SOME FACTORS OF NUMBERS OF THE FORM $b^n \pm 1$ FOUND USING ECM WITH NEW CLASSES OF CURVES

François Morain

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The elliptic curve method of factorization (ECM) [2] is one of the powerful algorithms able to find moderate size factors of large integers. The following tables list some unknown factors of composite numbers $b^n \pm 1$ for values of $13 \leq b \leq 10^4$, full tables of which are managed by Jonathan Crombie (continuing the work of Brent, Montgomery and te Riele, see `http://myfactors.mooo.com/`). This is an extension of the Cunningham project [1] for $2 \leq b \leq 12$. All the factors were found using GMP-ECM [4] (with the algorithms described in [3]).

More details on the computations and curves used will be given in the full version of the article.

Format of the tables: pretty straightforward. Remember that $b, n+$ (resp. $b, n-$) designates $b^n + 1$ (resp. $b^n - 1$). The last column contains a time stamp. For instance, 210919220236 should be decoded as 21/09/19:22.02.36 or 22h 02 min 36 sec on 19, September 2021. Numbers are sorted by increasing date of discovery. More factorizations will be added as time goes by.

References

[1] J. Brillhart, D. H. Lehmer, J. L. Selfridge, B. Tuckerman, and S. S. Wagstaff, Jr. *Factorizations of $b^n \pm 1$, $b = 2, 3, 5, 6, 7, 10, 11, 12$ up to high powers*. Number 22 in Contemporary Mathematics. AMS, 2 edition, 1988.

[2] H. W. Lenstra, Jr. Factoring integers with elliptic curves. *Ann. of Math. (2)*, 126:649–673, 1987.

[3] P. Zimmermann and B. Dodson. 20 years of ECM. In Florian Hess, Sebastian Pauli, and Michael Pohst, editors, *Algorithmic Number Theory – ANTS-VII*, volume 4076 of *Lecture Notes in Comput. Sci.*, pages 525–542. Springer-Verlag, 2006.

[4] P. Zimmermann et al. GMP-ECM (elliptic curve method for integer factorization). `https://gforge.inria.fr/projects/ecm/`, 2010.

Date: November 20, 2021.

rk	Nfactor of (dd)	$p \mid N$	dd	when
1	$89,136+(226dd)$	203670153440347727407773377	27	210919220236
2	$76,143-(226dd)$	10816735732755689569132534388740 31	34	210919220301
3	$91,143+(229dd)$	39097487455641709584561227838397	32	210919233345
4	$91,136+(238dd)$	99099698405587441901003199699 53	31	210920000550
5	$77,148+(246dd)$	61333841287145678508955313296 81	31	210920012712
6	$85,143-(228dd)$	63140596626898655914980038 69	28	210920023351
7	$91,136+(238dd)$	889178099098009729255560066482981031132 49	41	210920191115
8	$83,136+(224dd)$	12213113915078325375417795596071720 1	36	210920233543
9	$85,136+(244dd)$	48125100312765681815371320342730063156 81	40	210921034739
10	$90,136+(243dd)$	95029336145694838957824482284328662990992760116961	50	211001001346
11	$86,143+(226dd)$	12180238564481316453393696736485608413623556 3	45	211002060429
12	$89,136+(226dd)$	15072369402225607863860410617459024232106 09	43	211002233552
13	$91,143-(229dd)$	72384483510845642297891730646993695081321681523	47	211003103136
14	$93,136+(224dd)$	14436130601076702184923277185603187699028088 1	45	211004015217
15	$58,148+(245dd)$	11250688230662914901918046352 9	30	211014110840
16	$59,148+(246dd)$	26642397493750326276664956281	29	211014124008
17	$63,148+(239dd)$	24427916239531892701088033849	29	211014133356
18	$57,148+(243dd)$	16914812989931667675447915131 3	30	211014135020
19	$57,148+(243dd)$	42100938314013035363941726563752 9	33	211014143406
20	$62,148+(231dd)$	17448938927593549047809294275718948 9	36	211015013516
21	$63,148+(239dd)$	12677558719278361799687159315827170580 1	39	211015021559
22	$75,148+(239dd)$	28725666012993763214993379907201579 3	36	211015025728
23	$72,143+(219dd)$	25704656660343722651291158914 7	30	211016071715
24	$95,143+(223dd)$	10834648235840816020042259209742299612369 97	43	211016115307
25	$63,148+(239dd)$	27082173088072734297678991541749014073 93	40	211017013219
26	$80,148+(249dd)$	52800882843310449613581696315906641266752815440 1	48	211019054012
27	$59,148+(246dd)$	61536008911834513582886337182343516917554905021569 77	52	211019130348
28	$68,148+(226dd)$	62933356031386656737538694096688834389236280001 53	49	211023050826
29	$58,148+(245dd)$	19550654221097917652271879776019251180627791 3	45	211026193511
30	$59,148+(246dd)$	32549598165881659323145053260648245905788521	44	211027053852
31	$95,145-(222dd)$	20066555701966838882041978645704558724889134 1	45	211030231819
32	$88,143+(215dd)$	63581216002820212491674133733578362150121 53	43	211103190804
33	$74,145+(210dd)$	32422695234066058386096919125522608835030758770864 01	52	211103235736
34	$82,145+(211dd)$	86747896053466614248321331150730078446 71	40	211104111001
35	$69,145+(206dd)$	28650819345910522454509576 1	27	211104120058
36	$80,136+(208dd)$	14282340552001320145244125431697889	35	211104121931
37	$832,67-(191dd)$	51520601956502137111289483950008092540918327 89	46	211105073607
38	$82,145+(171dd)$	68474806501558942326539117461284822149654502216 1	48	211105104548
39	$908,67-(190dd)$	31492499252009518681370437918947002508314851388 9	48	211105120519
40	$97,143+(205dd)$	50292841684216910094749602933309975 01	37	211105133632
41	$85,143+(211dd)$	12567210297757156962579721322602100088448089 41	46	211106032159
42	$80,136+(173dd)$	17795867567307673051882685370072906684 49	40	211106123700
43	$78,145+(207dd)$	81072692624810646241556162224133216118 1	39	211106123941
44	$77,145+(204dd)$	53511294771052807007503502837693948073118 31	43	211106160032
45	$61,148+(206dd)$	53920836357327720337357889052702335826878373571 69	49	211107010418
46	$77,145+(161dd)$	15284105262481307713678612469222143741121	41	211107144237
47	$69,145+(180dd)$	51642735295054165114200696495014578655491621	44	211109025120
48	$78,143-(203dd)$	58906988500210848827148393647	29	211109124303
49	$74,145-(198dd)$	11569940943457975715006912231470590201582 1	42	211110124157
50	$74,145-(157dd)$	82680336163400607564958279513497353433701	41	211110131700

rk	Nfactor of (dd)	$p \mid N$	dd	when
51	$71,145+(200dd)$	3081838728839138646824667134834215417088225761	46	211110222511
52	$74,145-(116dd)$	67275789155815719762340857488362941681	39	211110222529
53	$737,67+(184dd)$	870917417466838788698821597667901172952093040299	48	211111104615
54	$682,67+(185dd)$	396043121436528788812914072249827239195762837l	46	211112150625
55	$68,145+(201dd)$	75952253272257363146490016886081179592454l	42	211112165156
56	$79,136+(212dd)$	16762571148898374633660104147430382656290l1	43	211113115453
57	$79,143+(201dd)$	617165874719529966578857601636808679159	39	211113125510
58	$69,143+(195dd)$	460327049908941630107996522456l9	32	211114031451
59	$564,67-(182dd)$	120063475026535559730912397815992519853724079l	46	211114070952
60	$97,145+(195dd)$	204750117086273654904215494721801599l1	38	211115132205
61	$75,145-(195dd)$	145820922799205411577526521075547603991	39	211115215945
62	$87,145+(194dd)$	594939672207712422337292028l1	29	211115224834
63	$76,136+(193dd)$	230638965208492675830062875068689	33	211115232601
64	$76,136+(161dd)$	15668064960179567518314166129l	30	211116040304
65	$98,145-(192dd)$	45381310620269686803871473639ll	31	211116115540
66	$76,136+(131dd)$	43864131470868999494580309441436401l	36	211116121717
67	$97,145+(158dd)$	203258396791080689425429576408186690l1	38	211116145935
68	$98,145-(161dd)$	10656547854550405978408562083619032l	36	211116155111
69	$88,143-(205dd)$	176631419599003306284596237977817343802484627	45	211116162947
70	$89,145-(192dd)$	24305667473968387305296796282225549729l	39	211116235218
71	$76,136+(96dd)$	74841304902050468479264138689615405766435l	42	211117023558
72	$89,145-(154dd)$	26769740020770035249479374829555278l	36	211117051016
73	$93,143+(185dd)$	11071437476096380864839337l9	28	211117102506
74	$71,143+(185dd)$	614761987798476480659645830099124399779l	40	211117143423
75	$54,148+(181dd)$	18737363826640569421257717903317685149l9	40	211117170211
76	$87,143-(186dd)$	869739041700065774535952087462183871	36	211117170847
77	$54,148+(142dd)$	47745297013416763673913373201	29	211117191122
78	$87,143-(150dd)$	13689994925606691795383634740569656473314l	42	211119071214
79	$84,145-(177dd)$	12809356786701789826746951683118l1	34	211119175550
80	$54,148+(113dd)$	79647427812727331528378671705173747075361	41	211119204943
81	$85,145-(174dd)$	20183280853192927742520254632997523660l	39	211120030246
82	$85,145-(136dd)$	18664749468129844199178503672496384l	36	211120034412