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# A Bayesian Framework for Poisson Process Characterization of Extremes with Objective Prior

### Théo Moins<sup>1</sup>, Julyan Arbel<sup>1</sup>, Anne Dutfoy<sup>2</sup>, Stéphane Girard<sup>1</sup>

<sup>1</sup>Statify, Inria Grenoble Rhône-Alpes

<sup>2</sup>EDF R&D dept. Périclès

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### Motivations

Let  $(X_1, \ldots, X_n)$  be i.i.d. r.v. and the associated point process  $N_n$ , evaluated on  $I_u = [u, +\infty)$ .

#### Theorem 1 (S. Coles 2001)

Under mild conditions and for a sufficiently large u,  $N_n$  can be approximated by a non-homogeneous Poisson process N of intensity measure  $\Lambda$  with parameters  $\theta = (\mu, \sigma, \xi) \in \mathbb{R} \times \mathbb{R}^+ \times \mathbb{R}$ , such that for all x > u,

$$\Lambda(I_x) = \int_x^{+\infty} \lambda(t) dt = \begin{cases} \left\{1 + \xi \left(\frac{x-\mu}{\sigma}\right)\right\}_+^{-\frac{1}{\xi}} & \text{if } \xi \neq 0, \\ \exp(-\frac{x-\mu}{\sigma}) & \text{if } \xi = 0. \end{cases}$$



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Likelihood: 
$$p_{PP}(\mathbf{x} \mid \mu, \sigma, \xi) = \exp\left\{-m\left(1 + \xi\left(\frac{u - \mu}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\} \sigma^{-n} \prod_{i=1}^{n} \left(1 + \xi\left(\frac{x_i - \mu}{\sigma}\right)\right)^{-\frac{1}{\xi} - 1}$$

Inference on  $\theta = (\mu, \sigma, \xi)$  using Bayesian algorithms such as MCMC.

The main advantages are (S. G. Coles et al. 1996):

- Consideration of expert information with informative prior,
- Can be used in any case, even when the likelihood is irregular (for  $\xi < -1$ ),
- Access to the posterior predictive distribution:

$$p(\tilde{x} \mid \mathbf{x}) = \int \underbrace{p(\tilde{x} \mid \boldsymbol{\theta})}_{\text{randomness}} \underbrace{p(\boldsymbol{\theta} \mid \mathbf{x})}_{\text{parameter}} d\boldsymbol{\theta}.$$

randomness of future obs.

parameter uncertainty

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Computational Bayesian issues: prior elicitation, reparameterization, etc.

## Uninformative prior for extremes

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### Theorem 2 (Moins et al.)

The uniform prior on  $(\mu, \log \sigma, \xi)$  leads to a proper posterior for the Poisson process model for extremes as soon as  $n \ge 3$ .

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#### Theorem 3 (Moins et al.)

Jeffreys prior for a Poisson process for extremes with parameters  $\theta = (\mu, \sigma, \xi)$  exists provided  $\xi > -1/2$ , and can be written as

$$\pi_{J,PP}(\mu,\sigma,\xi) \propto rac{\left(1+\xi\left(rac{u-\mu}{\sigma}
ight)
ight)^{-rac{3}{2\xi}-1}}{\sigma^2(1+\xi)(1+2\xi)^{rac{1}{2}}}.$$

It is improper and leads to a proper posterior as soon as  $\xi > -1/2$ .

Alternative parameterisation: instead of  $(\mu, \sigma, \xi)$ , run MCMC on an orthogonal parameterisation suggested for another purpose by (Chavez-Demoulin et al. 2005):

$$(r,\nu,\xi) = \left(m\left(1+\xi\left(\frac{u-\mu}{\sigma}\right)\right)^{-1/\xi},(1+\xi)(\sigma+\xi(u-\mu)),\xi\right).$$

*r*: intensity of PP, expected number of exceedances. This ensures full orthogonality, which means that  $I(r, \nu, \xi)$  is diagonal. Alternative parameterisation: instead of  $(\mu, \sigma, \xi)$ , run MCMC on an orthogonal parameterisation suggested for another purpose by (Chavez-Demoulin et al. 2005):

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Jeffreys prior:

$$\pi_{J,PP}(r,
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Our experiments show better convergence with orthogonal parameters.

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