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# Dissipative Systems Analysis and Control. Theory and Applications: Erratum/Addendum. Springer Nature Switzerland AG, Communications and Control Engineering, Third Edition, uncorrected printing, 2020

Bernard Brogliato\*

September 7, 2022

*(I will be glad to receive any comment which helps me to improve this Erratum/Addendum. Updates are published at [hal.inria.fr](http://hal.inria.fr)). This erratum corresponds to the uncorrected version of the third edition.*

- page 56, Lemma 2.64:  $H(s)$  is proper, so it is expandable in nonnegative powers of  $\frac{1}{s}$  near  $s = \infty$ .
- page 94 Lemma 3.16, 1:  $\mu_{\min}(L)$  is  $\lambda_{\min}(\frac{1}{2}(L + L^\top))$ .
- section 3.1.7: there exists a canonical form for descriptor variable systems which are passive, as shown in references [47,50] page 246. Let  $(E, A, B, C, D)$  in (3.55) have a transfer function which is positive real, and the state space realisation in (3.55) be minimal (see [50] for minimality conditions for descriptor variable systems). Then the canonical form is a special Weierstrass form which writes as:

$$\begin{cases} \dot{x}_1 = A_1 x_1 + B_1 u \\ \dot{x}_2 = x_3 + B_2 u \\ 0 \times \dot{x}_3 = 0 = x_2 + B_3 u \\ y = C_1 x_1 + C_2 x_2 + C_3 x_3 + Du, \end{cases} \quad (1)$$

where  $x_3$  is the algebraic variable. It can be shown that the quadruple  $(A_1, B_1, C_1, \hat{D})$  is passive, with  $\hat{D} = D - C_3 B_2 - C_2 B_3$ , and there exists a matrix  $P_3 = P_3^\top \succcurlyeq 0$  such that  $B_3^\top P_3 = -C_3$ . The system verifies a LMI as in (3.150) (with relevant matrices) for some  $P_1 = P_1^\top \succcurlyeq 0$ . If the system is strongly SPR and minimal then  $P_1 \succ 0$ . Storage functions of the descriptor variable system are of the form  $\frac{1}{2}x_1^\top P_1 x_1 + \frac{1}{2}x_2^\top P_3 x_2$ .

- page 107, line -3, replace “for all  $x \in W$ ” by “for all  $x \in \text{Im}(W)$ ”.
- page 142, line 4 inside Remark 3.64: replace “Item 4” by “item 2”.
- page 147, replace all “(3.165)” by (3.162) and all “(3.166)” by (3.163).

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- page 153, “... $(A, B)$  is controllable, is nonnegative if...”
- page 155, inside the frame, end of the second line, replace  $\preceq$  by  $\succcurlyeq$ .
- page 161, line 4 above Remark 3.88: delete “theoremFaurre”
- page 167, lemma 3.94: the Lemma as stated in the third and the second editions, is wrong and the first inequality in (3.217) is obviously incorrect in general.

**Lemma 1** (corrected Lemma 3.94) *Let  $D \succ 0$ , and let  $\Phi : \mathbb{R}^m \rightrightarrows \mathbb{R}^m$  be a maximal monotone mapping. Then the generalised equation  $y \in Cx - D\Phi(y)$  has a unique solution.*

**Proof:**  $y \in Cx - D\Phi(y) \Leftrightarrow D^{-1}y + \Phi(y) \ni D^{-1}Cx \Leftrightarrow (D^{-1} + \Phi)(y) \ni D^{-1}Cx \Leftrightarrow y \in (D^{-1} + \Phi)^{-1}(D^{-1}Cx)$ . There are several ways to prove the result. Consider the mapping  $A : y \mapsto A(y) = (D^{-1} + \Phi)(y)$ . Let  $x_1, x_2 \in \text{dom}(A)$ ,  $y_1 \in A(x_1)$ ,  $y_2 \in A(x_2)$ . Then  $\langle x_1 - x_2, y_1 - y_2 \rangle = \langle x_1 - x_2, D^{-1}(x_1 - x_2) \rangle + \langle x_1 - x_2, \xi_1 - \xi_2 \rangle$  where  $\xi_1 \in \Phi(x_1)$ ,  $\xi_2 \in \Phi(x_2)$ . Thus  $\langle x_1 - x_2, y_1 - y_2 \rangle \geq \langle x_1 - x_2, D^{-1}(x_1 - x_2) \rangle$  (using the monotonicity of  $\Phi(\cdot)$ ), and thus  $\langle x_1 - x_2, y_1 - y_2 \rangle \geq \frac{1}{2} \lambda_{\min}(D^{-1} + D^{-\top}) \langle x_1 - x_2, x_1 - x_2 \rangle$  (since  $D^{-1} \succ 0$  and thus  $D^{-1} + D^{-\top} \succ 0$ ). Therefore the mapping  $A(\cdot)$  is strongly monotone, and so is the mapping  $y \mapsto -D^{-1}Cx + A(y)$ . Using [3, Proposition 23.36, Corollary 23.37], it follows that the generalised equation  $0 \in -D^{-1}Cx + A(y)$  has a unique solution.

The result also follows from the application of [1, Theorem 3.3], that is itself a consequence of [6, Corollary 32.25]. It states that the generalised equation  $z \in F(y) + G(y)$  with  $F(\cdot)$  single-valued Lipschitz continuous and strongly monotone and  $G(\cdot)$  set-valued maximal monotone, always has a unique solution. Here the generalized equation is  $D^{-1}y + \Phi(y) \ni D^{-1}Cx$ , hence the result follows. When  $\Phi = \partial\phi$  for some proper convex lower semicontinuous function  $\phi(\cdot)$ , then the result follows from Proposition 1 in reference [278] page 255, which states that the operator  $(D^{-1} + \Phi)^{-1}(\cdot)$  is Lipschitz continuous, single-valued and well-defined.  $\square$

Notice that  $D$  needs not be symmetric for the Lemma to hold. Assume in addition that  $D = D^\top$ , and let  $D = R^2$ ,  $R = R^\top \succ 0$ . The generalised equation is equivalent to  $R^{-1}y \in R^{-1}Cx - R\Phi(RR^{-1}y)$ . Define  $z = R^{-1}y$ , then  $z \in R^{-1}Cx - R\Phi(Rz)$ . The mapping  $\bar{\Phi} : z \mapsto R\Phi(Rz) = R^\top\Phi(Rz)$  is maximal monotone. The generalised equation  $z \in R^{-1}Cx - R\Phi(Rz) \Leftrightarrow z + \bar{\Phi}(z) \ni R^{-1}Cx \Leftrightarrow z \in (I_m + \bar{\Phi})^{-1}(R^{-1}Cx)$ . If  $\bar{\Phi} = \partial\bar{\phi}$  for some proper, convex lower semicontinuous function  $\bar{\phi}(\cdot)$ , then Lemma A.98 (page 692) can be applied to conclude about uniqueness of the solution.

Another result can be stated:

**Lemma 2** (corrected Lemma 3.94) *Let  $D$  be a  $P$ -matrix, and let  $\Phi : \mathbb{R}^m \rightrightarrows \mathbb{R}^m$  be defined as  $\Phi(y) = \partial\psi_K(y)$ , where  $K = \mathbb{R}_+^m$ , and  $\psi_K(\cdot)$  is its indicator function. Then the generalised equation  $y \in Cx - D\Phi(y)$  has a unique solution.*

**Proof:** Similarly as in the previous proof, the generalised equation is rewritten equivalently  $D^{-1}y + \Phi(y) \ni D^{-1}Cx$ , where  $P^{-1}$  is also a P-matrix. Equivalently  $y \in (D^{-1} + \Phi)^{-1}(D^{-1}Cx)$ . Using Proposition 2 in reference [278] page 255, it follows that the mapping  $(D^{-1} + \Phi)^{-1}(\cdot)$  is Lipschitz continuous, single-valued and well-defined,  $y$  being the solution of a linear complementarity problem with P-matrix.  $\boxtimes$

Of course the above results hold true in case  $\Phi(\cdot)$  is single-valued and monotone (maximality then holds). Assume now that  $\Phi(\cdot)$  is hypomonotone with constant  $k$  (this holds if  $\Phi(\cdot)$  is differentiable, with bounded derivative, and sector bounded). Then the generalised equation is rewritten as  $y \in Cx - D(\Phi(y) + ky) + kDy$  where the mapping  $y \mapsto \Phi(y) + ky$  is maximal monotone. Consequently the generalised equation is equivalent to  $D^{-1}y \in D^{-1}Cx - (\Phi(y) + ky) + ky \Leftrightarrow (D^{-1} - kI_m)y \in D^{-1}Cx - (\Phi(y) + ky)$ . The condition here is that  $D^{-1} - kI_m \succ 0 \Leftrightarrow D^{-1} \succ kI_m \Leftrightarrow D \prec kI_m$ . It is noteworthy that these results are close in spirit to the results stated in [2] where  $\Phi(\cdot)$  is a saturation function (hence monotone mapping), though the equation considered in [2] is different as it takes the form  $y = Dx - (D - I_m)\Phi(y)$ .

The case with  $D \succcurlyeq 0$  is more tricky but more general since it encompasses  $D = 0$  that yields  $y = Cx$ . Extensions can be stated in particular cases like  $D = \text{blockdiag}(D_1, 0)$  with  $D_1 \succ 0$  an  $m_1 \times m_1$  matrix,  $m_1 \leq m$ , and if  $\Phi(y) = (\Phi_1(y_1), \Phi_2(y_2), \dots, \Phi_m(y_m))^T$ . For instance if  $\phi(y) = |y_1| + |y_2| + \dots + |y_m|$ , then  $\partial\phi(y) = (\text{sgn}(y_1), \dots, \text{sgn}(y_m))^T$ . The generalised equation takes the form  $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in Cx - \begin{pmatrix} D_1(\text{sgn}(y_1), \dots, \text{sgn}(y_{m_1}))^T \\ 0 \end{pmatrix}$ . Thus the generalised equation is equivalent to  $y_2 = C_2x$  and  $y_1 \in C_1x - D_1\Phi_1(y_1)$ : both Lemmae can be applied to this reduced generalised equation:

**Corollary 1** (corrected Lemma 3.94) Assume that  $D = \begin{pmatrix} D_1 & 0 \\ 0 & 0 \end{pmatrix}$  with  $D_1$  an  $m_1 \times m_1$  matrix,  $m_1 \leq m$ . Let  $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$   $y_1 \in \mathbb{R}^{m_1}$ ,  $y_2 \in \mathbb{R}^{m-m_1}$ , and  $C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$ ,  $C_1 \in \mathbb{R}^{m_1 \times n}$ ,  $C_2 \in \mathbb{R}^{(m-m_1) \times n}$ . Assume further that  $\Phi = \begin{pmatrix} \Phi_1(y_1) \\ \Phi_2(y) \end{pmatrix}$ ,  $\Phi_1 : \mathbb{R}^{m_1} \rightrightarrows \mathbb{R}^{m_1}$ . Then the generalised equation  $y \in Cx - D\Phi(y)$  is equivalent to  $y_2 = C_2x$  and the reduced generalised equation  $y_1 \in C_1x - D_1\Phi_1(y_1)$ , and both Lemmae 1 and 2 apply to the reduced generalised equation.

- page 441, first line in (6.53) has to be changed to:

$$\dot{x}(t) = J_s \nabla H_0(x(t)) - \sum_{i=1}^m \nabla H_i(x(t)) u_i(t)$$

- page 442, (6.56) has to be changed to:

$$\begin{pmatrix} \dot{q}(t) \\ \dot{p}(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} kq(t) \\ \frac{p(t)}{m} \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_1(t) - \begin{pmatrix} 0 \\ -1 \end{pmatrix} u_2(t)$$

- page 450, (6.78) and lines before have to be changed to: “...there exist interaction Hamiltonian functions such that:  $g_i(x) = -J(x)\nabla H_i(x)$ . The port conjugated outputs are then:  $y_i = \nabla H_0^\top(x)g_i(x) = -\nabla H_0^\top(x)J(x)\nabla H_i(x)$ . The natural outputs are:  $\tilde{y}_i = H_i(x)$ . Using the drift dynamics in (6.67), their derivatives are computed as:

$$\dot{\tilde{y}}_i(t) = dH_i^\top(x(t))\dot{x}(t) = y_i(t) - \sum_{j=1, j \neq i}^m u_j dH_i^\top(x(t))J(x(t))dH_j(x(t))$$

- page 602, line after (8.117): Corollary 4 is Corollary 2 in reference [5] page 603.
- page 670, **Proposition A.68**: the items 2 and 3 of the second part of the proposition, have to be deleted since they are not equivalent to  $M \succcurlyeq 0$ . Same has to be done with the corresponding result in Bernstein’s book (Fact 8.15.12 in the first edition, Fact 8.21.22 in the second edition [4], Fact 10.24.25 in the third edition [5]). The reason why is that item 2 does not imply that  $M_{12} = 0$  when  $M_{22} = 0$ , and item 3 does not imply that  $M_{12} = 0$  when  $M_{11} = 0$  (though it should, as can be deduced for instance from Lemma A.70). **Only items 1, 4 and 5 have to be used. Unfortunately I realized this after the corrected version of the third edition was published. It is therefore still present in the corrected version. Prof. Bernstein has been informed about this error.**
- page 670, Proposition A.68: In item 3 of the first part of the proposition:  $M_{12} = M_{12}M_{22}^\dagger M_{22}$ .
- page 691, Lemma A.96: it is  $\|x_1 - x_2\|^2$  (this typo is also present in the article [70]).
- page 692, Lemma A.98: The function  $\phi(\cdot)$  is proper, convex and lower semicontinuous.

## References

- [1] S. Adly, D. Goeleven, R. Oujja, Well-posedness of nonsmooth Lurie dynamical systems involving maximal monotone operators, in *Mathematical Analysis in Interdisciplinary Research*, Springer (I.N. Parasidis, E. Providis, T.M. Rassias, Eds.).
- [2] A.A. Adegbege, R. Levenson, Dynamic multivariable algebraic loop solver for input-constrained control, IEEE Transactions on Automatic Control, DoI: 10.1109/TAC.2021.3094743.
- [3] H.H. Bauscke, P.L. Combettes, *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*, Springer, Canadian Mathematical Society, Société Mathématique du Canada, 2011.
- [4] D.S. Bernstein, *Matrix Mathematics. Theory, Facts and Formulas*, 2nd ed., Princeton University Press, 2009.
- [5] D.S. Bernstein, *Scalar, Vector, and Matrix Mathematics: Theory, Facts, and Formulas - Revised and Expanded Edition*, 2018.
- [6] E. Zeidler, *Nonlinear Functional Analysis and its Applications. II/B. Nonlinear Monotone Operators*, Springer Science+Business Media New York, 1990.