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Stability analysis of high frequency nonlinear amplifiers via harmonic identification

L. Baratchart **S. Chevillard** **A. Cooman**
S. Fueyo **M. Olivi** **J.-B. Pomet** **F. Seyfert**

project-teams: **FACTAS** (ex APICS) and **McTAO**

Complex Days January 11, 2018

Motivation

- Amplifiers at high frequency are ubquitous (Cell phones, relays...), need to be quick to design and produced in large quantites.

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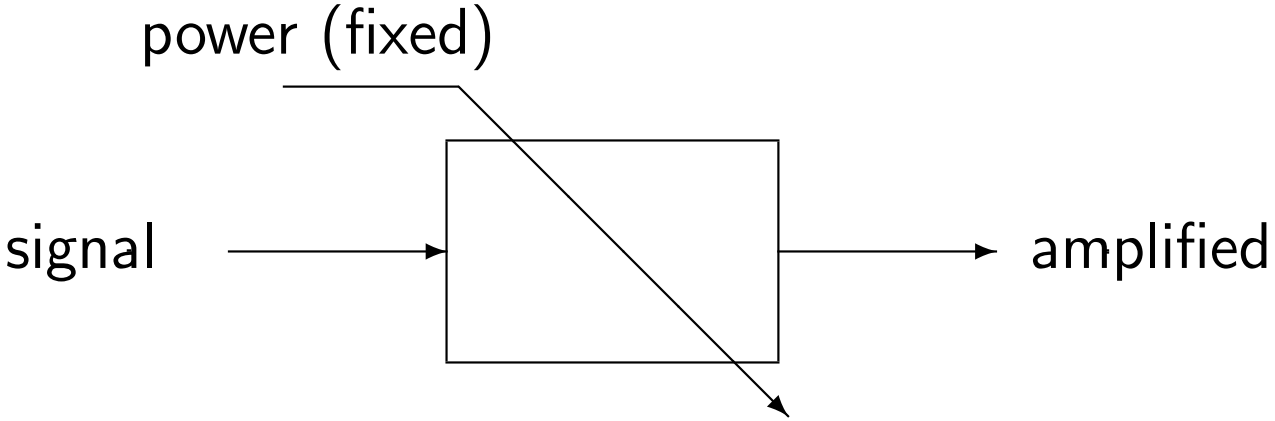
- Computer-assisted design and simulation before production.
- Powerful "frequency simulation" tools give a reliable prediction of the response, but that response might be unstable.
- **Need** for a tool to predict stability/unstability.

Amplifiers (seen at a distance)



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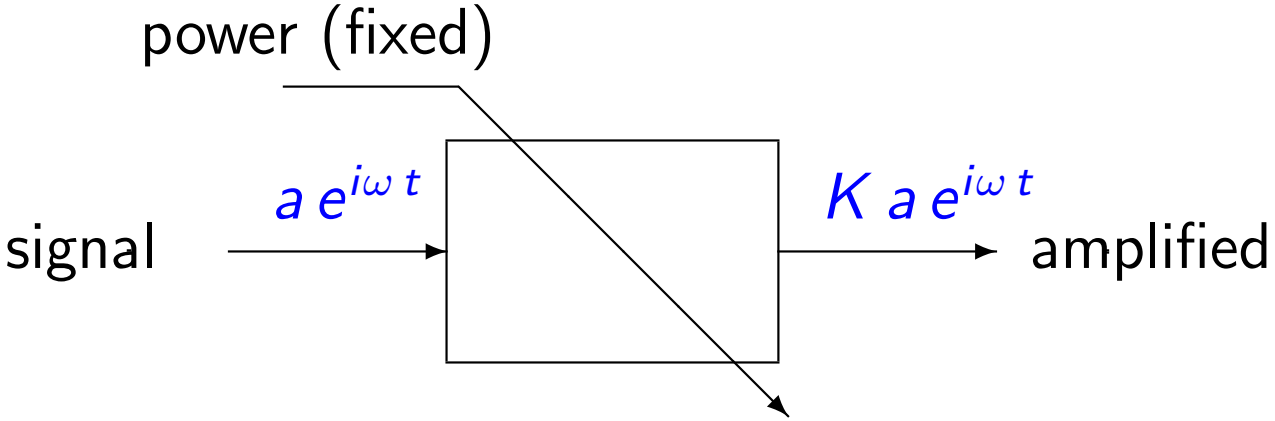

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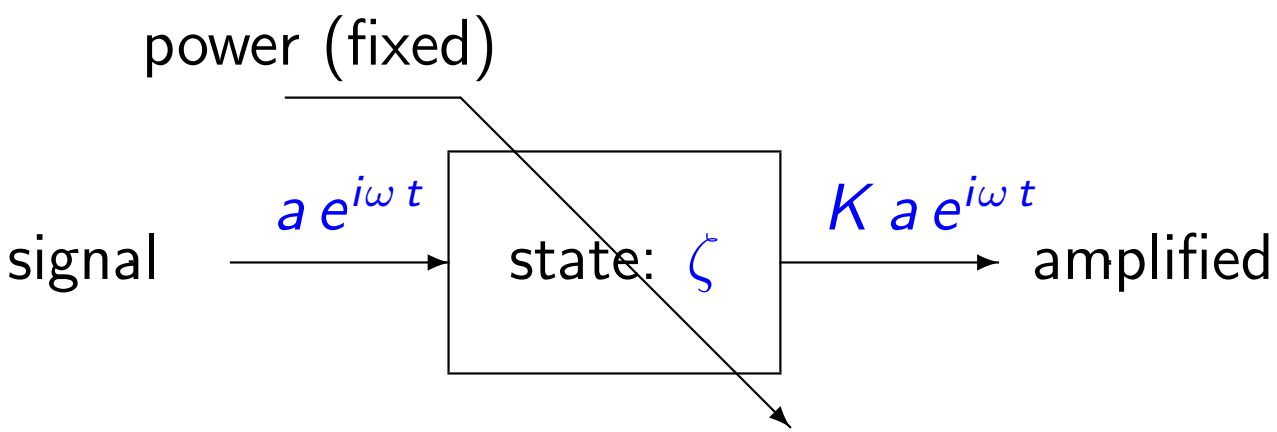

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possibly distributed (∞-dimensional)

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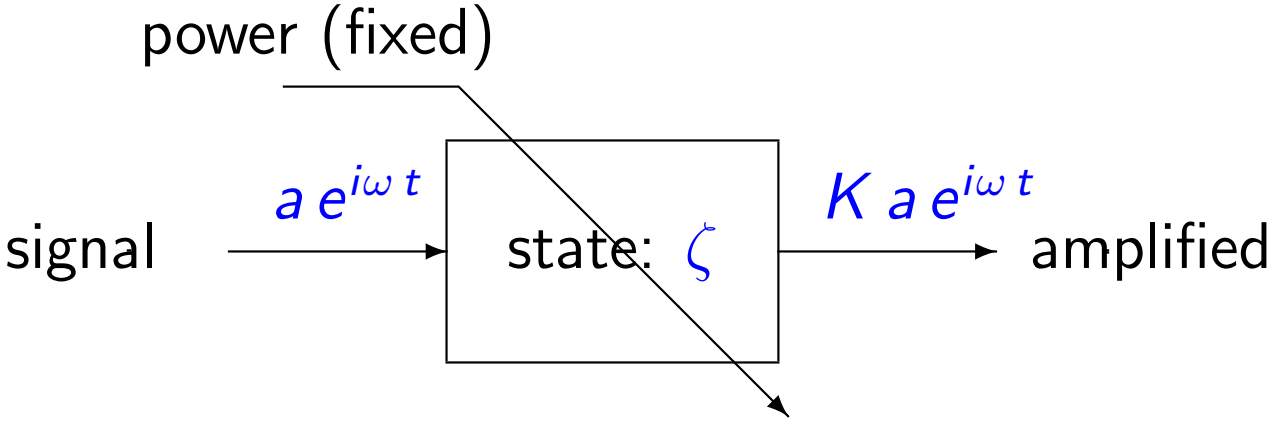

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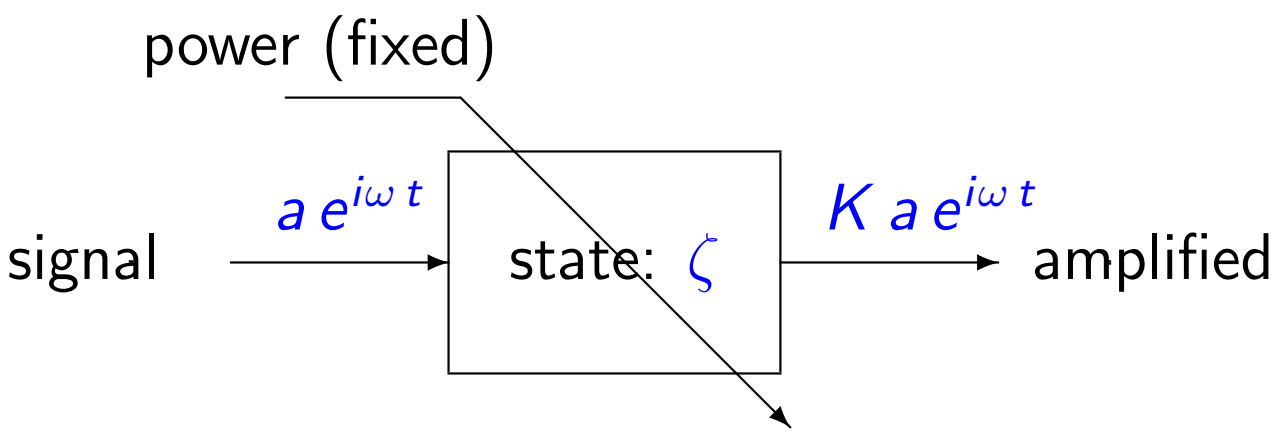

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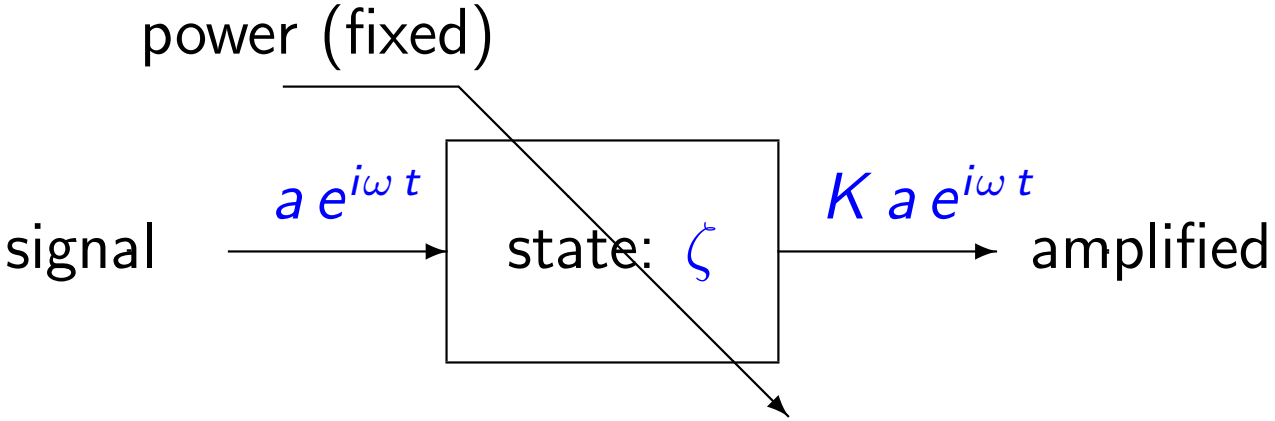

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▶▶ In particular, if zero signal,
$\dot{\zeta} = F_0(\zeta)$ must have a stable equilibrium ζ_0, with $g(\zeta_0) = 0$.

Inside

An amplifier is made of interconnected

- resistors, inductors, capacitors,
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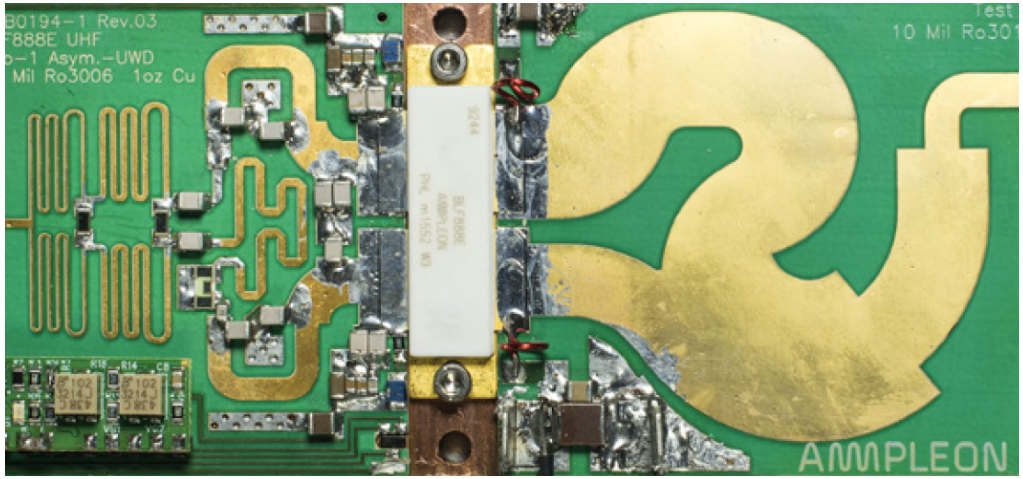
In fact,
transmission lines
have an important role
in the design.



Computer-assisted design

Harmonic Balance (HB): a reliable design and simulation tool.
Simulations take place in the frequency domain.

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- The nonlinear characteristic of the diods (and its linearization) are known numerically.

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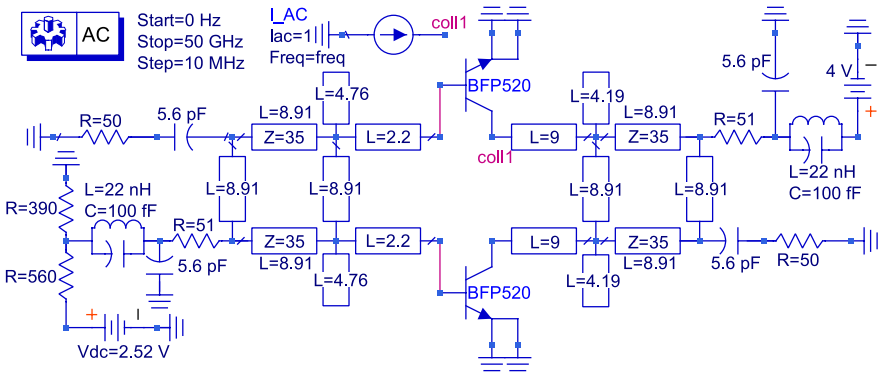
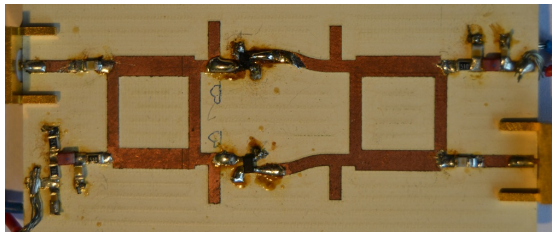


CAD tool



prototype

Stability from computer-assisted design?

$$\text{“small signal”} = 0 : \quad \dot{\zeta} = F_0(\zeta) \tag{1}$$

$$\text{“large signal”} = a\, e^{i\omega\, t} : \quad \dot{\zeta} = F_\omega(\zeta, t) \;\; (2\pi/\omega\text{-periodic}) \tag{2}$$

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Linearized system:

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Stability is given by the spectrum of $\frac{\partial F_0}{\partial \zeta}(\zeta_0)$,
or the time-var. counterpart (Floquet) of $\frac{\partial F_0}{\partial \zeta}(\zeta_{\mathrm{per}}(\cdot), \cdot)$.

These are out of reach, no “time-domain” modelisation.

Stability from computer-assisted design

However, *Harmonic Balance* may compute the frequency response of $u \to y$ as in

$$\dot{\xi} = \frac{\partial F_\omega}{\partial \zeta}(\zeta_{\mathrm{per}}(t), t)\,\xi + B\,u\,, \quad u = C\,\zeta \quad \text{or} \quad \cdots \qquad (*)$$

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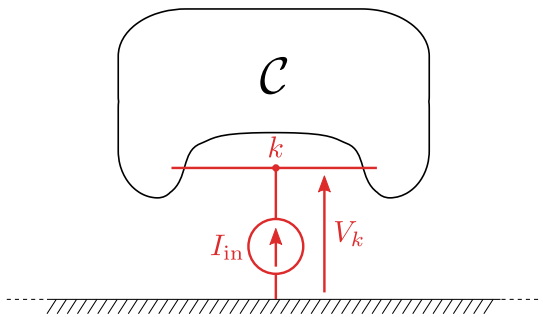
System (*) is built artificially by adding a fictive source of current or tension u and a measure y in the circuit.



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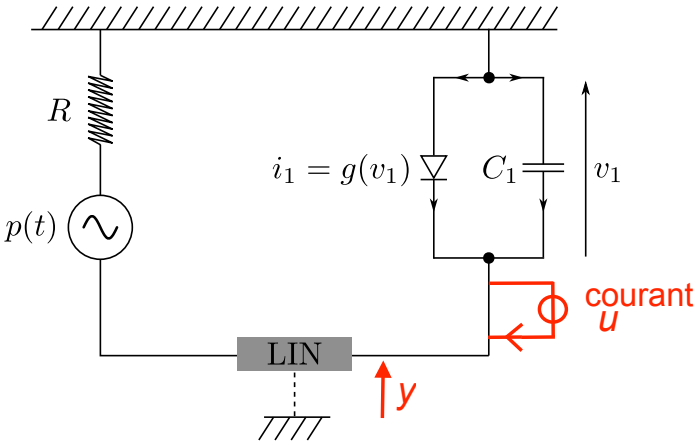
  transfer function is not meromorphic for "large signals" in general, conjecture: meromorphic in the right-half plane for real life circuits?

A simple circuit [Fueyo & al.]

Example due to [Hale & al 1993]



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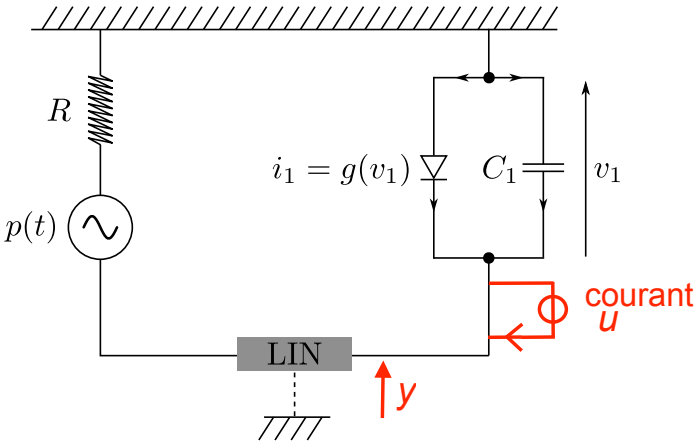

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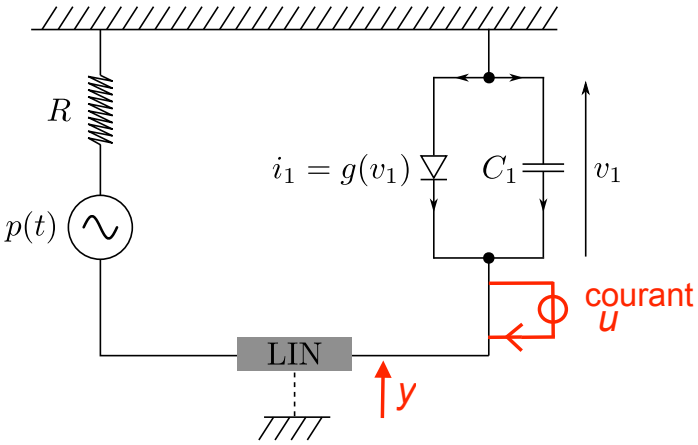

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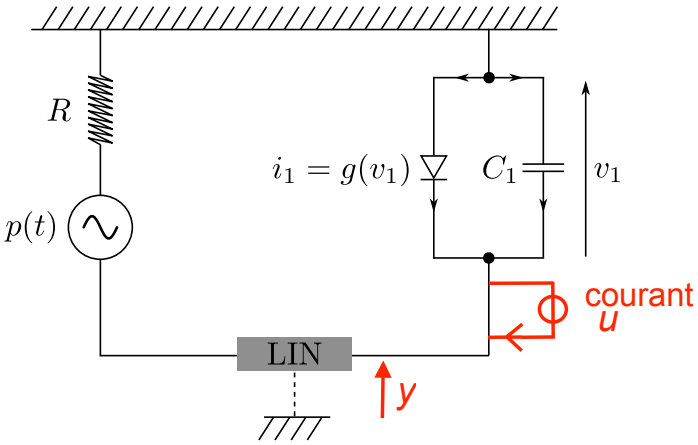

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- Modulo a minor lack of observability when some rational ratio, and at least right of a vertical axis with negative real part, the set of singularities of the harmonic transfer function is the spectrum of the monodromy operator.

Perspectives

- General circuits
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Thank you for your attention.