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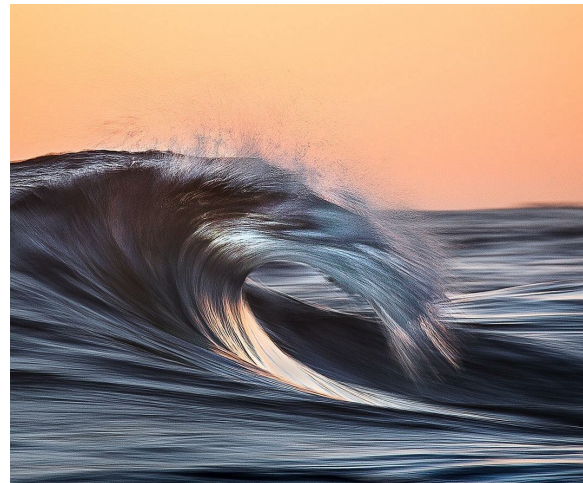
# Sensitivity analysis for two wave breaking models used by the Green-Naghdi equation

**Maria Kazolea**  
Subodh M. Joshi  
Mario Ricchiuto

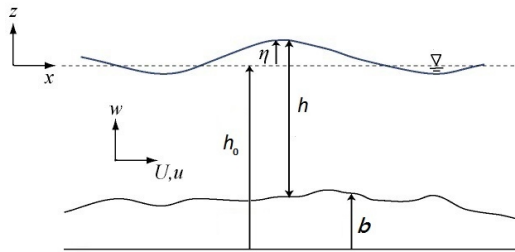
# Context and motivation

Near shore: wave transformation due to interaction with complex bathymetry

- Strong non-linearity, dispersion
- Refraction diffraction
- Shoaling
- **Breaking**
- Induced currents
- Run-up, inundation



# Modeling and discretisation



$$h_t + (hu)_x = 0$$

$$(hu)_t + (hu^2)_x + gh\eta_x = \phi$$

$$(I + \alpha\mathcal{T})\phi = \mathcal{W} - \mathcal{R} + (r_{wb})_x$$

$$\mathcal{W} = g\mathcal{T}(h_0\eta_x)$$

$$(r_{wb})_x = (\nu_t H u_x)_x$$

$$\mathcal{R} = h\mathcal{Q}(u)$$

$$\mathcal{T}(\cdot) = -\frac{1}{3}h^2(\cdot)_{xx} - \frac{1}{3}hh_x(\cdot)_x + \frac{1}{3}[h_x^2 + hh_{xx}](\cdot) + \left[b_x h_x + \frac{1}{2}hb_{xx} + b_x^2\right](\cdot)$$

$$\mathcal{Q}(\cdot) = 2hh_x(\cdot)_x^2 + \frac{4}{3}h^2(\cdot)_x(\cdot)_{xx} + b_x h(\cdot)_x^2 + b_{xx}h(\cdot)(\cdot)_x + \left[b_{xx}h_x + \frac{1}{2}hb_{xxx} + b_x b_{xx}\right](\cdot)^2$$

- Elliptic part: Continuous Galerkin FE method
- Hyperbolic part: Third order MUSCL FV method



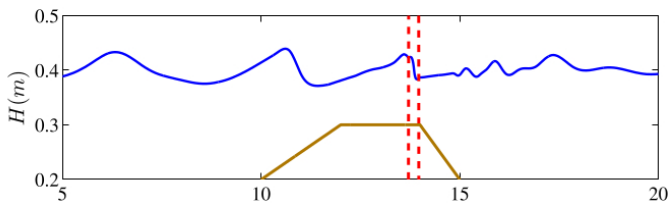
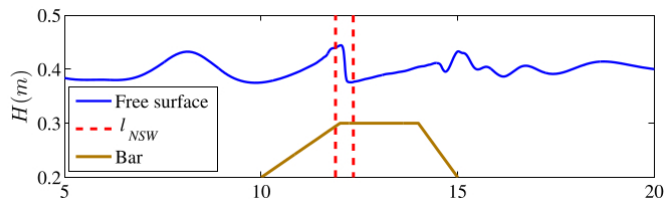
# Breaking closure: shallow water dissipation -Hybrid

1. Detect Breaking Regions
2. Remove dispersive terms
3. Shallow water shock
4. Dissipation



# Breaking closure: Shallow water dissipation -Hybrid

## 1. Detect and flag breaking regions



- Surface variation criterion:

$$|\eta_t| \geq \gamma \sqrt{gH}$$

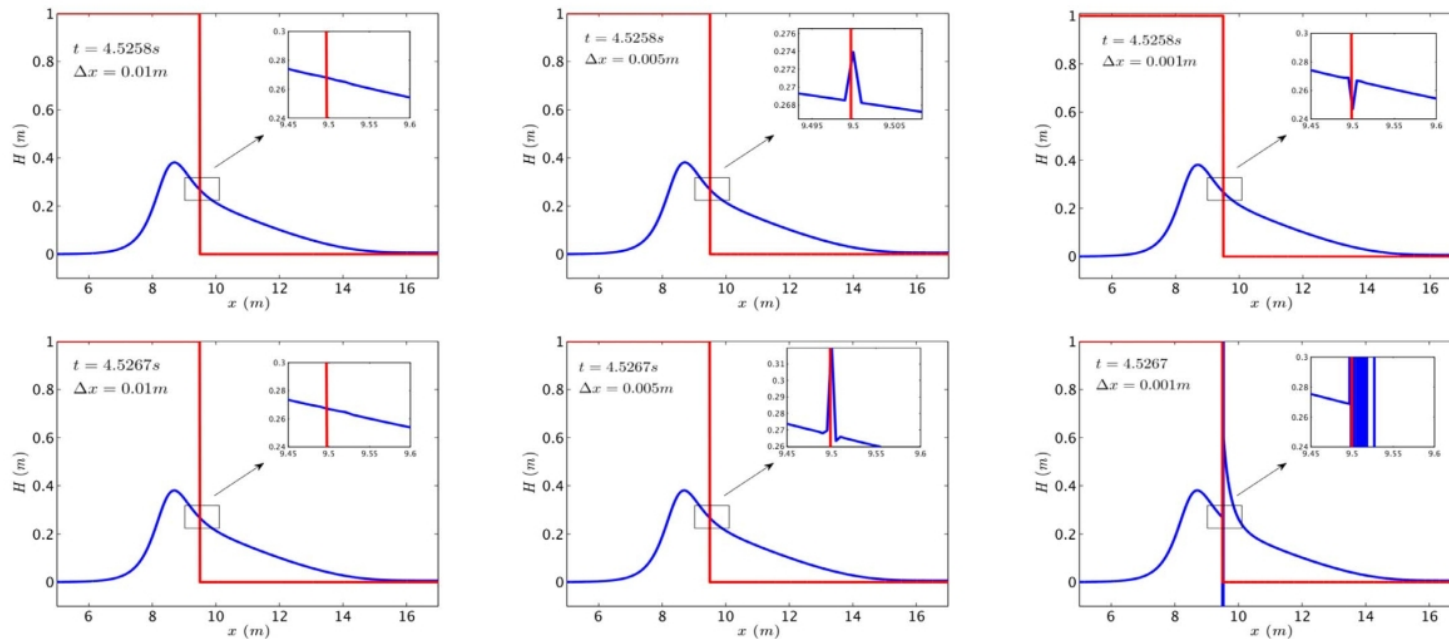
- Local slope angle criterion:

$$\|\nabla\eta\| \geq \tan\varphi_c$$

- The Froude number of each wave is computed and breaking is deactivated if  $Fr < 1.3$

# Breaking closure

- Codes using this closure: Funwave, BOSZ, Coulwave, Mike 21 Tucwave, Slows etc...
- Drawback: can not be used on variable meshes\*



Solitary wave breaking on a slope: hybrid treatment with order reduction at the coupling interface. Wave height at times  $t = 4.5258$  s,  $4.5267$  s (top and bottom rows), on mesh sizes (from left to right)  $\Delta x = 0.01$  m,  $0.005$  m,  $0.001$  m.

\*

- P. Bacigaluppi, M. Ricchiuto, P. Bonneton, *Implementation and Evaluation of Breaking Detection Criteria for a Hybrid Boussinesq model*, *Water Waves*, 2, 2020
- Kazolea, M., Ricchiuto, M.: *On wave breaking for Boussinesq-type models*. *Ocean Modelling* 123, 16–39 (2018)
- Shi, F., Kirby, J.T., Harris, J.C., Geiman, J.D., Grilli, S.T.: *A high-order adaptive time-stepping TVD solver for Boussinesq modeling of breaking waves and coastal inundation*. *Ocean Modelling* 43, 36–51 (2012)

# Breaking closure: Eddy viscosity model

via a PDE based TKE model

1. Detect breaking Regions (same as before)  $\gamma$ ,  $\varphi_c$
2. Dissipation required through the definition of artificial viscosity:

$$v_t = C_\nu \sqrt{\kappa_b \kappa_b + \kappa_j \kappa_j} A$$

$$k_t + uk_x = \mathcal{D} + \mathcal{P} - \mathcal{E}$$

$$\mathcal{D} = \sigma v_t k_{xx} \quad \mathcal{E} = -C_\nu^3 \frac{k^{3/2}}{l_t} \quad \mathcal{P} = B(t, x) \frac{l_t}{\sqrt{C_\nu^3}} (u^s)^3$$

$$l_t = \kappa H$$

$$\kappa, \sigma, C_\nu = \kappa_b, \sigma_b, C_{\nu_b} \quad \text{Bore}$$

$$\kappa, \sigma, C_\nu = \kappa_j, \sigma_j, C_{\nu_j} \quad \text{Hydraulic jump}$$

# Sensitivity analysis model

- Surrogate model – Kriging model
- ANOVA
  - Sobol decomposition

$$Y = F(\bar{\mathbf{x}}) = F_0 + \sum_{i=1}^M F_i(x_i) + \sum F_{ij}(x_i x_j) + \sum_{1 \leq i < j \leq M} F_{ij}(x_i, x_j) + \cdots + F_{1,2,\dots,M}(\bar{\mathbf{x}}) \quad \bar{\mathbf{x}} = [x_1, x_2, \dots, x_M]$$

$$F_0 = E(Y)$$

$$F_i(x_i) = E(Y | x_i) - F_0$$

$$F_{ij}(x_i, x_j) = E(Y | x_i, x_j) - F_0 - F_i - F_j$$

# Sensitivity analysis model

- First order indices

$$S_i = \frac{\text{Var}_{x_i}(P(Y|x_i))}{\text{Var}(Y)}$$

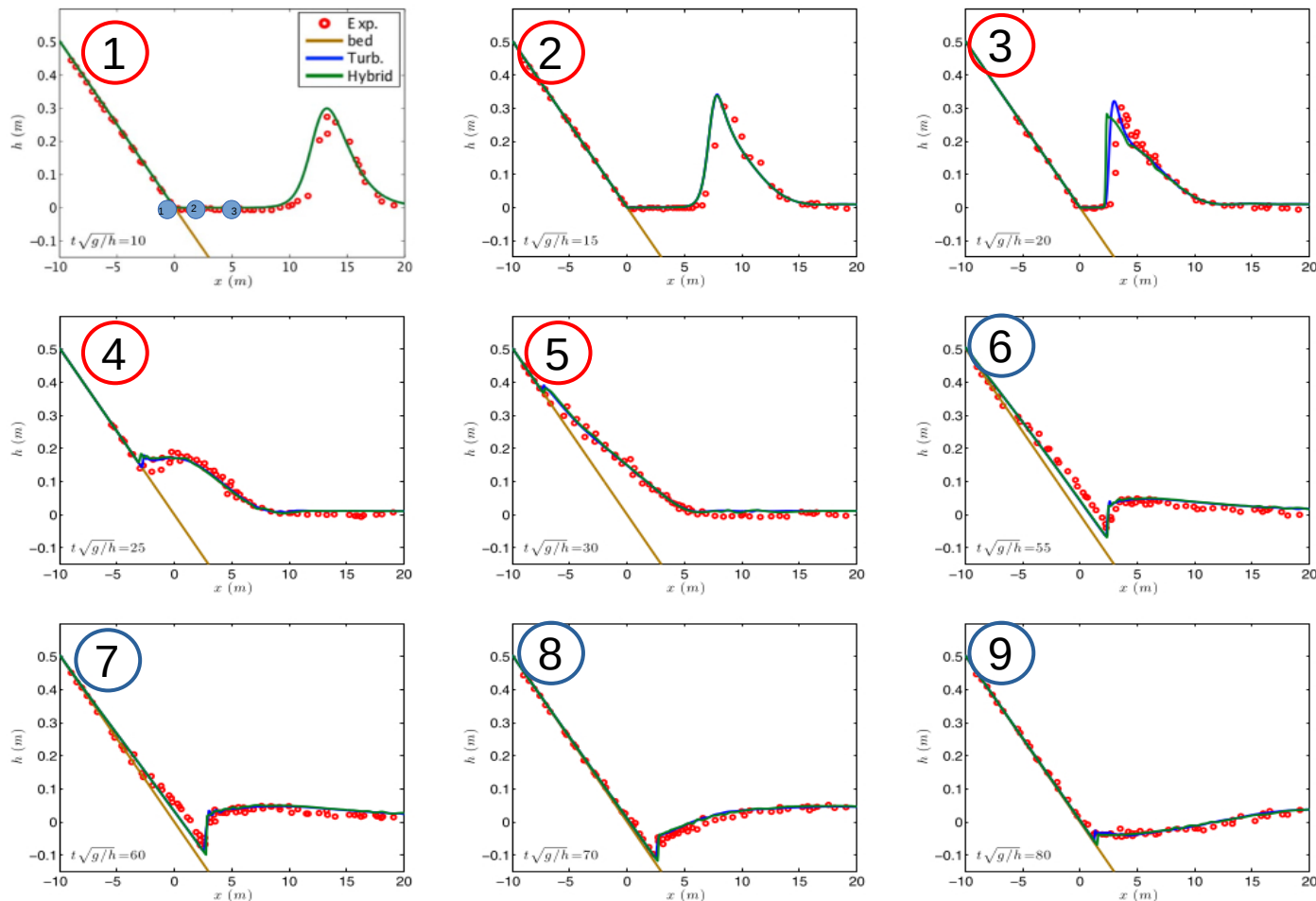
- Second order indices

$$S_{i,j} = \frac{\text{Var}_{x_i,x_j}(P(Y|x_i,x_j))}{\text{Var}(Y)}$$

- Total Sobol index of an input variable == sum of all Sobol indices involving this variable.

# Solitary wave on a plane beach

- Set up:  $A/d = 0.28\text{m}$ ,  $d=1\text{m}$ ,  $x=[-20, 80]\text{m}$ ,  $Nm=0.01$ ,  $WG_s: 0,2,5\text{m}$
- 1-5 : First phase of the flow (shoaling breaking and runup)
- 6-9 : Second phase of the flow (run-down and breaking )



Free surface elevation of solitary wave run-up on a plane beach for the GN model.

# Solitary wave on a plane beach: Input parameters

- Surface variation criterion:

$$|\eta_t| \geq \gamma \sqrt{gH} \quad \gamma \in [0.2, 0.6]$$

- Local slope angle criterion:

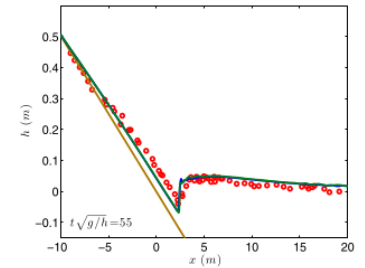
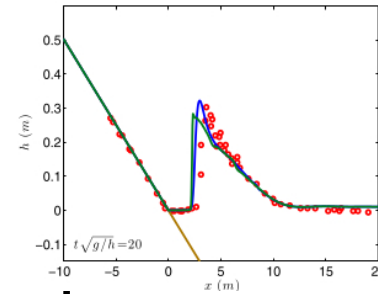
$$\|\nabla\eta\| \geq \tan \varphi_c \quad \varphi_c \in [0.3, 0.53]$$

- Constant parameters:  $\Delta x = 0.05$ ,  $Cfl = 0.01$ ,  $Fr < 1.3$



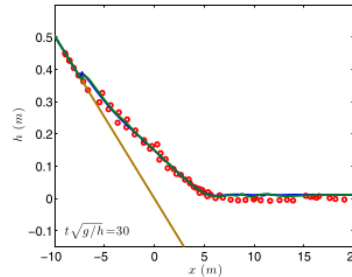
# Solitary wave on a plane beach: Output parameters

× Wave's breaking point location.

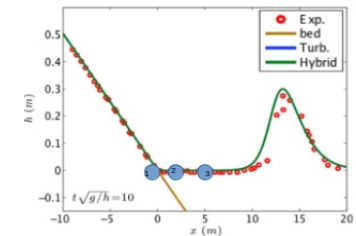


× Wave's height at the breaking point.

× Maximum run up.

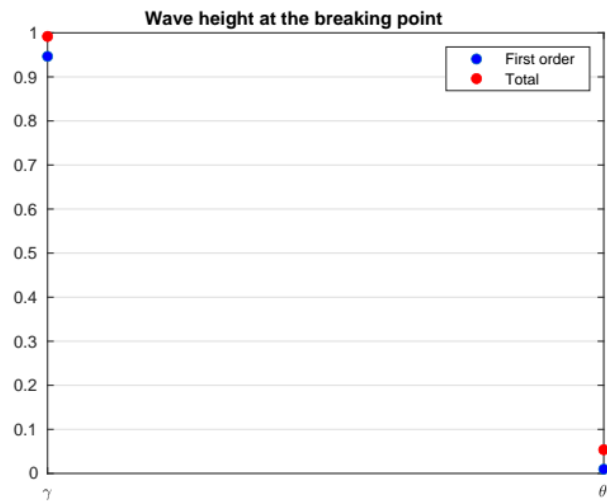


× Maximum wave height at wave gauges.



# Solitary wave on a plane beach (Hybrid)

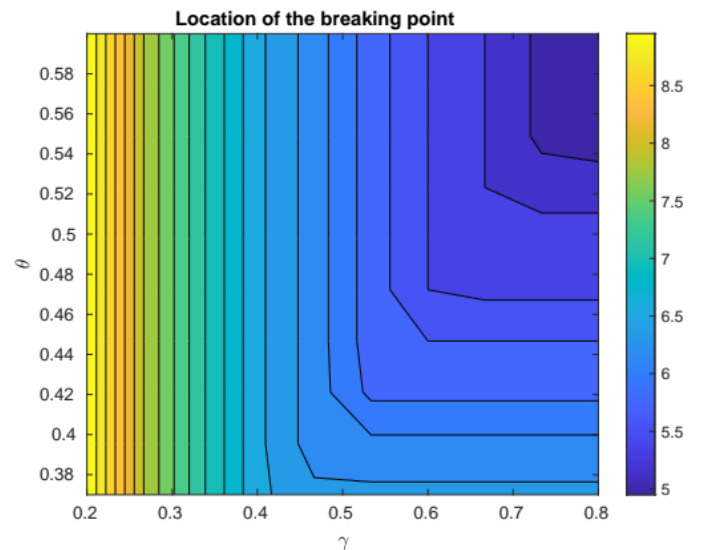
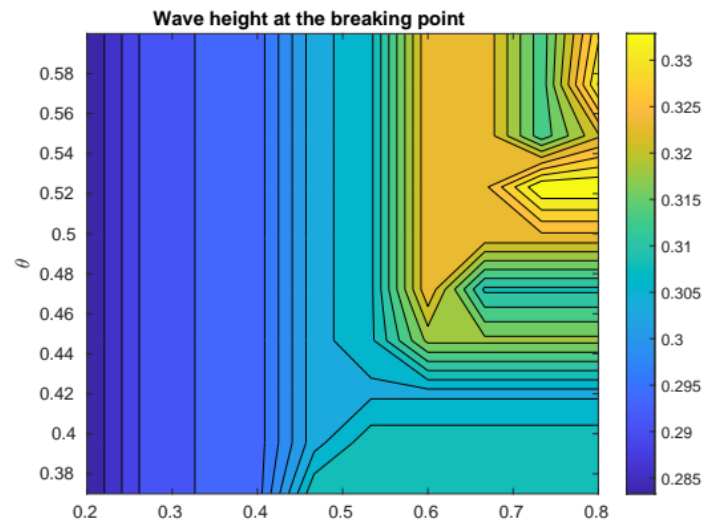
## Sobol's indices



## Location of the breaking point



## Contours



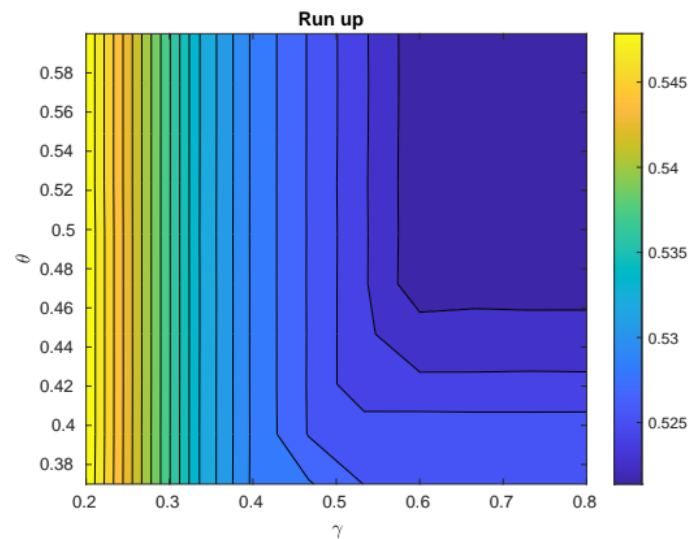
First phase of  
the flow 1-5

# Solitary wave on a plane beach (Hybrid)

Sobol's indices



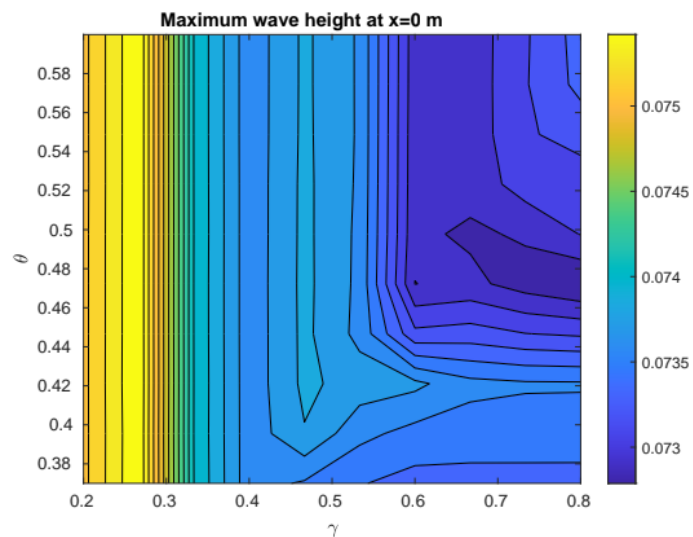
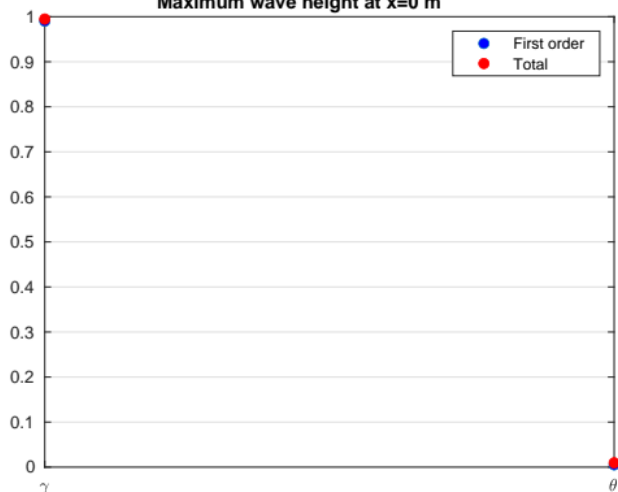
Contours



First phase of  
the flow 1-5

Range: 0.0278  
Std: 0.0087

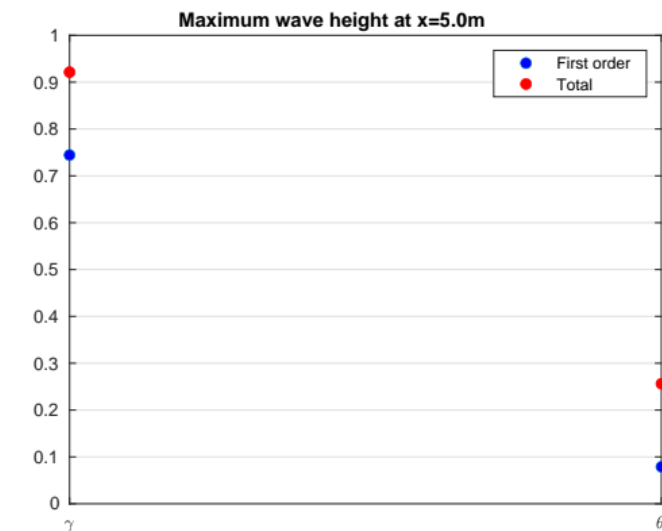
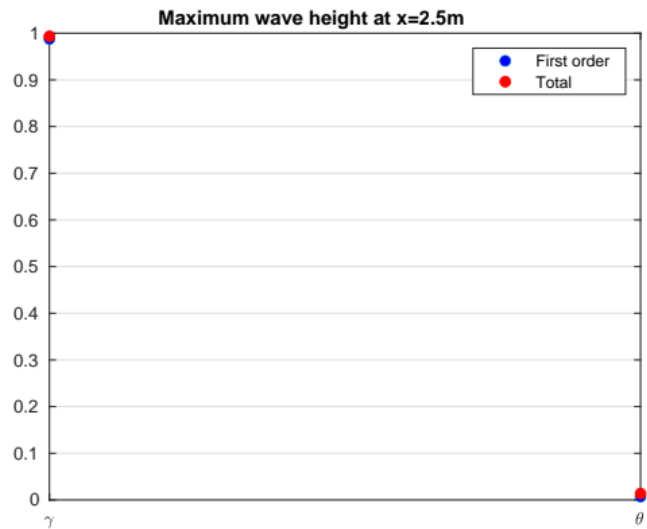
Maximum wave height at x=0 m



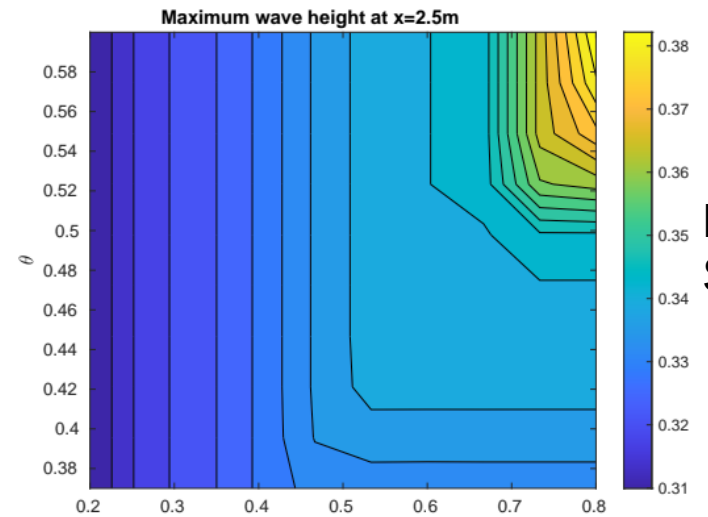
Range: 0.0028  
Std:  $8.1753 \cdot 10^{-4}$

# Solitary wave on a plane beach (Hybrid)

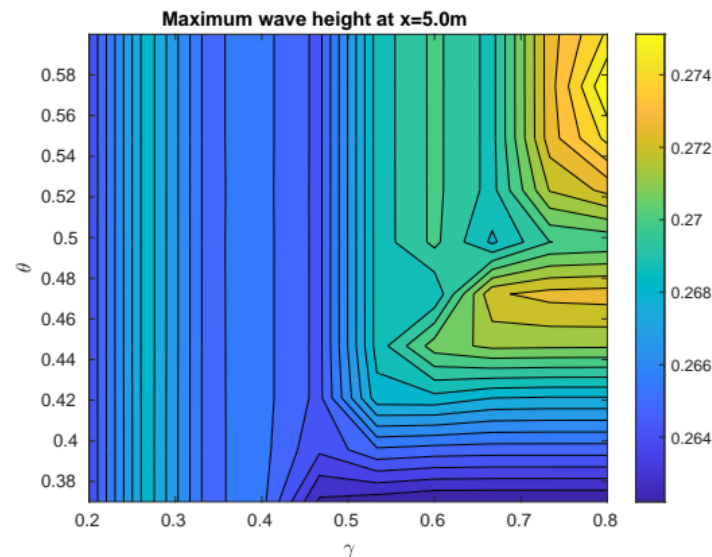
## Sobol's indices



## Contours



Range: 0.0758  
Std: 0.0151

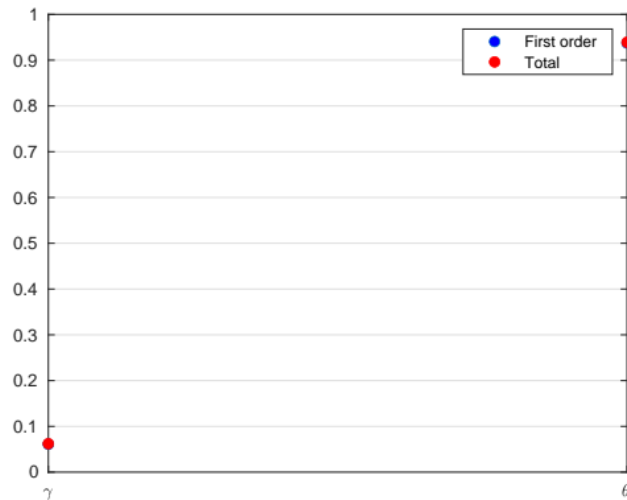


Range: 0.0135  
Std: 0.0032

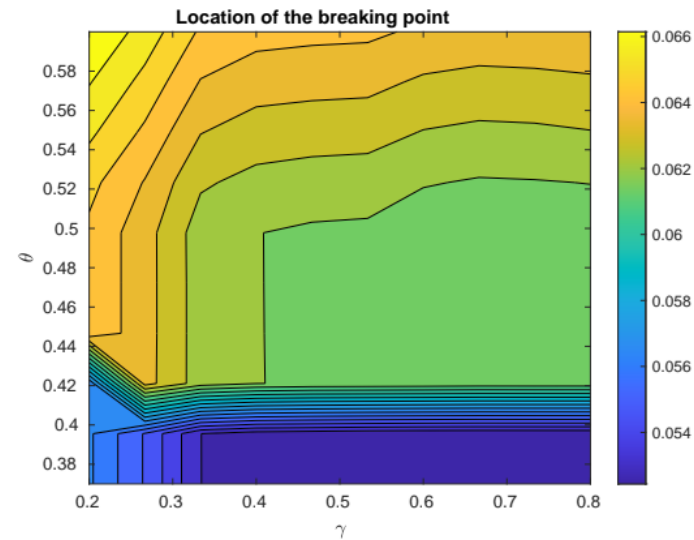
First phase of  
the flow 1-5

# Solitary wave on a plane beach (Hybrid)

Sobol's indices

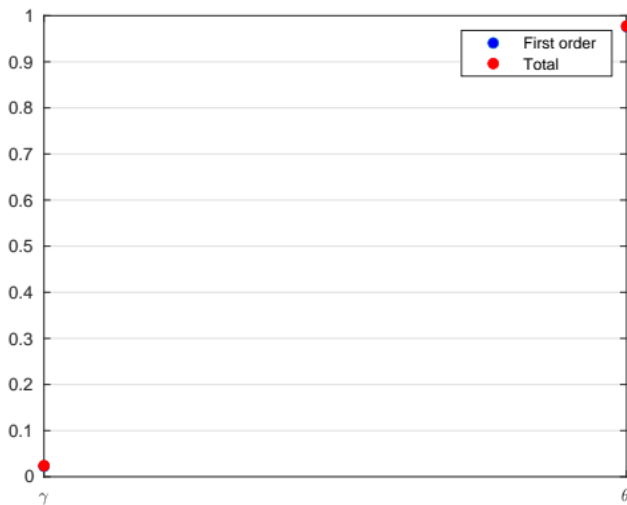


Contours

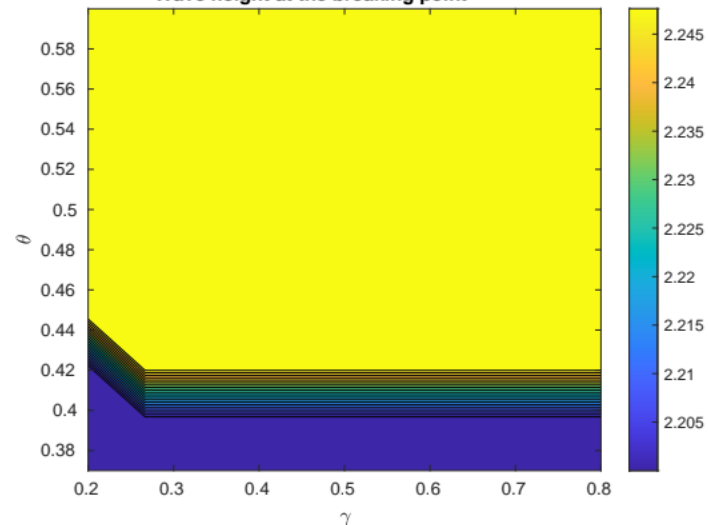


Second phase  
of the flow 6-9

Range: 0.0144  
Std: 0.0041



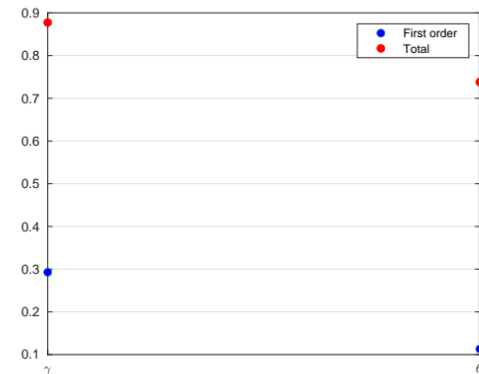
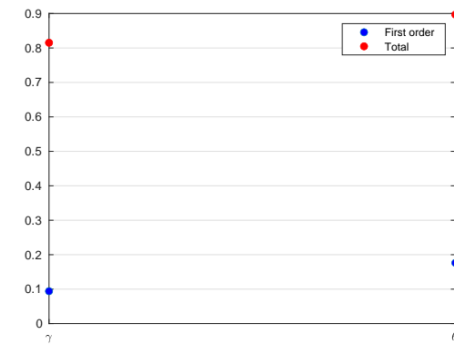
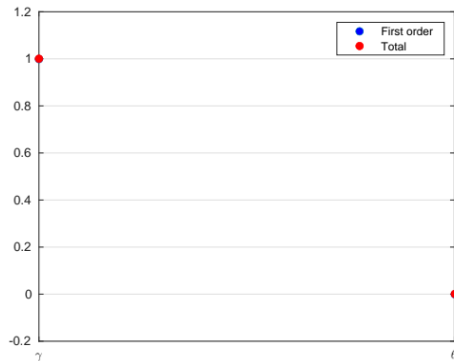
Wave height at the breaking point



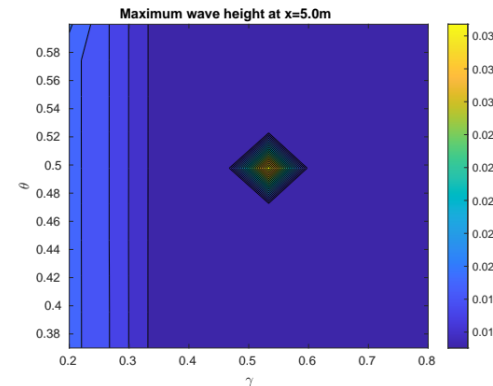
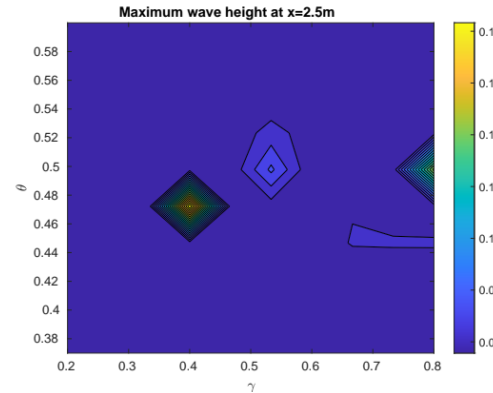
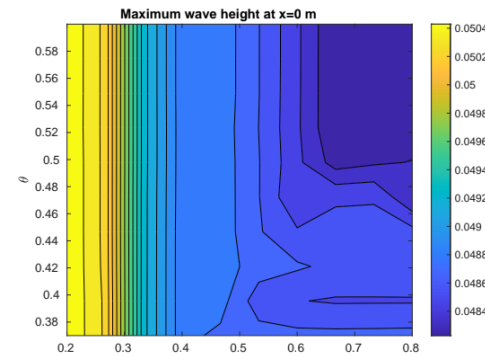
Range: 0.05  
Std: 0.0205

# Solitary wave on a plane beach (Hybrid)

Sobol's indices



Contours



Second phase  
of the flow 6-9

Range: 0.023  
Std:  $7.48 \times 10^{-4}$

Range: 0.01340  
Std: 0.0161

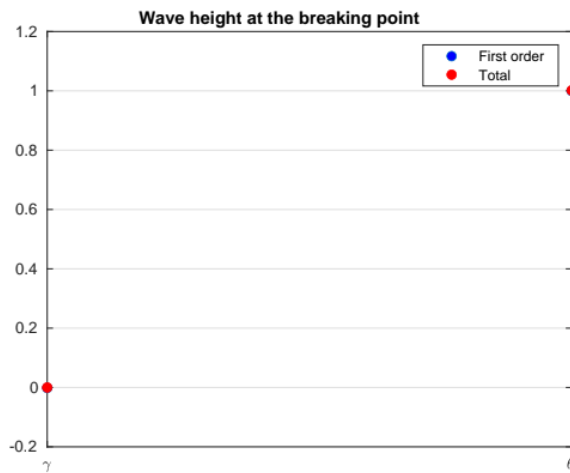
Range: 0.0207  
Std: 0.0024

# Solitary wave on a plane beach (Hybrid)

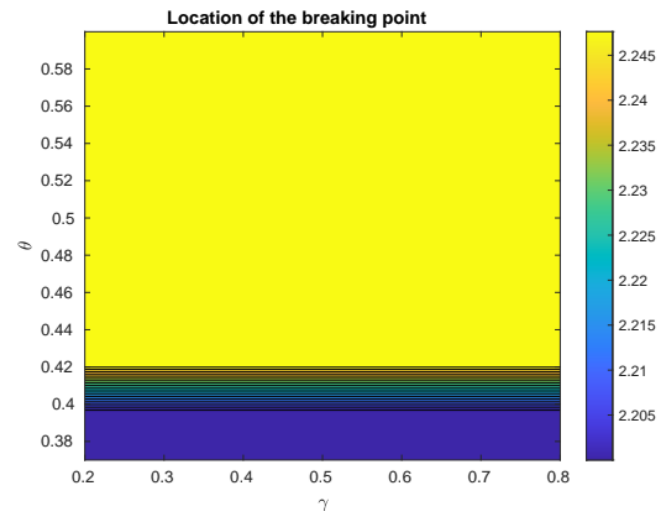
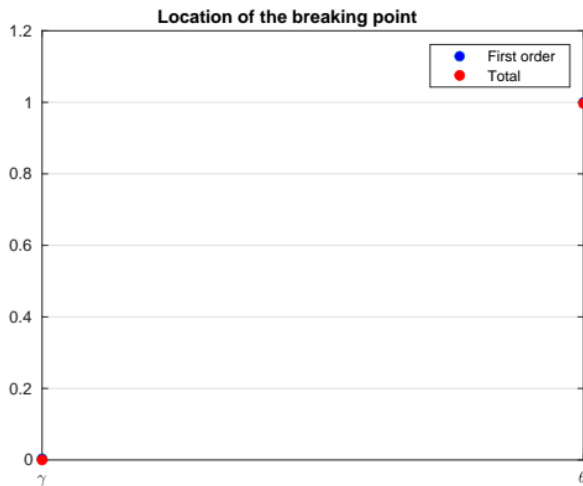
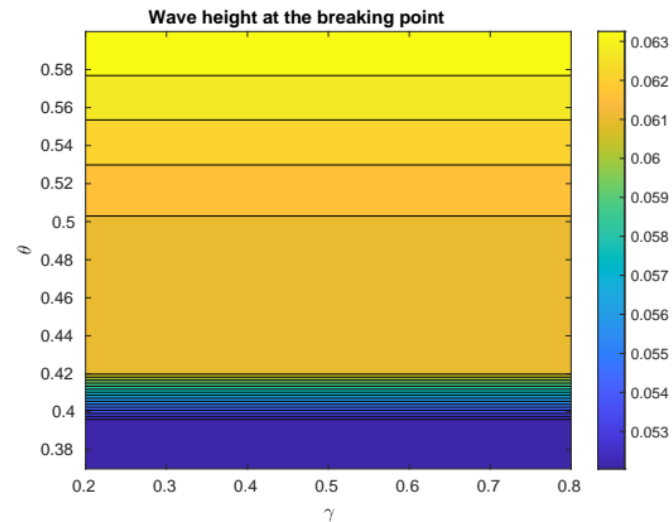
Second phase  
of the flow 6-9

Same initial  
condition

Sobol's indices



Contours

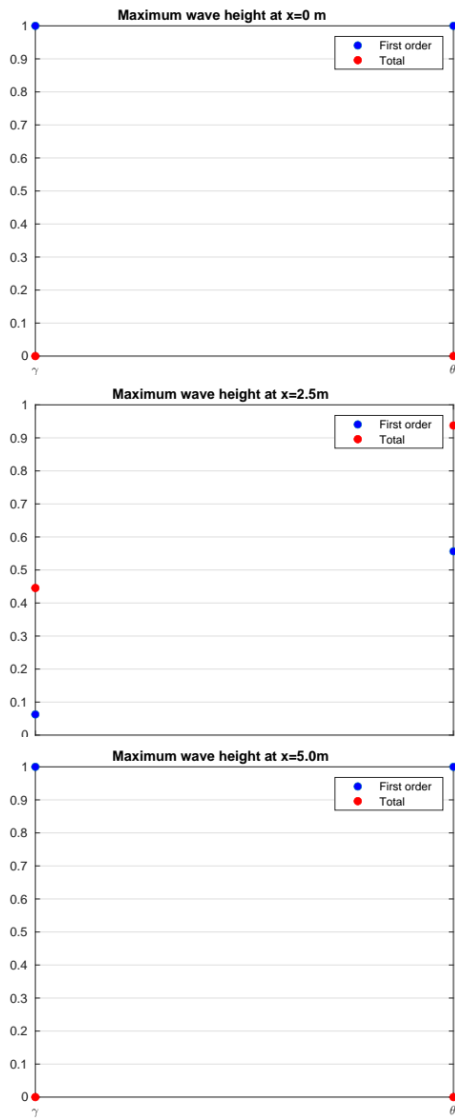


# Solitary wave on a plane beach (Hybrid)

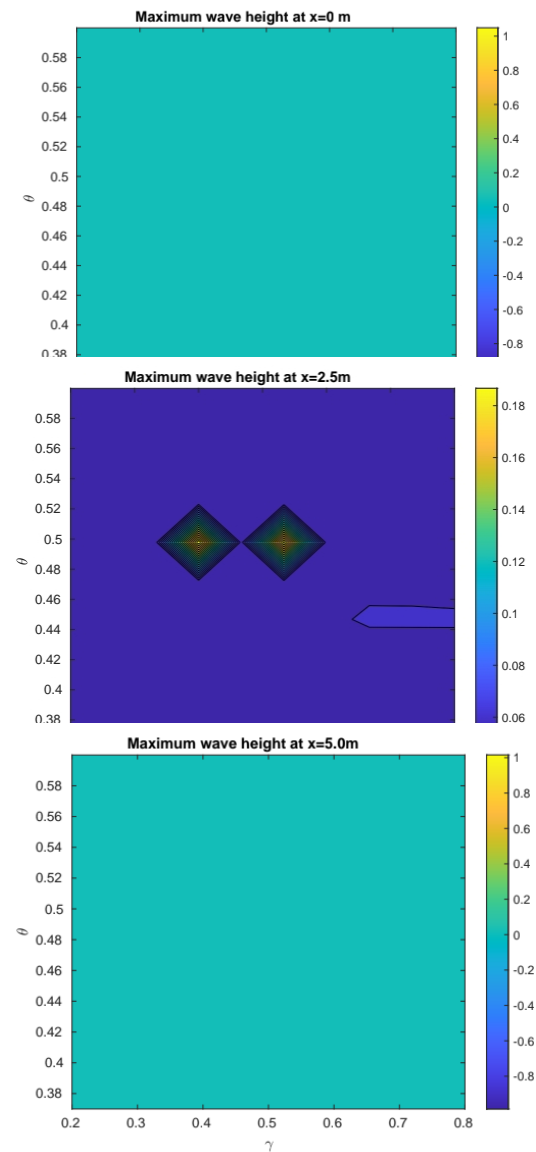
Second phase  
of the flow 6-9

Same initial  
condition

### Sobol's indices



### Contours



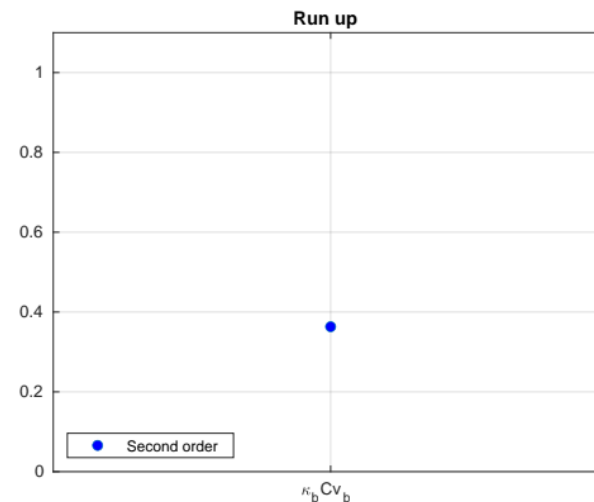
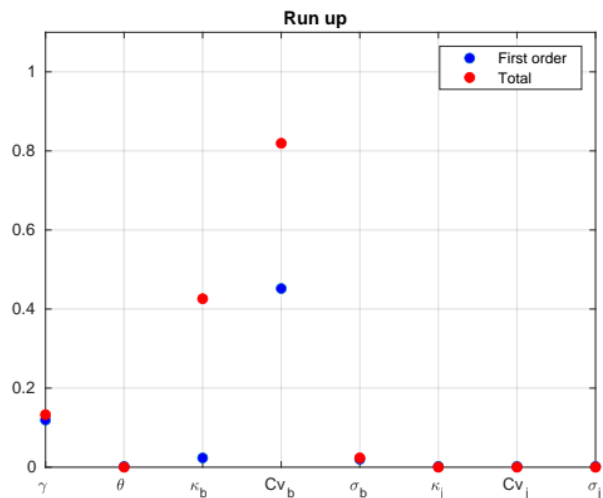
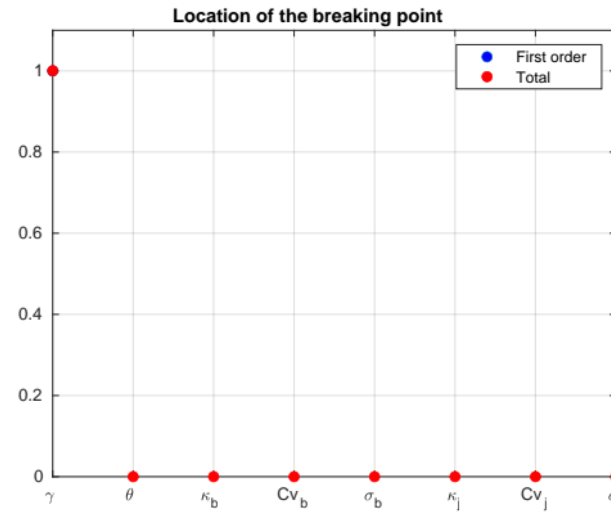
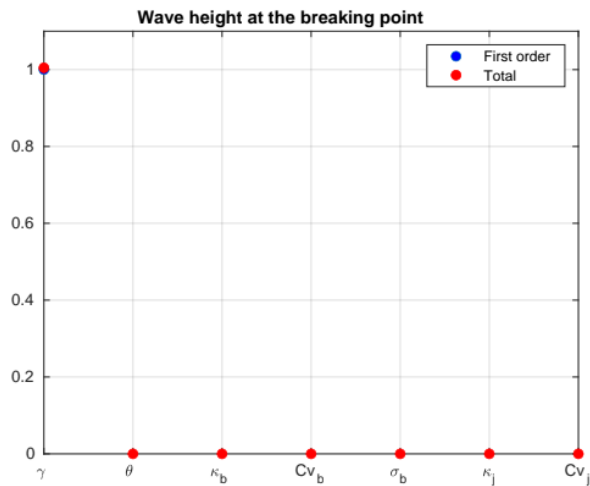
Range: 0.1402  
Std: 0.019



# Solitary wave on a plane beach (TKE)

First phase of the flow 1-5

## Sobol's indices

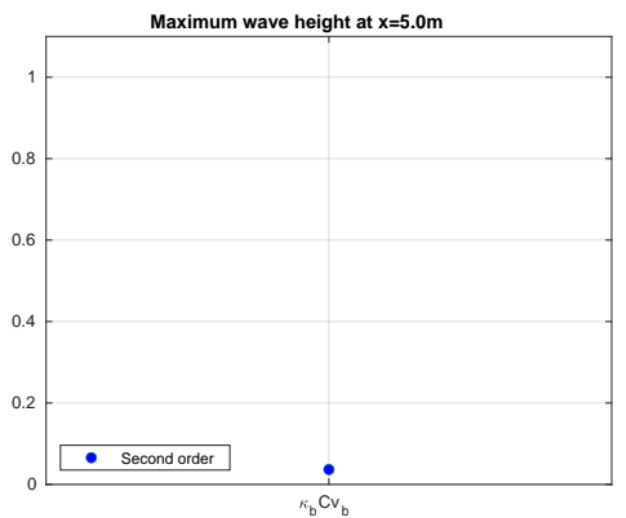
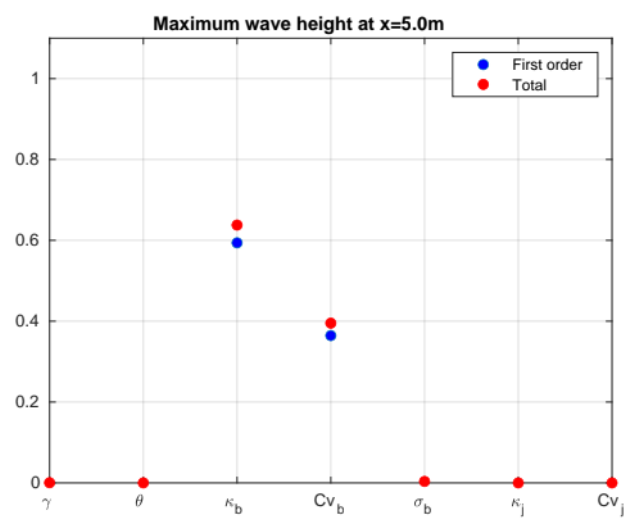
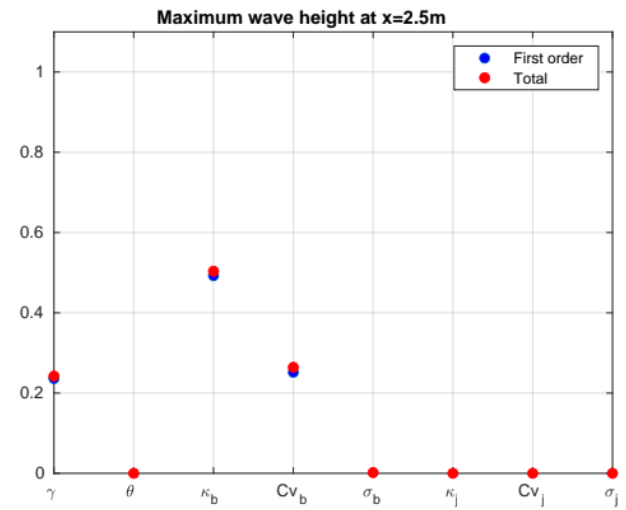
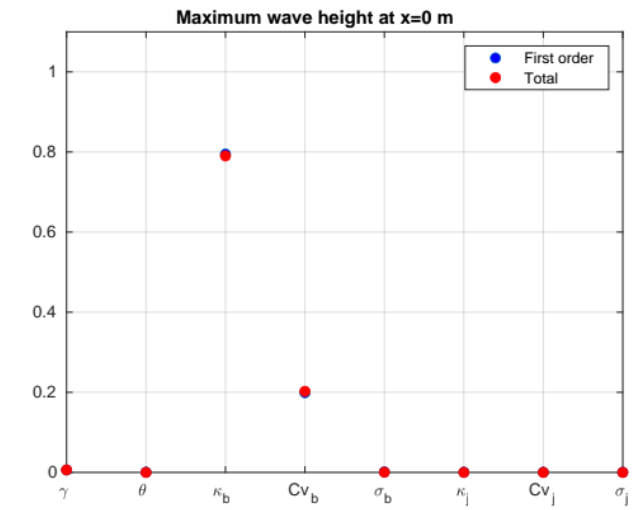


Range: 0.027  
Std: 0.0076

# Solitary wave on a plane beach (TKE)

First phase of the flow 1-5

## Sobol's indices-wave gauges

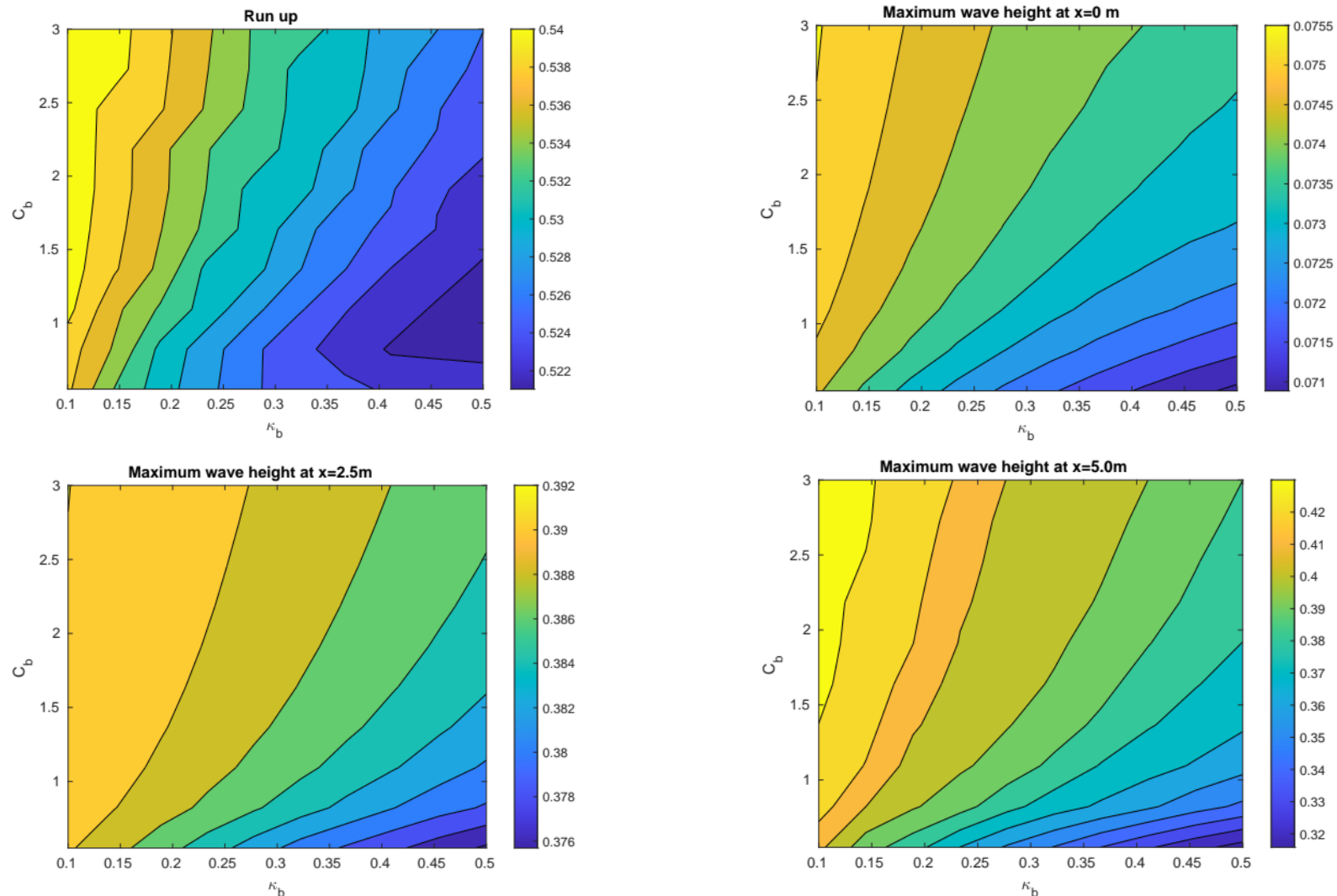


Range: 0.1501  
Std: 0.0515

# Solitary wave on a plane beach (TKE)

First phase of  
the flow 1-5

If we freeze all the input parameters except  $\kappa_b$  and  $c_b$

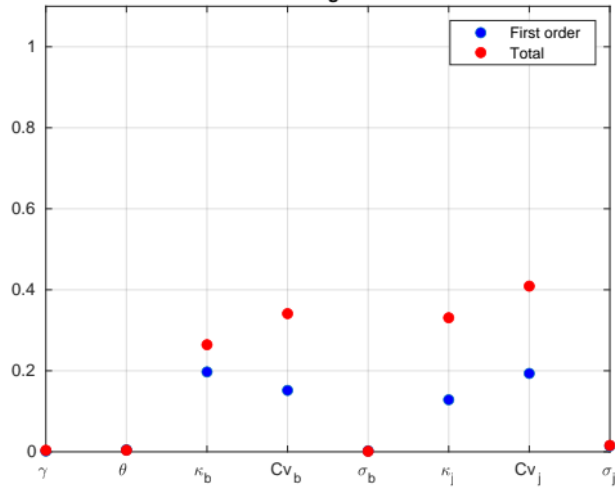


# Solitary wave on a plane beach (TKE)

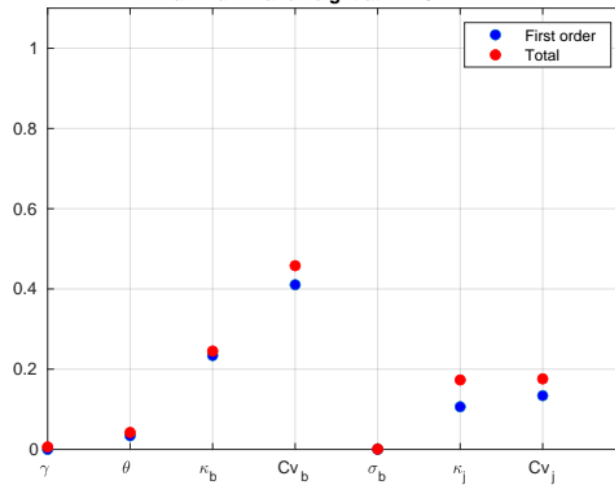
Second phase of the flow 6-9

Same initial condition

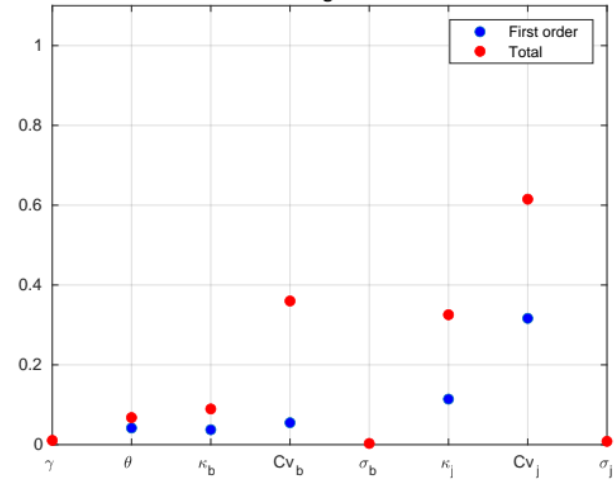
Maximum wave height at x=0 m



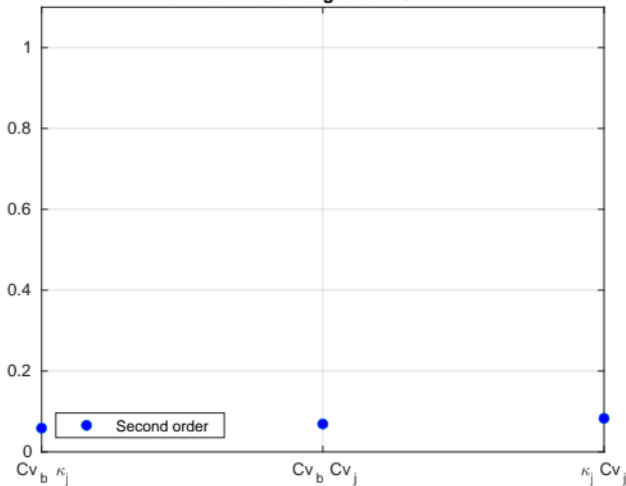
Maximum wave height at x=2.5m



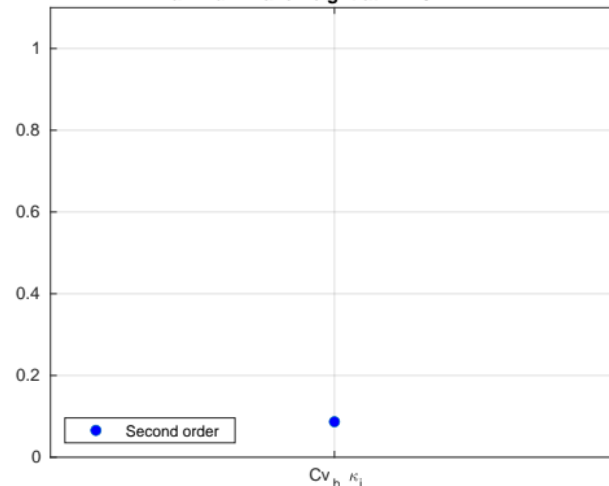
Maximum wave height at x=5.0m



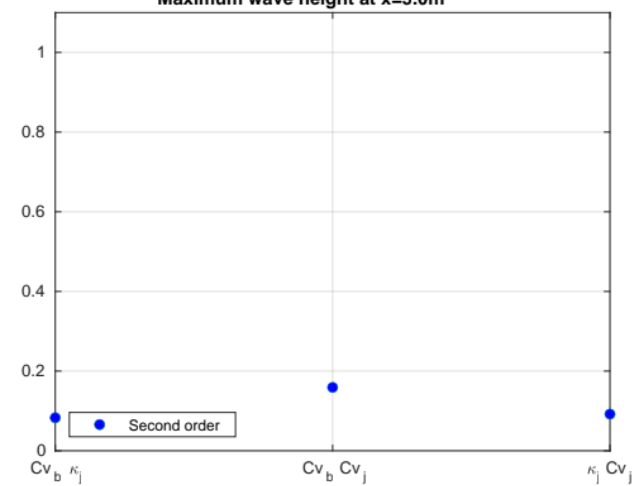
Maximum wave height at x=0 m



Maximum wave height at x=2.5m



Maximum wave height at x=5.0m

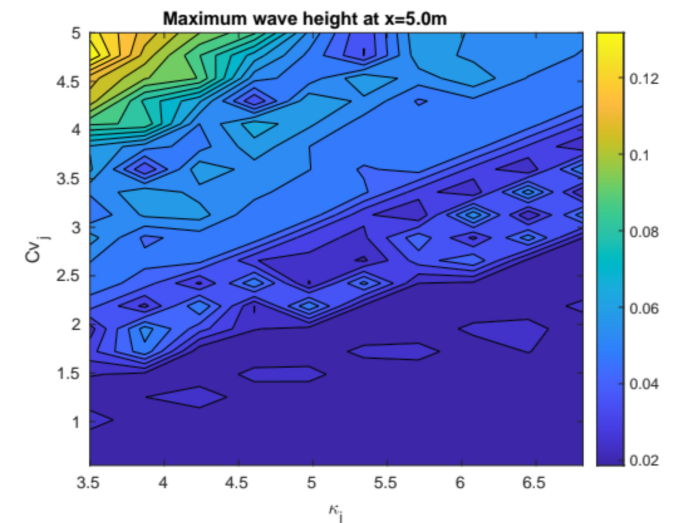
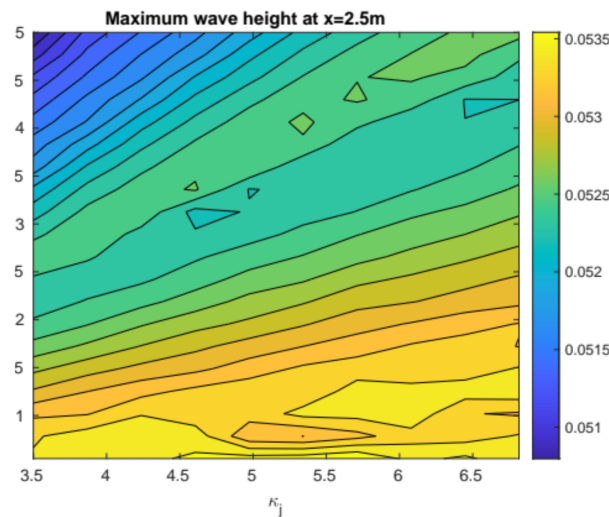
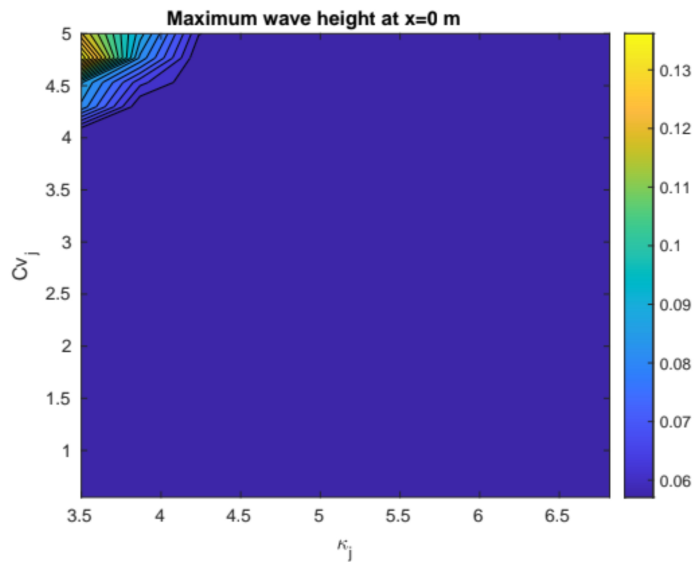
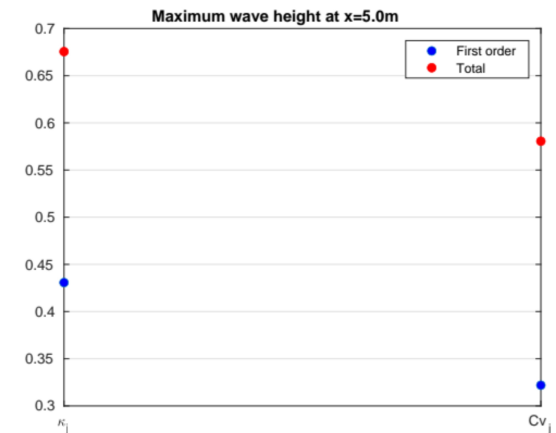
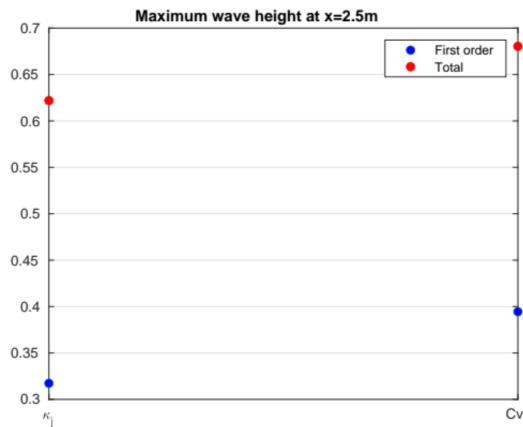
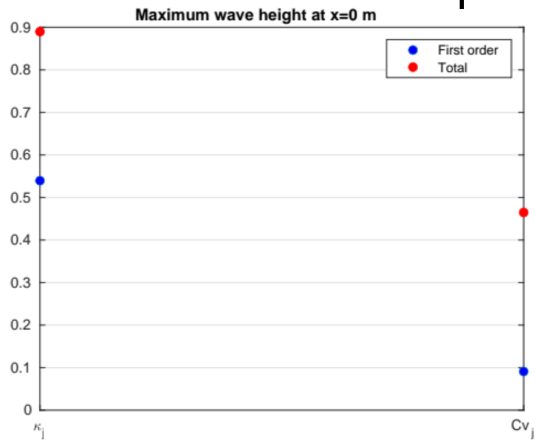


# Solitary wave on a plane beach (TKE)

Second phase of the flow 6-9

Same initial condition

If we freeze all the input parameters except  $\kappa_j$  and  $c_j$



# Conclusions

- Hybrid closure: when works , more robust
- Tke always works but many more parameters to tune
- The variation with TKE quite smooth /no discontinuity in the output parameters

*Thank you!!*