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Dissipative Systems Analysis and Control. Theory and Applications: Erratum/Addendum. Springer Nature Switzerland AG, Communications and Control Engineering, Third Edition, 2016

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October 5, 2021

(I will be glad to receive any comment which helps me to improve this Erratum/Addendum. Updates are published at hal.inria.fr).

- page 56, Lemma 2.64: $H(s)$ is proper, so it is expandable in nonnegative powers of $\frac{1}{s}$ near $s = \infty$.
- page 94 Lemma 3.16, 1: $\mu_{\min}(L)$ is $\lambda_{\min}(\frac{1}{2}(L + L^\top))$.
- section 3.1.7: there exists a canonical form for descriptor variable systems which are passive, as shown in references [47,50] page 246. Let (E, A, B, C, D) in (3.55) have a transfer function which is positive real, and the state space realisation in (3.55) be minimal (see [50] for minimality conditions for descriptor variable systems). Then the canonical form is a special Weierstrass form which writes as:

$$\begin{cases} \dot{x}_1 = A_1 x_1 + B_1 u \\ \dot{x}_2 = x_3 + B_2 u \\ 0 \times \dot{x}_3 = 0 = x_2 + B_3 u \\ y = C_1 x_1 + C_2 x_2 + C_3 x_3 + D u, \end{cases} \quad (1)$$

where x_3 is the algebraic variable. It can be shown that the quadruple (A_1, B_1, C_1, \hat{D}) is passive, with $\hat{D} = D - C_3 B_2 - C_2 B_3$, and there exists a matrix $P_3 = P_3^\top \succcurlyeq 0$ such that $B_3^\top P_3 = -C_3$. The system verifies a LMI as in (3.150) (with relevant matrices) for some $P_1 = P_1^\top \succcurlyeq 0$. If the system is strongly SPR and minimal then $P_1 \succ 0$. Storage functions of the descriptor variable system are of the form $\frac{1}{2} x_1^\top P_1 x_1 + \frac{1}{2} x_3^\top P_3 x_3$.

- page 107, line -3, replace “for all $x \in W$ ” by “for all $x \in \text{Im}(W)$ ”.
- page 142, line 4 inside Remark 3.64: replace “Item 4” by “item 2”.
- page 153, “... (A, B) is controllable, is nonnegative if...”
- page 155, inside the frame, end of the second line, replace \preccurlyeq by \succcurlyeq .

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- page 161, line 4 above Remark 3.88: delete “theoremFaurre”
- page 167, lemma 3.94: the Lemma as stated in the third and the second editions, is wrong and the first inequality in (3.217) is obviously incorrect in general.

Lemma 1 (corrected Lemma 3.94) *Let $D \succ 0$, and let $\Phi : \mathbb{R}^m \rightrightarrows \mathbb{R}^m$ be a maximal monotone mapping. Then the generalised equation $y \in Cx - D\Phi(y)$ has a unique solution.*

Proof: $y \in Cx - D\Phi(y) \Leftrightarrow D^{-1}y + \Phi(y) \ni D^{-1}Cx \Leftrightarrow (D^{-1} + \Phi)(y) \ni D^{-1}Cx \Leftrightarrow y \in (D^{-1} + \Phi)^{-1}(D^{-1}Cx)$. There are several ways to prove the result. Consider the mapping $A : y \mapsto A(y) = (D^{-1} + \Phi)(y)$. Let $x_1, x_2 \in \text{dom}(A)$, $y_1 \in A(x_1)$, $y_2 \in A(x_2)$. Then $\langle x_1 - x_2, y_1 - y_2 \rangle = \langle x_1 - x_2, D^{-1}(x_1 - x_2) \rangle + \langle x_1 - x_2, \xi_1 - \xi_2 \rangle$ where $\xi_1 \in \Phi(x_1)$, $\xi_2 \in \Phi(x_2)$. Thus $\langle x_1 - x_2, y_1 - y_2 \rangle \geq \langle x_1 - x_2, D^{-1}(x_1 - x_2) \rangle$ (using the monotonicity of $\Phi(\cdot)$), and thus $\langle x_1 - x_2, y_1 - y_2 \rangle \geq \frac{1}{2} \lambda_{\min}(D^{-1} + D^{-\top}) \langle x_1 - x_2, x_1 - x_2 \rangle$ (since $D^{-1} \succ 0$ and thus $D^{-1} + D^{-\top} \succ 0$). Therefore the mapping $A(\cdot)$ is strongly monotone, and so is the mapping $y \mapsto -D^{-1}Cx + A(y)$. Using [3, Proposition 23.36, Corollary 23.37], it follows that the generalised equation $0 \in -D^{-1}Cx + A(y)$ has a unique solution.

The result also follows from the application of [1, Theorem 3.3], that is itself a consequence of [4, Corollary 32.25]. It states that the generalised equation $z \in F(y) + G(y)$ with $F(\cdot)$ single-valued Lipschitz continuous and strongly monotone and $G(\cdot)$ set-valued maximal monotone, always has a unique solution. Here the generalized equation is $D^{-1}y + \Phi(y) \ni D^{-1}Cx$, hence the result follows. When $\Phi = \partial\phi$ for some proper convex lower semicontinuous function $\phi(\cdot)$, then the result follows from Proposition 1 in reference [278] page 255, which states that the operator $(D^{-1} + \Phi)^{-1}(\cdot)$ is Lipschitz continuous, single-valued and well-defined. \square

Notice that D needs not be symmetric for the Lemma to hold. Assume in addition that $D = D^\top$, and let $D = R^2$, $R = R^\top \succ 0$. The generalised equation is equivalent to $R^{-1}y \in R^{-1}Cx - R\Phi(RR^{-1}y)$. Define $z = R^{-1}y$, then $z \in R^{-1}Cx - R\Phi(Rz)$. The mapping $\bar{\Phi} : z \mapsto R\Phi(Rz) = R^\top\Phi(Rz)$ is maximal monotone. The generalised equation $z \in R^{-1}Cx - R\Phi(Rz) \Leftrightarrow z + \bar{\Phi}(z) \ni R^{-1}Cx \Leftrightarrow z \in (I_m + \bar{\Phi})^{-1}(R^{-1}Cx)$. If $\bar{\Phi} = \partial\bar{\phi}$ for some proper, convex lower semicontinuous function $\bar{\phi}(\cdot)$, then Lemma A.98 (page 692) can be applied to conclude about uniqueness of the solution.

Another result can be stated:

Lemma 2 (corrected Lemma 3.94) *Let D be a P-matrix, and let $\Phi : \mathbb{R}^m \rightrightarrows \mathbb{R}^m$ be defined as $\Phi(y) = \partial\psi_K(y)$, where $K = \mathbb{R}_+^m$, and $\psi_K(\cdot)$ is its indicator function. Then the generalised equation $y \in Cx - D\Phi(y)$ has a unique solution.*

Proof: Similarly as in the previous proof, the generalised equation is rewritten equivalently $D^{-1}y + \Phi(y) \ni D^{-1}Cx$, where P^{-1} is also a P-matrix. Equivalently $y \in (D^{-1} + \Phi)^{-1}(D^{-1}Cx)$. Using Proposition 2 in reference [278] page 255, it follows that the mapping $(D^{-1} + \Phi)^{-1}(\cdot)$ is

Lipschitz continuous, single-valued and well-defined, y being the solution of a linear complementarity problem with P-matrix. \boxtimes

Of course the above results hold true in case $\Phi(\cdot)$ is single-valued and monotone (maximality then holds). Assume now that $\Phi(\cdot)$ is hypomonotone with constant k (this holds if $\Phi(\cdot)$ is differentiable, with bounded derivative, and sector bounded). Then the generalised equation is rewritten as $y \in Cx - D(\Phi(y) + ky) + kDy$ where the mapping $y \mapsto \Phi(y) + ky$ is maximal monotone. Consequently the generalised equation is equivalent to $D^{-1}y \in D^{-1}Cx - (\Phi(y) + ky) + ky \Leftrightarrow (D^{-1} - kI_m)y \in D^{-1}Cx - (\Phi(y) + ky)$. The condition here is that $D^{-1} - kI_m \succ 0 \Leftrightarrow D^{-1} \succ kI_m \Leftrightarrow D \prec kI_m$. It is noteworthy that these results are close in spirit to the results stated in [2] where $\Phi(\cdot)$ is a saturation function (hence monotone mapping), though the equation considered in [2] is different as it takes the form $y = Dx - (D - I_m)\Phi(y)$.

The case with $D \succcurlyeq 0$ is more tricky but more general since it encompasses $D = 0$ that yields $y = Cx$. Extensions can be stated in particular cases like $D = \text{blockdiag}(D_1, 0)$ with $D_1 \succ 0$ an $m_1 \times m_1$ matrix, $m_1 \leq m$, and if $\Phi(y) = (\Phi_1(y_1), \Phi_2(y_2), \dots, \Phi_m(y_m))^T$. For instance if $\phi(y) = |y_1| + |y_2| + \dots + |y_m|$, then $\partial\phi(y) = (\text{sgn}(y_1), \dots, \text{sgn}(y_m))^T$. The generalised equation takes the form $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in Cx - \begin{pmatrix} D_1(\text{sgn}(y_1), \dots, \text{sgn}(y_{m_1}))^T \\ 0 \end{pmatrix}$. Thus the generalised equation is equivalent to $y_2 = C_2x$ and $y_1 \in C_1x - D_1\Phi_1(y_1)$: both Lemmae can be applied to this reduced generalised equation:

Corollary 1 (corrected Lemma 3.94) Assume that $D = \begin{pmatrix} D_1 & 0 \\ 0 & 0 \end{pmatrix}$ with D_1 an $m_1 \times m_1$ matrix, $m_1 \leq m$. Let $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ $y_1 \in \mathbb{R}^{m_1}$, $y_2 \in \mathbb{R}^{m-m_1}$, and $C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$, $C_1 \in \mathbb{R}^{m_1 \times n}$, $C_2 \in \mathbb{R}^{(m-m_1) \times n}$. Assume further that $\Phi = \begin{pmatrix} \Phi_1(y_1) \\ \Phi_2(y) \end{pmatrix}$, $\Phi_1 : \mathbb{R}^{m_1} \rightrightarrows \mathbb{R}^{m_1}$. Then the generalised equation $y \in Cx - D\Phi(y)$ is equivalent to $y_2 = C_2x$ and the reduced generalised equation $y_1 \in C_1x - D_1\Phi_1(y_1)$, and both Lemmae 1 and 2 apply to the reduced generalised equation.

- page 602, line after (8.117): Corollary 4 is Corollary 2 in reference [5] page 603.
- page 691, Lemma A.96: it is $\|x_1 - x_2\|^2$ (this typo is also present in the article [70]).
- page 692, Lemma A.98: The function $\phi(\cdot)$ is proper, convex and lower semicontinuous.

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