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Bernard Brogliato*

September 7, 2022

(I will be glad to receive any comment which helps me to improve this Erratum/Addendum. Updates are published at hal.inria.fr). This erratum corresponds to the uncorrected version of the third edition.

- page 56, Lemma 2.64: $H(s)$ is proper, so it is expandable in nonnegative powers of $\frac{1}{s}$ near $s = \infty$.
- page 94 Lemma 3.16, 1: $\mu_{\min}(L)$ is $\lambda_{\min}(\frac{1}{2}(L + L^T))$.
- section 3.1.7: there exists a canonical form for descriptor variable systems which are passive, as shown in references [47,50] page 246. Let $(E,A,B,C,D)$ in (3.55) have a transfer function which is positive real, and the state space realisation in (3.55) be minimal (see [50] for minimality conditions for descriptor variable systems). Then the canonical form is a special Weierstrass form which writes as:

$$\begin{align*}
\dot{x}_1 &= A_1x_1 + B_1u \\
\dot{x}_2 &= x_3 + B_2u \\
0 \times \dot{x}_3 &= 0 = x_2 + B_3u \\
y &= C_1x_1 + C_2x_2 + C_3x_3 + Du,
\end{align*}$$

where $x_3$ is the algebraic variable. It can be shown that the quadruple $(A_1, B_1, C_1, \hat{D})$ is passive, with $\hat{D} = D - C_3B_2 - C_2B_3$, and there exists a matrix $P_3 = P_3^T \succ 0$ such that $B_3^TP_3 = -C_3$. The system verifies a LMI as in (3.150) (with relevant matrices) for some $P_1 = P_1^T \succ 0$. If the system is strongly SPR and minimal then $P_1 \succ 0$. Storage functions of the descriptor variable system are of the form $\frac{1}{2}x_1^TP_1x_1 + \frac{1}{2}x_2^TP_3x_2$.

- page 107, line -3, replace “for all $x \in W$” by “for all $x \in \text{Im}(W)$”.
- page 142, line 4 inside Remark 3.64: replace “Item 4” by “item 2”.
- page 147, replace all “(3.165)” by (3.162) and all “(3.166)” by (3.163).

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• page 153, “...(A, B) is controllable, is nonnegative if...

• page 155, inside the frame, end of the second line, replace ≤ by ≥.

• page 161, line 4 above Remark 3.88: delete “theoremFauvre”

• page 167, lemma 3.94: the Lemma as stated in the third and the second editions, is wrong and the first inequality in (3.217) is obviously incorrect in general.

**Lemma 1** (corrected Lemma 3.94) Let \( D > 0 \), and let \( \Phi : \mathbb{R}^m \to \mathbb{R}^m \) be a maximal monotone mapping. Then the generalised equation \( y \in Cx - D\Phi(y) \) has a unique solution.

**Proof:** \( y \in Cx - D\Phi(y) \Leftrightarrow D^{-1}y + \Phi(y) \ni D^{-1}Cx \Leftrightarrow (D^{-1} + \Phi)(y) \ni D^{-1}Cx \Leftrightarrow y \in (D^{-1} + \Phi)^{-1}(D^{-1}Cx) \). There are several ways to prove the result. Consider the mapping \( A : y \mapsto A(y) = (D^{-1} + \Phi)(y) \). Let \( x_1, x_2 \in \text{dom}(A), y_1 \in A(x_1), y_2 \in A(x_2) \). Then \( \langle x_1 - x_2, y_1 - y_2 \rangle = \langle x_1 - x_2, D^{-1}(x_1 - x_2) + \Phi(x_1) - \Phi(x_2), where \xi_1 \in \Phi(x_1), \xi_2 \in \Phi(x_2) \). Thus \( \langle x_1 - x_2, y_1 - y_2 \rangle \geq \langle x_1 - x_2, D^{-1}(x_1 - x_2) \rangle \) (using the monotonicity of \( \Phi(\cdot) \)), and thus \( x_1 - x_2, y_1 - y_2 \rangle \geq \frac{1}{2} \lambda_{\text{min}}(D^{-1} + D^{-\top})\langle x_1 - x_2, x_1 - x_2 \rangle \) (since \( D^{-1} \succ 0 \) and thus \( D^{-1} + D^{-\top} \succ 0 \)). Therefore the mapping \( A(\cdot) \) is strongly monotone, and so is the mapping \( y \mapsto -D^{-1}Cx + A(y) \). Using [3, Proposition 23.36, Corollary 23.37], it follows that the generalised equation \( 0 \in -D^{-1}Cx + A(y) \) has a unique solution.

The result also follows from the application of [1, Theorem 3.3], that is itself a consequence of [6, Corollary 32.25]. It states that the generalised equation \( z \in F(y) + G(y) \) with \( F(\cdot) \) single-valued Lipschitz continuous and strongly monotone and \( G(\cdot) \) set-valued maximal monotone, always has a unique solution. Here the generalized equation is \( D^{-1}y + \Phi(y) \ni D^{-1}Cx \), hence the result follows. When \( \Phi = \partial \phi \) for some proper convex lower semicontinuous function \( \phi(\cdot) \), then the result follows from Proposition 1 in reference [278] page 255, which states that the operator \( (D^{-1} + \Phi)^{-1}(\cdot) \) is Lipschitz continuous, single-valued and well-defined. \( \square \)

Notice that \( D \) needs not be symmetric for the Lemma to hold. Assume in addition that \( D = D^\top \), and let \( D = R^2, R = R^\top \succ 0 \). The generalised equation is equivalent to \( R^{-1}y \in R^{-1}Cx - R\Phi(RR^{-1}y) \). Define \( z = R^{-1}y \), then \( z \in R^{-1}Cx - R\Phi(Rz) \). The mapping \( \tilde{\Phi} : z \mapsto R\Phi(Rz) = R^\top \Phi(Rz) \) is maximal monotone. The generalised equation \( z \in R^{-1}Cx - R\Phi(Rz) \Leftrightarrow z + \Phi(z) \ni R^{-1}Cx \Leftrightarrow z \in (I_m + \Phi)^{-1}(R^{-1}Cx) \). If \( \Phi = \partial \bar{\phi} \) for some proper, convex lower semicontinuous function \( \bar{\phi}(\cdot) \), then Lemma A.98 (page 692) can be applied to conclude about uniqueness of the solution.

Another result can be stated:

**Lemma 2** (corrected Lemma 3.94) Let \( D \) be a P-matrix, and let \( \Phi : \mathbb{R}^m \to \mathbb{R}^m \) be defined as \( \Phi(y) = \partial \psi_K(y) \), where \( K = \mathbb{R}_{+}^m \), and \( \psi_K(\cdot) \) is its indicator function. Then the generalised equation \( y \in Cx - D\Phi(y) \) has a unique solution.
Proof: Similarly as in the previous proof, the generalised equation is rewritten equivalently
\[ D^{-1}y + \Phi(y) \geq D^{-1}Cx, \] where \( P^{-1} \) is also a P-matrix. Equivalently \( y \in (D^{-1} + \Phi)^{-1}(D^{-1}Cx) \).
Using Proposition 2 in reference [278] page 255, it follows that the mapping \( (D^{-1} + \Phi)^{-1}(\cdot) \) is Lipschitz continuous, single-valued and well-defined, \( y \) being the solution of a linear complementarity problem with P-matrix. ∎

Of course the above results hold true in case \( \Phi(\cdot) \) is single-valued and monotone (maximality then holds). Assume now that \( \Phi(\cdot) \) is hypomonotone with constant \( k \) (this holds if \( \Phi(\cdot) \) is differentiable, with bounded derivative, and sector bounded). Then the generalised equation is rewritten as \( y \in Cx - D(\Phi(y) + ky) + kDy \) where the mapping \( y \mapsto \Phi(y) + ky \) is maximal monotone. Consequently the generalised equation is equivalent to \( D^{-1}y \in D^{-1}Cx - (\Phi(y) + ky) + ky \Leftrightarrow (D^{-1} - kI_m)y \in D^{-1}Cx - (\Phi(y) + ky) \). The condition here is that \( D^{-1} - kI_m \succ 0 \Leftrightarrow D^{-1} \succ kI_m \Leftrightarrow D \preceq kI_m \). It is noteworthy that these results are close in spirit to the results stated in [2] where \( \Phi(\cdot) \) is a saturation function (hence monotone mapping), though the equation considered in [2] is different as it takes the form \( y = Dx - (D - I_m)\Phi(y) \).

The case with \( D \succ 0 \) is more tricky but more general since it encompasses \( D = 0 \) that yields \( y = Cx \). Extensions can be stated in particular cases like \( D = \text{blockdiag}(D_1, 0) \) with \( D_1 \succ 0 \) an \( m_1 \times m_1 \) matrix, \( m_1 \leq m \), and if \( \Phi(y) = (\Phi_1(y_1), \Phi_2(y_2), \ldots, \Phi_m(y_m))^\top \). For instance if \( \phi(y) = |y_1| + |y_2| + \ldots + |y_m| \), then \( \partial \phi(y) = (\text{sgn}(y_1), \ldots, \text{sgn}(y_m))^\top \). The generalised equation takes the form \( \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in Cx - \begin{pmatrix} D_1(\text{sgn}(y_1), \ldots, \text{sgn}(y_m))^\top \\ 0 \end{pmatrix} \). Thus the generalised equation is equivalent to \( y_2 = C_2x \) and \( y_1 \in C_1x - D_1\Phi_1(y_1) \); both Lemmata can be applied to this reduced generalised equation:

**Corollary 1** (corrected Lemma 3.94) Assume that \( D = \begin{pmatrix} D_1 & 0 \\ 0 & 0 \end{pmatrix} \) with \( D_1 \) an \( m_1 \times m_1 \) matrix, \( m_1 \leq m \). Let \( y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \), \( y_1 \in \mathbb{R}^{m_1}, \ y_2 \in \mathbb{R}^{n-m_1}, \) and \( C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}, \ C_1 \in \mathbb{R}^{m_1 \times n}, \ C_2 \in \mathbb{R}^{(m-m_1) \times n} \). Assume further that \( \Phi = \begin{pmatrix} \Phi_1(y_1) \\ \Phi_2(y) \end{pmatrix}, \ \Phi_1 : \mathbb{R}^{m_1} \Rightarrow \mathbb{R}^{m_1}. \) Then the generalised equation \( y \in Cx - D\Phi(y) \) is equivalent to \( y_2 = C_2x \) and the reduced generalised equation \( y_1 \in C_1x - D_1\Phi_1(y_1) \), and both Lemmata 1 and 2 apply to the reduced generalised equation.

- page 441, first line in (6.53) has to be changed to:
  \[ \dot{x}(t) = J_s\nabla H_0(x(t)) - \sum_{i=1}^{m} \nabla H_i(x(t)) \ u_i(t) \]

- page 442, (6.56) has to be changed to:
  \[
  \begin{pmatrix} \dot{q}(t) \\ \dot{p}(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} kq(t) \\ p(t) \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_1(t) - \begin{pmatrix} 0 \\ -1 \end{pmatrix} u_2(t)
  \]
there exist interaction Hamiltonian functions such that:
\[ g_i(x) = -J(x)\nabla H_i(x). \]
The port conjugated outputs are then:
\[ y_i = \nabla H_0^\top(x)g_i(x) = -\nabla H_0^\top(x)J(x)\nabla H_i(x). \]
The natural outputs are:
\[ \tilde{y}_i = H_i(x). \]
Using the drift dynamics in (6.67), their derivatives are computed as:
\[
\dot{\tilde{y}}_i(t) = dH_i^\top(x(t))\dot{x}(t) = y_i(t) - \sum_{j=1,j\neq i}^{m} u_j dH_i^\top(x(t))J(x(t))dH_j(x(t)).
\]


- page 670, Proposition A.68: the items 2 and 3 of the second part of the proposition, have to be deleted since they are not equivalent to \( M \succ 0 \). Same has to be done with the corresponding result in Bernstein’s book (Fact 8.15.12 in the first edition, Fact 8.21.22 in the second edition [4], Fact 10.24.25 in the third edition [5]). The reason why is that item 2 does not imply that \( M_{12} = 0 \) when \( M_{22} = 0 \), and item 3 does not imply that \( M_{12} = 0 \) when \( M_{11} = 0 \) (though it should, as can be deduced for instance from Lemma A.70). \textbf{Only items 1, 4 and 5 have to be used. Unfortunately I realized this after the corrected version of the third edition was published. It is therefore still present in the corrected version. Prof. Bernstein has been informed about this error.}

- page 670, Proposition A.68: In item 3 of the first part of the proposition: \( M_{12} = M_{12}M_{22}^\top M_{22} \).

- page 691, Lemma A.96: it is \( ||x_1 - x_2||^2 \) (this typo is also present in the article [70]).

- page 692, Lemma A.98: The function \( \phi(\cdot) \) is proper, convex and lower semicontinuous.

References


