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# A Bayesian Framework for Poisson Process Characterization of Extremes with Objective Prior

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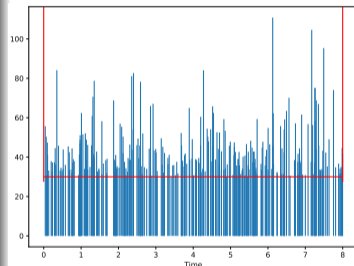
# Motivations

Let  $(X_1, \dots, X_n)$  be i.i.d. r.v. and the associated point process  $N_n$ , evaluated on  $I_u = [u, +\infty)$ .

## Theorem 1 (S. Coles 2001)

*Under mild conditions and for a sufficiently large  $u$ ,  $N_n$  can be approximated by a non-homogeneous Poisson process  $N$  of intensity measure  $\Lambda$  with parameters  $\theta = (\mu, \sigma, \xi) \in \mathbb{R} \times \mathbb{R}^+ \times \mathbb{R}$ , such that for all  $x > u$ ,*

$$\Lambda(I_x) = \int_x^{+\infty} \lambda(t) dt = \begin{cases} \left\{ 1 + \xi \left( \frac{x-\mu}{\sigma} \right) \right\}_+^{-\frac{1}{\xi}} & \text{if } \xi \neq 0, \\ \exp\left(-\frac{x-\mu}{\sigma}\right) & \text{if } \xi = 0. \end{cases}$$



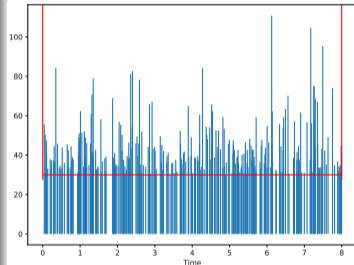
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$$\text{Likelihood: } p_{PPP}(\mathbf{x} \mid \mu, \sigma, \xi) = \exp \left\{ -m \left( 1 + \xi \left( \frac{u-\mu}{\sigma} \right) \right)^{-\frac{1}{\xi}} \right\} \sigma^{-n} \prod_{i=1}^n \left( 1 + \xi \left( \frac{x_i - \mu}{\sigma} \right) \right)^{-\frac{1}{\xi} - 1}$$

# The Bayesian Point of view

Inference on  $\theta = (\mu, \sigma, \xi)$  using Bayesian algorithms such as MCMC.

The main advantages are (S. G. Coles et al. 1996):

- Consideration of expert information with informative prior,
- Can be used in any case, even when the likelihood is irregular (for  $\xi < -1$ ),
- Access to the posterior predictive distribution:

$$p(\tilde{x} | \mathbf{x}) = \int \underbrace{p(\tilde{x} | \theta)}_{\text{randomness of future obs.}} \underbrace{p(\theta | \mathbf{x})}_{\text{parameter uncertainty}} d\theta.$$

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**Computational Bayesian issues:** prior elicitation, reparameterization, etc.

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*The uniform prior on  $(\mu, \log \sigma, \xi)$  leads to a proper posterior for the Poisson process model for extremes as soon as  $n \geq 3$ .*



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## Theorem 3 (Moins et al.)

*Jeffreys prior for a Poisson process for extremes with parameters  $\theta = (\mu, \sigma, \xi)$  exists provided  $\xi > -1/2$ , and can be written as*

$$\pi_{J,PP}(\mu, \sigma, \xi) \propto \frac{(1 + \xi (\frac{u-\mu}{\sigma}))^{-\frac{3}{2\xi}-1}}{\sigma^2(1 + \xi)(1 + 2\xi)^{\frac{1}{2}}}.$$

*It is improper and leads to a proper posterior as soon as  $\xi > -1/2$ .*

# An orthogonal parameterisation

**Alternative parameterisation:** instead of  $(\mu, \sigma, \xi)$ , run MCMC on an orthogonal parameterisation suggested for another purpose by (Chavez-Demoulin et al. 2005):

$$(r, \nu, \xi) = \left( m \left( 1 + \xi \left( \frac{u - \mu}{\sigma} \right) \right)^{-1/\xi}, (1 + \xi)(\sigma + \xi(u - \mu)), \xi \right).$$

$r$ : intensity of PP, expected number of exceedances.

This ensures full orthogonality, which means that  $I(r, \nu, \xi)$  is diagonal.

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**Our experiments show better convergence with orthogonal parameters.**

# References

- Chavez-Demoulin, V. and A.C. Davison (2005). “Generalized additive modelling of sample extremes”. In: *Journal of the Royal Statistical Society: Series C (Applied Statistics)* 54.1, pp. 207–222.
- Coles, S. G. and E. A. Powell (1996). “Bayesian Methods in Extreme Value Modelling: A Review and New Developments”. In: *International Statistical Review / Revue Internationale de Statistique* 64.1, pp. 119–136.
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