

The retina as a dynamical system Bruno Cessac

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The retina as a dynamical system

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The retina and its dynamics



« Encoder »

« Decoder »



« Encoder »





« Encoder »

« Decoder »







Most neurons in the retina do not spike (except Gcells) =>

« Analogic » computing.

Retina's layered structure



Visual cortex

Stimuli are non stationary, with spatio-temporal correlations.

A modeller point of view

- The retina is a *high dimensional, non autonomous and noisy* dynamical system, layered and structured, with *non stationary and spatially inhomogeneous entries* (visual scenes).
 - Local circuits of non linear cells, connected with non linear synapses efficiently process the local visual information. These local circuits are connected together, spanning the whole retina in a regular tiling.
- 3. The conjunction of non linearities and regular structure can produce a highly complex *spatio-temporal* dynamics.

4. The retina is sensitive to *changes in its physiology* (development, pharmacology, pathologies). These changes can be addressed from the point of view of *bifurcations theory*.

Retinal anticipation

Random connectivity and motion anticipation

S. Souihel (PhD 2016-2019), Biovision team, INRIA, Sophia Antipolis F. Chavane, INT, Marseille A. Destexhe, UNIC, Paris O. Marre, Institut de la Vision, Paris B. Cessac, Biovision team, INRIA, Sophia Antipolis



Project : ANR Trajectory.

Visual Anticipation

Anticipation is carried out by the primary visual cortex (V1) through an activation wave



Source : Benvenutti et al. 2015

Visual Anticipation

Anticipation also takes place in the retina



Source : Berry et al. 1999









Souihel S., Cessac B., "On the potential role of lateral connectivity in retinal anticipation", Journal of Mathematical Neuroscience, BioMed Central, 2021, 11, 10.1186/s13408-020-00101-z

(t),

$$\begin{cases} \frac{dV_{B_{i}}}{dt} = -\frac{1}{\tau_{B}}V_{B_{i}} + \sum_{j=1}^{N_{A}}W_{B_{i}}^{A_{j}}V_{A_{j}} + F_{B_{i}}\\ \frac{dV_{A_{j}}}{dt} = -\frac{1}{\tau_{A}}V_{A_{j}} + \sum_{i=1}^{N_{B}}W_{A_{j}}^{B_{i}}R_{B_{i}},\\ \frac{dA_{B_{i}}}{dt} = -\frac{A_{B_{i}}}{\tau_{a}} + h_{B}\mathcal{N}(V_{B_{i}}). \end{cases}$$





















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Linear regime

$$\frac{d\vec{\mathcal{X}}}{dt} = \mathcal{L}.\vec{\mathcal{X}} + \vec{\mathcal{F}}(t).$$

$$\mathcal{L} = \begin{pmatrix} -\frac{I_{N,N}}{\tau_B} & W_B^A & 0_{N,N} \\ W_A^B & -\frac{I_{N,N}}{\tau_A} & 0_{N,N} \\ h_B I_{N,N} & 0_{N,N} & -\frac{I_{N,N}}{\tau_a} \end{pmatrix}$$





Fig.7. Starbust atteories cells as attained with tucher yellow in wholemount rabbit telow.











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bving

Gain control

Figure 15: Effect of anticipatory mechanisms on a parabolic trajectory. A) Response to a dot moving along a parabolic trajectory. The first line shows the stimulus, the second line shows the GCells response with gain control, the third line presents the effect of lateral ACells connectivity with $w = 0.3 m s^{-1}$, and the last line shows the effect of asymmetric gap junctions with $v_{gap} = 9 mm/s$. B) Time course response of a cell responding to the dot near the trajectory turning point. Linear response corresponds to the response to the stimulus without gain control.







Mathematical extensions

- Non linear effects (gain control, rectification) maintain the dynamics in a compact set even when some eigenvalues of L are unstable ;
- This non linear dynamical system can be approximated by a piecewise linear one where dynamics is linear in convex domains spaning the phase space;
- The evolution under the influence of a time-dependent stimulus can be computed as a composition of (non commuting) linear operators labelled by their domains of continuity ;

Perspectives

• A retino-cortical (V1) model of anticipation (with F. Chavane, S. Chemla, A. Destexhe, S. Souihel, M. Di Volo).

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Figure 11: Comparing the spatio-temporal activity with the different drives. Left column figures display the spatiotemporal retinal activity with different anticipatory mechanisms, and the right column displays the corresponding spatio-temporal cortical activity. The red dashed lines denote the start and the end times of the LN retinal and cortical activities, as a reference base.

Logiciel Macular

Modèle rétino-cortical de Macular



Simulation du modèle de Macular



Perspectives

- A retino-cortical (V1) model of anticipation (with F. Chavane, S. Chemla, A. Destexhe, S. Souihel, M. Di Volo).
- Is our model able to anticipate more complex trajectories, with accelerations for instance ?
- How to better define connectivity using biology ?
- Would it be possible to design psycho-physical tests clearly showing the role of the retina in visual anticipation ?
- How do a moving object + lateral connectivity shape the spatiotemporal correlations in ganglion cells spike trains emitted toward the brain ?

Linear response for spiking neuronal networks with unbounded memory

R. Cofré, CIMFAV, Facultad de Ingenieria, Universidad de Valparaiso, Valparaiso, Chile

Ignacio Ampuero, Departamento de Informatica, Universidad Tecnica Federico Santa Maria, Valparaiso, Chile





30	
25	
20	
Neuron	
10	
5	







Sample averaging (non stationary)



How to compute the spatio-temporal response of a neuronal network to a time dependent stimulus ?

Can we compute the time variation of spike-observables as a function of the stimulus and spontaneous dynamics ?

B. Cessac, I. Ampuero, R. Cofre, Linear response for spiking neuronal networks with unbounded memory. Entropy, MDPI, 2021, 23 (2), pp.155.

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t,\omega) V_k = i_k(t,\omega).$$

- Linear in V.
- Spike history-dependent.

Dynamics integration

$$F_k(t_1, t, \omega) = e^{-\frac{1}{C_k} \int_{t_1 \vee \tau_k(t, \omega)}^t g_k(u, \omega) \, du}$$
$$V_k(t, \omega) = \underbrace{V_k^{(sp)}(t, \omega) + V_k^{(S)}(t, \omega)}_{V_k^{(det)}(t, \omega)} + V_k^{(noise)}(t, \omega)$$

$$V_k^{(sp)}(t,\omega) = V_k^{(syn)}(t,\omega) + V_k^{(L)}(t,\omega),$$

$$V_k^{(syn)}(t,\omega) = \frac{1}{C_k} \sum_{j=1}^N W_{kj} \int_{\tau_k(t,\omega)}^t \Gamma_k(t_1,t,\omega) \alpha_{kj}(t_1,\omega) dt_1$$

A conductance-based Integrate and Fire model

Variable length Markov chain with unbounded memory

$$P_n\left[\omega(n) \mid \omega_{-\infty}^{n-1}\right] \equiv \Pi\left(\omega(n), \frac{\theta - V_k^{(det)}(n-1,\omega)}{\sigma_k(n-1,\omega)}\right)$$



B. Cessac, I. Ampuero, R. Cofre, Linear response for spiking neuronal networks with unbounded memory. Entropy, MDPI, 2021, 23 (2), pp.155.

GIF model without stimulus \Rightarrow *stationary Gibbs distribution*.

time-dependent stimulus $S(t) \Rightarrow$ non stationary Gibbs distribution.

How is the average of an observable $f(\omega, t)$ affected by the stimulus ?

If S is weak enough: $\delta \mu [f(t)] = [\kappa_f * S](t)$, (linear response).

$$\kappa_{k,f}(t-t_{1}) = \frac{1}{C_{k}} \sum_{r=-\infty}^{n=[t]} \mathcal{C}^{(sp)}\left[f(t,\cdot), \frac{\mathcal{H}_{k}^{(1)}(r,\cdot)}{\sigma_{k}(r-1,\cdot)} \int_{\tau_{k}(r-1,\cdot)}^{r-1} S_{k}(t_{1})\Gamma_{k}(t_{1},r-1,\cdot)dt_{1}\right]$$

Firing rates

Computations have been done using the INRIA Pranas software, B. Cessac et al, Frontiers in Neuroinformatics, Vol 11, page 49, (2017).



Figure: Linear response of $f(\omega, n) = \omega_{k_c}(n)$ for different values of stimulus amplitude A.

Higher order observable

Computations have been done using the INRIA Pranas software, B. Cessac et al, Frontiers in Neuroinformatics, Vol 11, page 49, (2017).



Figure: Linear response of the observable $f(n, \omega) = \omega_{k_c-2}(n-3)\omega_{k_c}(n)$.

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