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# 1 Estimation of local failure in tensegrity using Interacting Particle-Ensemble 2 Kalman Filter

3 Neha Aswal<sup>a</sup>, Subhamoy Sen<sup>a,\*</sup>, Laurent Mevel<sup>b</sup>

4 <sup>a</sup>*Indian Institute of Technology Mandi, Mandi, HP, India*

5 <sup>b</sup>*Univ. Gustave Eiffel, Inria, Cosys-SII, I4S, Campus de Beaulieu, France*

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## 6 Abstract

7 Tensegrities form a special case of truss, wherein compression members (struts/bars) float within a network  
8 of tension members (cables). Tensegrities are characterized by the presence of at least one infinitesimal  
9 mechanism stabilized with member pre-stress to ensure equilibrium. Over prolonged usage, the cables may  
10 lose their pre-stress while the bars may buckle, get damaged, or corrode, affecting the structural stiffness  
11 leading to change in the measured dynamic properties. Upon loading, a tensegrity structure may change  
12 its form through altering its member pre-stress affecting its global stiffness, even in the absence of damage.  
13 This can potentially mask the effect of damage leading to a false impression of tensegrity health. This poses  
14 the major challenge in tensegrity health monitoring especially when the load is stochastic and unknown.

15 Present study proposes an output-only time-domain method that makes use of tensegrity vibrational  
16 responses within a Bayesian filtering-based approach to monitor the tensegrity health in the presence of  
17 uncertainties due to ambient force, model inaccuracy, and measurement noise. For this, an interacting  
18 strategy combining Particle Filter (PF) and Ensemble Kalman Filter (EnKF) has been adopted (Interacting  
19 particle-Ensemble Kalman Filter, IP-EnKF) in which the EnKF estimates the response states as ensembles  
20 while running within a PF envelop that estimates a set of location-based health parameters as particles.  
21 Furthermore, for a cheaper damage detection procedure, strain responses are used as measurements. The  
22 efficiency of the proposed methodology in terms of accuracy, computational cost, and robustness against noise  
23 contamination has been demonstrated using numerical experiments performed on two tensegrity modules:  
24 a simplex tensegrity and an extended-octahedron tensegrity.

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## 25 1. Introduction

26 Tensegrities form a special class of truss with dedicated tension and compression members, known as  
27 cables and struts, respectively, and/or bars which can take both tension and compression forces. Tensegrity  
28 structures derive their integrity from the pre-stress present in their members. Mention of this structure type

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\*Corresponding author; *E-mail address: subhamoy@iitmandi.ac.in*

29 finds its origin in the works of Ioganson (1920) and Snelson (1948) [54], where Ioganson's structure lacks  
30 one of the essential criteria for tensegrity, i.e., equilibrium without any external force [57]. While tensegrity  
31 was later formally introduced by Snelson as an architectural piece, its potential as a structure was promoted  
32 by Buckminster Fuller. Ever since its introduction, tensegrities have found applications in various fields:  
33 aerospace [68], bio-mechanics [34], robotics [44], etc. Although tensegrities demonstrate excellent utility for  
34 being deployable as well as aesthetically appealing [53, 64], the typical perspective of being considered as a  
35 light-weight structure has been debated in [26]. Nevertheless, the unique deployable attribute of tensegrities  
36 has found much acceptance in the field of controllable structures [62, 63, 65], aerospace application [21, 41, 61]  
37 and especially in robotics [6, 20, 35] wherein tension in the strings/cables are actively controlled by actuators  
38 to control the movement of the tensegrity robots. Accordingly, various methodologies have been developed  
39 to design [5, 67] and construct statically stable complex tensegrity structures [60, 72] that are easy to erect or  
40 deploy. The success with non-load bearing structures has quickly been adapted by the structural engineers  
41 as well and the tensegrity concept has been implemented for civil infrastructures in the form of roofs and  
42 bridges. The motivation comes from the fact that tensegrities can provide large column-free spaces allowing  
43 sufficient overhead clearance (advantageous for bridges to allow water vessels underneath) and unobstructed  
44 view (beneficial for the stadium roofs) [2, 23, 25]: Olympic Gymnastics Arena roof (Seoul, South Korea),  
45 Kurilpa bridge (Brisbane, Australia), etc. are some of the examples among many others.

46 Tensegrities are characterized by the presence of at least one infinitesimal mechanism [39] stiffened by  
47 the pre-stress present in the members due to their configuration. Of course, in the absence of these member  
48 pre-stresses, there would be no structure, thereby delineating tensegrities from other pre-stressed structures.  
49 The stability of tensegrity is therefore pre-stress dependant and conditioned on a particular configuration,  
50 known as self-stress configuration [69]. To accommodate a certain external load, tensegrity incur changes in  
51 its initial stable configuration. Tensegrities can thus have multiple self-stressed stable configurations under  
52 different external loading conditions [49, 69]. Modification in the shape, due to pre-stress levels as well  
53 as external forces, eventually, changes the stiffness properties, thereby altering the frequencies even in its  
54 undamaged condition. Moreover, tensegrities can have same shape with different stiffness and frequencies,  
55 because of different pre-stress levels [3, 4]. Tensegrities thus may exhibit different stiffness, dynamical  
56 properties, and spatial configurations even in its healthy state, which otherwise is anticipated only under  
57 damaged conditions for traditional structures. It should therefore be noted that stiffness alteration due to  
58 modification in member stress induced by force variability does not imply damage in a tensegrity.

59 As tensegrity does not belong to the category of traditional structures that are typically constructed with  
60 high levels of redundancy [25], the approach for monitoring its health is also not typical. Since tensegrities  
61 are substantially optimized [19, 56] from a design and construction point of view, a catastrophic failure may  
62 therefore occur if its health is not monitored rigorously. Moreover, their shape morphing attribute may lead  
63 to a false impression of damage when being dealt with traditional health monitoring techniques. Vibrational

64 properties of tensegrities are contrastingly less explored [21, 38, 41, 42] than their static performance.  
65 Accordingly, studies related to vibration-based structural health monitoring (SHM) for tensegrities are also  
66 insufficient [4, 66]. Assessment of health from global parameters like modal information is not an option  
67 for tensegrity health monitoring since modal information keeps on changing even in its normal operational  
68 condition, discussed later in this article. Hence, to identify/monitor possible damages in tensegrity, it is  
69 important to have an SHM approach specific for tensegrity that takes into account its nature [49]. Yet  
70 literature available in this field is of an insignificant volume.

71 Three methods for tensegrity damage detection have been compared in [66], namely, frequency analysis,  
72 error-domain model falsification (EDMF) using node position measurement, and moving-window principal  
73 component analysis (MWPCA) using strain measurements. It has been observed that, for tensegrities,  
74 natural frequencies and mode shapes can not be considered as features sensitive only to damage (further  
75 demonstrated later in this article). The slacking scenario in the cables substantially impacts the first natural  
76 frequency, which however differs from one scenario to another. Hence to detect this reduction in tension,  
77 individual monitoring of damage induced frequency alterations has been suggested in [4]. Although it  
78 has been perceived that for tensegrities with forces unknown, modal domain SHM is no longer an option.  
79 Results obtained from EDMF [66] were observed to be sensitive to ambient uncertainty. Also, EDMF tends  
80 to become costly when tracking positions at sub-millimeter resolution. MWPCA [66] has an advantage over  
81 the other mentioned methods since it uses inexpensive strain gauges. It has been observed to be efficient with  
82 low to moderate noise levels but has been reported to perform poorly for high levels of noise contamination.  
83 Satisfactory performance for damage assessment using dynamic strain measurements has also been observed  
84 by [11]. Electro-mechanical impedance (EMI) measures are also considered as measurements for this study,  
85 which has been analysed for high frequency signatures (in kHz) as damage sensitive feature. The study  
86 further compares the performances of EMI and dynamic strain as measurements and concludes that the  
87 dynamic strain measurement-based approach is more cost-effective than the former.

88 Evidently, most of the works on tensegrity SHM have been cast in the deterministic domain. Nevertheless,  
89 any typical model-based SHM approach for a real tensegrity needs to deal with uncertainties due to modeling  
90 error, ambient forcing, and measurement noise. Yet these sources of uncertainties are mostly left unaccounted  
91 for with deterministic SHM approaches. Force is an important aspect of tensegrity stiffness and should thus  
92 be known for the deterministic tensegrity SHM approach to alienate a force-induced change in structural  
93 response from a damaged induced anomaly. For tensegrities, subjected to ambient force, the problem gets  
94 aggravated since an explicit knowledge of ambient forces is rarely available. Real-life tensegrities, therefore,  
95 need a special SHM approach capable of dealing with the forcing uncertainties efficiently.

96 In this context, Bayesian filters have proved their merit in SHM research dealing with the mentioned  
97 uncertainties. With Bayesian filter-based SHM approaches, the uncertainties due to force and modeling  
98 inaccuracies are dealt with a probabilistic process model while a measurement model deals with the sensor

99 noise uncertainties separately. Within the process model, the dynamics of the structure is defined in state-  
100 space with a set of internal unobserved variables, called states. The dynamics of the system are then  
101 defined in terms of system state propagation in time following a Chapman-Kolmogorov formulation. These  
102 unobserved variables are further observed through the measured responses (e.g. acceleration, strain, etc.)  
103 employing a measurement model/equation involving uncertainties due to sensor noise. Although, the system  
104 dynamics can be better defined in the continuous time domain, to facilitate estimation using discretely  
105 sampled sensor measurements, both physical models are transformed into discrete time domain.

106 Depending on the nature of the formulated process and/or measurement model, several filter types have  
107 been proposed in the literature. For linear time invariant (LTI) systems (linear process and measurement  
108 model), Kalman filter (KF) can be identified as the most employed approach. On the introduction of  
109 non-linearity in either of the models (process and/or measurement) or time variability in the system, the  
110 inability of KF redirects to the usage of non-linear filter variants like Extended (EKF) [28], Unscented  
111 (UKF) [31, 37], Ensemble (EnKF) [22] KFs or Particle filter (PF) [24]. Non-linearity in the process model  
112 may also be caused due to non-linearity in the system itself; tensegrities being one such example manifesting  
113 geometric non-linearity. For linear/non-linear time variant (LTV/NLTV) systems, the system estimation is  
114 proceeded with first parameterizing the system and subsequently estimating them alongside as additional  
115 parameter states,  $\theta_k$ . This, however, renders the assessed system to be non-linear due to the non-linear  
116 mapping of  $\theta_k$  to the corresponding measurements.

117 In the context of SHM, a set of location-based health indices (**HI**s) is employed for parameterizing the  
118 system health which are then estimated/monitored as the additional parameter states,  $\theta_k$ . Estimation  
119 of the **HI**s can further be approached either jointly [36] or conditionally [17, 51] with respect to the real  
120 system states. The relative efficiency of the conditional over joint estimation approach has already been  
121 corroborated in several articles [13] and upon further introduction of interacting strategies by [32], the focus  
122 has strongly shifted to the use of individual filters for states or parameter estimation, like in Interacting  
123 Particle-Kalman filter (IPKF) [52, 71], Dual KF [9], Dual EKF (DEKF) [51], etc.

124 Within the context of tensegrity SHM, the self-stiffening property [49] can be accounted for by considering  
125 geometric non-linearity in the tensegrity dynamics [33]. Eventually, with the non-linear tensegrity dynamics  
126 defined through this process model, the model is axiomatically non-linear. With  $\theta_k$  as additional states to  
127 be observed through measurements, the measurement model is also non-linear. Hence a major challenge  
128 in tensegrity SHM is to handle these non-linearities simultaneously and efficiently. PF has been successful  
129 in dealing with highly non-linear systems [8, 12, 14], although at the expense of high computational cost.  
130 To overcome the cost issue, IPKF was introduced [71] in which KF deals with the linear state estimation,  
131 while PF is employed for non-linear parameter estimation. Nevertheless, the dynamic model pertaining to  
132 tensegrity SHM is non-linear, invalidating the KF. The replacement can be chosen from the available filter  
133 variants. Of them, EnKF has been proved to be efficient in the propagation of non-linear system states [27]

134 while allowing the entire health monitoring approach to be parallelized together with the PF. An Interacting  
135 Particle Ensemble Kalman Filter (IP-EnKF) has therefore been employed to estimate tensegrity health.

136 The algorithm has been formulated to make use of only strain gauge response as the measured data to  
137 the proposed IP-EnKF, since strain gauges are cheaper than accelerometers while being reported as more  
138 sensitive towards the presence of damage [50]. Detailed discussion on the tensegrity modeling and simulating  
139 dynamic responses have been demonstrated in Section 2, with details of the state-space definition of the  
140 tensegrity dynamics (Section 2.2). The proposed IP-EnKF algorithm is further explained in Section 3  
141 followed by a numerical validation study detailed in Section 4 that demonstrates the application of the  
142 proposed approach on a simplex tensegrity (ST) and an expanded-octahedron tensegrity (EOT) modules.

## 143 2. Tensegrity model and dynamic response

144 While modeling a tensegrity, suitable internal force inequalities should be added to the model to account  
145 for the nature of the dedicated tension cables or compression struts or bars that can take up both tension and  
146 compression forces, if present in the structure. This makes modeling of the tensegrities different from that  
147 of the typical truss structures. The design and identification of self-stressed configuration for tensegrities is  
148 a separate and much-explored field of research, not in the scope of this article. Yet for the sake of clarity,  
149 this article details the form-finding algorithm (Algorithm 2) employed in this study to identify the initial  
150 stable form of the tensegrity. Special measures are further taken to ensure that no local failure conditions  
151 (bar buckling and/or cable slacking) occur while finding the initial stable configuration of the tensegrities  
152 through constraining the member pre-stress levels.

153 To account for the large deformations of tensegrity members under external loading, geometric non-  
154 linearity is introduced in the model. It has been observed that tensegrity with low pre-stress levels, mani-  
155 fests stronger non-linearity compared to tensegrities with higher pre-stress levels [40]. Consequent to load  
156 application and related changes in the configuration, the current strain-displacement relationship becomes  
157 an implicit problem involving the ever-evolving tensegrity configuration. With the finite element modeling  
158 (FEM) approach to discretize the spatial domain, the aspect of geometric non-linearity can be invoked  
159 without much complexity. Nevertheless, the implicit nature of the problem needs substantial computation  
160 within a recursive estimation approach which might render the involved SHM, although accurate, slow.  
161 Since with the Bayesian approach, the model inaccuracy can be complemented with recursive inferencing  
162 from the data, in this article an explicit representation of the strain-displacement relationship is adopted  
163 powered by explicit Newmark-beta method [15, 43]. The modeling is detailed in the following.

### 164 2.1. Geometric non-linear finite element model

165 Modeling of tensegrity with a geometric non-linear FEM approach exists in literature [33]. Except for  
166 geometric non-linearity, this article does not consider any other source of non-linearity, like material or

167 boundary non-linearity. The initial form (involving coordinate position and member pre-stress levels) of the  
 168 tensegrity is required to be identified through the form-finding approach following Algorithm 2. Algorithm  
 169 2 presents a force-density based optimization approach for tensegrity form-finding that has been adopted  
 170 to identify the initial coordinates and related pre-stress levels with constraints on the member pre-stress  
 171 ensuring no tension/compression member is slacking/buckling, respectively. The resulting stable form and  
 172 related data are presented in Figures 2 and 4 and Tables 1 and 2, respectively.

173 Next, at any arbitrary time instant  $t$ , for each of the  $m^{th}$  member/element of the self-stressed tensegrity,  
 174 the associated global coordinates (defined in the global coordinate system (GCS),  $xyz$ ),  $\mathbf{q}^m(t)_{6 \times 1} \subset \mathbf{q}(t)$ ,  
 175 are transformed to their counterparts,  $\mathbf{q}^{m,l}(t)_{2 \times 1}$ , in the local coordinate system (LCS),  $\bar{x}\bar{y}\bar{z}$  (cf. Figure  
 176 (1)), with the help of member-specific transformation matrix  $\mathbf{T}^m(t)$ . Here  $\mathbf{q}(t)$  denotes the entire global  
 177 coordinate set of all the tensegrity nodes.

$$\mathbf{q}^{m,l}(t) = \mathbf{T}^m(t)\mathbf{q}^m(t) \quad (1)$$

178 where,  $\mathbf{T}^m(t) = \begin{bmatrix} \cos\theta_x^m(t) & \cos\theta_y^m(t) & \cos\theta_z^m(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta_x^m(t) & \cos\theta_y^m(t) & \cos\theta_z^m(t) \end{bmatrix}$ ,

179  $\mathbf{q}^{m,l}(t) = \{q_1^l(t) \quad q_2^l(t)\}^{mT}$  and  $\mathbf{q}^m(t) = \{q_{1x}(t) \quad q_{1y}(t) \quad q_{1z}(t) \quad q_{2x}(t) \quad q_{2y}(t) \quad q_{2z}(t)\}^{mT}$ .

180  $\cos\theta_x^m(t)$ ,  $\cos\theta_y^m(t)$  and  $\cos\theta_z^m(t)$  are time varying angular positions of the member  $m$  with respect to GCS.

181 A schematic for the assumed element is demonstrated in Figure 1.

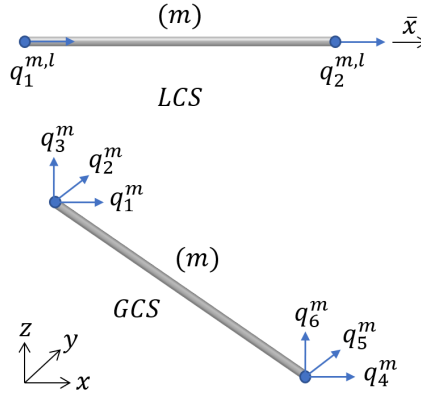


Figure 1: LCS and GCS for bar element type

182 The deformation,  $\mathbf{u}^m(r, t)$ , at any point within the element  $m$  can further be described using shape  
 183 functions ( $N_1(r)$  and  $N_2(r)$ ) and local nodal displacements ( $\mathbf{q}^{m,l}(t)$ ).

$$\mathbf{u}^m(r, t) = [N_1(r) \quad N_2(r)] \mathbf{q}^{m,l}(t) \quad (2)$$

184 where the shape functions are described in natural coordinate system,  $N_1(r) = (1 - r)/2$  and  $N_2(r) =$

185  $(1+r)/2$ , for the ease of integration.  $r$  being the natural variable defined within the range  $-1 \leq r \leq 1$ .  
 186 To incorporate geometric non-linearity in strain, a second order relationship between Green's strain and  
 187 displacement fields has been considered in this study (as per [33]),

$$\varepsilon^m(r, t) = \frac{\partial \mathbf{u}^m(r, t)}{\partial x} + \frac{1}{2} \left( \frac{\partial \mathbf{u}^m(r, t)}{\partial x} \right)^2 \quad (3)$$

188 with  $\mathbf{u}^m(r, t)$ , as defined in Equation (2), the member strain field  $\varepsilon^m(r, t)$  can further be expressed introduc-  
 189 ing linear ( $\mathbf{B}_L^m$ ) and non-linear ( $\mathbf{B}_{NL}^m$ ) strain-displacement matrices with  $\mathbf{B}_{NL}^m(\mathbf{q}^{m,l}(t))$  being a non-linear  
 190 function of  $\mathbf{q}^{m,l}(t)$ . The functional representation of  $\mathbf{B}_{NL}^m$  to demonstrate its dependence on the  $\mathbf{q}^{m,l}(t)$  is  
 191 although dropped from here on for the sake of compactness.

$$\varepsilon^m(r, t) = \mathbf{B}_L^m \mathbf{q}^{m,l}(t) + \mathbf{B}_{NL}^m \mathbf{q}^{m,l}(t) \quad (4)$$

192 where  $\mathbf{B}_L^m = \left[ \frac{\partial N_1(r)}{\partial x} \quad \frac{\partial N_2(r)}{\partial x} \right]$  and  $\mathbf{B}_{NL}^m = \frac{1}{2} \mathbf{q}^{m,l}(t)^T \begin{bmatrix} \frac{\partial N_1(r)}{\partial x} \\ \frac{\partial N_2(r)}{\partial x} \end{bmatrix} \begin{bmatrix} \frac{\partial N_1(r)}{\partial x} & \frac{\partial N_2(r)}{\partial x} \end{bmatrix}$ .

193 The element tangent stiffness matrix can further be obtained by applying the principle of virtual work, i.e.,  
 194 minimizing the difference (i.e. virtual work,  $\delta W$ ) between the work done by the internal forces (second  
 195 Piola-Kirchhoff stress,  $\sigma^m(r, t)$ ) undergoing incremental Green's strain  $\delta \varepsilon^m(r, t)$  and the work done by the  
 196 external forces undergoing virtual displacement  $\delta \mathbf{q}^m(t)$  integrated over the entire volume,  $V^m$  [29]. The  
 197 virtual work can therefore be defined as,

$$\delta W = \int_{V^m} \delta \varepsilon^m(r, t)^T \sigma^m(r, t) dV - \delta \mathbf{q}^m(t)^T \mathbf{F}(t) \quad (5)$$

198 where,  $\sigma^m(r, t)$  is obtained from the constitutive relation,  $\sigma^m(r, t) = \mathbf{E}^m \varepsilon^m(r, t)$  with  $\mathbf{E}^m$  being the consti-  
 199 tutive matrix. The above equation is further expanded as follows,

$$\delta W = \int_{V^m} \delta \mathbf{q}^m(t)^T \mathbf{T}^{mT} \mathbf{B}^{mT} \mathbf{E}^m \mathbf{B}^m \mathbf{T}^m \mathbf{q}^m(t) dV - \delta \mathbf{q}^m(t)^T \mathbf{F}(t) \quad (6)$$

200 where,  $\mathbf{B}^m = \mathbf{B}_L^m + \mathbf{B}_{NL}^m$ , making  $\mathbf{B}^m$  a function of  $\mathbf{q}^{m,l}(t)$  as well. Further, ignoring the trivial part of  
 201 the solution (i.e,  $\delta \mathbf{q}^m(t) \neq 0$ ), and taking derivative of the internal force with respect to  $\mathbf{q}^m(t)$ , element  
 202 tangential stiffness matrix  $\mathbf{K}^m(t)$  can be defined in compact form as,

$$\mathbf{K}^m(t) = \frac{A^m l^m}{2} \int_{-1}^1 \frac{\partial (\mathbf{B}^{mT} \sigma^m(r, t))}{\partial \mathbf{q}^m(t)} dr \quad (7)$$

assuming a uniform cross section  $A^m$  over the entire length  $l^m$  of element  $m$ . Numerical integration of  
 the above integral can be obtained through Gauss-Quadrature method with one Gauss-point. The tangen-  
 tial stiffness matrix ( $\mathbf{K}^m(t)$ ) can further be splitted into material ( $\mathbf{K}_M^m(t)$ ), geometric ( $\mathbf{K}_G^m(t)$ ) and initial  
 displacement ( $\mathbf{K}_U^m(t)$ ) stiffness matrices [29, 60]:

$$\mathbf{K}^m(t) = \mathbf{K}_M^m(t) + \mathbf{K}_G^m(t) + \mathbf{K}_U^m(t) \quad (8)$$



203 where,  $\mathbf{K}_M^m(t)$ ,  $\mathbf{K}_G^m(t)$  and  $\mathbf{K}_U^m(t)$  are given by Equations (9), (10) and (11), respectively

$$\mathbf{K}_M^m(t) = \frac{\mathbf{E}^m A^m l^m}{2} \int_{-1}^1 \mathbf{T}^{mT} \mathbf{B}_L^{mT} \mathbf{B}_L^m \mathbf{T}^m dr \quad (9)$$

$$\mathbf{K}_G^m(t) = \frac{A^m l^m}{2} \int_{-1}^1 \frac{\partial \mathbf{B}_{NL}^{mT}}{\partial \mathbf{q}^m(t)} \sigma^m(r, t) dr \quad (10)$$

$$\mathbf{K}_U^m(t) = \frac{\mathbf{E}^m A^m l^m}{2} \int_{-1}^1 \mathbf{T}^{mT} (\mathbf{B}_L^{mT} \mathbf{B}_{NL}^m + \mathbf{B}_{NL}^{mT} \mathbf{B}_L^m + \mathbf{B}_{NL}^{mT} \mathbf{B}_{NL}^m) \mathbf{T}^m dr \quad (11)$$

204 Further, global tangential stiffness matrix  $\mathbf{K}(t)$  can be obtained by assembling the elemental stiffness matrices  
 205 and applying natural boundary conditions. Similarly, the mass matrix  $\mathbf{M}$  can be obtained by following the  
 206 consistent mass matrix assumption. The global tangent stiffness matrix  $\mathbf{K}(t)$  is determined taking basis on  
 207 updated Lagrange formulation that defines the stiffness at current time. For that, the initial displacement  
 208 matrix has been recursively re-calibrated taking displacements from the last step.

## 209 2.2. State space formulation of tensegrity dynamics

210 Dynamics of typical truss structures can be defined with a linear second-order governing differential equa-  
 211 tion (*gde*). However, the embedded geometric non-linearity in the tensegrity model requires the dynamics  
 212 to be defined using non-linear *gde* as,

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}(t)\dot{\mathbf{q}}(t) + \mathbf{P}(\mathbf{q}(t)) = \mathbf{F}(t) \quad (12)$$

213 Clearly, the inelastic resisting force,  $\mathbf{P}(\mathbf{q}(t))$ , is non-linear and time-dependent due to the consideration of  
 214 non-linear geometry. Specific to the tensegrity SHM problems under consideration, time dependency in  
 215  $\mathbf{P}(\mathbf{q}(t))$  is also due to the varying health condition of the tensegrity. Suitable damping model for tensegrity  
 216 is a well researched topic [58, 59] weighing the proportional and non-proportional damping models as op-  
 217 tions. It has been perceived in general, that compared to non-proportional damping models, proportional  
 218 damping models are computationally inexpensive [1], although may lack accuracy sometimes [58]. The rel-  
 219 ative modeling inaccuracies can however be complemented with recursive Bayesian estimation approach in  
 220 which the additional process noise can take care of this modeling uncertainty while benefiting the algorithm  
 221 with promptness. Rayleigh damping has therefore been assumed for this tensegrity simulation. This is a  
 222 classical viscous damping model assuming damping to be linearly proportional to mass and stiffness, as  
 223  $\mathbf{C}(t) = a_0(t)\mathbf{M} + a_1(t)\mathbf{K}(t)$  where  $\mathbf{K}(t)$  is the locally linearized tangent stiffness matrix. Although classical  
 224 approach assumes the damping to be constant all through out, for non-linear systems with varying tangent  
 225 stiffness matrix, updated stiffness is suggested to be employed along with varying proportionality coefficients  
 226 (i.e.  $a_0(t)$  and  $a_1(t)$ ) instead of initial stiffness matrix [16, 30, 45]. Further assumptions are imposed on first  
 227 two modes being equally damped in order to estimate time varying coefficients  $a_0(t)$  and  $a_1(t)$ . The details

228 of Rayleigh damping model can be found in [15]. Eventually, damping force being defined using Rayleigh's  
 229 damping model, is also time dependent. Nevertheless, any other damping model can also be used instead  
 230 [46].

231 The mass matrix is, however, considered to be time invariant. The structure is subjected to externally  
 232 applied ambient forcing  $\mathbf{F}(t)$  which is assumed to be not known explicitly, yet can be modeled as zero mean  
 233 white Gaussian noise (WGN) of known stationary statistics  $\mathbf{Q}$ , as  $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{Q})$ .

234 The system dynamics can further be defined with displacement ( $\mathbf{q}(t)$ ), velocity ( $\dot{\mathbf{q}}(t)$ ) and acceleration  
 235 ( $\ddot{\mathbf{q}}(t)$ ) as system states observed through a set of strain measurements,  $\{\varepsilon_k^m\}$ , sampled in discrete time from  
 236 the strain gauges patched on to the surface of the bars at their midpoints ( $r = 0.5$ ).  $\varepsilon_k^m$  is the discrete  
 237 counterpart corresponding to its continuous time entity,  $\varepsilon^m(r, t)$ , with  $k$  being the time instant at which the  
 238 strain is sampled. To accommodate such discrete measurement, the non-linear state transition function has  
 239 to be defined in discrete time state space formulation as,

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{M}, \mathbf{K}_k, \mathbf{C}_k, dt, \mathbf{v}_k) , \text{ where } \mathbf{v}_k \sim \mathcal{N}(0, \mathbf{Q}) \quad (13)$$

240 Here,  $\mathbf{x}_k = [\mathbf{q}_k \quad \dot{\mathbf{q}}_k \quad \ddot{\mathbf{q}}_k]^T$ , i.e. the discrete definition of the system states evolving over the non-linear  
 241 state propagation function  $f(\bullet)$ .  $\mathbf{q}_k, \dot{\mathbf{q}}_k, \ddot{\mathbf{q}}_k, \mathbf{M}, \mathbf{K}_k, \mathbf{C}_k$  are the respective discrete quantities corresponding  
 242 to their continuous definitions.  $dt$  is the time step for discretization.  $\mathbf{v}_k$  has additionally been incorporated  
 243 to collectively account for the uncertainties originating from the unavoidable model inaccuracies and ambient  
 244 WGN force,  $\mathbf{P}_k$ . This WGN model is assumed with constant covariance  $\mathbf{Q}$ , same as the variance of the  
 245 ambient force. Subsequently, the measurement equation can be defined as,

$$\varepsilon_k = \mathbf{H}\mathbf{B}(\mathbf{x}_k) + \mathbf{w}_k , \text{ where } \mathbf{w}_k \sim \mathcal{N}(0, \mathbf{R}) \quad (14)$$

246 where,  $\mathbf{B}(\bullet)$  denotes the global non-linear strain-displacement relationship for all members with  $\mathbf{x}_k$  being its  
 247 argument.  $\mathbf{B}(\mathbf{x}_k)$  is acting here as a non-linear measurement function to map the unobserved states  $\mathbf{x}_k$  to  
 248 the measurement space.  $\varepsilon_k$  consists of all the recorded member strains i.e.,  $\varepsilon_k = \{\varepsilon_k^m, m \in m^o\}$ , where  $m^o$  is  
 249 the measured subset of  $\mathbf{m}$ , ( $\mathbf{m} = \cup \{m^o; m^u\}$ ) that are instrumented with strain gauges at their midpoints.  
 250 Naturally,  $m^u$  denotes the unobserved subset of  $\mathbf{m}$ . Accordingly,  $\mathbf{H}$  stands for the selection matrix that  
 251 isolates the measured member strains from all of the predicted set.  $\mathbf{w}_k \sim \mathcal{N}(0, \mathbf{R})$  accounts for the sensor  
 252 noise modeled as WGN process of constant covariance  $\mathbf{R}$ .

253 For system simulation, Newmark-beta method has been employed in its explicit formulation. The method  
 254 is proven to have acceptable accuracy with non-linear dynamic simulations [10, 15, 43]. This approach  
 255 takes its basis on an incremental equilibrium equation corresponding to the original dynamic equation (cf.

256 Equation (12)) to solve for the discrete non-linear structural response variables, i.e.,  $\ddot{\mathbf{q}}_k$ ,  $\dot{\mathbf{q}}_k$  and  $\mathbf{q}_k$ ,

$$\mathbf{M}\Delta\ddot{\mathbf{q}}_k + \mathbf{C}_k\Delta\dot{\mathbf{q}}_k + \mathbf{K}_k\Delta\mathbf{q}_k = \Delta\mathbf{F}_k \quad (15)$$

257 Operator  $\Delta$  denotes the corresponding increment over each time step. Due to the non-linear geometry,  
258 the incremental equation is by nature implicit, for which iterative approach has to be adopted for accurate  
259 solution. Although, without compromising the accuracy by a substantial extent, Equation (15) can be solved  
260 using explicit formulation of Newmark-beta algorithm, detailed in Appendix B. This in turn facilitates with  
261 improved promptness of the damage detection by reducing computation in state propagation. Further, the  
262 method shows an unconditional stability for average constant acceleration assumption with  $\gamma = 0.5$  and  
263  $\beta = 0.25$ , as adopted in this article.

### 264 2.3. Non-linearity in tensegrity dynamics

265 In the following, the non-linearity of a tensegrity is investigated. For this, an EOT type tensegrity  
266 module has been selected (cf. Figure 2). The nodal positions, elemental connectivity and initial tension  
267 coefficients are presented in Table 1. For the simulation, the bars are assumed to act as compression as well  
268 as tension members, while cables take up only tension. The member connections are idealized as friction-less  
269 pin-joints. The assumptions made for the simulations are further presented here for lucid comprehension.

- 270 1. Members are connected by friction-less pin-joints.
- 271 2. Bars act as compression as well as tension members, while cables take up only tension.
- 272 3. Only geometric non-linearity is considered for the modeling.
- 273 4. In line with [70] the considered tensegrities are assumed to be constrained at certain nodes to a fixed  
274 base which minimizes the flexibility of bars due to Coriolis effect hence the Coriolis terms can be  
275 neglected in Equation (12).
- 276 5. Newmark-beta algorithm assumes average acceleration method which is known to be unconditionally  
277 stable.
- 278 6. Rayleigh's proportional damping model is used to model damping in tensegrity.

279 The tensegrity is excited with a sinusoidal force ( $= 750\sin(4t)N$ ) at its  $3^{rd}$  node in x-direction. The  
280 related hysteresis and phase plane diagram curves are plotted in Figure 3. The hysteresis plot proves the  
281 existence of the non-linear relationship between displacement (as output) and forces (as input). For the  
282 phase plane diagram, displacement and velocity response of third node at its x *dof* is plotted. It is evident  
283 from the figures 3a and 3b that the simulated tensegrity dynamics is demonstrating a non-linear behaviour.  
284 The phase plane diagram also establishes the dynamic stability of the assumed tensegrity under all assumed  
285 specifications of the tensegrity simulation.

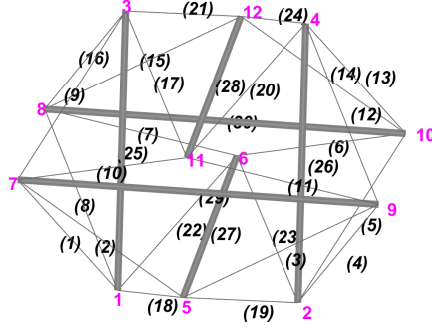
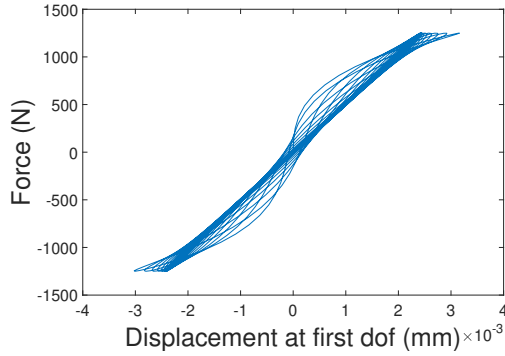


Figure 2: Expanded-octahedron tensegrity (EOT) configuration

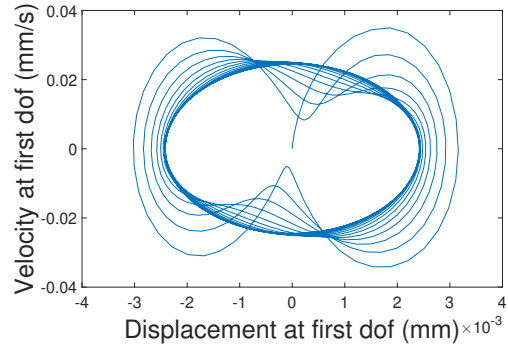
Table 1: Nodal coordinates, elemental connectivity and initial tension coefficients of expanded-octahedron tensegrity (EOT) (with *c*:cable and *b*:bar)

	Node	1	2	3	4	5	6	7	8	9	10	11	12
EOT	X	0	0	0.548	0.548	-1.726	2	-0.658	1.205	-0.657	1.205	-1.452	2.274
	Y	1	-1	0.999	-0.999	0	0	1.999	1.999	-1.999	-1.999	0	0
	Z	-2	-2	0.904	0.904	-1.548	-1	-0.685	-0.411	-0.685	-0.411	-0.096	0.45

EOT	Element	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Node 1	1	5	5	2	2	6	6	1	3	7	9	4	4	10	8
	Node 2	7	7	9	9	10	10	8	8	7	11	11	9	10	12	12
	Type	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>
	Initial tension coeff. (N/m)	0.8782														
		16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Node 1	3	3	1	2	4	3	1	2	4	1	2	5	11	7	8
	Node 2	8	11	5	5	11	12	6	6	12	3	4	6	12	9	10
	Type	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>
	Initial tension coeff. (N/m)	0.8782										-1.3173				



(a) Tensegrity hysteresis for EOT



(b) Velocity - displacement plot for EOT

Figure 3: Non-linear behaviour of tensegrity system - EOT

### 286 3. Proposed approach

287 The system equation and simulation approaches for tensegrity structures have been demonstrated in  
 288 Section 2. The approach for tensegrity SHM will further be detailed in this section. An IP-EnKF approach  
 289 has been adopted for this in which the PF approaches the non-linear health parameter estimation while the  
 290 EnKF estimates the non-linearly evolving system states  $\mathbf{x}_k$  (as per Equation (13)). IP-EnKF can therefore  
 291 be considered as an improvisation of IPKF [52] wherein EnKF replaces KF to extend the reach of the

292 algorithm to non-linear systems. It further facilitates with the option to parallelize the entire computation.  
 293 The major aspect of this approach is that with IP-EnKF, compute-intensive PF handles only the severely  
 294 non-linear parameter estimation problem while for the rest of the non-linear estimation, EnKF is employed.  
 295 The pertinent interacting strategy between these two filter types is demonstrated later in this section.

296 For quantifying the health of the tensegrity, a set of location-based health indices (**HI**s) are devised.  
 297 These **HI**s track the health of each individual members by a value within the range of 1 and 0 with 1  
 298 denoting healthy and 0 signifying completely damaged conditions. These time varying **HI**s are estimated  
 299 with a vector  $\boldsymbol{\theta}_k$  parameterizing the process model.

300 Provided that the process model of the system is known (at least as a sufficiently accurate model),  
 301 error in the predicted output can be attributed to incorrect estimate of the model parameters  $\boldsymbol{\theta}_k$ . In the  
 302 context of system health estimation, reduction in model parameter estimates can in turn signify a change  
 303 in structural stiffness. Typically, structural stiffness is defined by its material (elasticity, cross-section, etc.)  
 304 and geometric (configuration, pre-stress, etc.) stiffnesses. With the proposed algorithm, geometric stiffness  
 305 of tensegrity is taken care of by introducing geometric non-linearity in the finite element model. Eventually,  
 306 the prediction error can be attributed to a possible change in the material stiffness. Hence, for modeling  
 307 purpose, damage in the members can be replicated through reduction in their initial elasticity,  $\mathbb{E}_0$ , using  
 308 health indices,  $\boldsymbol{\theta}_k$  as:

$$\mathbb{S}_k(\boldsymbol{\theta}_k) = \langle \mathbb{S}_0 \cdot \boldsymbol{\theta}_k \rangle \quad (16)$$

309 where,  $\mathbb{S}_0 = [(\mathbf{E}_0^1 A_0^1), \dots, (\mathbf{E}_0^m A_0^m)]$  is the vector encompassing the initial axial stiffness of all the tensegrity  
 310 members,  $[S_0^1, S_0^2, \dots, S_0^m]$ . The reduced axial stiffness  $\mathbb{S}_k$  of all the members at time step  $k$ , is thus a function  
 311 of the health parameters  $\boldsymbol{\theta}_k$ .  $\boldsymbol{\theta}_k$ , therefore, traces the alteration in the material stiffness of all the members  
 312 of tensegrity, thereby detecting damage.

313 At any arbitrary time step  $k$ , PF propagates a set of  $N_p$  parameter particles,  $\boldsymbol{\Xi}_k = [\boldsymbol{\xi}_k^1, \boldsymbol{\xi}_k^2, \dots, \boldsymbol{\xi}_k^{N_p}]_{m_s \times N_p}$ ,  
 314 in time as realizations of the random variable  $\boldsymbol{\theta}_k$ . Each  $j^{th}$  particle,  $\boldsymbol{\xi}_k^j$ , lists  $m_s \times 1$  individual parame-  
 315 ter realizations for **HI**s corresponding to  $m_s$  members being monitored. This numerical approximation  
 316 helps avoiding an explicit analytical integration over the entire parameter space,  $\boldsymbol{\theta}_k$ . The adopted particle  
 317 evolution in time is basically a Gaussian perturbation around the current estimate of the particle  $\boldsymbol{\xi}_{k-1}^j$ ,

$$\boldsymbol{\xi}_k^j = \alpha \boldsymbol{\xi}_{k-1}^j + \mathcal{N}(\delta \boldsymbol{\xi}_k; \boldsymbol{\sigma}_k^\xi) \quad (17)$$

318 where a Gaussian blurring is performed on  $\boldsymbol{\xi}_{k-1}^j$  with a shift  $\delta \boldsymbol{\xi}_k = (1 - \alpha) \bar{\boldsymbol{\xi}}_{k-1}$  and a spread of  $\boldsymbol{\sigma}_k^\xi$ .  $\alpha$  is a  
 319 hyper-parameter that controls the turbulence in the estimation. Upon prediction for the particle estimate  
 320 in current time, the correction is performed according to the likelihood of the particle estimates against the  
 321 measured data, detailed next.

322 Eventually, the evolution of the parameter particles is automated and conditioned on their likelihood  
323 against measurement only, avoiding the requirement of any specific initial distribution for the particle space.  
324 To estimate the likelihood, the propagated particles are further put through the nested EnKF for state  
325 estimation. Within EnKF,  $N_e$  state ensembles are propagated through the system (cf. Equation (13)). For  
326 this, current estimate for the stiffness matrix  $\mathbf{K}_k$  is required. As per the current tensegrity configuration,  
327 extracted from the current estimates for the state ensembles,  $\mathbf{x}_{k-1|k-1}^{i,j}$ , the member lengths,  $l^m$ , and trans-  
328 formation matrices,  $\mathbf{T}^m$ , are updated. Next, with current parameter particles,  $\boldsymbol{\xi}_k^j$ , and state ensembles,  
329  $\mathbf{x}_{k-1|k-1}^{i,j}$ , the current estimate for stiffness matrix,  $\mathbf{K}_{k|k-1}^{i,j}$ , is calculated.  $\mathbf{K}_{k|k-1}^{i,j}$  is associated to  $i^{th}$  ensem-  
330 ble,  $\mathbf{x}_{k-1|k-1}^{i,j}$ , and  $j^{th}$  particle,  $\boldsymbol{\xi}_k^j$ . Thus combining Equations (7) and (16), the current estimate for  $\mathbf{K}_{k|k-1}^{i,j}$   
331 can be obtained as,

$$\mathbf{K}_{k|k-1}^{i,j} = \mathcal{M}(\boldsymbol{\xi}_k^j, \mathbf{x}_{k-1|k-1}^{i,j}) \quad (18)$$

332 where,  $\mathcal{M}(\bullet)$  is the stiffness calibration function that takes basis on the current tensegrity configuration. The  
333 prior state ensembles  $\mathbf{x}_{k-1|k-1}^{i,j}$  are further propagated to the next time step as propagated ensembles,  $\mathbf{x}_{k|k-1}^{i,j}$ ,  
334 as per Equation (13). Subsequently, these propagated ensembles are observed as measurement predictions,  
335  $\mathbf{y}_{k|k-1}^{i,j}$ , following Equation (14). The process and measurement equation for the system is presented in the  
336 following.

$$\begin{aligned} \mathbf{x}_{k|k-1}^{i,j} &= f(\mathbf{x}_{k-1|k-1}^{i,j}, \mathbf{K}_{k|k-1}^{i,j}, \mathbf{M}, dt, \mathbf{v}_k^{i,j}), \text{ where } \mathbf{v}_k^{i,j} \sim \mathcal{N}(0, \mathbf{Q}) \\ \mathbf{y}_{k|k-1}^{i,j} &= \mathbf{HB}(\mathbf{x}_{k|k-1}^{i,j}) + \mathbf{w}_k^{i,j}, \text{ where } \mathbf{w}_k^{i,j} \sim \mathcal{N}(0, \mathbf{R}). \end{aligned} \quad (19)$$

337 Next, the predicted measurement,  $\mathbf{y}_{k|k-1}^{i,j}$ , is compared with the actual measurement obtained from  
338 the sensors. Innovation  $\epsilon_k^{i,j}$  can be obtained as the deviation of  $\mathbf{y}_{k|k-1}^{i,j}$  from the corresponding actual  
339 measurements  $\mathbf{y}_k$ . The innovation statistics is further quantified with an ensemble innovation mean  $\bar{\epsilon}_k^j =$   
340  $\frac{1}{N_e} \sum_{i=1}^{N_e} \epsilon_k^{i,j}$ . Next, the ensemble mean of propagated state estimates,  $\bar{\mathbf{x}}_{k|k-1}^j$ , and predicted measure-  
341 ments,  $\bar{\mathbf{y}}_{k|k-1}^j$ , are obtained as  $\bar{\mathbf{x}}_{k|k-1}^j = \frac{1}{N_e} \sum_{i=1}^{N_e} \mathbf{x}_{k|k-1}^{i,j}$  and  $\bar{\mathbf{y}}_{k|k-1}^j = \frac{1}{N_e} \sum_{i=1}^{N_e} \mathbf{y}_{k|k-1}^{i,j}$ , respectively. Cross-  
342 covariance between state and measurement prediction,  $C_k^{j,xy}$ , and the measurement prediction covariance,  
343  $C_k^{j,yy}$ , can further be computed as per [18].

$$\begin{aligned} C_k^{j,xy} &= \frac{1}{N_e - 1} \sum_{i=1}^{N_e} (\mathbf{x}_{k|k-1}^j - \bar{\mathbf{x}}_{k|k-1}^j)(\mathbf{y}_{k|k-1}^j - \bar{\mathbf{y}}_{k|k-1}^j)^T \\ C_k^{j,yy} &= \frac{1}{N_e - 1} \sum_{i=1}^{N_e} (\mathbf{y}_{k|k-1}^j - \bar{\mathbf{y}}_{k|k-1}^j)(\mathbf{y}_{k|k-1}^j - \bar{\mathbf{y}}_{k|k-1}^j)^T \end{aligned} \quad (20)$$

344 The innovation error covariance,  $\mathbf{S}_k^j$ , and EnKF gain,  $\mathbf{G}_k^j$ , are then obtained as  $\mathbf{S}_k^j = C_k^{j,yy} + \mathbf{R}$  and

345  $\mathbf{G}_k^j = C_k^{j,xy}(\mathbf{S}_k^j)^{-1}$ . With this gain, the state ensembles are updated as,

$$\mathbf{x}_{k|k}^{i,j} = \mathbf{x}_{k|k-1}^{i,j} + \mathbf{G}_k^j \epsilon_k^{i,j} \quad (21)$$

346 Finally, likelihood of each particle, i.e.  $\mathcal{L}(\boldsymbol{\xi}_k^j)$ , is calculated based on the innovation mean,  $\epsilon_k^j$ , and  
347 co-variance,  $\mathbf{S}_k^j$  as,

$$\mathcal{L}(\boldsymbol{\xi}_k^j) = \frac{1}{(2\pi)^n \sqrt{|\mathbf{S}_k^j|}} e^{-0.5 \epsilon_k^j T \mathbf{S}_k^{j-1} \epsilon_k^j} \quad (22)$$

348 The normalized weight for each  $j^{th}$  particle is further obtained using corresponding likelihood,

$$w(\boldsymbol{\xi}_k^j) = \frac{w(\boldsymbol{\xi}_{k-1}^j) \mathcal{L}(\boldsymbol{\xi}_k^j)}{\sum_{j=1}^N w(\boldsymbol{\xi}_{k-1}^j) \mathcal{L}(\boldsymbol{\xi}_k^j)} \quad (23)$$

349 The particle approximations for the states and parameters are then estimated as,

$$\mathbf{x}_{k|k} = \sum_{j=1}^{N_p} w(\boldsymbol{\xi}_k^j) \mathbf{x}_{k|k}^j \quad \text{and} \quad \boldsymbol{\theta}_{k|k} = \sum_{j=1}^{N_p} w(\boldsymbol{\xi}_k^j) \boldsymbol{\xi}_k^j \quad (24)$$

350 For better understanding of the IP-EnKF algorithm used for tensegrity SHM, a pseudo code has been  
351 provided in Algorithm 1.

---

**Algorithm 1** IP-EnKF algorithm for tensegrity SHM

---

```

1: procedure IP-ENKF( $\mathbf{y}_k, \mathbf{Q}, \mathbf{R}$ )                                     ▷ Process and measurement noise covariances
2:   Initialize particles,  $\{\boldsymbol{\xi}_0^j\}$ , and state estimates,  $\{\mathbf{x}_{0|0}^{i,j}\}$                                      ▷ Initialization
3:   for <each  $k^{th}$  measurement  $\mathbf{y}_k$ > do
4:     procedure IP-ENKF( $\{\boldsymbol{\xi}_{k-1}^j\}, \{\mathbf{x}_{k-1|k-1}^{i,j}\}$ )
5:       for <each particle  $\boldsymbol{\xi}_{k-1}^j$ > do
6:         Evolve  $\{\boldsymbol{\xi}_{k-1}^j\} \rightarrow \{\boldsymbol{\xi}_k^j\}$                                      ▷ Particle evolution, as per Equation (17)
7:         procedure ENKF( $\boldsymbol{\xi}_k^j, \{\mathbf{x}_{k-1|k-1}^{i,j}\}, \mathbf{y}_k, \mathbf{Q}_k^P$ )                                     ▷ For  $j^{th}$  particle
8:           for <each ensemble  $\mathbf{x}_{k-1|k-1}^{i,j}$ > do
9:             Define external force,  $\mathbf{P}_k^{i,j}$  as  $\mathcal{N}(0, \mathbf{Q}_k^P)$ 1
10:            Obtain current stiffness,  $\mathbf{K}_{k|k-1}^{i,j}$                                      ▷ see Equation (18)
11:            Predict  $\mathbf{x}_{k|k-1}^{i,j}$  and  $\mathbf{y}_{k|k-1}^{i,j}$                                      ▷ see Equation (19)
12:          end for
13:          Calculate  $\mathbf{x}_{k|k-1}^j, \mathbf{Y}_{k|k-1}^j, \boldsymbol{\epsilon}_{k|k-1}^{i,j}, C_k^{j,xy}, C_k^{j,yy}$  and  $\bar{\epsilon}_{k|k-1}^j$                                      ▷ as per Section 2.2
14:          Compute innovation error covariance ( $\mathbf{S}_k^j$ ) and EnKF gain ( $\mathbf{G}_k^j$ )                                     ▷ as per Section 2.2
15:          Obtain corrected predicted state estimate,  $\mathbf{x}_{k|k}^{i,j}$                                      ▷ see Equation (21)
16:        end procedure
17:      end for
18:      procedure PARTICLE RE-SAMPLING( $\{\boldsymbol{\xi}_k^j\}$ )
19:        Calculate  $w(\boldsymbol{\xi}_k^j)$  for each  $\boldsymbol{\xi}_k^j$  and re-sample                                     ▷ see Equation (23)
20:        Calculate, updated state estimate,  $\mathbf{x}_{k|k}$  and parameter estimate,  $\bar{\boldsymbol{\xi}}_k$                                      ▷ see Equation (24)
21:      end procedure
22:    end procedure
23:  end for
24: end procedure

```

---

352 **4. Numerical Experiment**

353 Large scale tensegrity structures are typically designed or built as assemblage of several modular units  
 354 as the basis of design and construction [47, 48]. These modular units are connected to each other by  
 355 tension mechanism (cables). To check the efficacy of the proposed algorithm for tensegrity SHM, it has been  
 356 numerically tested on two of the most common tensegrity modules: simplex tensegrity (ST) and expanded-  
 357 octahedron tensegrity (EOT). These modules are first numerically simulated for strain responses under a  
 358 WGN forcing. However, prior to the numerical simulation, their initial forms are estimated following the  
 359 process detailed in Algorithm 2.

360 In the following, a dynamic simulation is performed and strain data is collected from all the members  
 361 that are not fixed. The responses from strain gauges are sampled at a fixed sampling frequency of 100 Hz  
 362 [15, 55] for 5 seconds. Although average acceleration technique is unconditionally stable for all  $dt$  values, the  
 363 study used a  $dt$  value that is also consistent with the explicit central difference scheme ( $\omega\Delta t \leq 2$ ). To mimic  
 364 real-life sensor data, the computed strain data is contaminated by adding (1%/2%/5%/10%) SNR WGN.  
 365 Henceforth, the contaminated strain data is used as the actual measured data,  $\mathbf{y}_k$ , for IP-EnKF algorithm  
 366 that has been used for tensegrity SHM.

367 For both the aforementioned cases (ST and EOT), the effect of measurement noise level on damage  
 368 detection accuracy along with the extent of damage that can be detected with precision, has been studied.  
 369 The ability of the algorithm to detect multiple damages in a tensegrity has also been tested. The initial  
 370 self-stressed configurations of tensegrity modules, ST (cf. Figure 4) and EOT (cf. Figure 2) have been  
 371 obtained through force density-based form-finding algorithm (Appendix A) and are presented in Tables 2  
 372 and 1, respectively, in terms of self-stress coordinates, member connectivity and initial tension coefficients.

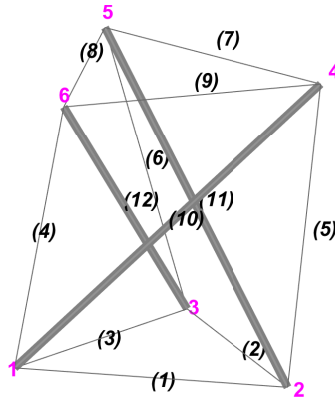


Figure 4: Simplex Tensegrity configuration

373 Adopted ST is a cylindrical tensegrity with 3 bars and 6 cables, whereas the EOT is a spherically  
 374 symmetric tensegrity with 6 bars and 24 cables. The algorithm (Appendix A) to obtain initial statically



375 stable coordinates has been verified with [7]. All the members, cables and bars, of both the tensegrities are  
376 assumed to be made of steel (*modulus of elasticity* = 200GPa). The diameters of bar and cable members  
377 are taken as 20mm and 5mm, respectively. For dynamic analysis, each of them is connected to a fixed base  
378 at three nodes: (1-3) for ST (cf. Figure 4) and (1,2,6) for EOT (cf. Figure 2). The stable form of the  
379 tensegrities is further excited with an ambient Gaussian force (elaborated later for each case) applied to ST  
380 and EOT on the fourth and the third node, respectively, in x-direction.

Table 2: Nodal coordinates, elemental connectivity and initial tension coefficients of simplex tensegrity (ST) (*with c:cable and b:bar*)

	Node	1	2	3	4	5	6
ST	X	0.577	-0.244	-0.266	-0.452	0.0094	0.509
	Y	0	0.5	-0.461	0.301	-0.542	0.279
	Z	0	0	0	0.919	0.919	0.919

	Element	1	2	3	4	5	6	7	8	9	10	11	12
ST	Node 1	1	2	3	1	2	3	4	5	6	1	2	3
	Node 2	2	3	1	6	4	5	5	6	4	4	5	6
	Type	c	c	c	c	c	c	c	c	c	b	b	b
	Initial tension coeff. (N/m)	0.6834			1.1837			0.6835			-1.1838		

381 For both the tensegrity modules, damage is induced in their members 0.5s after the simulation starts.  
382 The initial distribution type for the parameter particles,  $\theta_k$  (HIs) is set to be Gaussian, with their mean set  
383 as 1 assuming an undamaged condition and a standard deviation of 0.02, with  $\alpha$  chosen as 0.90 (cf. Equation  
384 (17)). For consistency and understanding, the HIs of damaged members are compared to the HI (= 1) of  
385 undamaged member 10 for all the cases.

#### 386 4.1. Effect of external load on vibrational properties of undamaged tensegrity

387 As already discussed in the article, upon load application, the vibrational properties of a tensegrity  
388 change due to a change in the tensegrity stiffness owing to the change in the pre-stress. This has been  
389 demonstrated through an example case study on the ST subjected to an external WGN load of variance  
390  $1.25 \times 10^4 N^2$ . In the case study, the WGN is applied on the fourth node along its x-direction (cf. Figure  
391 4). No member is damaged and the system is simulated for 5 seconds. The responses are recorded at a  
392 sampling frequency of 100 Hz. It is observed that under varying external load, natural frequencies of ST  
393 change considerably even in the absence of any damage. Figure 5 demonstrates the relative change in first  
394 three natural frequencies ( $\omega_1, \omega_2$  and  $\omega_3$ ) in time under a time varying load in comparison to their values  
395 ( $\omega_1^0, \omega_2^0$  and  $\omega_3^0$ ) corresponding to a stable form. Clearly, this establishes that modal comparison is not an  
396 option for tensegrity SHM and establishes the necessity for time domain approaches. Further, since the  
397 tensegrity stiffness is a function of force, a tensegrity with unknown force can not evidently be estimated  
398 with a deterministic approach. This emphasizes the need for probabilistic approaches in which the system  
399 health can be estimated with a probabilistic measure and thereby justifies the employment of the proposed  
400 Bayesian filtering-based algorithm.

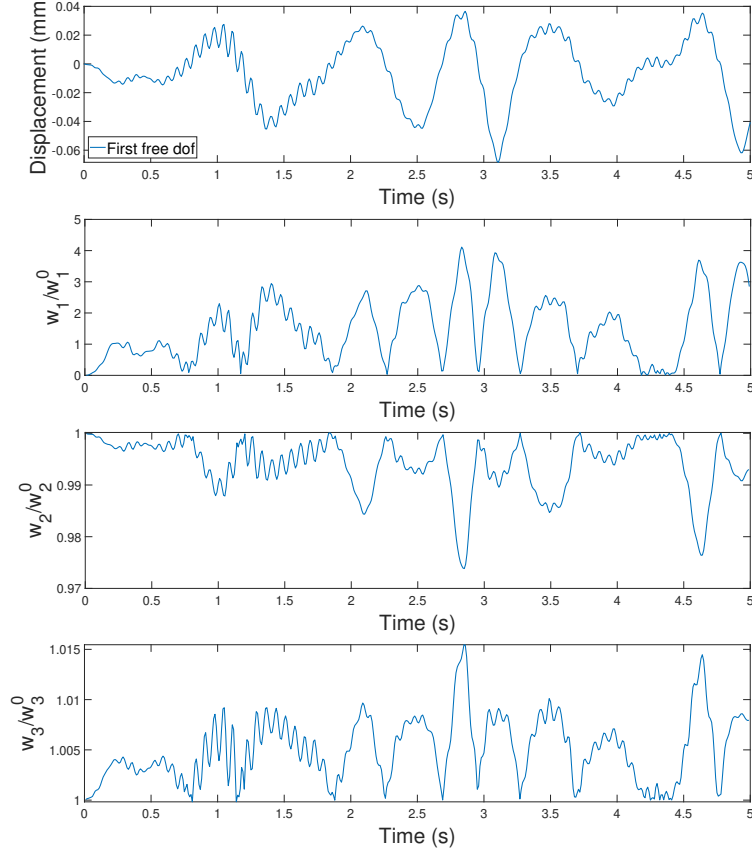


Figure 5: Variation in frequency ( $1^{st}$ ,  $2^{nd}$  &  $3^{rd}$ ) of simplex (undamaged) under varying load

401 *4.2. Calibration of particle and ensemble pool*

402 A calibration study has been performed on ST to identify the minimum number of particles and ensembles  
 403 that can be utilized to efficiently identify the damage induced. The details of tensegrity configuration, force  
 404 statistics as well as simulation specification have been kept the same as specified in section 4.1. Further  
 405 a 90% damage is induced in the 11<sup>th</sup> member of the simplex. A set of numerical experiments are further  
 406 performed targeting evaluation of the optimal number of particles and ensembles to be utilized for the rest  
 407 of the numerical experiments based on the algorithm's performance for accuracy and computational time.

408 Firstly, the number of particles are varied as 500, 1000, 2500 and 5000 for an ensemble pool size of 50  
 409 ensembles (cf. Figure 6a). It has been observed that, beyond a particle pool size of 2500, the accuracy is not  
 410 improving any further while only the computational expense is increasing substantially. Thus, a pool size of  
 411 2500 particles is chosen for PF. Next, optimal number of ensembles has been tested for EnKF with ensemble  
 412 pool sizes of 75 and 100 (cf. Figure 6). Again it has been observed that an ensemble pool of 100 ensembles  
 413 is sufficient to achieve desired accuracy while being within a manageable computational demand. A lower  
 414 value of ensemble number (here, 50) decreases the overall accuracy of the algorithm for all particle sizes  
 415 (500/1000/2500/5000). It should also be noted that increasing the particle size improves the promptness

416 in detection for the algorithm while increasing the computational cost of the algorithm as well. Thus for  
 417 estimation of the tensegrity health with proposed IP-EnKF, 2500 filter particles are selected for the PF  
 418 while 100 ensembles are chosen for the EnKF.

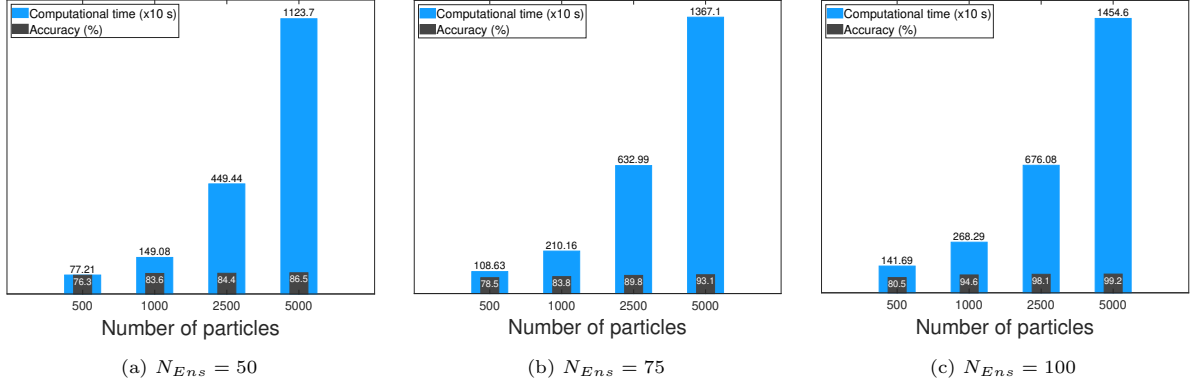


Figure 6: Effect of number of particles and ensembles ( $N_{En_s}$ ) on accuracy and computational time of the algorithm

#### 419 4.3. Simplex tensegrity (ST)

420 In the following, the proposed algorithm is tested on an ST module (cf. figure 4) while keeping the  
 421 force statistics, application node, and other simulation specifications, the same as provided in section 4.1  
 422 for the sake of consistency. Again a damage is induced in its 11<sup>th</sup> member (bar) by numerically reducing  
 423 its stiffness by 90%, 0.5s after the start of simulation. Strain measurements are collected from all the  
 424 unrestricted members of ST, i.e, members {4 – 12}, under various SNR levels.

425 The proposed algorithm is tested for its sensitivity against measurement noise contamination. Four  
 426 SNR levels are selected for this comparison: 1%, 2%, 5% and 10%. Damages have been detected, localized  
 427 and quantified for all noise levels (cf. Figure 7) with acceptable accuracy; although the promptness is not  
 428 observed with noise of 10% SNR (cf. Figure 7). Clearly, this states that the proposed algorithm is sufficiently  
 429 accurate with practical noise contamination levels.

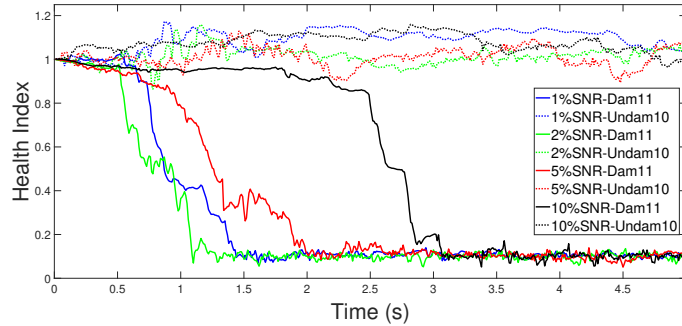


Figure 7: Measurement noise sensitivity of proposed approach - ST

430 To check the capability of the proposed approach to detect multiple damage in a tensegrity, a 90%

431 damage ( $\mathbf{HI} = 0.1$ ) is introduced to 8<sup>th</sup> and 11<sup>th</sup> member of the ST (cable and bar, respectively). The  
 432 damage is induced simultaneously after 0.5s of simulation. The simulated strain data is contaminated with  
 433 1% SNR WGN. The algorithm is capable to detect multiple damages, irrespective of the type of member  
 434 (cable or bar), with equal promptness and precision (cf. Figure 8).

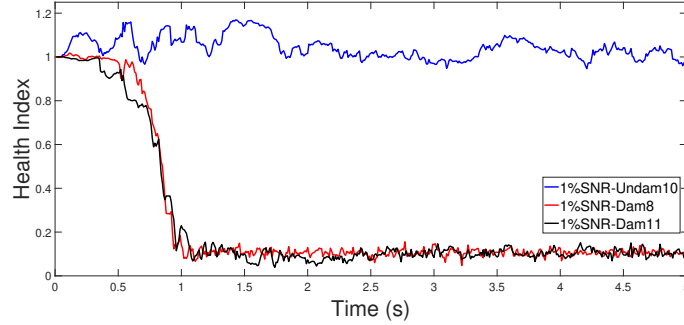


Figure 8: Multiple damage detection by the proposed approach - ST

435 Figure 9, shows the ability of the proposed approach to identify various damage levels (10%, 20%, 30%,  
 436 40% and 90%) with corresponding  $\mathbf{HI}s = \{0.9, 0.8, 0.7, 0.6 \text{ and } 0.1\}$ . It has been observed that for the  
 437 lower damage levels (10%), the algorithms output might confuse the investigator since the accuracy of the  
 438 estimation may get masked within the estimation variation. However, for moderate or high levels of damage,  
 439 demarcation of damaged state is quite straightforward with the proposed algorithm. It has been experienced  
 440 that the proposed algorithm can effectively demarcate a damaged member having a damage level as small  
 441 as 20% ( $\mathbf{HI} = 0.8$ ) without any confusion.

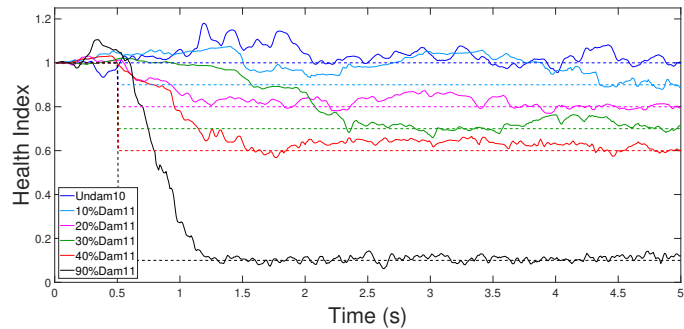


Figure 9: Detection of various damage levels by the proposed approach - ST

442 The minimum number of strain gauges required by the proposed approach to detect damage in ST, has  
 443 been further investigated (cf. Figure 10). Following cases have been included, i) 9 strain gauges {4 – 12},  
 444 ii) 6 strain gauges {4 5 7 8 10 11}, iii) 3 strain gauges {5 8 11}, and iv) 1 strain gauge {11}. A 90% damage  
 445 is induced in the 11<sup>th</sup> member for all the above cases. It is observed that the algorithm is able to detect the  
 446 damage with acceptable level of accuracy, even with a single strain gauge. Notably, the placement of sensor  
 447 plays a major role in precision and promptness of the algorithm: sensors in the vicinity of the damages always

448 alleviate the effort to detect them. This has been exhibited by the proposed approach as well. Nevertheless,  
 449 with the increasing numbers of sensors, this problem is observed to attenuate. This aspect is however very  
 450 much system specific. Accordingly, this case study can only give an idea about minimum sensors required  
 451 and as such can not help to interpret the efficacy of the proposed algorithm.

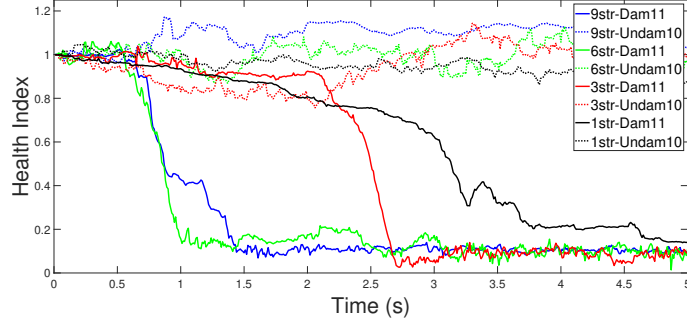


Figure 10: Performance of proposed approach under varying number of strain gauges - ST

452 *4.4. Expanded-octahedron tensegrity (EOT)*

453 Further, similar numerical experiments are performed on an EOT module, three times larger in *dofs*  
 454 than the ST tested before. The objective is to check the efficacy of the algorithm with larger systems.  
 455 An external WGN load of variance  $1.25 \times 10^4 N^2$  has been applied on the third node along its x-direction  
 456 (cf. Figure 2). 0.5 s from the start of the simulation, a 90% damage level in the 11<sup>th</sup> member (cable) of  
 457 EOT is simulated. The sizes of particle and ensemble pool were selected as 2500 and 100, respectively.  
 458 Strain measurements are collected from all the unrestricted members, i.e, members  $\{1 - 21, 24 - 30\}$  of the  
 459 EOT, under various SNR (1%, 2%, 5% and 10%). As was observed for ST, the estimation is found to be  
 460 prompt and accurate till noise contamination level of 5% SNR WGN (cf. Figure 11), beyond which (10%  
 461 SNR) promptness is compromised while estimation still being accurate. It has further been realized that for  
 462 highly noisy systems, promptness can be regained by increasing the number of particles, ensembles or both,  
 463 which however comes at a higher computational cost.

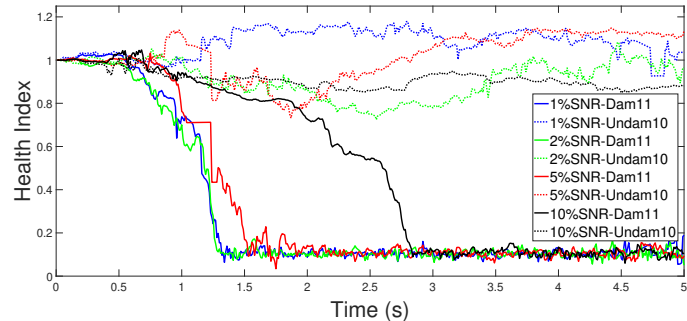


Figure 11: Measurement noise sensitivity of proposed approach - EOT

464 To investigate the efficacy of the algorithm for multiple damage cases in EOT, two cable members  
 465 (11 and 24) are simultaneously damaged to 90% damage level. A noise of 1% SNR level is added to the  
 466 strain data. It has been observed (cf. Figure 12) that the algorithm is able to detect damage with equal  
 467 precision and promptness, even if same member types (cables in this case) are damaged.

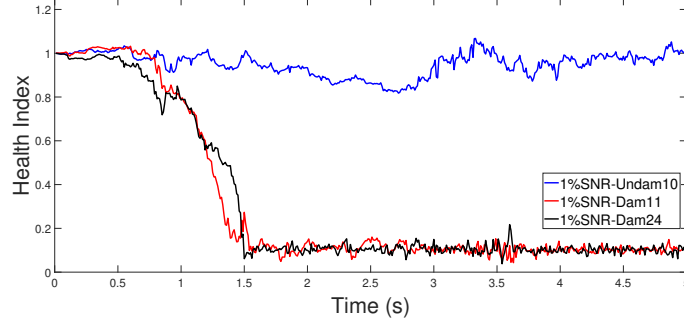


Figure 12: Multiple damage detection by the proposed approach - EOT

468 The algorithm is further tested to determine the extent of damage level that can be estimated for EOT  
 469 (cf. Figure 13). Five different damage levels: 10%, 20%, 30%, 40% and 90%, are tested in this endeavor.  
 470 The algorithm precisely detects a damage level of 20%, corresponding to an  $\mathbf{HI} = 0.8$ .

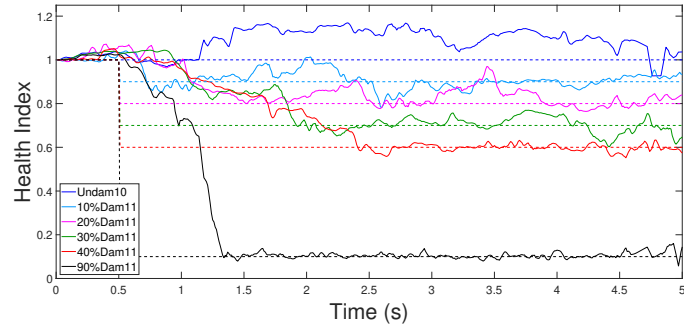


Figure 13: Detection of various damage levels by the proposed approach - EOT

471 As observed for ST, the proposed algorithm is able to accurately detect damage with strain gauge  
 472 number as low as one (cf. Figure 14). The observation has been made by applying a 90% damage in the  
 473 11<sup>th</sup> member of EOT, for each of the following cases, i) 28 strain gauges {1 – 21, 24 – 30}, ii) 15 strain  
 474 gauges {1 3 4 6 7 10 11 14 17 19 20 25 26 28 29}, iii) 6 strain gauges {4 11 19 24 26 28}, and iv) 1 strain  
 475 gauge {11}. A decrease in the employed number of sensors is observed to affect the promptness of detection.  
 476 Further, a few false positives (for damages below 40%) have also been observed for lower sensor number.  
 477 This is although expected since compared to ST, EOT is defined with higher *dofs* and therefore needs more  
 478 sensors to get monitored.

479 Finally, it has been observed that for both the tensegrities, the poor detection performance of the algo-  
 480 rithm, owing to higher noise contamination and/or weaker damage levels, can still be improved by employing

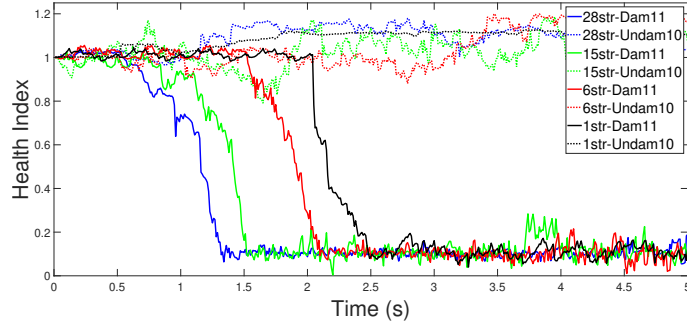
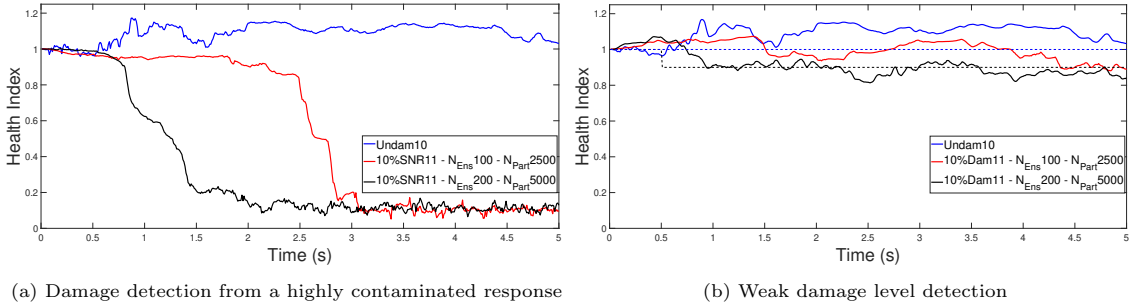


Figure 14: Performance of proposed approach under varying number of strain gauges - EOT

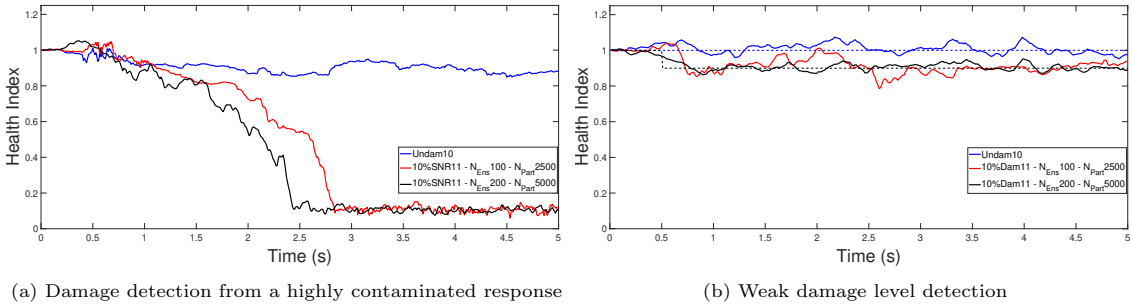
481 bigger particle and/or ensemble pools. Two sets of experiments are performed on ST and EOT specifically  
 482 for those cases for which the algorithm performed poorly (i.e. cases with 10% SNR noise contamination and  
 483 10% damage). For both ST and EOT, the loss of promptness due to high level of noise contamination is  
 484 regained (cf. 15a and 16a) after enhancing the particle and ensemble pools to 5000 and 200 respectively.  
 485 The enhanced pool sizes also improved the precision and stability for the estimation of weak damages (cf.  
 486 Figures 15b and 16b). This in turn enables the algorithm to handle more complicated problems using  
 487 compute-intensive approaches.



(a) Damage detection from a highly contaminated response

(b) Weak damage level detection

Figure 15: Effect of selecting a bigger particle and/or ensemble pools - ST



(a) Damage detection from a highly contaminated response

(b) Weak damage level detection

Figure 16: Effect of selecting a bigger particle and/or ensemble pools - EOT

## 488 5. Conclusion

489 A novel interacting filtering based damage detection approach has been proposed for tensegrity structures.  
490 The approach successfully estimates the health parameters, through PF, along with the system states,  
491 through EnKF nested inside the PF. Proposed probabilistic approach enables monitoring the tensegrity  
492 health as long as a precise model of tensegrity dynamics is available and the input forcing statistics is known  
493 to the investigator. No explicit knowledge of input time history is required for the estimation. The method  
494 is found to be efficient in accurate detection and localization of the tensegrity damages and sufficiently  
495 robust against practical levels of measurement noise. The algorithm is observed to perform even with sparse  
496 instrumentation. Multiple damage cases were also detected without any confusion. Promptness and precision  
497 is observed to be affected for the weak damage cases and/or highly contaminated signals. Nevertheless, it  
498 has also been observed that the performance for such cases can be rectified by employing bigger particle  
499 and/or ensemble pools at a higher computational cost. The algorithm however restricts itself for tensegrities  
500 subjected to stationary Gaussian forcing only. Further research is required in order to develop tensegrity  
501 SHM approaches that are robust against input forcing.

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## 650 Appendix A. Form-finding of statically stable tensegrity

651 To find the initial statically stable configuration of tensegrity, a Force Density Method based algorithm  
 652 is utilized that optimizes force density coefficients,  $\mathbf{p}$ , of the member elements to obtain initial coordinates,  
 653  $\mathbf{X}_{est}$ , of the tensegrity. Along with the optimization of force density coefficients, global stability criteria  
 654 [72] (cf. lines 12-15, Algorithm 2) are also introduced to obtain a stable tensegrity configuration. While  
 655 constructing a physical tensegrity it has been noticed that the bars tend to buckle under self-stress. To avoid  
 656 such a situation, local stability criteria (cf. lines 18-19, Algorithm 2) of buckling failure ( $\mathbf{p}_{bars} < \mathbf{p}_{critical}$ )

657 as well as cable slackening ( $\mathbf{p}_{cables} > 0$ ) have been added to the optimization.

---

**Algorithm 2** Form-finding algorithm to obtain initial stable configuration of tensegrity

---

```

1: Define connectivity of members, coordinates of known fixed dofs, member type (bar/cable), material properties
2: Initialize force density coefficients,  $\mathbf{p}$ 
3: procedure STATICALLY STABLE FORM-FINDING (Optimizing with stability criteria)
4:   procedure OPTIMIZE FORCE DENSITY COEFFICIENTS USING fmincon (IN-BUILT MATLAB FUNCTION) ( $\mathbf{p}_{est}$ )
5:     Find force density matrix,  $\mathbf{D}$ 
6:     Optimise  $\mathbf{p}$  such that atleast 4 eigen values of  $\mathbf{D} = 0$  and  $\mathbf{D}$  is positive semi-definite, for a 3 dimensional
tensegrity ▷ For details see [69]
7:   end procedure
8:   Calculate force density matrix,  $\mathbf{D}$  from estimated  $\mathbf{p}_{est}$ 
9:   Calculate nodal coordinates of the unknown free dofs,  $\mathbf{X}_{est}$  from the null space of  $\mathbf{D}$  by performing eigenvalue
decomposition ▷ For details see [69]
10:  Calculate Equilibrium matrix,  $\mathbf{A}$  and Geometric matrix  $\mathbf{G}$  ▷ For details see [69]
11:  Global stability checks: ▷ For details see [72]
12:  1.  $rank(\mathbf{D}) \leq n - (d + 1)$ ; for 3-d tensegrity  $d = 3$ 
13:  2.  $eig(\mathbf{D}) \geq 0$ 
14:  3.  $rank(\mathbf{G}) = d(d + 1)/2$ ; for 3-d tensegrity  $d = 3$ 
15:  4.  $rank(\mathbf{A}) <$  total number of members present in tensegrity
16:  Calculate  $\mathbf{p}_{critical} = \frac{\pi^2 EI}{l^3}$  (pin-pin connections) to incorporate local buckling criteria for bars
17:  Local stability checks:
18:  5.  $\mathbf{p}_{bars} < \mathbf{p}_{critical}$ 
19:  6.  $\mathbf{p}_{cables} > 0$ 
20:  if All the above six criteria are met then break
21:  else  $\mathbf{p} = \mathbf{p}_{est}$ ; GO TO STEP 3
22:  end if
23: end procedure

```

---

658 **Appendix B. Explicit Newmark-beta method: incremental formulation**

659 Algorithm 3 presents the pseudo-code for explicit Newmark-beta method [15] utilized in this study.

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**Algorithm 3** Explicit Newmark-beta method: incremental formulation

---

```

1: Average acceleration assumptions:  $\beta = 0.25; \gamma = 0.5$ 
2: for <for each time step  $k$ > do
3:   procedure STATE PROPAGATION( $\mathbf{M}, \mathbf{K}_{k-1}, dt, \mathbf{Q}, \mathbf{q}_{k-1}, \dot{\mathbf{q}}_{k-1}, \ddot{\mathbf{q}}_{k-1}$ )
4:     Re-calibrate  $\mathbf{K}_{k-1}$  as  $\mathbf{K}_k$  as a function of  $\mathbf{q}_{k-1}$  ▷ See Section 2.1
5:     Calculate  $\mathbf{C}_k$  as a function of  $\mathbf{K}_k$  and  $\mathbf{M}$  ▷ as per Rayleigh damping model
6:     Realize  $\mathbf{P}_k$  from the noise process  $\mathcal{N}(0, \mathbf{Q})$  ▷ White Gaussian noise forcing
7:      $a_1 = \frac{\mathbf{M}}{\beta dt^2} + \frac{\gamma \mathbf{C}_k}{\beta dt}$ 
8:      $a_2 = \frac{\mathbf{M}}{\beta dt} + (\frac{\gamma}{\beta} - 1)\mathbf{C}_k$ 
9:      $a_3 = (\frac{1}{2\beta} - 1)\mathbf{M} + dt\mathbf{C}_k(\frac{\gamma}{2\beta} - 1)$ 
10:     $\hat{\mathbf{K}}_k = \mathbf{K}_k + a_1$ 
11:     $\hat{\mathbf{P}}_k = \mathbf{P}_k + a_1\mathbf{q}_{k-1} + a_2\dot{\mathbf{q}}_{k-1} + a_3\ddot{\mathbf{q}}_{k-1}$ 
12:     $\mathbf{q}_k = \hat{\mathbf{K}}_k^{-1}\hat{\mathbf{P}}_k$ 
13:     $\dot{\mathbf{q}}_k = \frac{\gamma}{\beta dt}(\mathbf{q}_k - \mathbf{q}_{k-1}) + (1 - \frac{\gamma}{\beta})\dot{\mathbf{q}}_{k-1} + (1 - \frac{\gamma}{2\beta})dt\ddot{\mathbf{q}}_{k-1}$ 
14:     $\ddot{\mathbf{q}}_k = \frac{1}{\beta dt^2}(\mathbf{q}_k - \mathbf{q}_{k-1}) - \frac{\dot{\mathbf{q}}_{k-1}}{\beta dt} + (\frac{1}{2\beta} - 1)\ddot{\mathbf{q}}_{k-1}$ 
15:  end procedure
16: end for

```

---