$

In Fig. 15, we lift the accessible determinization to $\text{SHA}[-, @]$. In Fig. 16, the extension of the schema-based cleaning for $\text{SHA}[-, @]$ is shown. In Fig. 17, we show how to extend schema-based determinization to $\text{SHA}[-, @]$.

10 Adding Typed Else Rules

We consider typed alphabets Σ such that each letter can be given several types. For this, we assume a finite set T of types and for each type $\tau \in T$ a subsets $[[\tau]] \subseteq \Sigma$ of letters that can be given type τ .

$$\frac{Q@_ \rightarrow Q' \in \Delta^A \quad \exists P. P \sim r \wedge Q@P \rightarrow Q' \in \Delta^{exp(A)} \quad Q \sim s}{s@_r \rightarrow s' \in \Delta^{\Delta^{exp(S)}} \quad s' \notin sink(S)} \quad Q@_ \rightarrow Q' \in \Delta^{det_S(A)} \quad Q' \sim s'$$

Fig. 17: Schema-based determinization for $\text{SHA}[-, @_]$.

$$\frac{q_1 \xrightarrow{a} q \in \Delta^A \quad s \notin sink(S) \quad q_1 \sim s_1 \quad q_1 \xrightarrow{a} q_2 \in \Delta^A \quad a \in \llbracket \tau \rrbracket}{s_1 \xrightarrow{a} s \in \Delta^{exp(S)} \quad q_1 \xrightarrow{a} q \in \Delta^{scl_S(A)} \quad q \sim s} \quad \frac{q_1 \xrightarrow{a} q_2 \in \Delta^A \quad a \in \llbracket \tau \rrbracket}{q_1 \xrightarrow{a} q_2 \in \Delta^{scl_S(A)}} \quad \frac{q_1 \xrightarrow{\neg\tau} q \in \Delta^A \quad \neg\exists q_2. q_1 \xrightarrow{a} q_2 \in \Delta^A \quad q_1 \sim s_1}{s_1 \xrightarrow{a} s \in \Delta^{exp(S)} \quad s \notin sink(S) \quad a \in \llbracket \tau \rrbracket} \quad \frac{q_1 \xrightarrow{\neg\tau} q \in \Delta^{scl_S(A)} \quad q \sim s}{q_1 \xrightarrow{\neg\tau} q \in \Delta^{scl_S(A)} \quad q \sim s}$$

Fig. 18: Schema-based cleaning of $\text{SHAS}[-:T, @_]$.

Definition 5. An $\text{SHA}[-:T, @_]$ is a tuple $A = (\Sigma, \mathcal{Q}, \mathcal{P}, \mathcal{P}_{else}, \Delta, I, F)$ such that $\Delta = (\Delta', _{}^\Delta)$ and $A' = (\Sigma, \mathcal{Q}, \mathcal{P}, \Delta', I, F)$ is a $\text{SHA}[@_]$ and $_{}^\Delta \subseteq \mathcal{Q} \times T \times \mathcal{Q}$ a set of typed else rules. We call A deterministic or a $\text{dSHA}[-:T, @_]$ if A' is a $\text{dSHA}[@_]$ and for each state $q \in \mathcal{P}$ and letter $a \in \Sigma$ there is at most one rule $(q, \tau, q') \in _{}^\Delta$ such that $a \in \llbracket \tau \rrbracket$.

Instead of $(q, \tau, q') \in _{}^\Delta$, we will write $q \xrightarrow{\neg\tau} q' \in \Delta$ and draw the typed else rule as $\textcircled{q} \xrightarrow{\neg\tau} \textcircled{q'}$. In order to simulate untyped else rules, we chose T containing the universal type $all \in T$ which satisfies $\llbracket all \rrbracket = \Sigma$. We can always add the universal type to T without loss of generality since any letter can be given several types. Typed else rules $q \xrightarrow{\neg\tau} q' \in \Delta$ can then be used for representing untyped else rules $q \rightarrow q'$.

The expansion of $\text{SHA}[-, @_]$ can then be adapted to $\text{SHAS}[-:T, @_]$ as follows:

$$\frac{q \xrightarrow{\neg\tau} q' \in \Delta^A \quad a \in \llbracket \tau \rrbracket \quad \neg\exists q'' \in \mathcal{Q}^A. q \xrightarrow{a} q'' \in \Delta^A}{q \xrightarrow{a} q' \in \Delta^{exp(A)}} \quad \frac{q \xrightarrow{a} q' \in \Delta^A}{q \xrightarrow{a} q' \in \Delta^{exp(A)}}$$

Schema-based cleaning of $\text{SHA}[-, @_]$ can be adapted to $\text{SHA}[-:T, @_]$ based on expansion as shown in Fig. 18.

Since a same letter can be given different type, we will use intersection types for determinization. The set of all intersection types over T is defined as follows:

$$I, I' \in \mathcal{I}_T ::= \tau \mid \neg\tau' \mid I \cap I' \quad \text{where } \tau, \tau' \in T$$

Each intersection type $I \in \mathcal{I}_T$ specifies a finite set of letters $\llbracket I \rrbracket \subseteq \Sigma$ as follows:

$$\llbracket I \cap I' \rrbracket = \llbracket I \rrbracket \cap \llbracket I' \rrbracket \quad \llbracket \neg I \rrbracket = \Sigma \setminus \llbracket I \rrbracket$$

We then consider the class $\text{SHA}[-:\mathcal{I}_T, @_]$ of symbolic automata with else rules typed by intersection types, and lift our algorithms to this class. Accessible determinization is lifted in Fig. 19 and schema-based determinization in Fig. 20.

$$\begin{array}{c}
\frac{Q_1 \xrightarrow{a} Q_2 \in \Delta^{\det(\text{exp}(A))} \quad \exists q_1 \in Q_1 \exists q \in \mathcal{Q}^A. q_1 \xrightarrow{a} q \in \Delta^A}{Q_1 \xrightarrow{a} Q_2 \in \Delta^{\det(A)}} \\
\frac{Q_1 \in \mathcal{Q}^{\det(\text{exp}(A))} \quad Q_2 = \{q_2 \in \mathcal{Q}^A \mid q_1 \xrightarrow{\neg I'} q_2 \in \Delta^A, q_1 \in Q_1, \exists I'' . I' \cap I'' = I\} \neq \emptyset \\ I \in \mathcal{I}_T \text{ complete intersection} \quad \exists a \in \llbracket I \rrbracket. \forall q_1 \in Q_1 \neg \exists q' \in \mathcal{Q}^A. q_1 \xrightarrow{a} q' \in \Delta^A}{Q_1 \xrightarrow{\neg I} Q_2 \in \Delta^{\det(A)}}
\end{array}$$

Fig. 19: Accessible determinization for SHAS $[-:\mathcal{I}_T, @_-]$.

$$\begin{array}{c}
\frac{Q_1 \xrightarrow{a} Q \in \Delta^{\det(A)} \quad s_1 \xrightarrow{a} s \in \Delta^{\text{exp}(S)} \quad s \notin \text{sink}(S) \quad Q_1 \sim s_1}{Q_1 \xrightarrow{a} Q \in \Delta^{\det_S(A)} \quad Q \sim s} \quad \frac{Q_1 \xrightarrow{a} Q \in \Delta^{\det(A)} \quad a \in \llbracket I \rrbracket}{Q_1 \xrightarrow{\neg I} Q_2 \in \Delta^{\det_S(A)}} \\
\frac{Q_1 \xrightarrow{\neg I} Q \in \Delta^{\det(A)} \quad \neg \exists Q_2. Q_1 \xrightarrow{a} Q_2 \in \Delta^{\det(A)} \quad Q_1 \sim s_1 \\ s_1 \xrightarrow{a} s \in \Delta^{\text{exp}(S)} \quad s \notin \text{sink}(S) \quad a \in \llbracket I \rrbracket}{Q_1 \xrightarrow{\neg I} Q \in \Delta^{\det_S(A)} \quad Q \sim s}
\end{array}$$

Fig. 20: Schema-based determinization for SHAS $[-:\mathcal{I}_T, @_-]$.

The relevant case for our applications are types sets $T = \{\tau_1, \dots, \tau_n, \text{all}\}$ with quasi disjoint types, so that $\llbracket \tau_i \rrbracket \cap \llbracket \tau_j \rrbracket = \emptyset$ for all $1 \leq i < j \leq n$ and $\llbracket \text{all} \rrbracket = \Sigma$. In the case of *XML* documents we used the quasi disjoint signature:

$$\{\text{node-type}, \text{namespace}, \text{name}, \text{char}, \text{var}, \text{all}\}$$

We next argue that any SHA $[-:\mathcal{I}_T, @_-]$ can be rewritten to a SHA $[-:T, @_-]$ such that determinism is preserved, if $T = \{\tau_1, \dots, \tau_n, \text{all}\}$ is a signature with quasi disjoint types. Note that in this case:

$$\llbracket \neg \tau_i \rrbracket = \tau_1 \cup \dots \cup \tau_{i-1} \cup \tau_{i+1} \cup \dots \cup \tau_n$$

As a consequence, any intersection type $I \in \mathcal{I}_T$ can be rewritten into a union of types from T . For instance consider $n = 4$ and $I = \neg \tau_2 \cap \neg \tau_4 \cap \text{all}$. Then $\llbracket I \rrbracket = \llbracket \tau_1 \rrbracket \cup \llbracket \tau_3 \rrbracket$ so that $q \xrightarrow{\neg I} q'$ can be replaced equivalently by $q \xrightarrow{\neg \tau_1} q'$ and $q \xrightarrow{\neg \tau_3} q'$. Clearly, determinism is preserved by this rewriting. Hence, any $d\text{SHA}[-:\mathcal{I}_T, @_-]$ with a set of quasi disjoint types T can be rewritten to an equivalent $d\text{SHA}[-:T, @_-]$.

This can be used to determinize any SHAS $[-:T, @_-]$ with a set of quasi disjoint types T by using intersection types as follows. First notice that any SHAS $[-:T, @_-]$ A is also an SHA $[-:\mathcal{I}_T, @_-]$ and that we presented determinization algorithms for this class above. The determinization algorithm in Fig. 19 yields the $d\text{SHA}[-:\mathcal{I}_T, @_-]$ $\det(A)$ and the schema-based determinization from Fig. 20 the $d\text{SHA}[-:\mathcal{I}_T, @_-]$ $\det_S(A)$. These automata, in turn, can be rewritten into the class $d\text{SHAS}[-:T, @_-]$ since we are assuming that T contains quasi disjoint types.

Q	A= <i>sha</i> (Q)	<i>det</i> (A)	<i>scl</i> _{<i>one</i>^{<i>x</i>}} (<i>det</i> (A))	<i>det</i> _{<i>one</i>^{<i>x</i>}} (A)	<i>scl</i> _{<i>x-doc</i>} (<i>scl</i> _{<i>one</i>^{<i>x</i>}} (<i>det</i> (A)))	<i>det</i> _{<i>one</i>^{<i>x</i>} × <i>x-doc</i>} (A)
A1	135 (57)	149 (37)	149 (37)	149 (37)	107 (37)	107 (37)
A2	106 (42)	468 (58)	187 (36)	187 (36)	96 (32)	96 (32)
A3	113 (46)	332 (46)	211 (35)	211 (35)	116 (34)	116 (34)
A4	162 (67)	171 (42)	171 (42)	171 (42)	124 (42)	124 (42)
A5	181 (71)	440 (57)	420 (57)	420 (57)	259 (55)	259 (55)
A6	239 (91)	193 (45)	193 (45)	193 (45)	145 (45)	145 (45)
A7	161 (65)	174 (41)	170 (41)	170 (41)	126 (41)	126 (41)
A8	896 (268)	753 (124)	643 (118)	643 (118)	528 (118)	528 (118)

Fig. 21: Regular Path queries of XPathMark benchmark

11 Other Benchmarks

For our first set of experiments, we used the forward fragment that can be compiled to SHA, of the usual XML benchmark XPathMark [10]. The results are shown in Fig. 21. The queries A1-A8 are first compiled to their SHA equivalents, their respective size and number of states are presented in the second column, namely A . Then, we determinized, schema-cleaned $\mathit{det}(A)$ and schema-based determinized A , firstly with one^x alone, then with one^x and $x\text{-doc}$. The results are shown in the consequent four columns and the size of the resultant automata were relatively small. The test on this collection confirmed that the integration of schema-based cleaning into determinization yields the same outcome that schema-cleaning after determinization with $x\text{-doc}$ and one^x , as reported previously in [16]. Since the size of the automata remains relatively small, the gain remains quite limited for this collection.

A second collection of XPATH queries that we devised is the following $Q_{n,m}$ that can be scaled with for all naturals n and m :

```
(Qn,m)    /*[self::a0 or ... or self::an]
           [descendant::*[self::b0 or ... or self::bm]]">
```

This query pattern select all elements which name is a_0 or a_1, \dots, a_n under the condition that these elements have some descendants b_0 or b_1, \dots, b_m . An example of such query is **Q2.2** : `//*[self::a0 or self::a1][descendant::*[self::b0 or self::b1]]`. We performed the same steps as per the first collection and the results for the queries **Q2.1**, \dots , **Q3.4** with schemas one^x and $\mathit{one}^x \times x\text{-doc}$ are reported in Fig. 22. It is clear that size of the deterministic automata $\mathit{det}(A)$ is growing relatively to the number of the disjunctions in the query and quickly, the schema-cleaning process deems inefficient. This is shown for queries **Q2.3**, **Q2.4**, **Q3.2**, **Q3.3** and **Q3.4** where schema-based cleaning could no more be applied afterwards. With schema-based determinization, however, we could obtain small deterministic automata without problems. One could think of scaling up to test the limits of schema-based determinization in practice, but it is clear that even with our killer example **QN7** of size more than 1.6 million, we

Q	A= <i>sha</i> (Q)	<i>det</i> (A)	<i>scl</i> _{<i>one</i>^{<i>x</i>}} (<i>det</i> (A))	<i>det</i> _{<i>one</i>^{<i>x</i>}} (A)	<i>scl</i> _{<i>x-doc</i>} (<i>scl</i> _{<i>one</i>^{<i>x</i>}} (<i>det</i> (A)))	<i>det</i> _{<i>one</i>^{<i>x</i>} × <i>x-doc</i>} (A)
Q2.1	165 (63)	2302 (125)	588 (67)	588 (67)	391 (67)	391 (67)
Q2.2	196 (74)	5290 (231)	1288 (117)	1288 (117)	973 (117)	973 (117)
Q2.3	227 (85)	11878 (433)		2668 (203)		2187 (203)
Q2.4	258 (96)	26650 (835)		5436 (361)		4705 (361)
Q3.1	205 (77)	4341 (193)	735 (82)	735 (82)	488 (82)	488 (82)
Q3.2	243 (90)	9849 (363)		1595 (144)		1210 (144)
Q3.3	281 (103)	22097 (697)		3345 (256)		2760 (256)
Q3.4	300 (112)	57170 (2219)		9324 (613)		8302 (613)
QN7	322 (132)	1633790 (10003)		499 (72)		248 (72)

Fig. 22: A scaling collection of path queries with schema *one*^{*x*} and *x-doc* × *one*^{*x*}. The open fields could not be computed within 30 minutes or ran out of memory.

$$\begin{array}{c}
\frac{\diamond^{\Delta^S} = \{s\}}{\diamond^{\Delta^A} \in \diamond^{\Delta^{scl_S(A)}}} \quad \frac{q \rightarrow p \in \Delta^A \quad s \rightarrow r \in \Delta^S \quad r \notin \text{sink}(S) \quad q \sim s}{q \rightarrow p \in \Delta^{scl_S(A)} \quad p \sim r} \\
\frac{q_1 @ q_2 \rightarrow q' \in \Delta^A \quad s_1 @ s_2 \rightarrow s' \in \Delta^S \quad s' \notin \text{sink}(S) \quad q_1 \sim s_1 \quad q_2 \sim s_2}{q_1 @ q_2 \rightarrow q' \in \Delta^{scl_S(A)} \quad q' \sim s'}
\end{array}$$

Fig. 23: Schema-based cleaning of SHAS[.] extending on Fig. 8.

were still able to determinize it with both *one*^{*x*} and *one*^{*x*} × *x-doc*. QN7 query is the following:

(QN7) /a/b//(* | @* | comment() | text())

It selects all nodes of an XML document that are descendants of a *b*-element below an *a*-element at the root. The node may have any XML type: element, attribute, comment, or text. The nondeterministic SHA has 132 states and an overall size of 322. Its determinization however leads to an automaton with 10.003 states and an overall size of 1.633.790.