



HAL
open science

Dissipative Systems Analysis and Control. Theory and Applications: Erratum/Addendum. Springer Nature Switzerland AG, Communications and Control Engineering, Third Edition, 2016

Bernard Brogliato

► **To cite this version:**

Bernard Brogliato. Dissipative Systems Analysis and Control. Theory and Applications: Erratum/Addendum. Springer Nature Switzerland AG, Communications and Control Engineering, Third Edition, 2016. 2021. hal-03364924v1

HAL Id: hal-03364924

<https://inria.hal.science/hal-03364924v1>

Preprint submitted on 5 Oct 2021 (v1), last revised 7 Sep 2022 (v3)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Dissipative Systems Analysis and Control. Theory and Applications: Erratum/Addendum. Springer Nature Switzerland AG, Communications and Control Engineering, Third Edition, 2016

Bernard Brogliato*

October 5, 2021

(I will be glad to receive any comment which helps me to improve this Erratum/Addendum. Updates are published at hal.inria.fr).

- page 56, Lemma 2.64: $H(s)$ is proper, so it is expandable in nonnegative powers of $\frac{1}{s}$ near $s = \infty$.
- page 94 Lemma 3.16, 1: $\mu_{\min}(L)$ is $\lambda_{\min}(\frac{1}{2}(L + L^\top))$.
- section 3.1.7: there exists a canonical form for descriptor variable systems which are passive, as shown in references [47,50] page 246. Let (E, A, B, C, D) in (3.55) have a transfer function which is positive real, and the state space realisation in (3.55) be minimal (see [50] for minimality conditions for descriptor variable systems). Then the canonical form is a special Weierstrass form which writes as:

$$\begin{cases} \dot{x}_1 = A_1 x_1 + B_1 u \\ \dot{x}_2 = x_3 + B_2 u \\ 0 \times \dot{x}_3 = 0 = x_2 + B_3 u \\ y = C_1 x_1 + C_2 x_2 + C_3 x_3 + D u, \end{cases} \quad (1)$$

where x_3 is the algebraic variable. It can be shown that the quadruple (A_1, B_1, C_1, \hat{D}) is passive, with $\hat{D} = D - C_3 B_2 - C_2 B_3$, and there exists a matrix $P_3 = P_3^\top \succcurlyeq 0$ such that $B_3^\top P_3 = -C_3$. The system verifies a LMI as in (3.150) (with relevant matrices) for some $P_1 = P_1^\top \succcurlyeq 0$. If the system is strongly SPR and minimal then $P_1 \succ 0$. Storage functions of the descriptor variable system are of the form $\frac{1}{2} x_1^\top P_1 x_1 + \frac{1}{2} x_3^\top P_3 x_3$.

- page 107, line -3, replace “for all $x \in W$ ” by “for all $x \in \text{Im}(W)$ ”.
- page 142, line 4 inside Remark 3.64: replace “Item 4” by “item 2”.
- page 153, “... (A, B) is controllable, is nonnegative if...”
- page 155, inside the frame, end of the second line, replace \preccurlyeq by \succcurlyeq .

*INRIA Grenoble, Université Grenoble-Alpes, 655 avenue de l'Europe, Inovallée, 38334 Saint-Ismier, France. bernard.brogliato@inria.fr

- page 161, line 4 above Remark 3.88: delete “theoremFaurre”
- page 167, lemma 3.94: the Lemma as stated in the third and the second editions, is wrong and the first inequality in (3.217) is obviously incorrect in general.

Lemma 1 (corrected Lemma 3.94) *Let $D \succ 0$, and let $\Phi : \mathbb{R}^m \rightrightarrows \mathbb{R}^m$ be a maximal monotone mapping. Then the generalised equation $y \in Cx - D\Phi(y)$ has a unique solution.*

Proof: $y \in Cx - D\Phi(y) \Leftrightarrow D^{-1}y + \Phi(y) \ni D^{-1}Cx \Leftrightarrow (D^{-1} + \Phi)(y) \ni D^{-1}Cx \Leftrightarrow y \in (D^{-1} + \Phi)^{-1}(D^{-1}Cx)$. There are several ways to prove the result. Consider the mapping $A : y \mapsto A(y) = (D^{-1} + \Phi)(y)$. Let $x_1, x_2 \in \text{dom}(A)$, $y_1 \in A(x_1)$, $y_2 \in A(x_2)$. Then $\langle x_1 - x_2, y_1 - y_2 \rangle = \langle x_1 - x_2, D^{-1}(x_1 - x_2) \rangle + \langle x_1 - x_2, \xi_1 - \xi_2 \rangle$ where $\xi_1 \in \Phi(x_1)$, $\xi_2 \in \Phi(x_2)$. Thus $\langle x_1 - x_2, y_1 - y_2 \rangle \geq \langle x_1 - x_2, D^{-1}(x_1 - x_2) \rangle$ (using the monotonicity of $\Phi(\cdot)$), and thus $\langle x_1 - x_2, y_1 - y_2 \rangle \geq \frac{1}{2} \lambda_{\min}(D^{-1} + D^{-\top}) \langle x_1 - x_2, x_1 - x_2 \rangle$ (since $D^{-1} \succ 0$ and thus $D^{-1} + D^{-\top} \succ 0$). Therefore the mapping $A(\cdot)$ is strongly monotone, and so is the mapping $y \mapsto -D^{-1}Cx + A(y)$. Using [3, Proposition 23.36, Corollary 23.37], it follows that the generalised equation $0 \in -D^{-1}Cx + A(y)$ has a unique solution.

The result also follows from the application of [1, Theorem 3.3], that is itself a consequence of [4, Corollary 32.25]. It states that the generalised equation $z \in F(y) + G(y)$ with $F(\cdot)$ single-valued Lipschitz continuous and strongly monotone and $G(\cdot)$ set-valued maximal monotone, always has a unique solution. Here the generalized equation is $D^{-1}y + \Phi(y) \ni D^{-1}Cx$, hence the result follows. When $\Phi = \partial\phi$ for some proper convex lower semicontinuous function $\phi(\cdot)$, then the result follows from Proposition 1 in reference [278] page 255, which states that the operator $(D^{-1} + \Phi)^{-1}(\cdot)$ is Lipschitz continuous, single-valued and well-defined. \square

Notice that D needs not be symmetric for the Lemma to hold. Assume in addition that $D = D^\top$, and let $D = R^2$, $R = R^\top \succ 0$. The generalised equation is equivalent to $R^{-1}y \in R^{-1}Cx - R\Phi(RR^{-1}y)$. Define $z = R^{-1}y$, then $z \in R^{-1}Cx - R\Phi(Rz)$. The mapping $\bar{\Phi} : z \mapsto R\Phi(Rz) = R^\top\Phi(Rz)$ is maximal monotone. The generalised equation $z \in R^{-1}Cx - R\Phi(Rz) \Leftrightarrow z + \bar{\Phi}(z) \ni R^{-1}Cx \Leftrightarrow z \in (I_m + \bar{\Phi})^{-1}(R^{-1}Cx)$. If $\bar{\Phi} = \partial\bar{\phi}$ for some proper, convex lower semicontinuous function $\bar{\phi}(\cdot)$, then Lemma A.98 (page 692) can be applied to conclude about uniqueness of the solution.

Another result can be stated:

Lemma 2 (corrected Lemma 3.94) *Let D be a P-matrix, and let $\Phi : \mathbb{R}^m \rightrightarrows \mathbb{R}^m$ be defined as $\Phi(y) = \partial\psi_K(y)$, where $K = \mathbb{R}_+^m$, and $\psi_K(\cdot)$ is its indicator function. Then the generalised equation $y \in Cx - D\Phi(y)$ has a unique solution.*

Proof: Similarly as in the previous proof, the generalised equation is rewritten equivalently $D^{-1}y + \Phi(y) \ni D^{-1}Cx$, where P^{-1} is also a P-matrix. Equivalently $y \in (D^{-1} + \Phi)^{-1}(D^{-1}Cx)$. Using Proposition 2 in reference [278] page 255, it follows that the mapping $(D^{-1} + \Phi)^{-1}(\cdot)$ is

Lipschitz continuous, single-valued and well-defined, y being the solution of a linear complementarity problem with P-matrix. \boxtimes

Of course the above results hold true in case $\Phi(\cdot)$ is single-valued and monotone (maximality then holds). Assume now that $\Phi(\cdot)$ is hypomonotone with constant k (this holds if $\Phi(\cdot)$ is differentiable, with bounded derivative, and sector bounded). Then the generalised equation is rewritten as $y \in Cx - D(\Phi(y) + ky) + kDy$ where the mapping $y \mapsto \Phi(y) + ky$ is maximal monotone. Consequently the generalised equation is equivalent to $D^{-1}y \in D^{-1}Cx - (\Phi(y) + ky) + ky \Leftrightarrow (D^{-1} - kI_m)y \in D^{-1}Cx - (\Phi(y) + ky)$. The condition here is that $D^{-1} - kI_m \succ 0 \Leftrightarrow D^{-1} \succ kI_m \Leftrightarrow D \prec kI_m$. It is noteworthy that these results are close in spirit to the results stated in [2] where $\Phi(\cdot)$ is a saturation function (hence monotone mapping), though the equation considered in [2] is different as it takes the form $y = Dx - (D - I_m)\Phi(y)$.

The case with $D \succcurlyeq 0$ is more tricky but more general since it encompasses $D = 0$ that yields $y = Cx$. Extensions can be stated in particular cases like $D = \text{blockdiag}(D_1, 0)$ with $D_1 \succ 0$ an $m_1 \times m_1$ matrix, $m_1 \leq m$, and if $\Phi(y) = (\Phi_1(y_1), \Phi_2(y_2), \dots, \Phi_m(y_m))^T$. For instance if $\phi(y) = |y_1| + |y_2| + \dots + |y_m|$, then $\partial\phi(y) = (\text{sgn}(y_1), \dots, \text{sgn}(y_m))^T$. The generalised equation takes the form $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in Cx - \begin{pmatrix} D_1(\text{sgn}(y_1), \dots, \text{sgn}(y_{m_1}))^T \\ 0 \end{pmatrix}$. Thus the generalised equation is equivalent to $y_2 = C_2x$ and $y_1 \in C_1x - D_1\Phi_1(y_1)$: both Lemmae can be applied to this reduced generalised equation:

Corollary 1 (corrected Lemma 3.94) Assume that $D = \begin{pmatrix} D_1 & 0 \\ 0 & 0 \end{pmatrix}$ with D_1 an $m_1 \times m_1$ matrix, $m_1 \leq m$. Let $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ $y_1 \in \mathbb{R}^{m_1}$, $y_2 \in \mathbb{R}^{m-m_1}$, and $C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$, $C_1 \in \mathbb{R}^{m_1 \times n}$, $C_2 \in \mathbb{R}^{(m-m_1) \times n}$. Assume further that $\Phi = \begin{pmatrix} \Phi_1(y_1) \\ \Phi_2(y) \end{pmatrix}$, $\Phi_1 : \mathbb{R}^{m_1} \rightrightarrows \mathbb{R}^{m_1}$. Then the generalised equation $y \in Cx - D\Phi(y)$ is equivalent to $y_2 = C_2x$ and the reduced generalised equation $y_1 \in C_1x - D_1\Phi_1(y_1)$, and both Lemmae 1 and 2 apply to the reduced generalised equation.

- page 602, line after (8.117): Corollary 4 is Corollary 2 in reference [5] page 603.
- page 691, Lemma A.96: it is $\|x_1 - x_2\|^2$ (this typo is also present in the article [70]).
- page 692, Lemma A.98: The function $\phi(\cdot)$ is proper, convex and lower semicontinuous.

References

- [1] S. Adly, D. Goeleven, R. Oujja, Well-posedness of nonsmooth Lurie dynamical systems involving maximal monotone operators, in *Mathematical Analysis in Interdisciplinary Research*, Springer (I.N. Parasidis, E. Providis, T.M. Rassias, Eds.).
- [2] A.A. Adegbege, R. Levenson, Dynamic multivariable algebraic loop solver for input-constrained control, IEEE Transactions on Automatic Control, DoI: 10.1109/TAC.2021.3094743.

- [3] H.H. Bauscke, P.L. Combettes, *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*, Springer, Canadian Mathematical Society, Société Mathématique du Canada, 2011.
- [4] E. Zeidler, *Nonlinear Functional Analysis and its Applications. II/B. Nonlinear Monotone Operators*, Springer Science+Business Media New York, 1990.