



**HAL**  
open science

## Stability analysis of high frequency nonlinear amplifiers via harmonic identification

Laurent Baratchart, Sylvain Chevillard, Adam Cooman, Sébastien Fueyo,  
Martine Olivi, Jean-Baptiste Pomet, Fabien Seyfert

### ► To cite this version:

Laurent Baratchart, Sylvain Chevillard, Adam Cooman, Sébastien Fueyo, Martine Olivi, et al.. Stability analysis of high frequency nonlinear amplifiers via harmonic identification. Complex Days 2018, without insertion in the proceedings, Université Côte d'Azur, Jan 2018, Nice, France. hal-03526549

**HAL Id: hal-03526549**

**<https://inria.hal.science/hal-03526549>**

Submitted on 14 Jan 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Stability analysis of high frequency nonlinear amplifiers via harmonic identification

L. Baratchart   S. Chevillard   A. Cooman  
S. Fueyo   M. Olivi   J.-B. Pomet   F. Seyfert

project-teams: **FACTAS** (ex APICS) and **McTAO**



## Motivation

- Amplifiers at high frequency are ubiquitous (Cell phones, relays...), need to be quick to design and produced in large quantities.

## Motivation

- Amplifiers at high frequency are ubiquitous (Cell phones, relays...), need to be quick to design and produced in large quantities.
- Computer-assisted design and simulation before production.

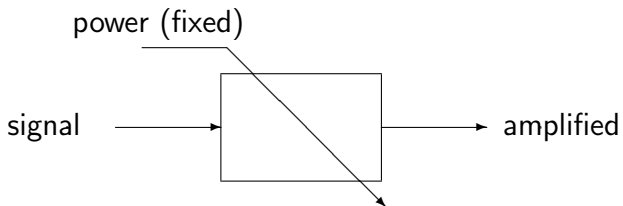
## Motivation

- Amplifiers at high frequency are ubiquitous (Cell phones, relays...), need to be quick to design and produced in large quantities.
- Computer-assisted design and simulation before production.
- Powerful “frequency simulation” tools give a reliable prediction of the response, but that response might be unstable.

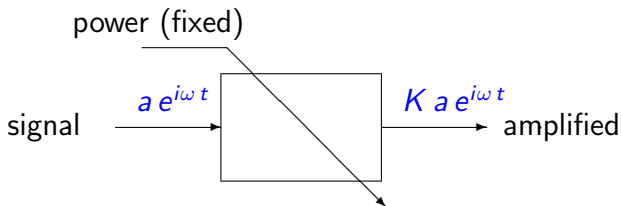
## Motivation

- Amplifiers at high frequency are ubiquitous (Cell phones, relays...), need to be quick to design and produced in large quantities.
- Computer-assisted design and simulation before production.
- Powerful “frequency simulation” tools give a reliable prediction of the response, but that response might be unstable.
- **Need** for a tool to predict stability/unstability.

## Amplifiers (seen at a distance)

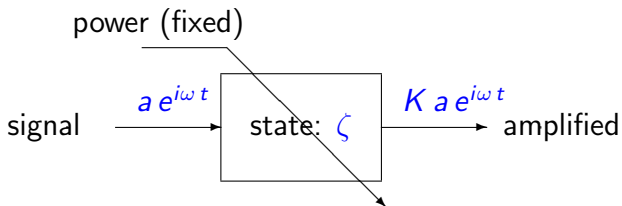


## Amplifiers (seen at a distance)



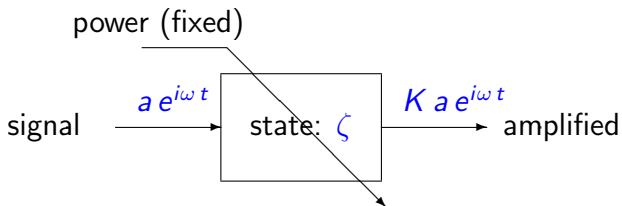


## Amplifiers (seen at a distance)



State  $\zeta$  contains many currents and tensions,  
possibly distributed ( $\infty$ -dimensional)

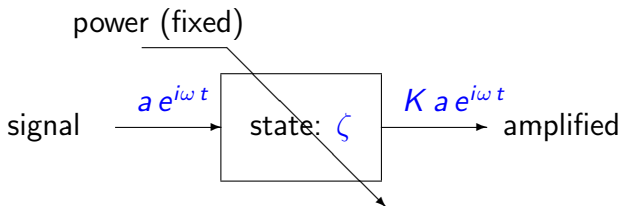
## Amplifiers (seen at a distance)



State  $\zeta$  contains many currents and tensions,  
possibly distributed ( $\infty$ -dimensional)

$$\dot{\zeta} = F_{\text{signal}}(\zeta, t), \quad \text{amplified} = g(\zeta) \quad (*)$$

## Amplifiers (seen at a distance)

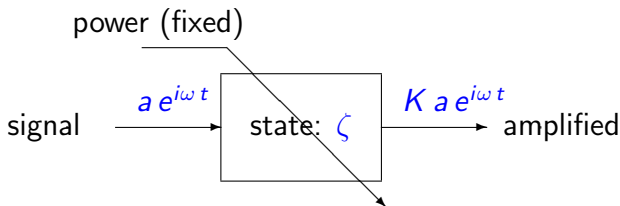


State  $\zeta$  contains many currents and tensions,  
possibly distributed ( $\infty$ -dimensional)

$$\dot{\zeta} = F_{\text{signal}}(\zeta, t), \quad \text{amplified} = g(\zeta) \quad (*)$$

►► **stable** periodic solution of (\*), yielding desired amplified signal.

## Amplifiers (seen at a distance)



State  $\zeta$  contains many currents and tensions,  
possibly distributed ( $\infty$ -dimensional)

$$\dot{\zeta} = F_{\text{signal}}(\zeta, t), \quad \text{amplified} = g(\zeta) \quad (*)$$

- ▶▶ **stable** periodic solution of (\*), yielding desired amplified signal.
- ▶▶ In particular, if zero signal,  
 $\dot{\zeta} = F_0(\zeta)$  must have a stable equilibrium  $\zeta_0$ , with  $g(\zeta_0) = 0$ .

# Inside

An amplifier is made of interconnected

- resistors, inductors, capacitors,
- diodes/transistors,
- transmission lines.

An amplifier is made of interconnected

- resistors, inductors, capacitors,
- diodes/transistors, ▶ **nonlinear**
- transmission lines.
  - ▶ **infinite dimension** (delays, telegrapher equation...)  
cannot be neglected at high frequencies

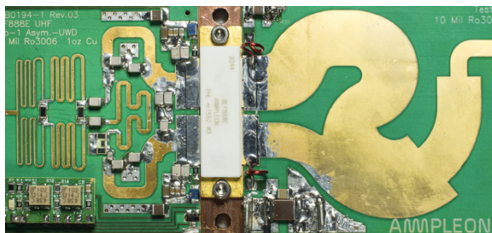
# Inside

An amplifier is made of interconnected

- resistors, inductors, capacitors,
- diodes/transistors, ▶ **nonlinear**
- transmission lines.

▶ **infinite dimension** (delays, telegrapher equation...)  
cannot be neglected at high frequencies

In fact,  
transmission lines  
have an important role  
in the design.



# Computer-assisted design

**Harmonic Balance (HB):** a reliable design and simulation tool.

*Simulations take place in the frequency domain.*

- Transmission lines are described by their frequency response, sometimes experimental.
- The nonlinear characteristic of the diodes (and its linearization) are known numerically.





## Stability from computer-assisted design?

$$\text{"small signal"} = 0 : \quad \dot{\zeta} = F_0(\zeta) \quad (1)$$

$$\text{"large signal"} = a e^{i\omega t} : \quad \dot{\zeta} = F_\omega(\zeta, t) \quad (2\pi/\omega\text{-periodic}) \quad (2)$$

## Stability from computer-assisted design?

“small signal” = 0 :  $\dot{\zeta} = F_0(\zeta)$  (1)

“large signal” =  $a e^{i\omega t}$  :  $\dot{\zeta} = F_\omega(\zeta, t)$  ( $2\pi/\omega$ -periodic) (2)

*Harmonic Balance* computes an equilibrium point  $\zeta_0$  of (1)  
or a periodic solution  $t \mapsto \zeta_{\text{per}}(t)$  of (2)

## Stability from computer-assisted design?

$$\text{"small signal"} = 0 : \quad \dot{\zeta} = F_0(\zeta) \quad (1)$$

$$\text{"large signal"} = a e^{i\omega t} : \quad \dot{\zeta} = F_\omega(\zeta, t) \quad (2\pi/\omega\text{-periodic}) \quad (2)$$

*Harmonic Balance* computes an equilibrium point  $\zeta_0$  of (1)  
or a periodic solution  $t \mapsto \zeta_{\text{per}}(t)$  of (2)

*Linearized system:*

$$\dot{\xi} = \frac{\partial F_0}{\partial \zeta}(\zeta_0) \xi \quad \text{or} \quad \dot{\xi} = \frac{\partial F_0}{\partial \zeta}(\zeta_{\text{per}}(t), t) \xi.$$

## Stability from computer-assisted design?

$$\text{"small signal"} = 0 : \quad \dot{\zeta} = F_0(\zeta) \quad (1)$$

$$\text{"large signal"} = a e^{i\omega t} : \quad \dot{\zeta} = F_\omega(\zeta, t) \quad (2\pi/\omega\text{-periodic}) \quad (2)$$

*Harmonic Balance* computes an equilibrium point  $\zeta_0$  of (1)  
or a periodic solution  $t \mapsto \zeta_{\text{per}}(t)$  of (2)

*Linearized system:*

$$\dot{\xi} = \frac{\partial F_0}{\partial \zeta}(\zeta_0) \xi \quad \text{or} \quad \dot{\xi} = \frac{\partial F_0}{\partial \zeta}(\zeta_{\text{per}}(t), t) \xi.$$

Stability is given by the spectrum of  $\frac{\partial F_0}{\partial \zeta}(\zeta_0)$ ,  
or the time-var. counterpart (Floquet) of  $\frac{\partial F_0}{\partial \zeta}(\zeta_{\text{per}}(\cdot), \cdot)$ .

These are out of reach, no "time-domain" modelisation.

## Stability from computer-assisted design

However, *Harmonic Balance* may compute the frequency response of  $u \rightarrow y$  as in

$$\dot{\xi} = \frac{\partial F_{\omega}}{\partial \zeta}(\zeta_{\text{per}}(t), t) \xi + B u, \quad u = C \zeta \quad \text{or} \quad \dots \quad (*)$$

for suitable linear operators  $B$ ,  $C$ . Hence a possibility to reconstruct the singularities of the (harmonic) transfer function of this linear system.

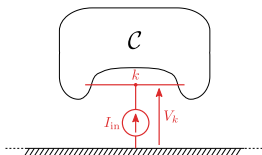
## Stability from computer-assisted design

However, *Harmonic Balance* may compute the frequency response of  $u \rightarrow y$  as in

$$\dot{\xi} = \frac{\partial F_{\omega}}{\partial \zeta}(\zeta_{\text{per}}(t), t) \xi + B u, \quad u = C \zeta \quad \text{or} \quad \dots \quad (*)$$

for suitable linear operators  $B$ ,  $C$ . Hence a possibility to reconstruct the singularities of the (harmonic) transfer function of this linear system.

System (\*) is built artificially by adding a fictive source of current or tension  $u$  and a measure  $y$  in the circuit.



## Program

**State of the art** in electronics is to use the frequency response to perform rational approximation of the (harmonic) transfer function on frequency intervals (it is not rational!), check the location of poles and their stability [Jugo & al 2001, Suarez 2015].



# Program

**State of the art** in electronics is to use the frequency response to perform rational approximation of the (harmonic) transfer function on frequency intervals (it is not rational!), check the location of poles and their stability [Jugo & al 2001, Suarez 2015].

**Our program** is to improve the estimation of the singularities and to further analyse stability to justify/understand its link with these.

# Program

**State of the art** in electronics is to use the frequency response to perform rational approximation of the (harmonic) transfer function on frequency intervals (it is not rational!), check the location of poles and their stability [Jugo & al 2001, Suarez 2015].

**Our program** is to improve the estimation of the singularities and to further analyse stability to justify/understand its link with these.

- [Baratchart & al 2017, Cooman & al]: singularities can be determined without rational approximation;

## Program

**State of the art** in electronics is to use the frequency response to perform rational approximation of the (harmonic) transfer function on frequency intervals (it is not rational!), check the location of poles and their stability [Jugo & al 2001, Suarez 2015].

**Our program** is to improve the estimation of the singularities and to further analyse stability to justify/understand its link with these.

- [Baratchart & al 2017, Cooman & al]: singularities can be determined without rational approximation; in the “small signal” case, singularities are poles.

# Program

**State of the art** in electronics is to use the frequency response to perform rational approximation of the (harmonic) transfer function on frequency intervals (it is not rational!), check the location of poles and their stability [Jugo & al 2001, Suarez 2015].

**Our program** is to improve the estimation of the singularities and to further analyse stability to justify/understand its link with these.

- [Baratchart & al 2017, Cooman & al]: singularities can be determined without rational approximation; in the “small signal” case, singularities are poles.
- [Fueyo, PhD]: time-domain modelisation;

# Program

**State of the art** in electronics is to use the frequency response to perform rational approximation of the (harmonic) transfer function on frequency intervals (it is not rational!), check the location of poles and their stability [Jugo & al 2001, Suarez 2015].

**Our program** is to improve the estimation of the singularities and to further analyse stability to justify/understand its link with these.

- [Baratchart & al 2017, Cooman & al]: singularities can be determined without rational approximation;  
in the “small signal” case, singularities are poles.
- [Fueyo, PhD]: time-domain modelisation;  
link between: local stability / the spectrum of the generator / the singularities of (harmonic) transfer function;

# Program

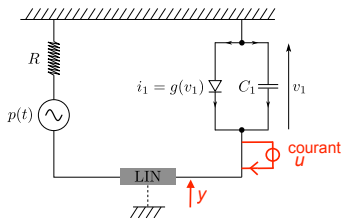
**State of the art** in electronics is to use the frequency response to perform rational approximation of the (harmonic) transfer function on frequency intervals (it is not rational!), check the location of poles and their stability [Jugo & al 2001, Suarez 2015].

**Our program** is to improve the estimation of the singularities and to further analyse stability to justify/understand its link with these.

- [Baratchart & al 2017, Cooman & al]: singularities can be determined without rational approximation; in the “small signal” case, singularities are poles.
- [Fueyo, PhD]: time-domain modelisation; link between: local stability / the spectrum of the generator / the singularities of (harmonic) transfer function; transfer function is not meromorphic for “large signals” in general, conjecture: meromorphic in the right-half plane for real life circuits?

## A simple circuit [Fueyo & al.]

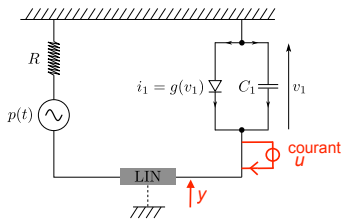
Example due  
to [Hale & al 1993]



Model " $\dot{\zeta} = F_{\omega}(\zeta, t)$ "  
with  $\zeta$   $\infty$ -dimension.  
Assume a periodic solution.

## A simple circuit [Fueyo & al.]

Example due  
to [Hale & al 1993]



Model " $\dot{\zeta} = F_{\omega}(\zeta, t)$ "

with  $\zeta$   $\infty$ -dimension.

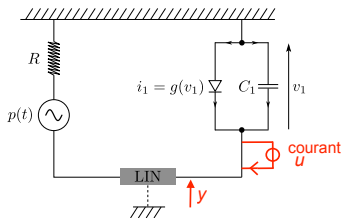
Assume a periodic solution.

- The spectrum of the monodromy operator along the periodic solution is made of:
  - a continuous component or eigenvalues with infinite multiplicity
  - plus a finite number of eigenvalues with finite multiplicity,



## A simple circuit [Fueyo & al.]

Example due  
to [Hale & al 1993]



Model " $\dot{\zeta} = F_{\omega}(\zeta, t)$ "

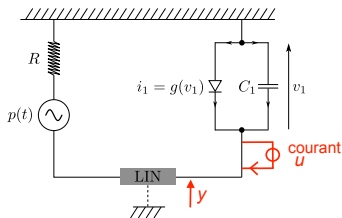
with  $\zeta$   $\infty$ -dimension.

Assume a periodic solution.

- The spectrum of the monodromy operator along the periodic solution is made of:
  - a continuous component or eigenvalues with infinite multiplicity
  - plus a finite number of eigenvalues with finite multiplicity,
- It does give local stability of the nonlinear circuit.

## A simple circuit [Fueyo & al.]

Example due  
to [Hale & al 1993]



Model " $\dot{\zeta} = F_{\omega}(\zeta, t)$ "

with  $\zeta$   $\infty$ -dimension.

Assume a periodic solution.

- The spectrum of the monodromy operator along the periodic solution is made of:
  - a continuous component or eigenvalues with infinite multiplicity
  - plus a finite number of eigenvalues with finite multiplicity,
- It does give local stability of the nonlinear circuit.
- Modulo a minor lack of observability when some rational ratio, and at least right of a vertical axis with negative real part, the set of singularities of the harmonic transfer function is the spectrum of the monodromy operator.

# Perspectives

- General circuits
- Algorithms
- Transfer

# Perspectives

- General circuits
- Algorithms
- Transfer

**Thank you for your attention.**