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(I will be glad to receive any comment which helps me to improve this Erratum/Addendum. Updates are published at hal.inria.fr). This erratum corresponds to the uncorrected version of the third edition.

- page 56, Lemma 2.64: \( H(s) \) is proper, so it is expandable in nonnegative powers of \( \frac{1}{s} \) near \( s = \infty \).
- page 94 Lemma 3.16, 1: \( \mu_{\min}(L) \) is \( \lambda_{\min}(\frac{1}{2}(L + L^\top)) \).
- section 3.1.7: there exists a canonical form for descriptor variable systems which are passive, as shown in references [47,50] page 246. Let \((E,A,B,C,D)\) in (3.55) have a transfer function which is positive real, and the state space realisation in (3.55) be minimal (see [50] for minimality conditions for descriptor variable systems). Then the canonical form is a special Weierstrass form which writes as:

\[
\begin{align*}
\dot{x}_1 &= A_1 x_1 + B_1 u \\
\dot{x}_2 &= x_3 + B_2 u \\
0 \times \dot{x}_3 &= 0 = x_2 + B_3 u \\
y &= C_1 x_1 + C_2 x_2 + C_3 x_3 + D u,
\end{align*}
\]

(1)

where \( x_3 \) is the algebraic variable. It can be shown that the quadruple \((A_1, B_1, C_1, \hat{D})\) is passive, with \( \hat{D} = D - C_3 B_2 - C_2 B_3 \), and there exists a matrix \( P_3 = P_3^\top \succ 0 \) such that \( B_3^\top P_3 = -C_3 \). The system verifies a LMI as in (3.150) (with relevant matrices) for some \( P_1 = P_1^\top \succ 0 \). If the system is strongly SPR and minimal then \( P_1 \succ 0 \). Storage functions of the descriptor variable system are of the form \( \frac{1}{2} x_1^\top P_1 x_1 + \frac{1}{2} x_2^\top P_3 x_2 \).

- page 107, line -3, replace “for all \( x \in W \)” by “for all \( x \in \text{Im}(W) \)”.
- page 142, line 4 inside Remark 3.64: replace “Item 4” by “item 2”.
- page 147, replace all “(3.165)” by (3.162) and all “(3.166)” by (3.163).

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Lemma 1 (corrected Lemma 3.94) Let $D > 0$, and let $\Phi : \mathbb{R}^m \rightarrow \mathbb{R}^m$ be a maximal monotone mapping. Then the generalised equation $y \in Cx - D\Phi(y)$ has a unique solution.

Proof: $y \in Cx - D\Phi(y) \iff D^{-1}y + \Phi(y) \ni D^{-1}Cx \iff (D^{-1} + \Phi)(y) \ni D^{-1}Cx \iff y \in (D^{-1} + \Phi)^{-1}(D^{-1}Cx)$. There are several ways to prove the result. Consider the mapping $A : y \mapsto A(y) = (D^{-1} + \Phi)(y)$. Let $x_1, x_2 \in \text{dom}(A)$, $y_1 \in A(x_1)$, $y_2 \in A(x_2)$. Then $\langle x_1-x_2, y_1-y_2 \rangle \geq \langle x_1-x_2, D^{-1}(x_1-x_2) \rangle + \langle x_1-x_2, \xi_1-\xi_2 \rangle$ where $\xi_1 \in \Phi(x_1), \xi_2 \in \Phi(x_2)$. Thus $\langle x_1-x_2, y_1-y_2 \rangle \geq \frac{1}{2}\lambda_{\text{min}}(D^{-1} + D^{-\top})(x_1-x_2)^\top(x_1-x_2)$ (since $D^{-1} > 0$ and thus $D^{-1} + D^{-\top} > 0$). Therefore the mapping $A(\cdot)$ is strongly monotone, and so is the mapping $y \mapsto -D^{-1}Cx + A(y)$. Using [3, Proposition 23.36, Corollary 23.37], it follows that the generalised equation $0 \in -D^{-1}Cx + A(y)$ has a unique solution.

The result also follows from the application of [1, Theorem 3.3], that is itself a consequence of [6, Corollary 32.25]. It states that the generalised equation $z \in F(y) + G(y)$ with $F(\cdot)$ single-valued Lipschitz continuous and strongly monotone and $G(\cdot)$ set-valued maximal monotone, always has a unique solution. Here the generalized equation is $D^{-1}y + \Phi(y) \ni D^{-1}Cx$, hence the result follows. When $\Phi = \partial \phi$ for some proper convex lower semicontinuous function $\phi(\cdot)$, then the result follows from Proposition 1 in reference [278] page 255, which states that the operator $(D^{-1} + \Phi)^{-1}(\cdot)$ is Lipschitz continuous, single-valued and well-defined.

Notice that $D$ needs not be symmetric for the Lemma to hold. Assume in addition that $D = D^\top$, and let $D = R^2$, $R = R^\top > 0$. The generalised equation is equivalent to $R^{-1}y \in R^{-1}Cx - R\Phi(RR^{-1}y)$. Define $z = R^{-1}y$, then $z \in R^{-1}Cx - R\Phi(Rz)$. The mapping $\hat{\Phi} : z \mapsto R\Phi(Rz) = R^\top \Phi(Rz)$ is maximal monotone. The generalised equation $z \in R^{-1}Cx - R\Phi(Rz) \iff z + \hat{\Phi}(z) \ni R^{-1}Cx \iff z \in (I_m + \hat{\Phi})^{-1}(R^{-1}Cx)$. If $\Phi = \partial \phi$ for some proper, convex lower semicontinuous function $\hat{\phi}(\cdot)$, then Lemma A.98 (page 692) can be applied to conclude about uniqueness of the solution.

Another result can be stated:

Lemma 2 (corrected Lemma 3.94) Let $D$ be a $P$-matrix, and let $\Phi : \mathbb{R}^m \Rightarrow \mathbb{R}^m$ be defined as $\Phi(y) = \partial \psi_K(y)$, where $K = \mathbb{R}^m_+$, and $\psi_K(\cdot)$ is its indicator function. Then the generalised equation $y \in Cx - D\Phi(y)$ has a unique solution.
\section*{Proof:} Similarly as in the previous proof, the generalised equation is rewritten equivalently\( D^{-1} y + \Phi(y) \ni D^{-1} C x \), where \( P^{-1} \) is also a P-matrix. Equivalently \( y \in (D^{-1} + \Phi)^{-1} (D^{-1} C x) \). Using Proposition 2 in reference [278] page 255, it follows that the mapping \((D^{-1} + \Phi)^{-1} (\cdot)\) is Lipschitz continuous, single-valued and well-defined, \( y \) being the solution of a linear complementarity problem with P-matrix. \( \Box \)

Of course the above results hold true in case \( \Phi(\cdot) \) is single-valued and monotone (maximality then holds). Assume now that \( \Phi(\cdot) \) is hypomonotone with constant \( k \) (this holds if \( \Phi(\cdot) \) is differentiable, with bounded derivative, and sector bounded). Then the generalised equation is rewritten as \( y \in C x - D(\Phi(y) + ky) + k Dy \) where the mapping \( y \mapsto \Phi(y) + ky \) is maximal monotone. Consequently the generalised equation is equivalent to \( D^{-1} y \in D^{-1} C x - (\Phi(y) + ky) + ky \Leftrightarrow (D^{-1} - kI_m)y \in D^{-1} C x - (\Phi(y) + ky) \). The condition here is that \( D^{-1} - kI_m \succ 0 \Leftrightarrow \exists I \in kI_m \Rightarrow D \prec kI_m \). It is noteworthy that these results are close in spirit to the results stated in [2] where \( \Phi(\cdot) \) is a saturation function (hence monotone mapping), though the equation considered in [2] is different as it takes the form \( y = Dx - (D - I_m) \Phi(y) \).

The case with \( D \succ 0 \) is more tricky but more general since it encompasses \( D = 0 \) that yields \( y = C x \). Extensions can be stated in particular cases like \( D = \text{blockdiag}(D_1, 0) \) with \( D_1 \succ 0 \) an \( m_1 \times m_1 \) matrix, \( m_1 \leq m \), and if \( \Phi(y) = (\Phi_1(y_1), \Phi_2(y_2), \ldots, \Phi_m(y_m))^T \). For instance if \( \phi(y) = |y_1| + |y_2| + \ldots + |y_m| \), then \( \partial \phi(y) = (\text{sgn}(y_1), \ldots, \text{sgn}(y_m))^T \). The generalised equation takes the form \( (y_1, y_2) \in C x - (D_1(\text{sgn}(y_1), \ldots, \text{sgn}(y_m))^T, 0) \). Thus the generalised equation is equivalent to \( y_2 = C_2 x \) and \( y_1 \in C_1 x - D_1 \Phi_1(y_1) \): both Lemmae can be applied to this reduced generalised equation.

\begin{corollary}[corrected Lemma 3.94] Assume that \( D = \begin{pmatrix} D_1 & 0 \\ 0 & 0 \end{pmatrix} \) with \( D_1 \) an \( m_1 \times m_1 \) matrix, \( m_1 \leq m \). Let \( y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \) \( y_1 \in \mathbb{R}^{m_1}, \ y_2 \in \mathbb{R}^{m-m_1}, \ \text{and} \ C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}, \ C_1 \in \mathbb{R}^{m_1 \times n}, \ C_2 \in \mathbb{R}^{(m-m_1) \times n} \). Assume further that \( \Phi = \begin{pmatrix} \Phi_1(y_1) \\ \Phi_2(y) \end{pmatrix}, \ \Phi_1 : \mathbb{R}^{m_1} \rightrightarrows \mathbb{R}^{m_1}. \) Then the generalised equation \( y \in C x - D \Phi(y) \) is equivalent to \( y_2 = C_2 x \) and the reduced generalised equation \( y_1 \in C_1 x - D_1 \Phi_1(y_1) \), and both Lemmae 1 and 2 apply to the reduced generalised equation.
\end{corollary}

- page 441, first line in (6.53) has to be changed to:
\[ \dot{y}(t) = J_y \nabla H_0(x(t)) - \sum_{i=1}^m \nabla H_i(x(t)) \ u_i(t) \]

- page 442, (6.56) has to be changed to:
\[ \begin{pmatrix} \dot{q}(t) \\ \dot{p}(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} kq(t) \\ p(t) \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_1(t) - \begin{pmatrix} 0 \\ -1 \end{pmatrix} u_2(t) \]
...there exist interaction Hamiltonian functions such that: \( g_i(x) = -J(x)\nabla H_i(x) \). The port conjugated outputs are then: \( y_i = \nabla H_0^\top(x)g_i(x) = -\nabla H_0^\top(x)J(x)\nabla H_i(x) \). The natural outputs are: \( \tilde{y}_i = H_i(x) \). Using the drift dynamics in (6.67), their derivatives are computed as:

\[
\dot{\tilde{y}}_i(t) = dH_i^\top(x(t))\dot{x}(t) = y_i(t) - \sum_{j=1,j\neq i}^m u_jdH_i^\top(x(t))J(x(t))dH_j(x(t))
\]

- page 670, Proposition A.68: In item 3 of the first part of the proposition: \( M_{12} = M_{12}M_2^\top M_{22} \).
- page 691, Lemma A.96: it is \( ||x_1 - x_2||^2 \) (this typo is also present in the article [70]).
- page 692, Lemma A.98: The function \( \phi(\cdot) \) is proper, convex and lower semicontinuous.

References


