Flow simulations in large scale discrete fracture networks with the hybrid high-order (HHO) method.

Alexandre Ern, Florent Hédin, Géraldine Pichot, Nicolas Pignet

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Work in collaboration with

Patrick Laug (Inria GAMMA3, Saclay)

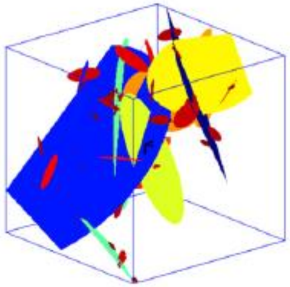
Simon Legrand (Inria SED Paris)

Caroline Darcel and Romain Le Goc (ITASCA S.A.S)

Philippe Davy (Geosciences Rennes, CNRS)

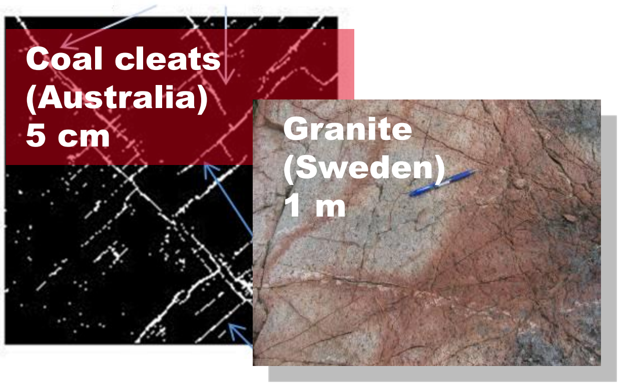
UFM (Unified Fracture Model)

Davy, P., R. Le Goc, and C. Darcel (2013), A model of fracture nucleation, growth and arrest, and consequences for fracture density and scaling, *Journal of Geophysical Research: Solid Earth*, *118*(4), 1393-1407

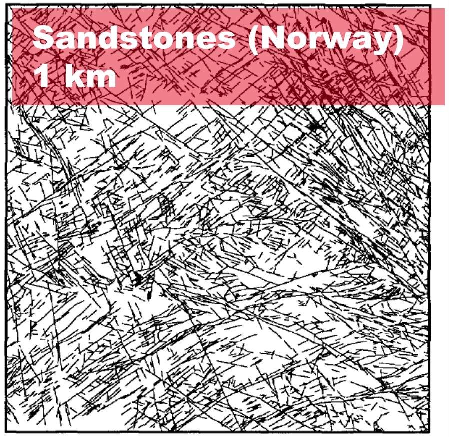


In 2012,
6,845 fractures

de Dreuzy, J.-R., et al., Synthetic benchmark for modeling flow in 3D fractured media. Computers & Geosciences (2012),

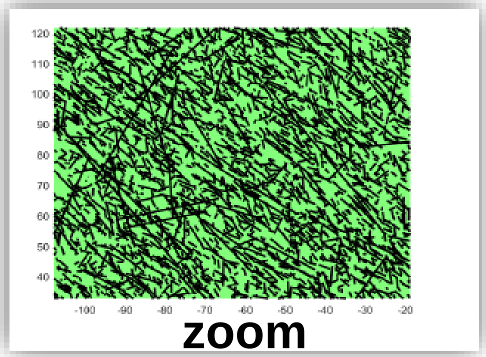


MAP 7: H = 370m, Area = 720m x 720m

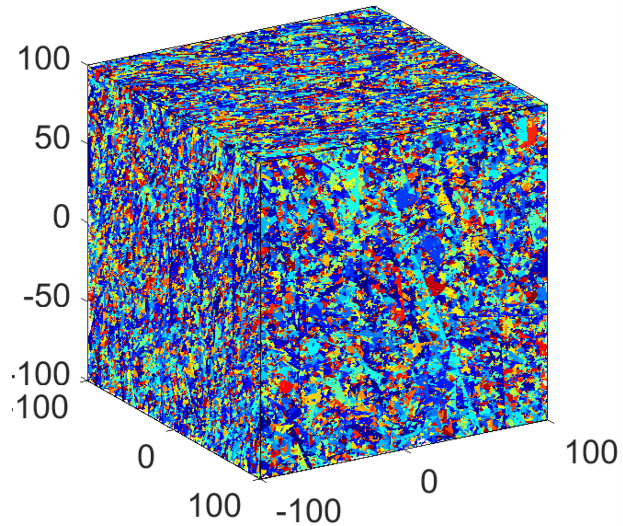


DFN test cases provided by the LabCom fractory

L=200 m
1,176,566 fractures
2,410,539 intersections
Transmissivity range (one value per fracture):
[3.35-06; 25.8]



The largest fracture:
11,710 intersections



The flow problem

J. Erhel, J.-R. de Dreuzy, and B. Poirriez. SIAM J. Sci. Comput. (2009)

J. Maryska, O. Severyn, and M. Vohralik, Computational Geosciences (2004)

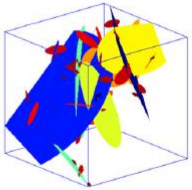
Assumptions:

- The rock matrix is impervious: flow is only simulated in the fractures
- Study of steady state flow
- There is no longitudinal flux in the intersections of fractures

Flow equations within each fracture f_i with κ_i a positive definite tensor

$$\nabla \cdot \mathbf{u}_i(\mathbf{x}) = g_i(\mathbf{x}), \quad \text{for } \mathbf{x} \in f_i, \qquad \textit{(Continuity equation)}$$

$$\mathbf{u}_i(\mathbf{x}) = -\kappa_i \nabla p_i(\mathbf{x}), \quad \text{for } \mathbf{x} \in f_i. \qquad \textit{(Poiseuille's law)}$$



$$p_i(\mathbf{x}) = p_i^D(\mathbf{x}), \quad \text{for } \mathbf{x} \in \Gamma^D \cap \Gamma_i, \qquad \textit{(Dirichlet boundary conditions)}$$

$$\mathbf{u}_i(\mathbf{x}) \cdot \mathbf{n} = q_i^N(\mathbf{x}), \quad \text{for } \mathbf{x} \in \Gamma^N \cap \Gamma_i, \qquad \textit{(Neumann boundary conditions)}$$

$$\mathbf{u}_i(\mathbf{x}) \cdot \mathbf{n} = 0, \quad \text{for } \mathbf{x} \in \Gamma_i \backslash (\Gamma_i \cap (\Gamma_D \cup \Gamma_N)). \qquad \textit{(Impervious rock matrix)}$$

Continuity conditions at each intersection I_m with I_m the m^{th} intersection, $m = 1, \ldots, N_I$

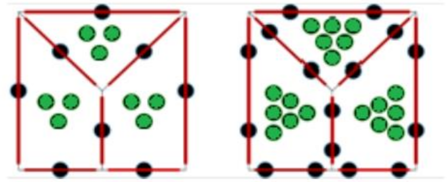
N_I total number of intersections

$$p_k^{m,+} = p_l^{m,+} = p_k^{m,-} = p_l^{m,-}, \quad \text{on } I_m, \qquad \textit{(Continuity of the hydraulic heads)}$$

$$\sum_{j \in S_m} \mathbf{u}_j \cdot \mathbf{n}_{j,m}^{+} + \mathbf{u}_j \cdot \mathbf{n}_{j,m}^{-} = 0, \quad \text{on } I_m. \qquad \textit{(Conservation of fluxes)}$$

The Hybrid High Order (HHO) method

$l=k+1$



$k = 0$ $k = 1$

Di Pietro, D.A and Ern, A. and Lemaire, S. Computational Methods in Applied Mathematics (2014)

Cicuttin, M. and Di Pietro, D.A. and Ern, A. Journal of Computational and Applied Mathematics (2018)

- Discrete unknowns on each finite element are:
  - On mesh faces
  - Inside the elements
- Algebraically, we obtain a Schur complement system for only the faces unknowns, and known to be symmetric positive definite.
- Local gradient reconstruction from cell and face unknowns
- A stabilization term ensures that the traces of the cell unknows and the face unknowns match in a least-squares sense.
- Conservative fluxes are built from the sum of two terms
  - ✓ the normal component of the reconstructed gradient
  - ✓ A correction term coming from the stabilization

Cockburn, B. and Di Pietro, D.A. and Ern, A. ESAIM: M2AN (2016)

- Main advantages of HHO:
  - ✓ **Feature 1**: Globally and locally **mass conservative method** (whatever $k \geq 0$)
  - ✓ **Feature 2**: Face polynomials of **order $k \geq 0$**
  - ✓ **Feature 3**: Support **general meshes** (polygonal/polyhedral cells)

✓ Software:
- ❑ **Library Disk++ for HHO (open-source)**
- ❑ **NEF++ (Inria) for an application to DFNs**
- ❑ **Prune (Inria) for automatic parameter studies**

Choice of the basis functions

- HHO unknowns are **polynomial coefficients**
- **On mesh faces:** scaled monomials x_F^{α} with $\alpha = 0, \ldots, k$ and

$$x_F = 2\frac{(x - \bar{x}_F)}{|F|}$$

- **On every mesh cell T:** we compare two choices of monomials

$$\alpha, \beta = 0, \ldots, l = k + 1, \alpha + \beta \leq l$$

[Cartesian] $x_T^{\alpha} y_T^{\beta}$

$$x_T = 2\frac{(x - \bar{x}_T)}{h_x^T}, \qquad y_T = 2\frac{(y - \bar{y}_T)}{h_y^T}$$

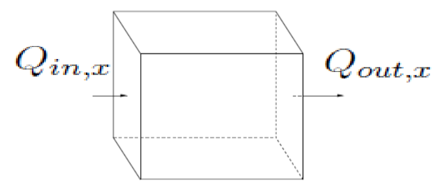
[Rotated] $\xi_T^{\alpha} \zeta_T^{\beta}$

$$\xi_T = 2\frac{(x - \bar{x}_T, y - \bar{y}_T)\cdot a_1^T}{h_1^T}$$

$$\zeta_T = 2\frac{(x - \bar{x}_T, y - \bar{y}_T)\cdot a_2^T}{h_2^T}$$

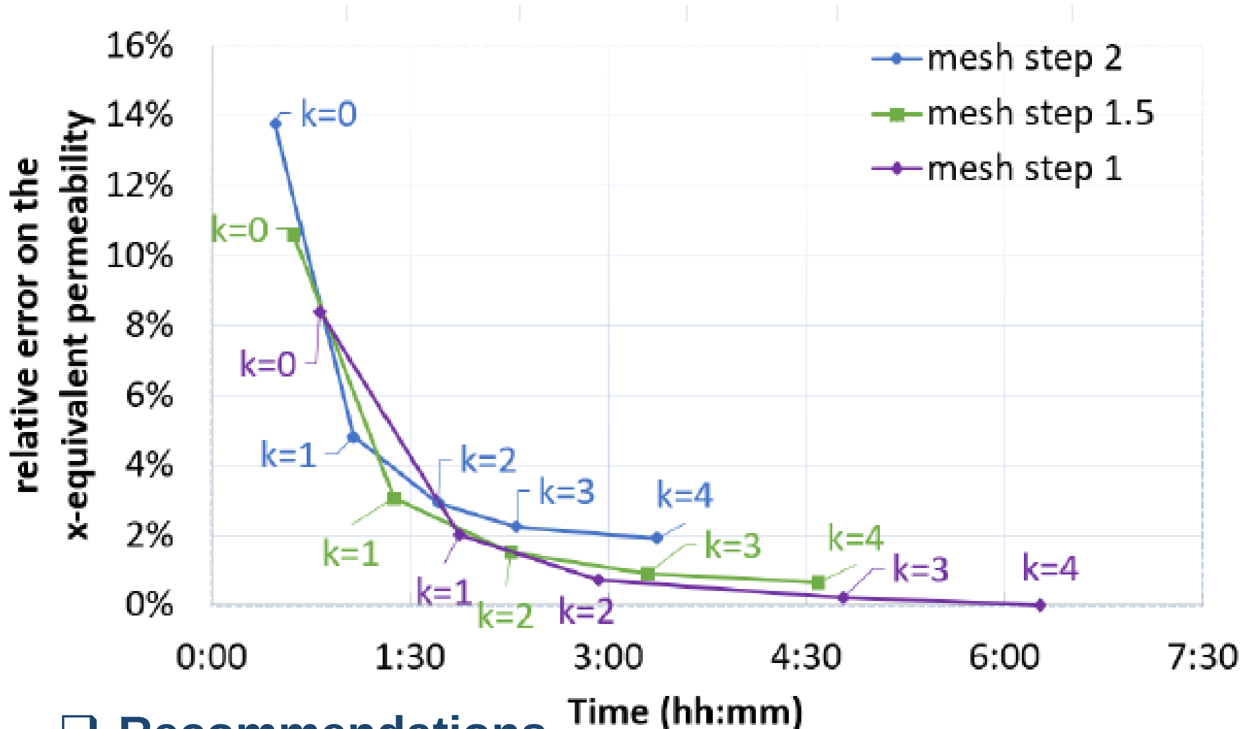
		$\lvert Q_{in,x} + Q_{out,x}\rvert$	
	k	[Cartesian]	[Rotated]
L100	0	**1.44e-07**	**9.82e-11**
	1	**1.66e-4**	**5.26e-10**
	2	**2.31e-2**	**4.63e-09**
	3	**15.21**	**4.20e-08**
L150	0	**3.12E-07**	**7.32e-10**
	1	**1.58e-3**	**1.31e-08**
	2	**38.22**	**1.85e-08**
	3	**45.73**	**4.85e-08**

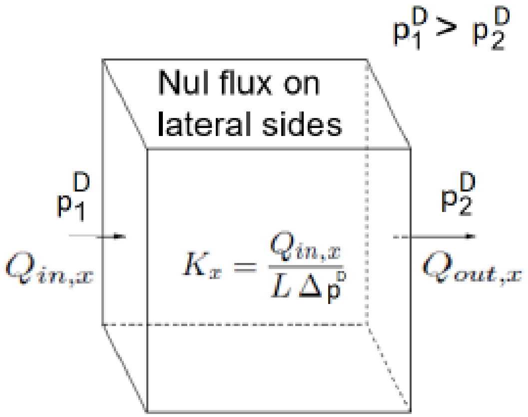
- **Mass conservation**
  - ✗ **[Cartesian]**
  - ✓ **[Rotated]**
    - **Global and local mass conservation ensured whatever k**



Trade-off between polynomial order and mesh refinement

➤ Evaluation of the equivalent permeability

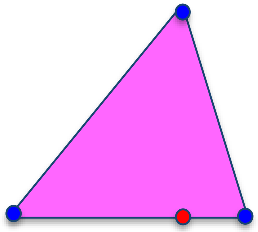




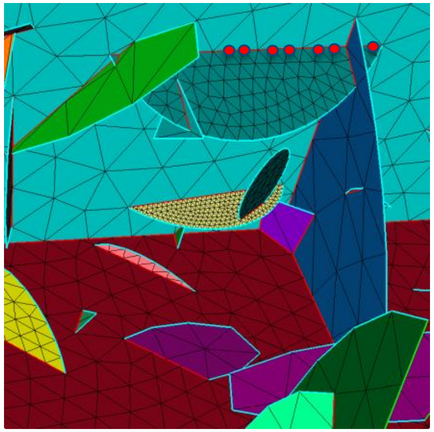
❑ **Recommendations**

- ✓ **use a face polynomial order k at least equal to 1, instead of refining a mesh while keeping $k = 0$**
- ✓ **choose $k = 1$ on a mesh fine rather than $k > 1$ on a coarser mesh**

How to take advantage of polygonal cells?



- Vertices of the triangles
- Extra vertices to create a polygon



20,522,575 triangles
All fractures → mesh **step of 1.5**

k	#dofs	Total Time (hh:mm:ss)	RAM (GiB)	Rel. Error *Kx*
0	**31,711,430**	**1:28:45**	**100**	**10.7%**
1	**63,422,860**	**3:16:14**	**246**	**2.6%**
2	**95,134,290**	**6:19:46**	**481**	**0.9%**
3	**126,845,720**	**9:44:50**	**784**	**0.3%**
4	**158,557,150**	**14:39:47**	**1186**	**0.0%**

18,648,084 triangles **(-9%)**
Most contributing fractures → step of 1.5
The others → mesh **step of 2.**

k	#dofs	Total Time (hh:mm:ss)	RAM (GiB)	Rel. Error *Kx*
0	**28,685,198**	**01:18:01**	**93**	**11.7%**
1	**57,370,396**	**03:12:49***	**223**	**3.3%**
2	**86,055,594**	**05:24:48**	**435**	**1.5%**
3	**114,740,792**	**08:16:42**	**732**	**0.8%**
4	**143,425,990**	**12:10:58**	**1087**	**0.5%**

Comparison with the fine mesh flow simulation: -12% to -16% time and -7% to -10% RAM

Conclusion

- We have **successfully** tested the **HHO method on large scale DFN** (more than **1 million of fractures**)
- The implementation of the method is **locally and globally conservative** (whatever k ≥0) thanks to the basis functions
- The use of high order is an advantage to compute a **more accurate** flow in DFN
- We have shown that the use of non-matching meshes, the channelling effect of DFN flow and the polygonal feature of the **HHO method allows to reduce the number of #dofs and therefore the RAM requirements** in DFN flow simulations.

Perpectives

- Further reduction of the #dofs by using the **additional recent features of HHO (Unfitted HHO; merging triangles** in **polygons)**
- Studying **an iterative solver to reduce the RAM requirements**

E. Burman and A. Ern, An unfitted hybrid high-order method for elliptic interface problems, SIAM J. Numer. Anal., 56(3), 1525-1546 (2018)

A. Miraçi, J. Papež, and M. Vohralík (2020)
A Multilevel Algebraic Error Estimator and the Corresponding Iterative Solver with *p*-Robust Behavior. SIAM J. Numer. Anal., 58(5), 2856–2884.

Thanks a lot for your attention !