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The retina as a dynamical system

Bruno Cessac

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The retina as a dynamical system

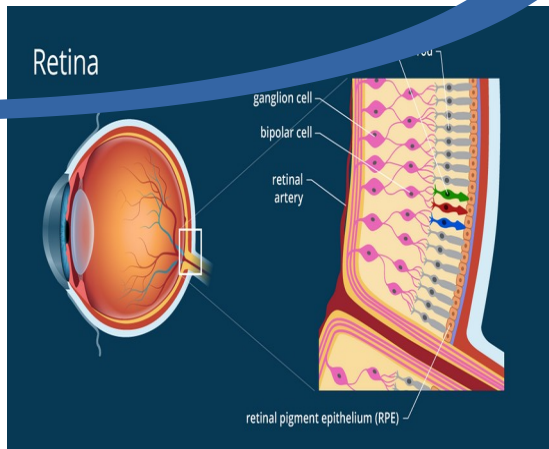
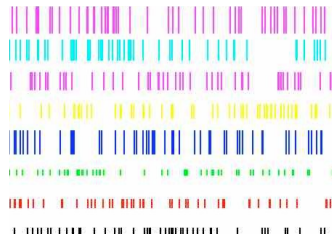
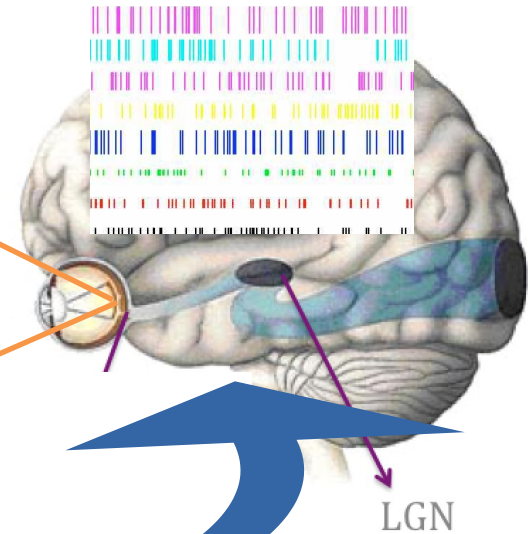
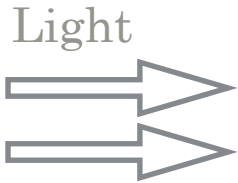
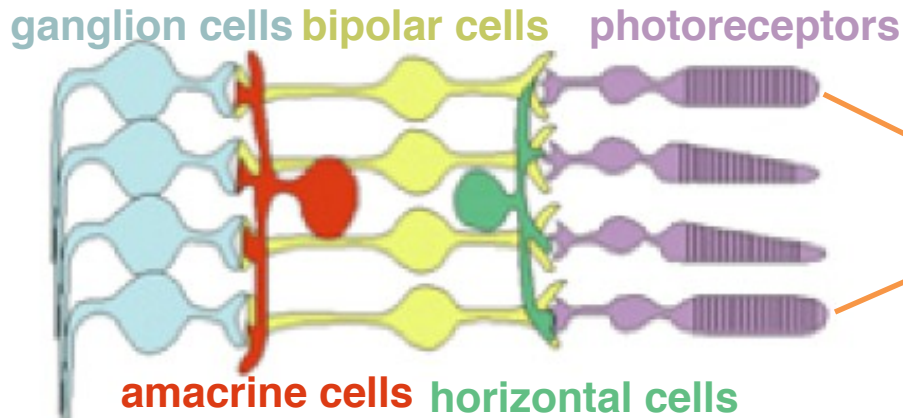
B. Cessac, Université Côte d'Azur, INRIA,
Biovision team and Neuromod Institute
Sophia Antipolis, France



The retina and its dynamics

The visual system

Retina's layered structure

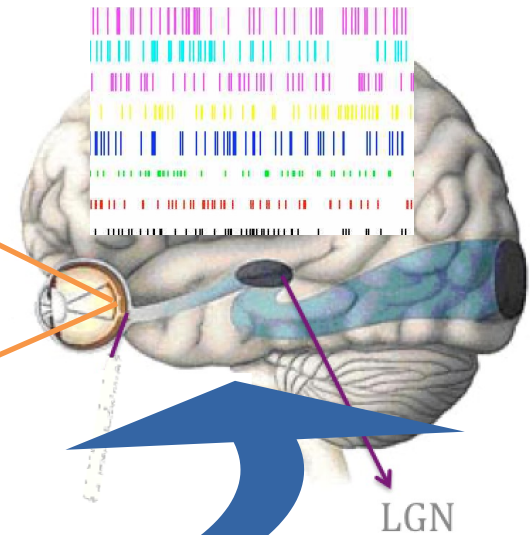
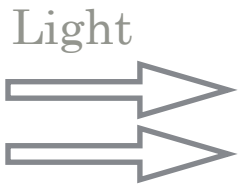
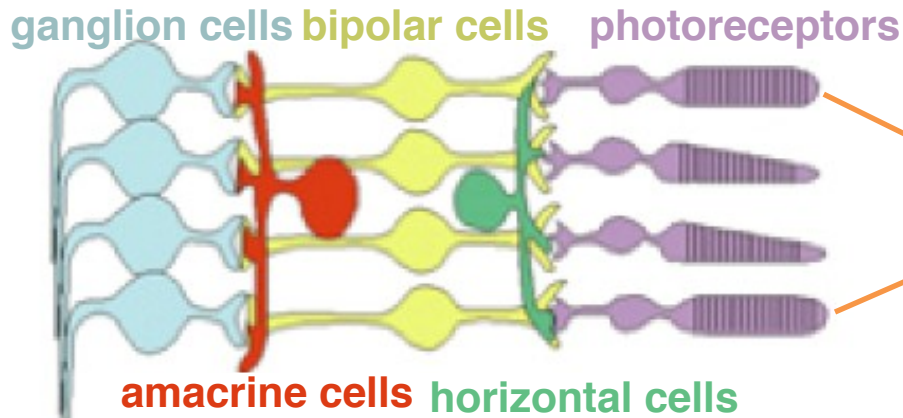


The visual system

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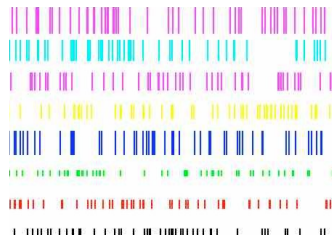
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Retina's layered structure



Visual cortex

LGN



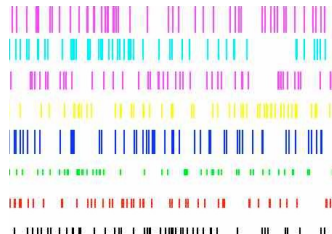
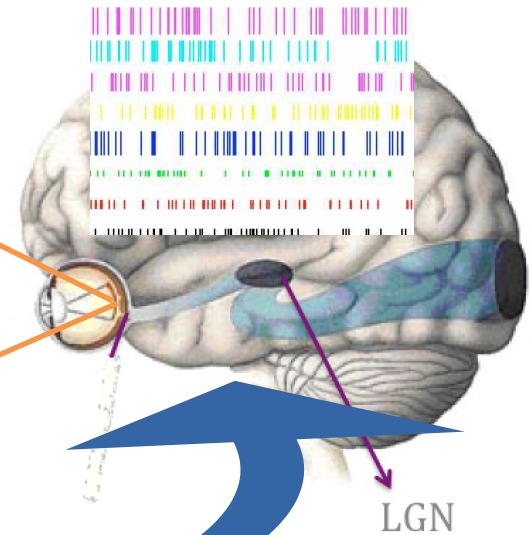
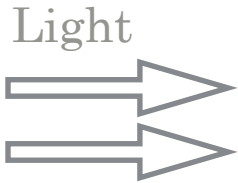
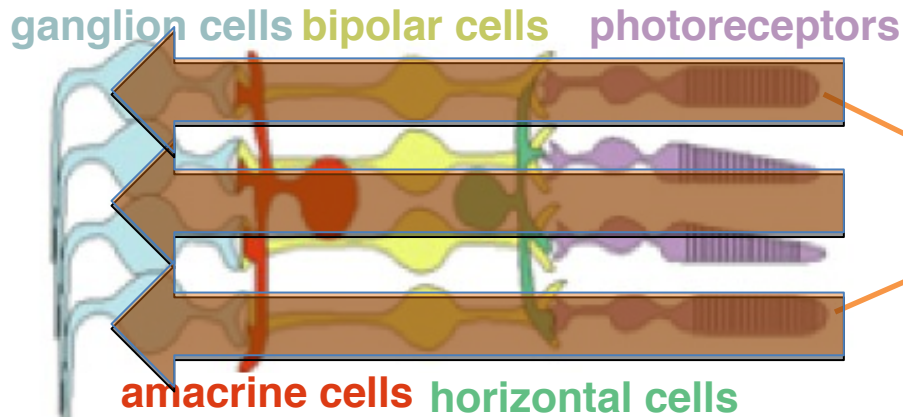
« Information » flux

The visual system

« Encoder »

« Decoder »

Retina's layered structure



« Information » flux

The visual system

« Encoder »

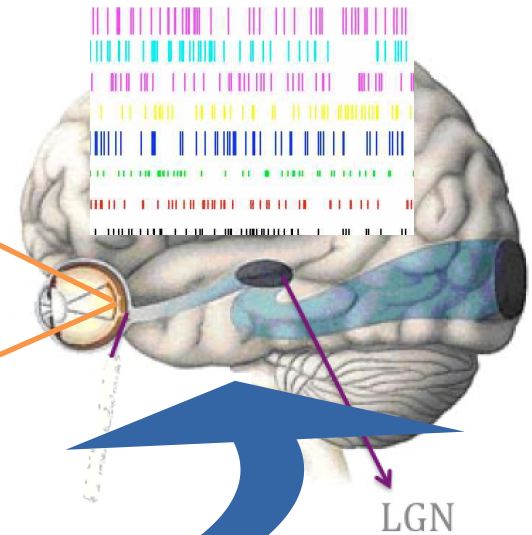
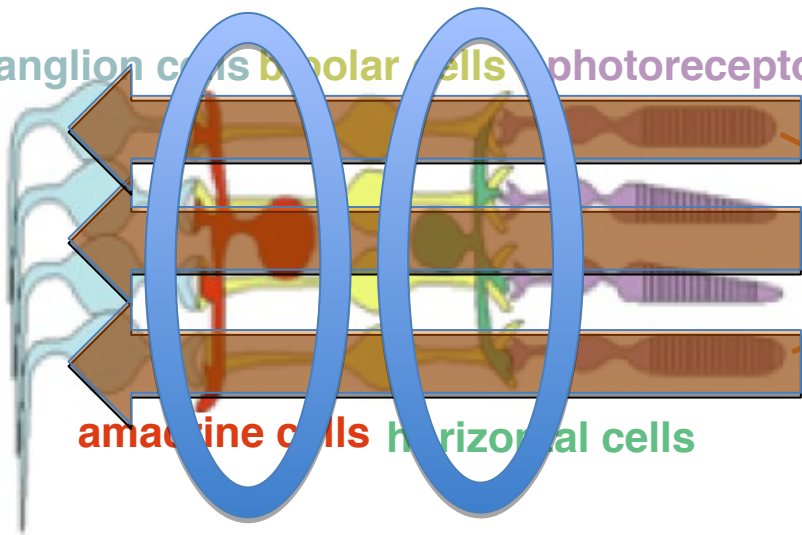
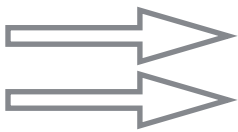
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Retina's layered structure

ganglion cells bipolar cells photoreceptors

amacrine cells horizontal cells

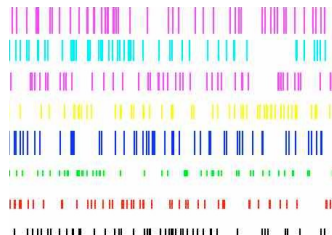
Light



Visual cortex

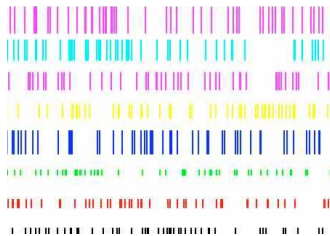
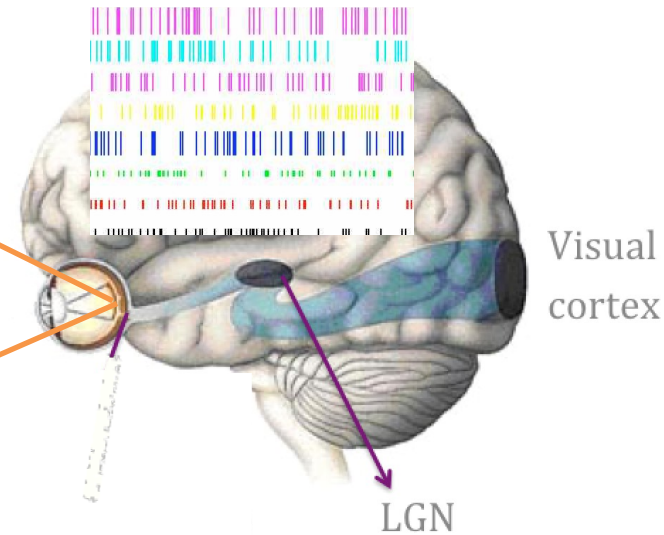
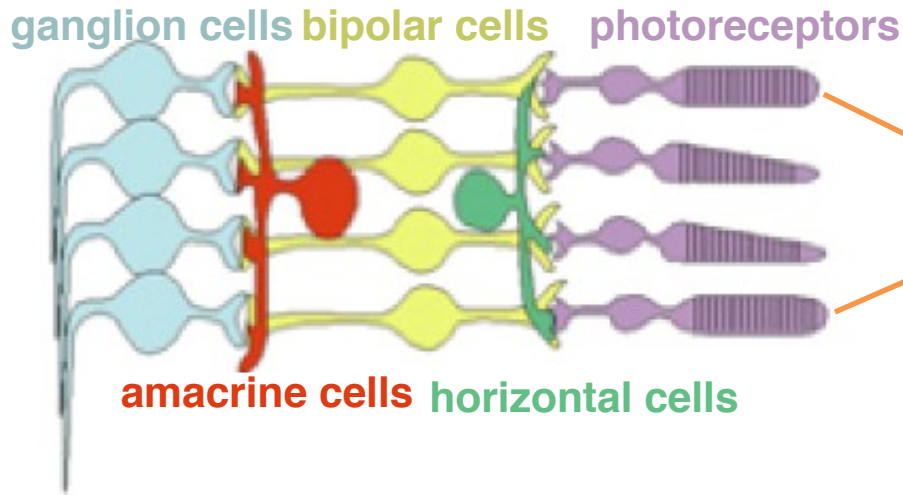
LGN

« Information » flux



The visual system

Retina's layered structure

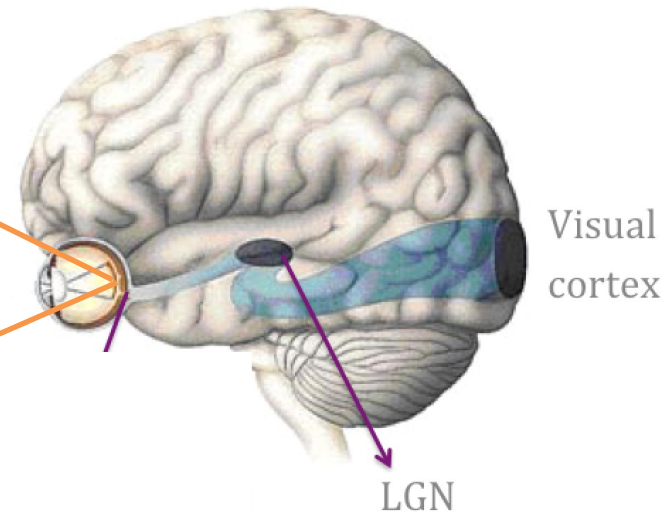
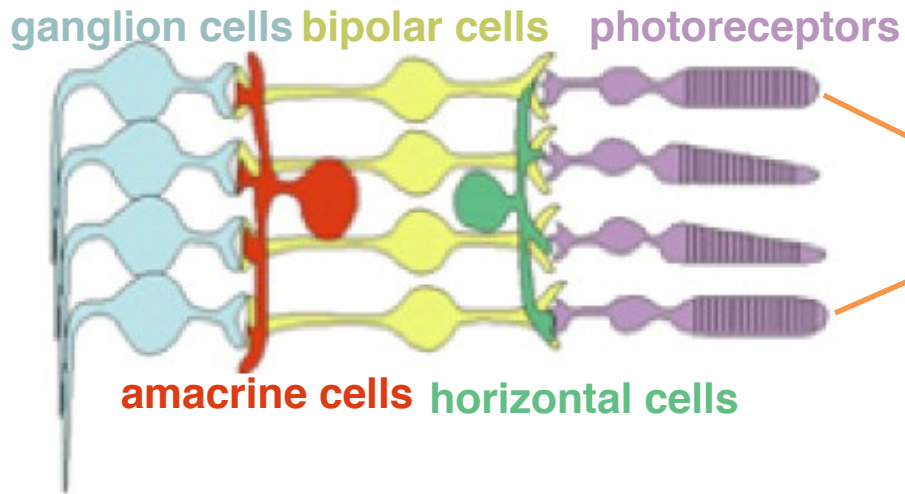


Most neurons in the retina do not spike (except Gcells) =>

« Analogic » computing.

The visual system

Retina's layered structure



Stimuli are non stationary, with spatio-temporal correlations.

A modeller point of view

1. The retina is a ***high dimensional, non autonomous and noisy*** dynamical system, layered and structured, with ***non stationary and spatially inhomogeneous entries*** (visual scenes).
2. ***Local circuits*** of non linear cells, connected with non linear synapses efficiently process the ***local visual information***. These local circuits are connected together, spanning the whole retina in a ***regular tiling***.
3. The conjunction of non linearities and regular structure can produce a highly complex ***spatio-temporal*** dynamics.
4. The retina is sensitive to ***changes in its physiology*** (development, pharmacology, pathologies). These changes can be addressed from the point of view of ***bifurcations theory***.

Retinal anticipation

Random connectivity and motion anticipation

S. Souihel (PhD 2016-2019), Biovision team,
INRIA, Sophia Antipolis

F. Chavane, INT, Marseille

A. Destexhe, UNIC, Paris

O. Marre, Institut de la Vision, Paris

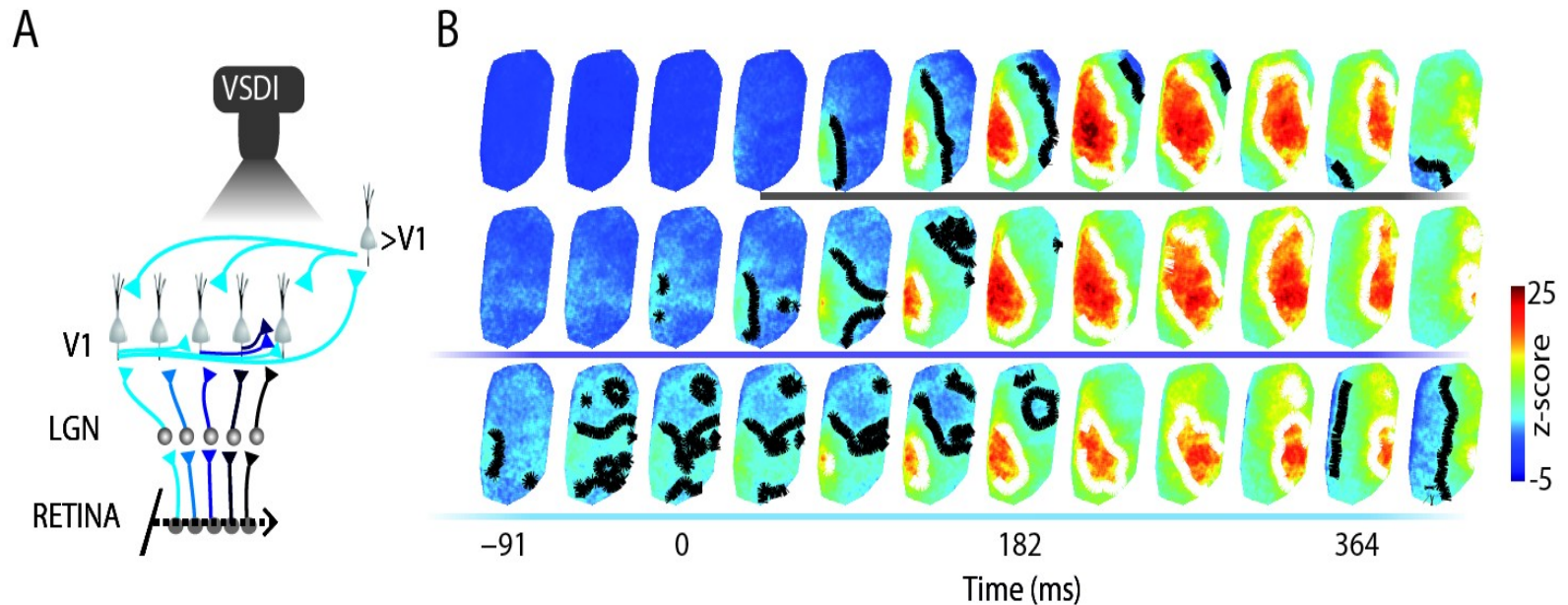
B. Cessac, Biovision team, INRIA, Sophia Antipolis



Project : ANR Trajectory.

Visual Anticipation

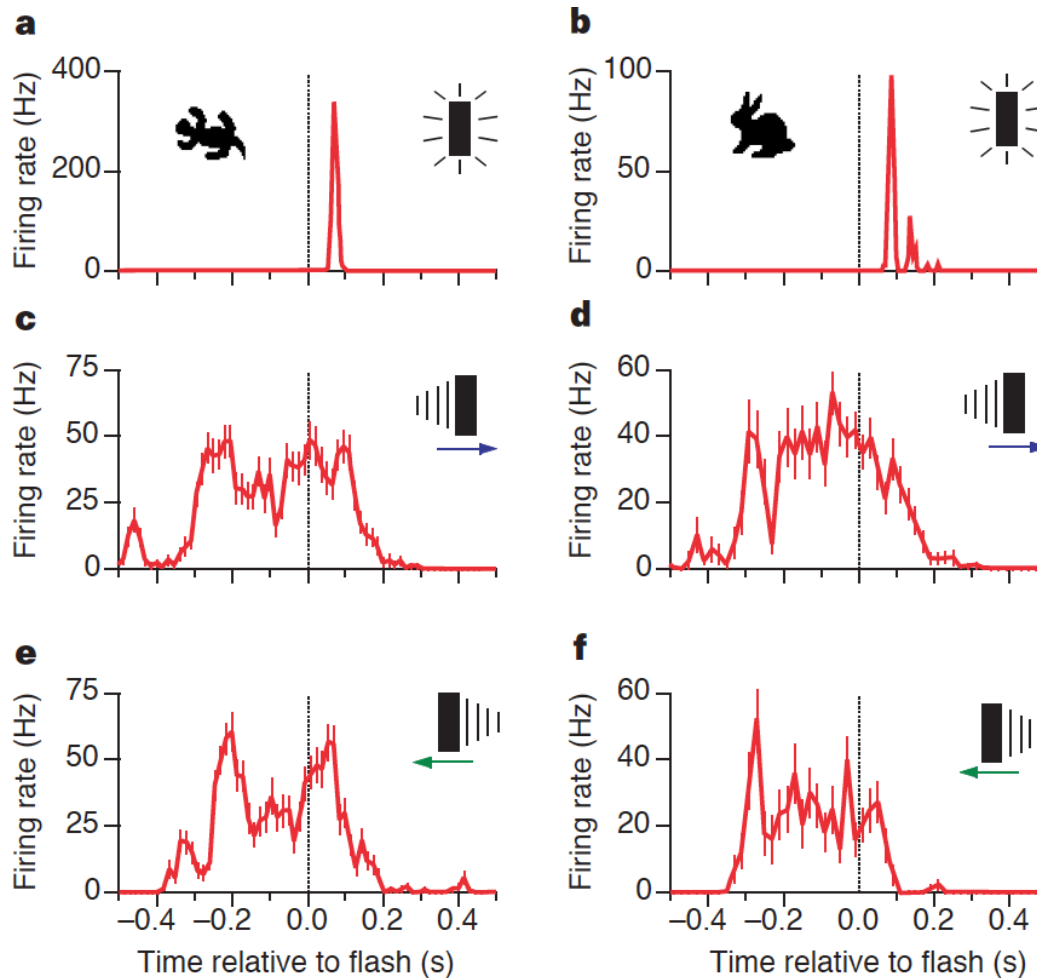
Anticipation is carried out by the primary visual cortex (V1) through an activation wave



Source : Benvenutti et al. 2015

Visual Anticipation

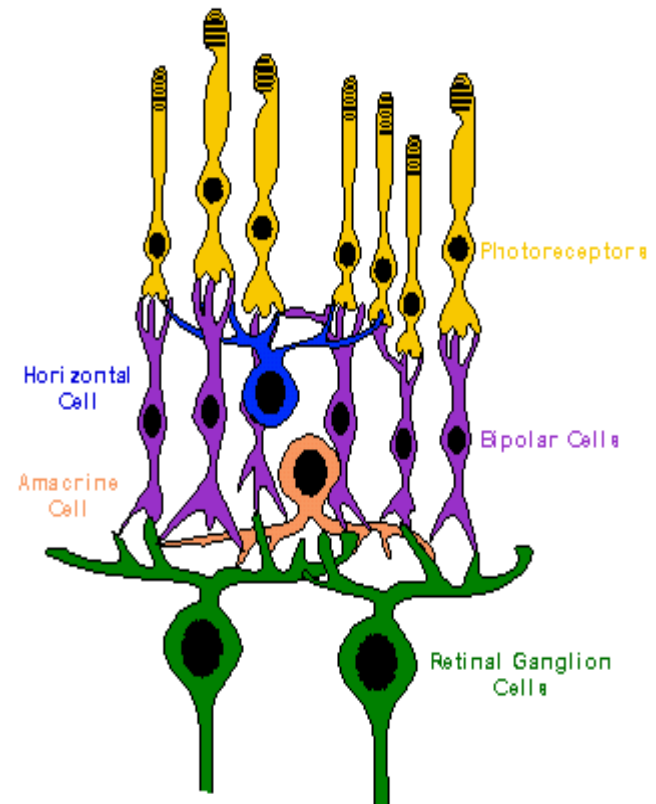
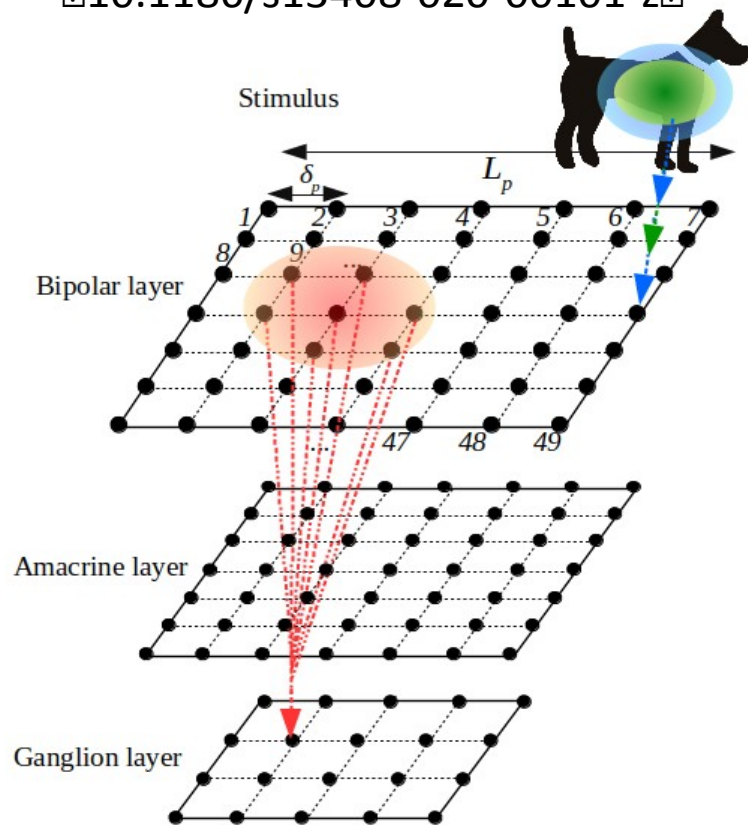
Anticipation also takes place in the retina



Source :
Berry et al.
1999

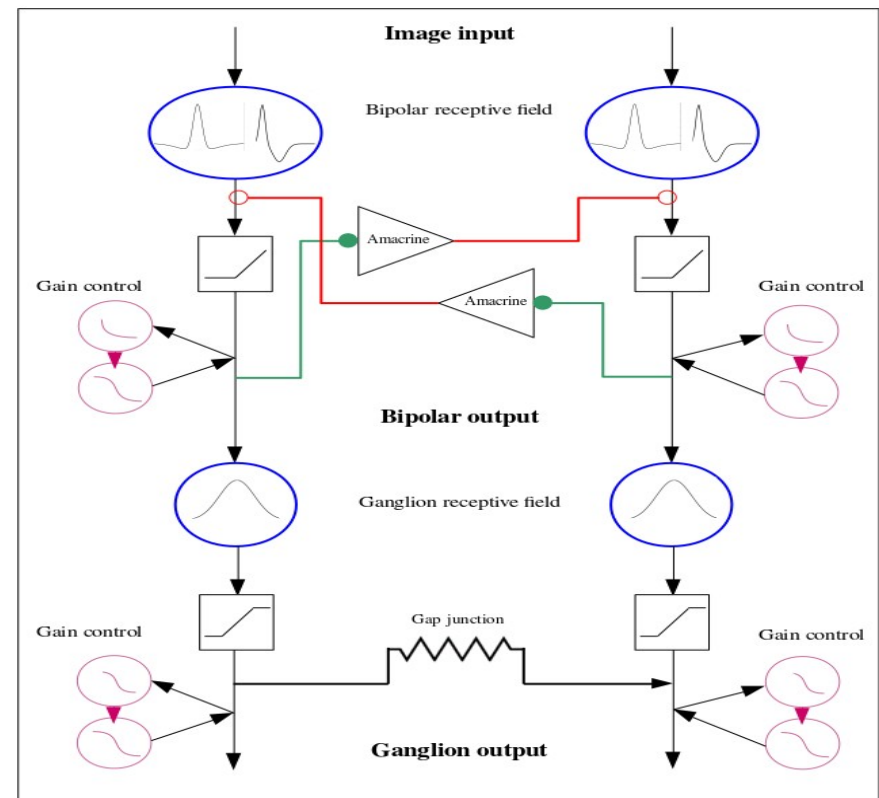
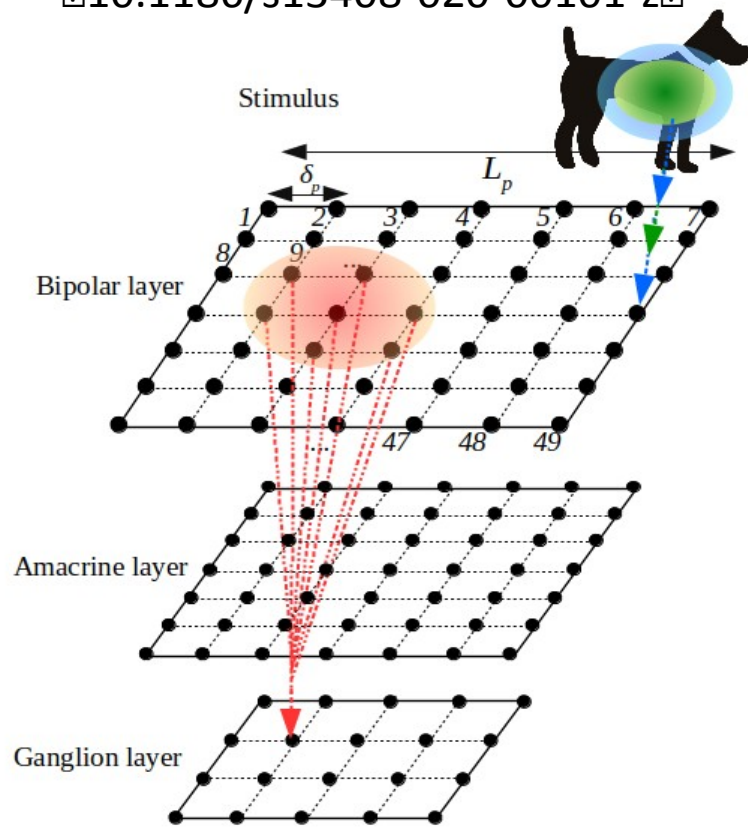
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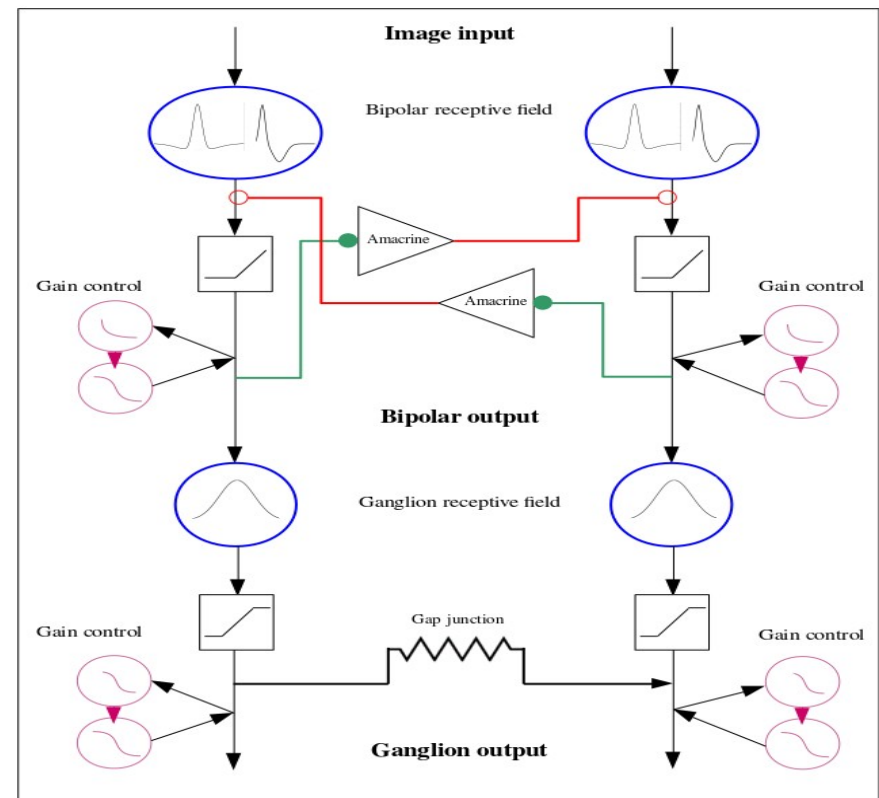
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$$\begin{cases} \frac{dV_{B_i}}{dt} = -\frac{1}{\tau_B}V_{B_i} + \sum_{j=1}^{N_A} W_{B_i}^{A_j} V_{A_j} + F_{B_i}(t), \\ \frac{dV_{A_j}}{dt} = -\frac{1}{\tau_A}V_{A_j} + \sum_{i=1}^{N_B} W_{A_j}^{B_i} R_{B_i}, \\ \frac{dA_{B_i}}{dt} = -\frac{A_{B_i}}{\tau_a} + h_B \mathcal{N}(V_{B_i}). \end{cases}$$

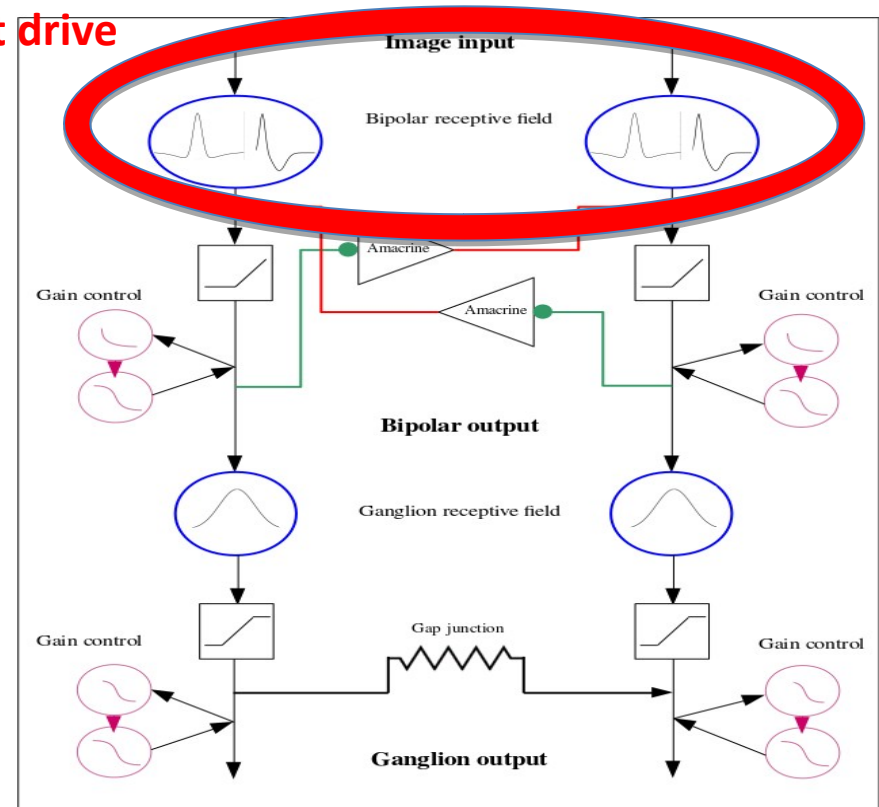


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Stimulus dependent drive

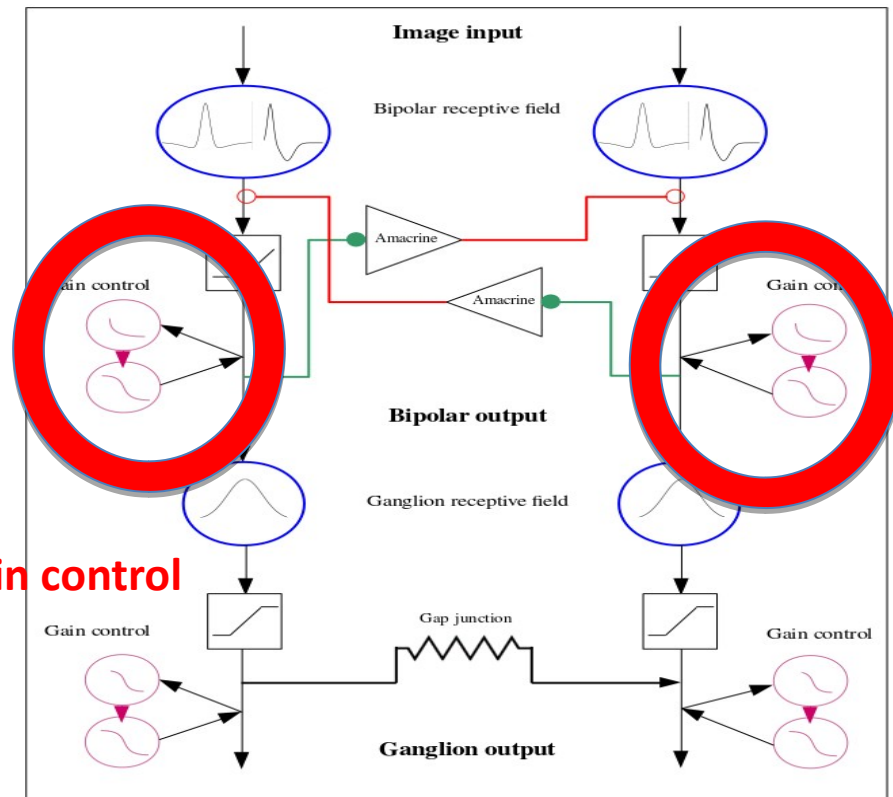
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Non linear gain control

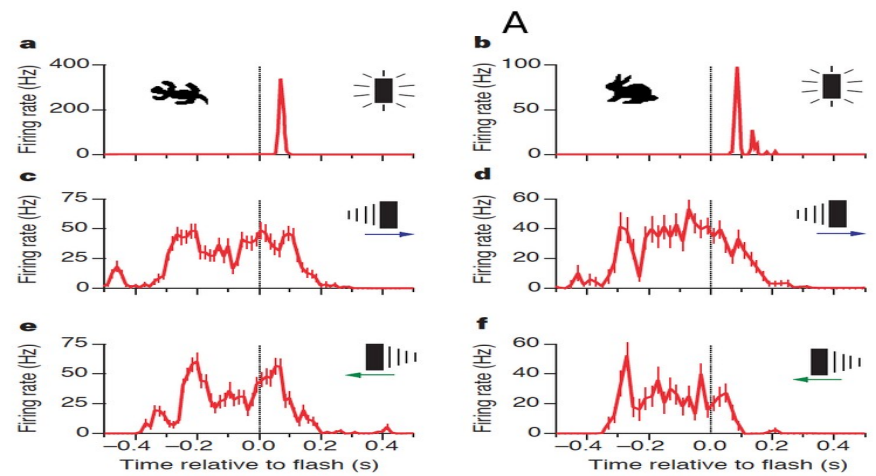
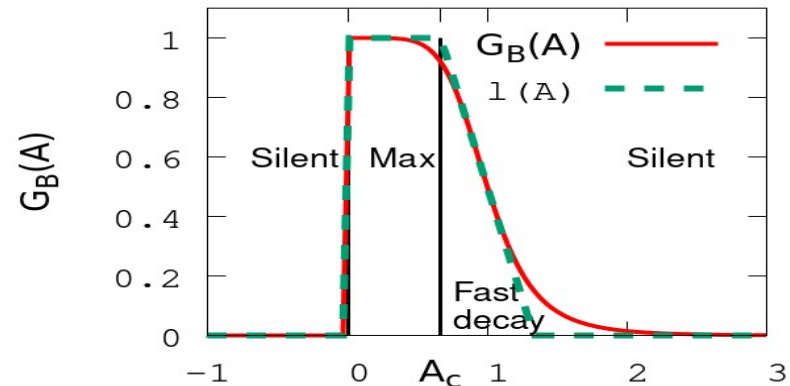
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Non linear gain control

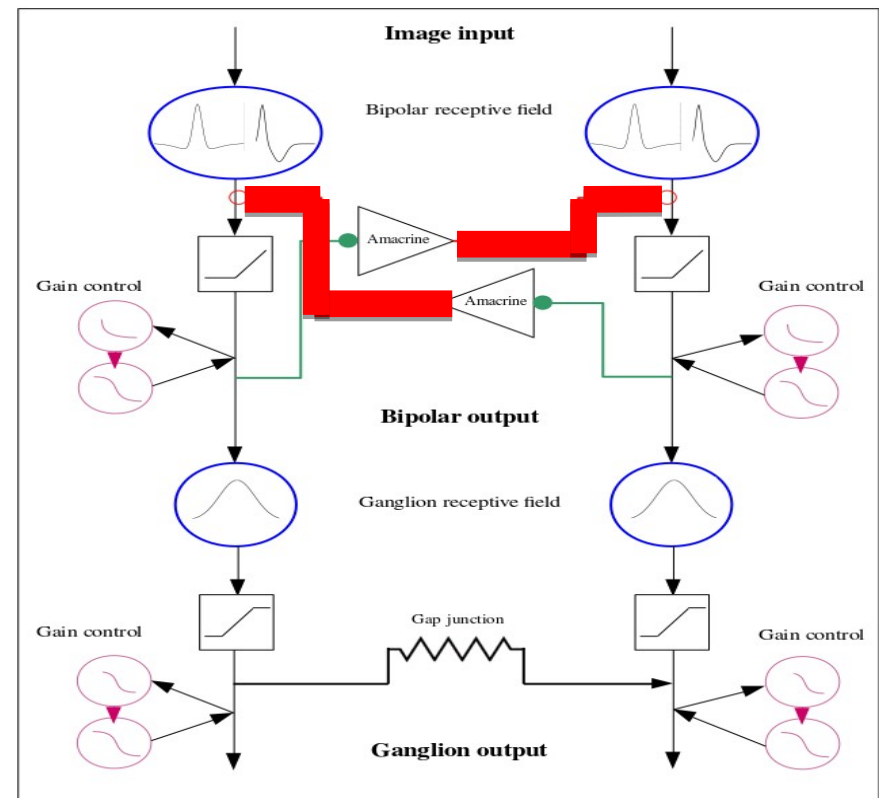


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Inhibition

$$\begin{cases} \frac{dV_{B_i}}{dt} = -\frac{1}{\tau_B}V_{B_i} + \sum_{j=1}^N W_{B_i}^{A_j} V_{A_j} + F_{B_i}(t), \\ \frac{dV_{A_j}}{dt} = -\frac{1}{\tau_A}V_{A_j} + \sum_{i=1}^{N_B} W_{A_j}^{B_i} R_{B_i}, \\ \frac{dA_{B_i}}{dt} = -\frac{A_{B_i}}{\tau_a} + h_B \mathcal{N}(V_{B_i}). \end{cases}$$

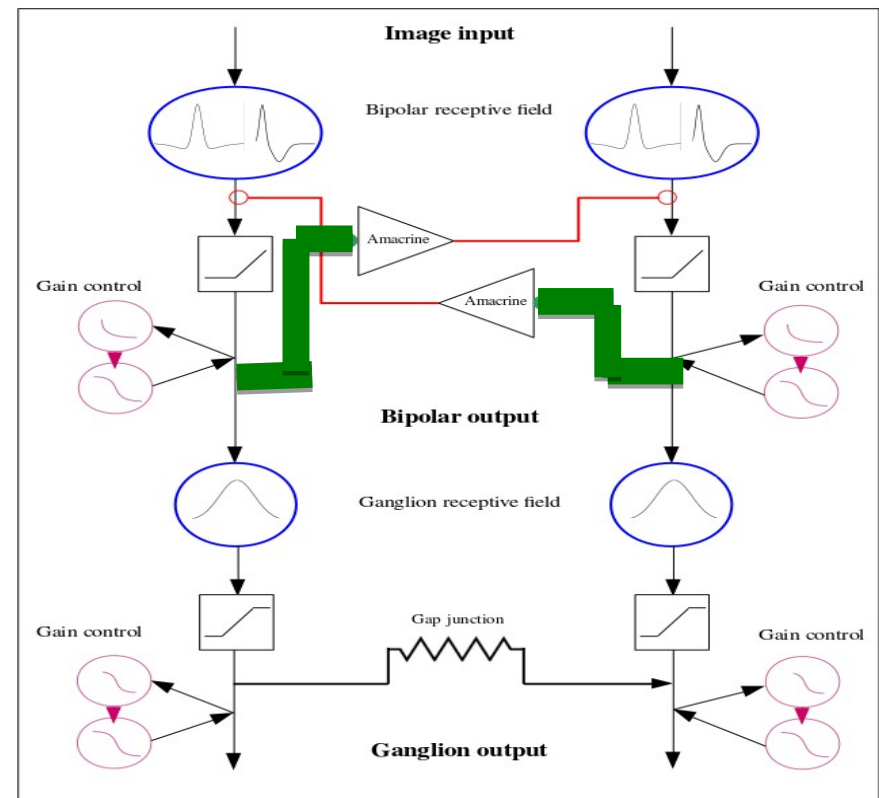


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Excitation



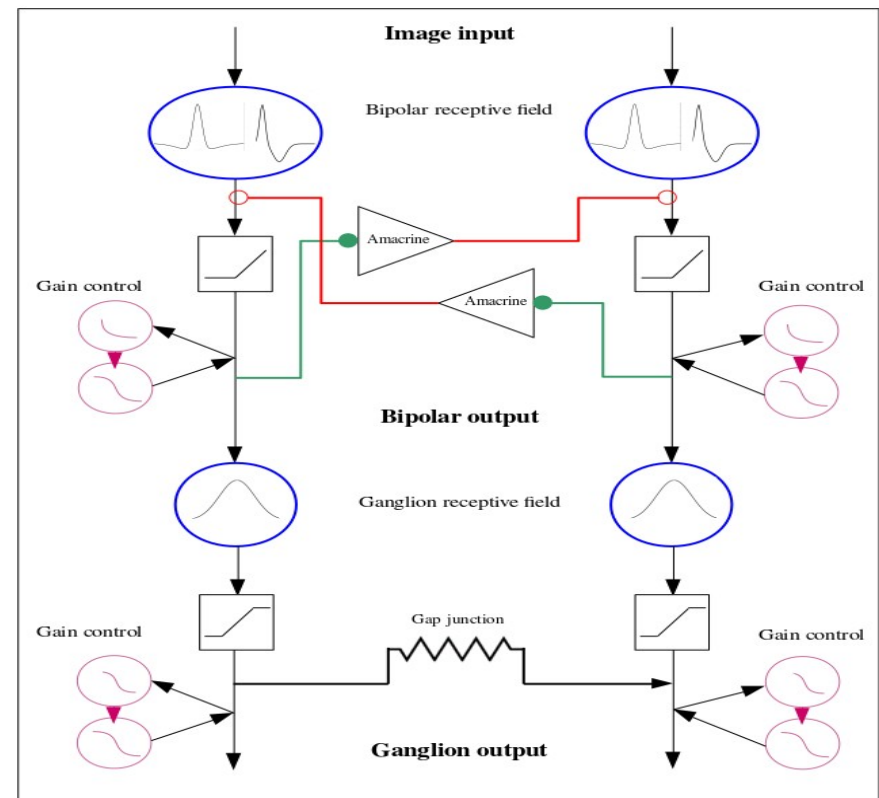
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Linear regime

$$\frac{d\vec{\chi}}{dt} = \mathcal{L} \cdot \vec{\chi} + \vec{F}(t).$$

$$\mathcal{L} = \begin{pmatrix} -\frac{I_{N,N}}{\tau_B} & W_B^A & 0_{N,N} \\ W_A^B & -\frac{I_{N,N}}{\tau_A} & 0_{N,N} \\ h_B I_{N,N} & 0_{N,N} & -\frac{I_{N,N}}{\tau_a} \end{pmatrix}$$



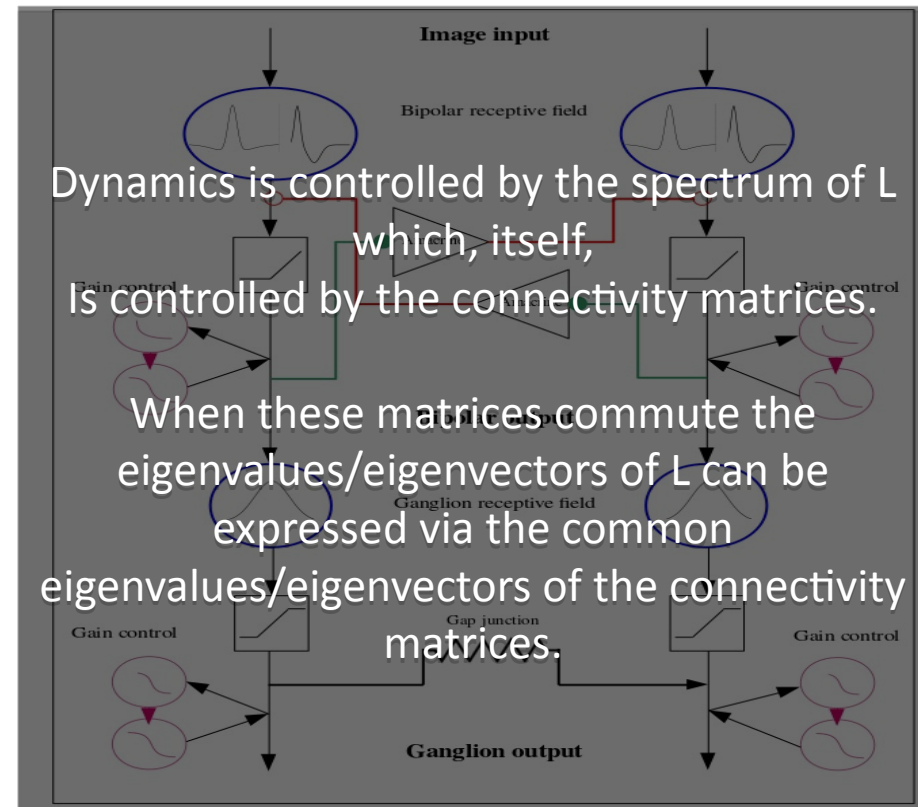
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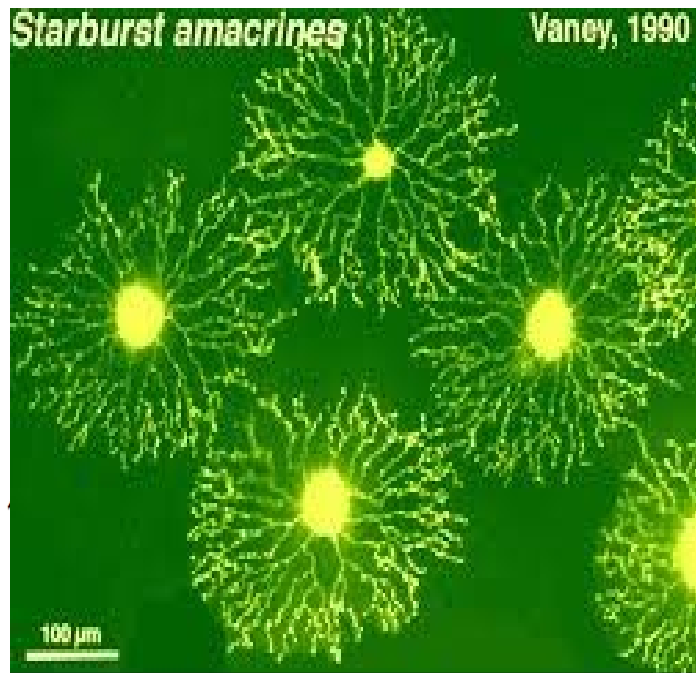
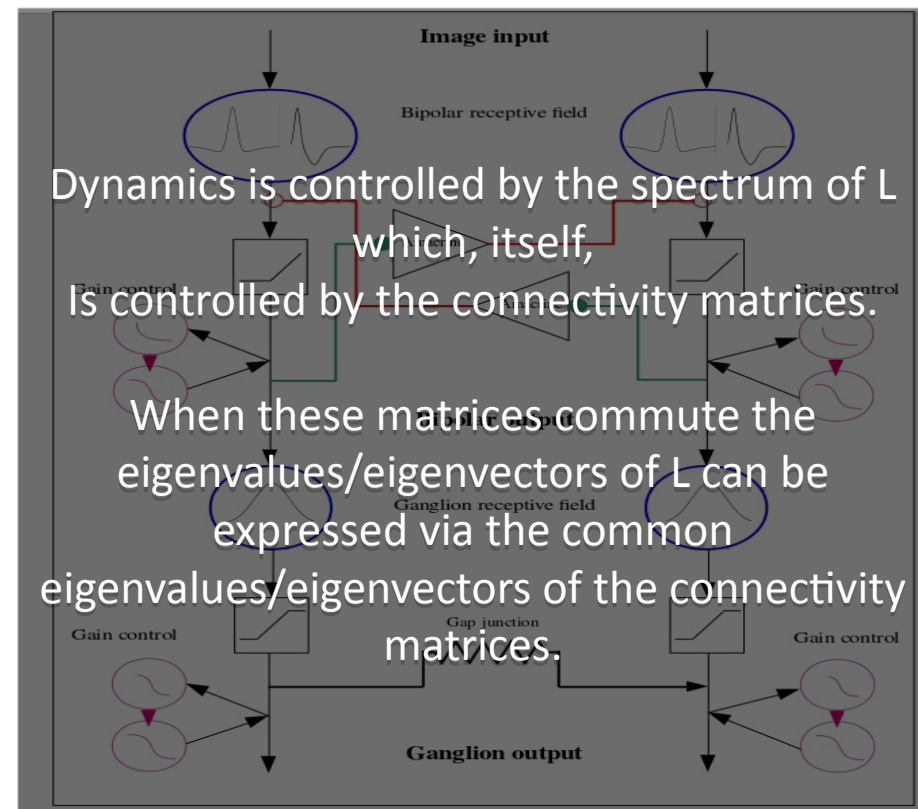
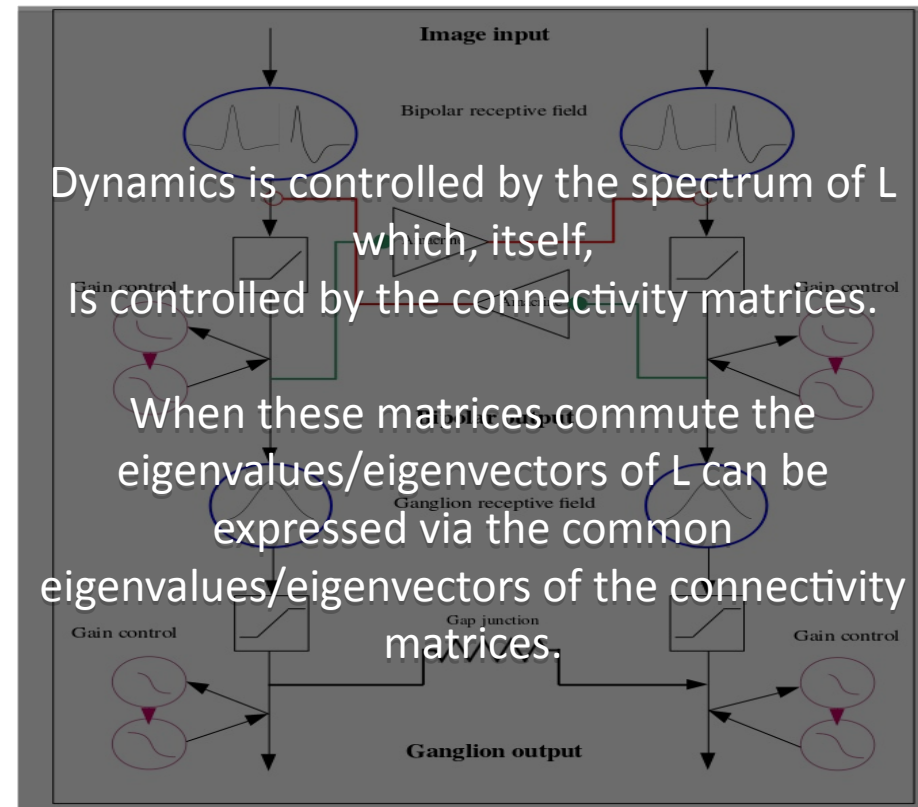
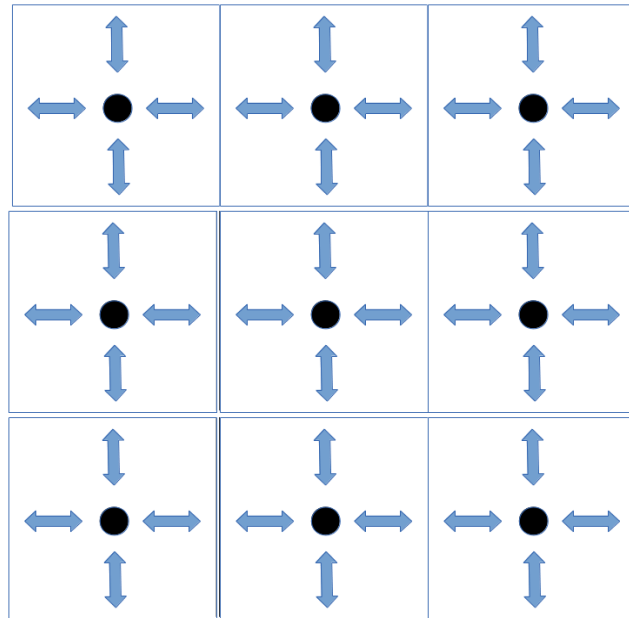


Fig. 7. Starburst amacrine cells as stained with Lucifer yellow in whole-mount rabbit retina.



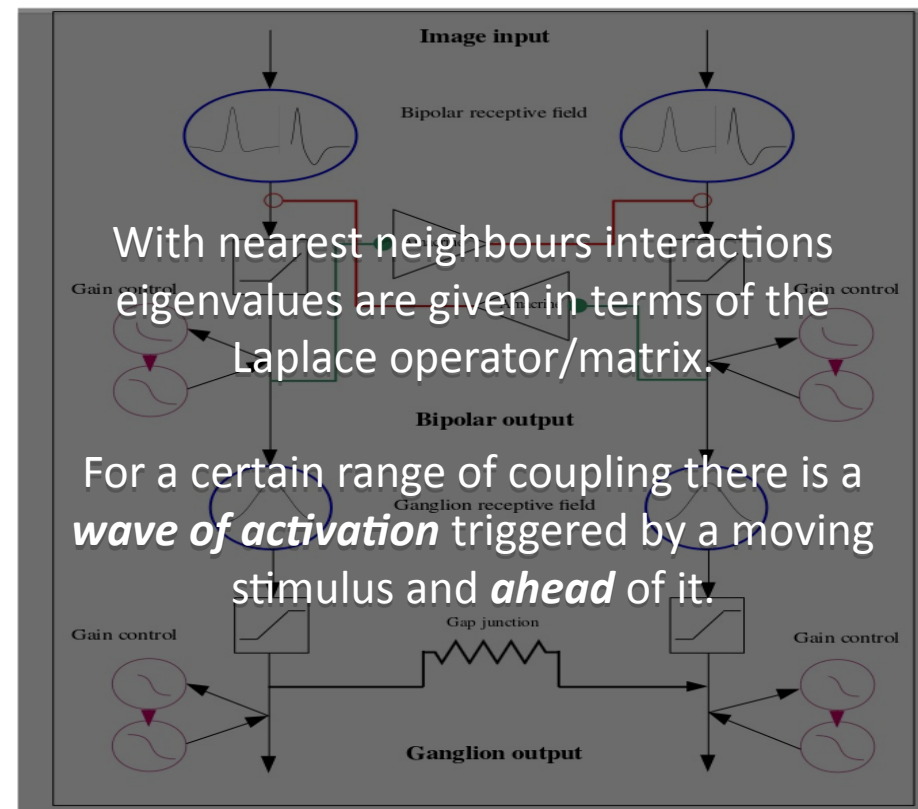
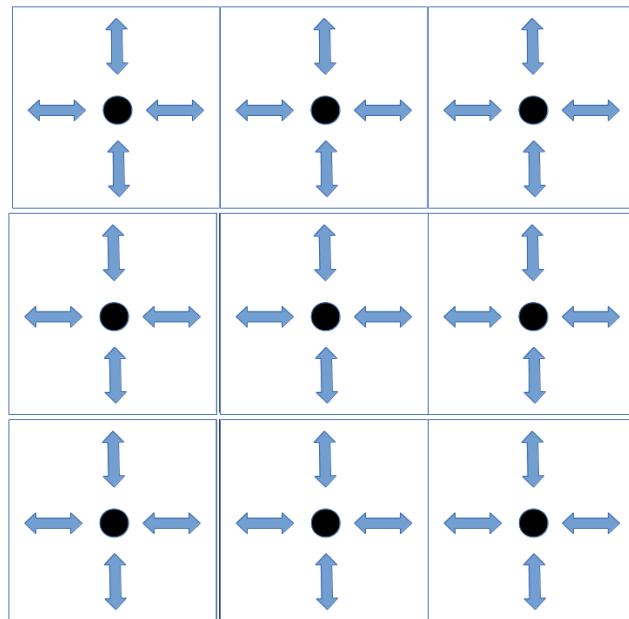
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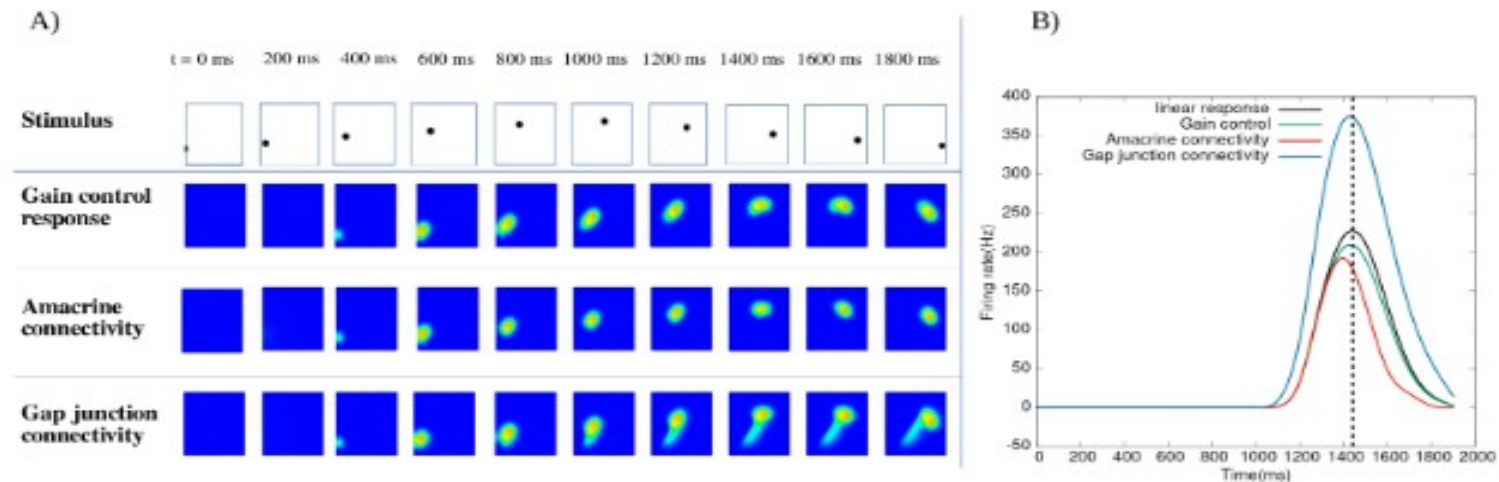
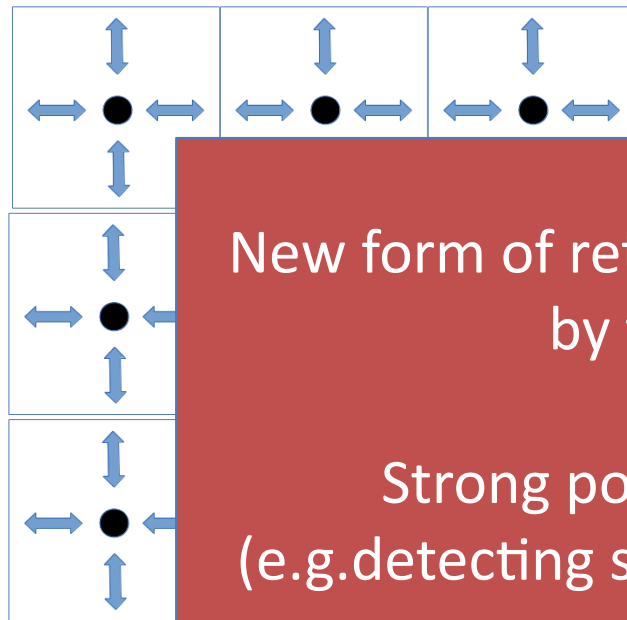


Figure 15: **Effect of anticipatory mechanisms on a parabolic trajectory.** A) Response to a dot moving along a parabolic trajectory. The first line shows the stimulus, the second line shows the GCells response with gain control, the third line presents the effect of lateral ACells connectivity with $w = 0.3 \text{ ms}^{-1}$, and the last line shows the effect of asymmetric gap junctions with $v_{gap} = 9 \text{ mm/s}$. B) Time course response of a cell responding to the dot near the trajectory turning point. Linear response corresponds to the response to the stimulus without gain control.



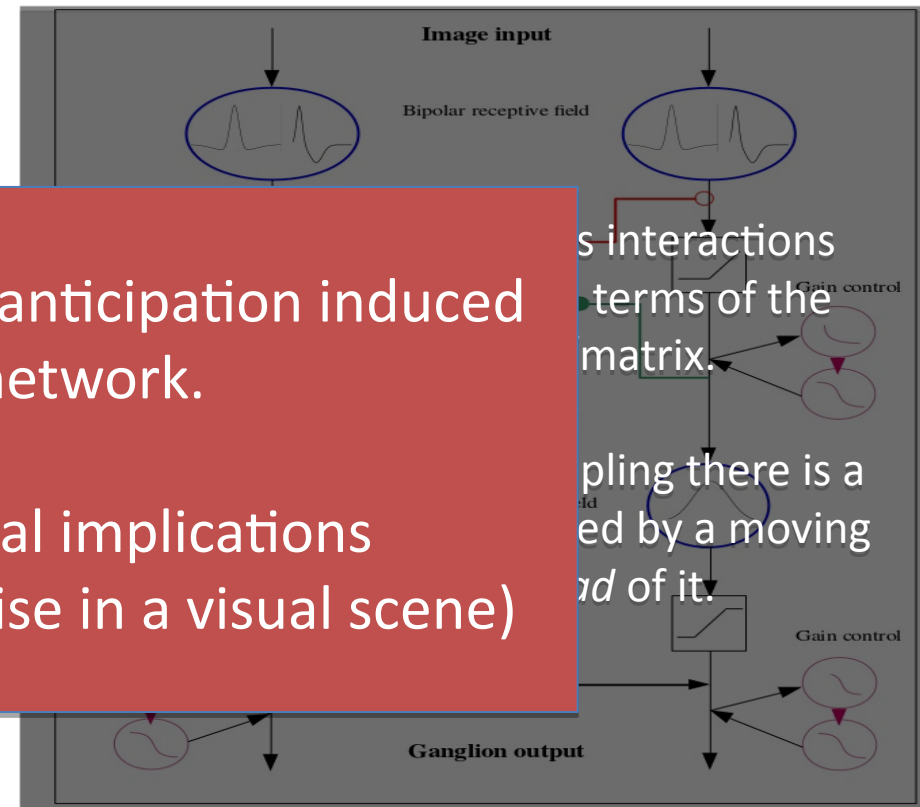
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New form of retinal anticipation induced by the network.

Strong potential implications (e.g. detecting surprise in a visual scene)



Building a 2D retina model for motion anticipation

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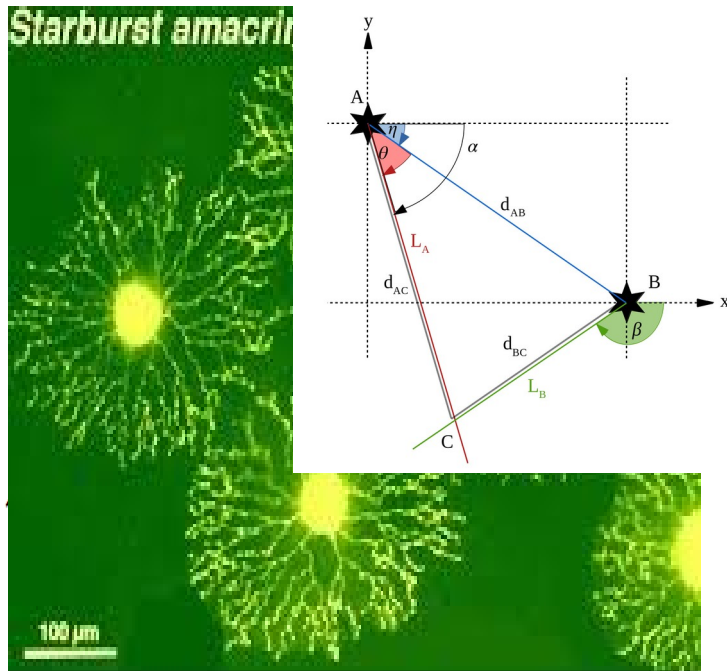
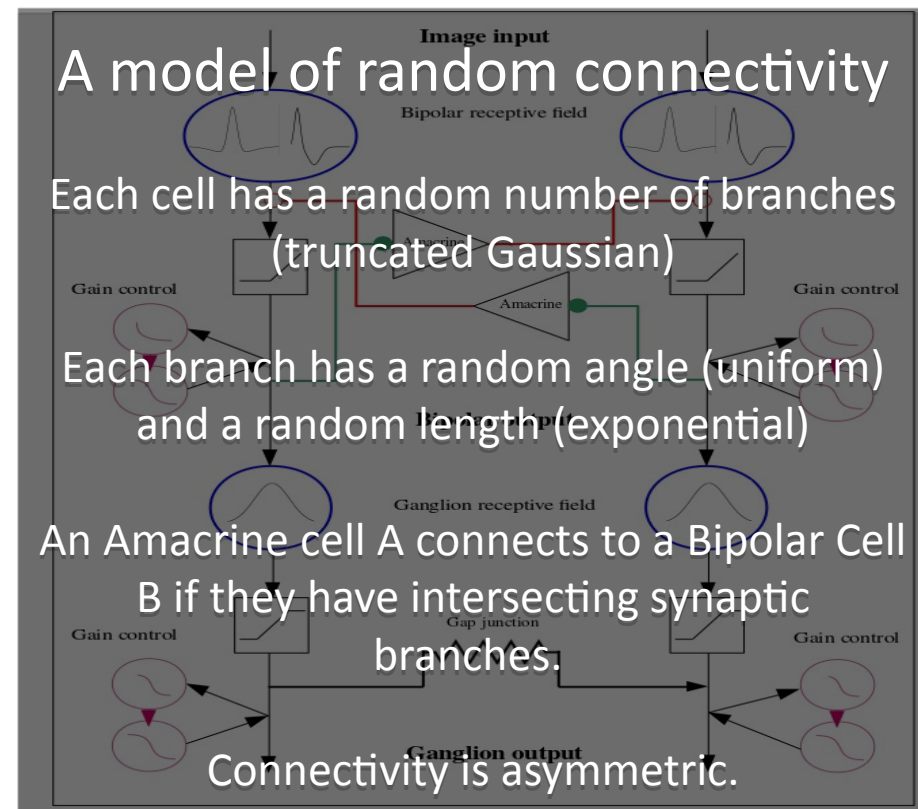
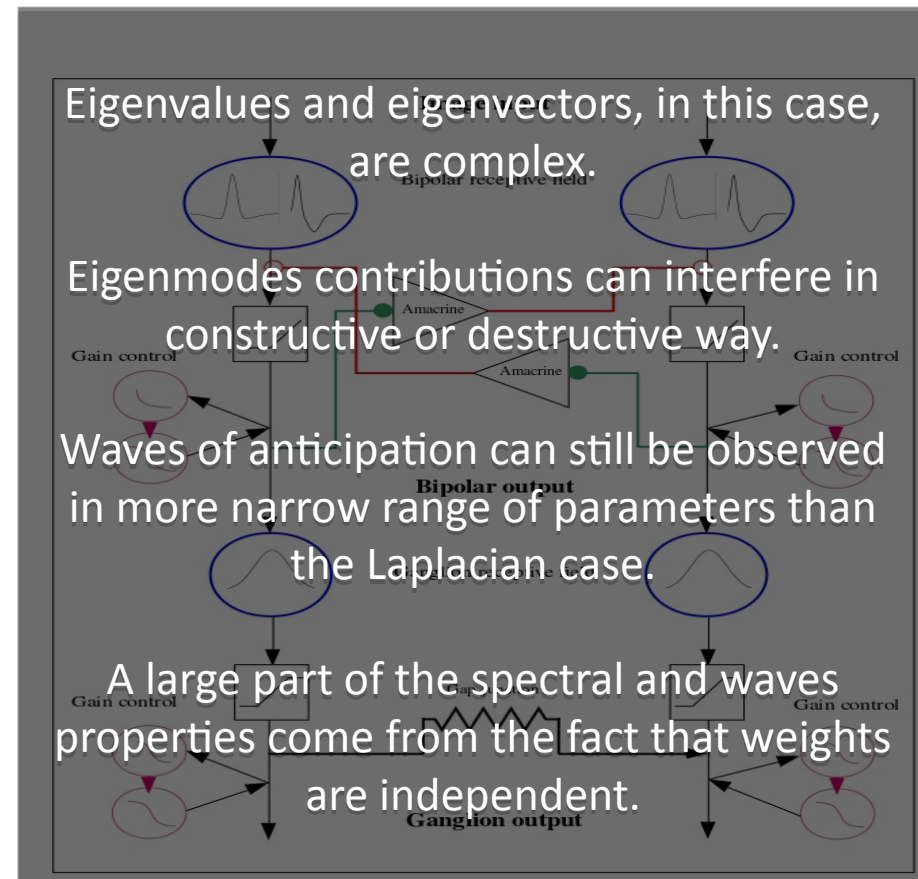
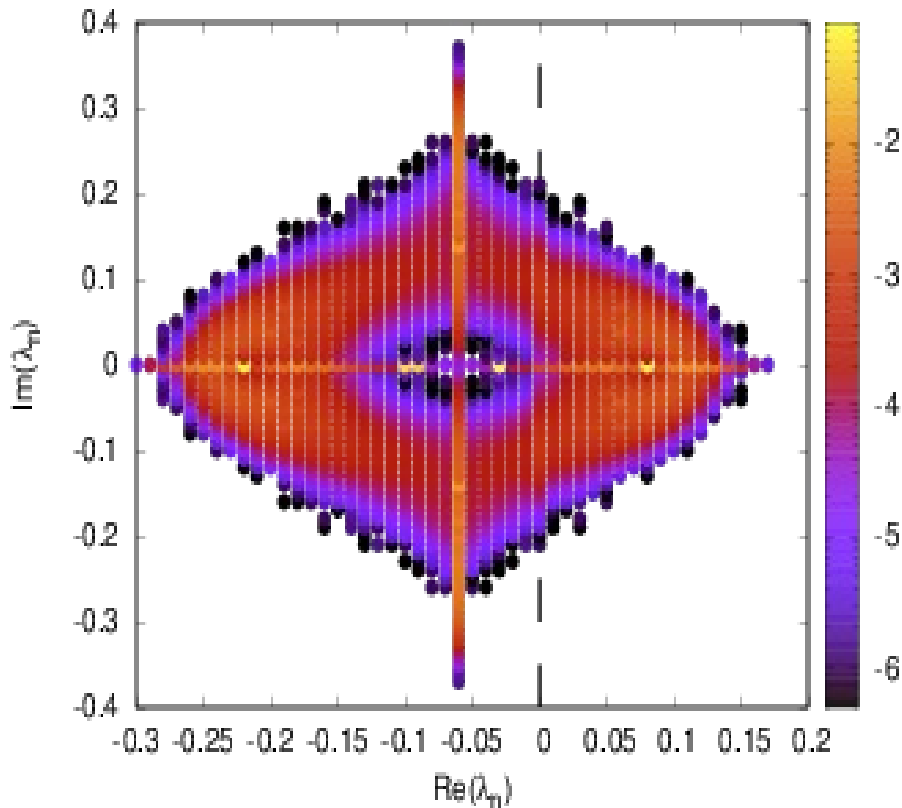


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Mathematical extensions

- Non linear effects (gain control, rectification) maintain the dynamics in a compact set even when some eigenvalues of L are unstable ;
- This non linear dynamical system can be approximated by a piecewise linear one where dynamics is linear in convex domains spanning the phase space;
- The evolution under the influence of a time-dependent stimulus can be computed as a composition of (non commuting) linear operators labelled by their domains of continuity ;

Perspectives

- A retino-cortical (V1) model of anticipation (with F. Chavane, S. Chemla, A. Destexhe, S. Souihel, M. Di Volo).

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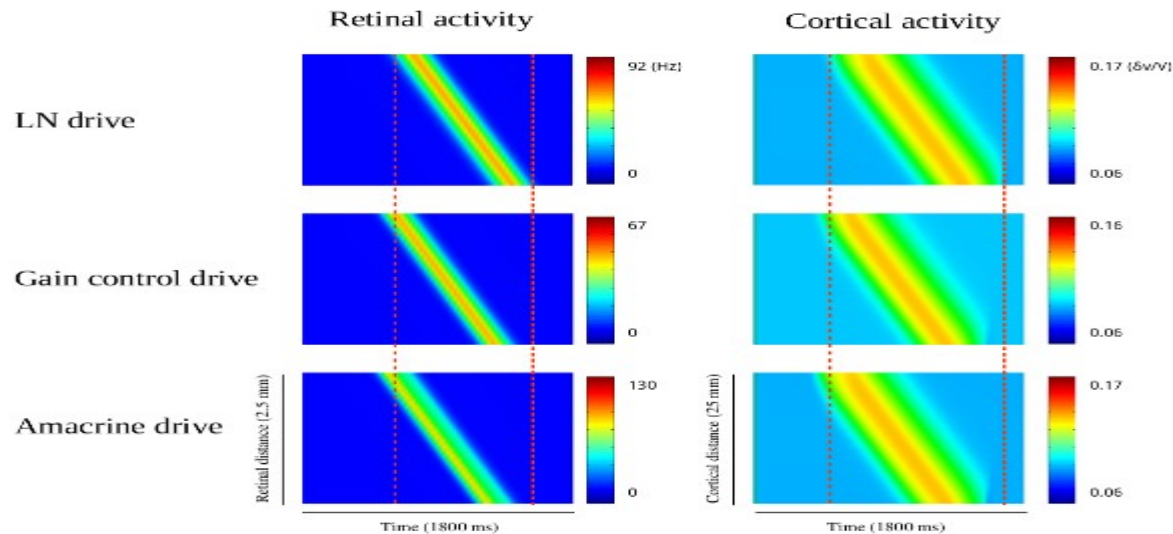
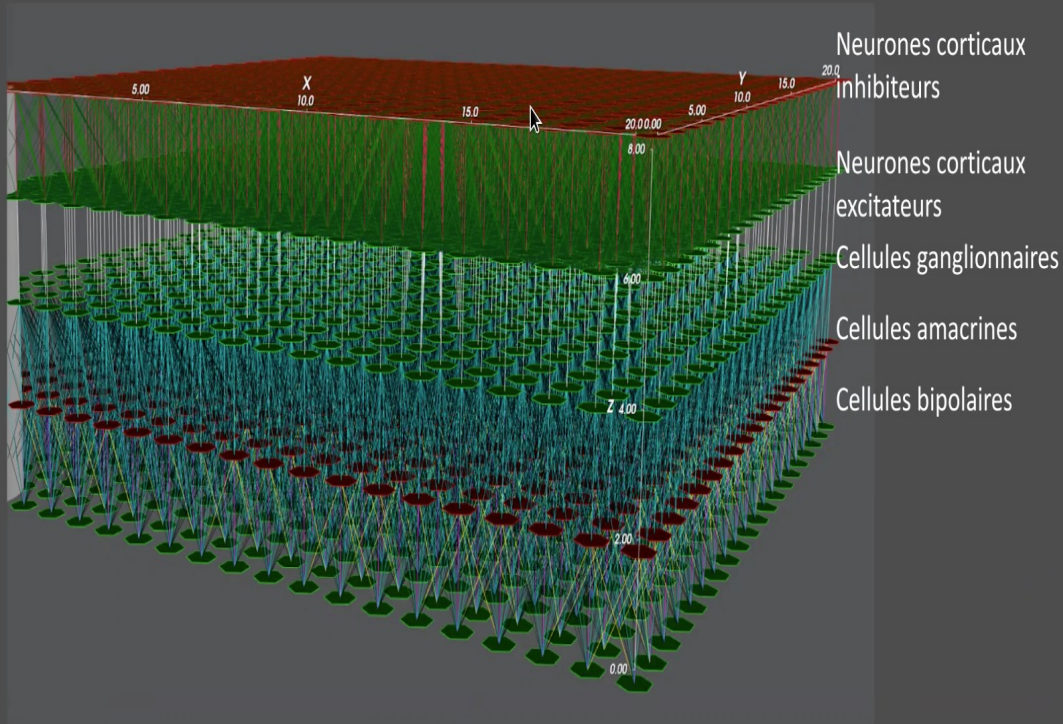


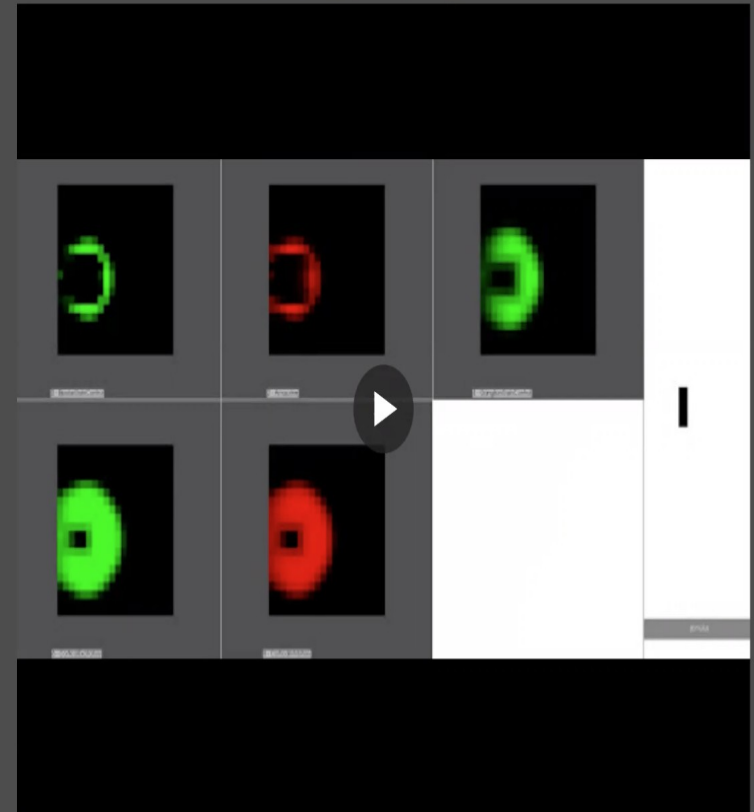
Figure 11: Comparing the spatio-temporal activity with the different drives. Left column figures display the spatio-temporal retinal activity with different anticipatory mechanisms, and the right column displays the corresponding spatio-temporal cortical activity. The red dashed lines denote the start and the end times of the LN retinal and cortical activities, as a reference base.

Logiciel Macular

Modèle rétinocortical de Macular



Simulation du modèle de Macular



Perspectives

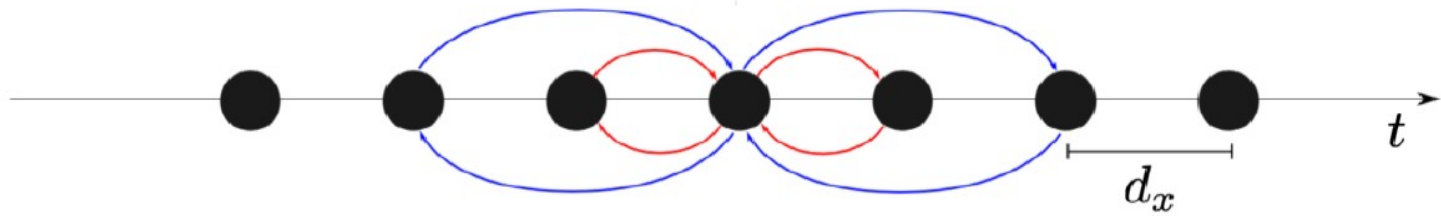
- A retino-cortical (V1) model of anticipation (with F. Chavane, S. Chemla, A. Destexhe, S. Souihel, M. Di Volo).
- Is our model able to anticipate more complex trajectories, with accelerations for instance ?
- How to better define connectivity using biology ?
- Would it be possible to design psycho-physical tests clearly showing the role of the retina in visual anticipation ?
- **How do a moving object + lateral connectivity shape the spatio-temporal correlations in ganglion cells spike trains emitted toward the brain ?**

Linear response for spiking neuronal networks with unbounded memory

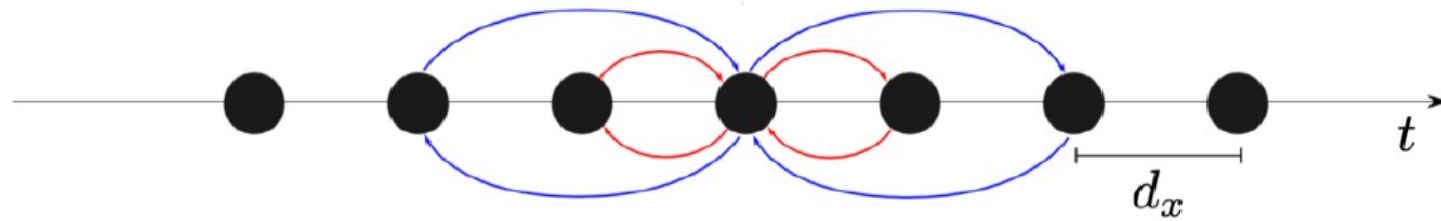
R. Cofré, CIMFAV, Facultad de Ingenieria,
Universidad de Valparaiso, Valparaiso, Chile

Ignacio Ampuero, Departamento de Informatica,
Universidad Tecnica Federico Santa Maria,
Valparaiso, Chile

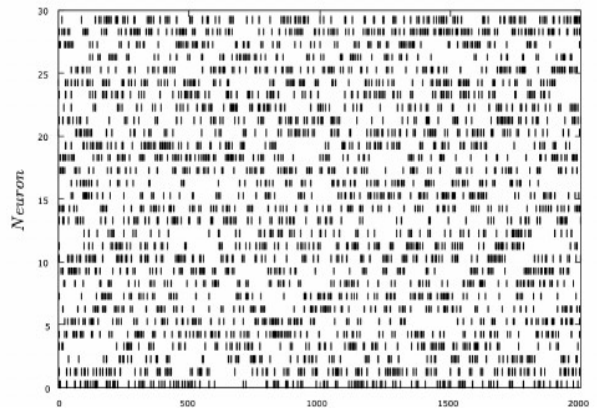
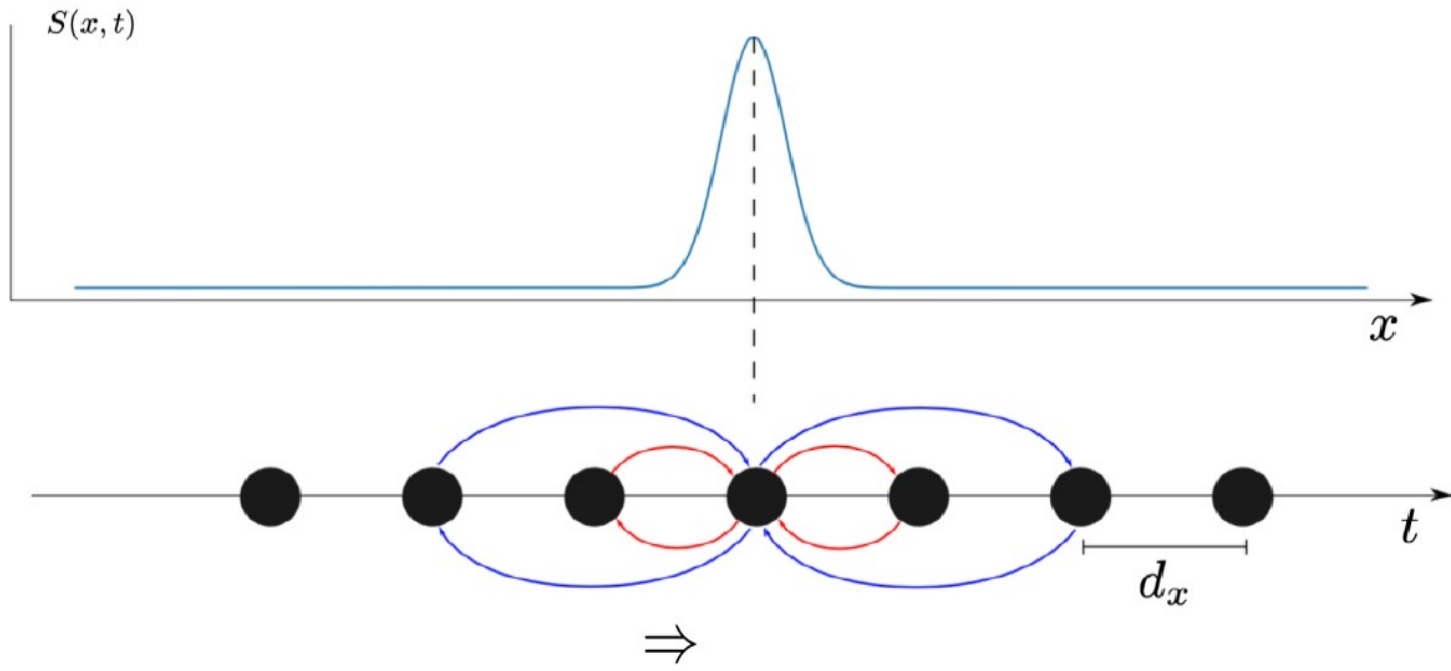
Spontaneous spiking activity



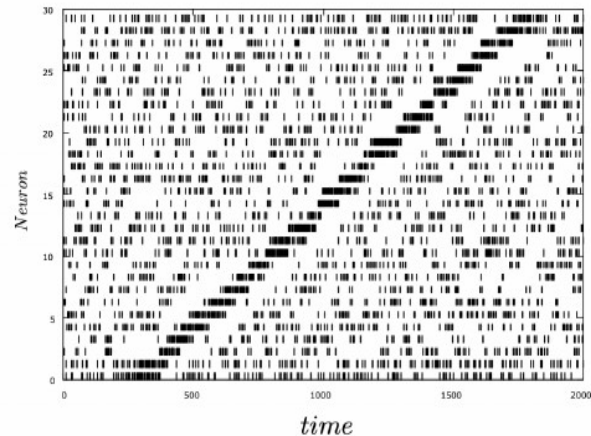
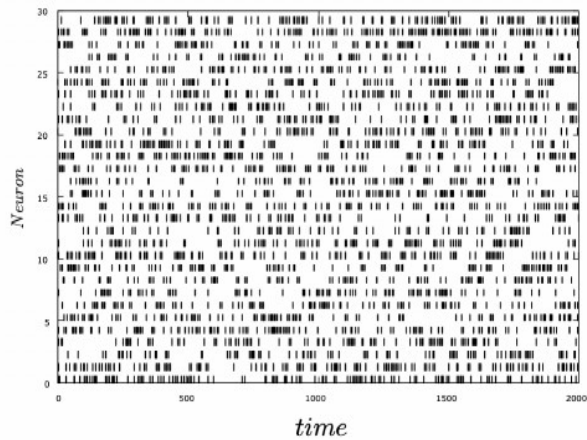
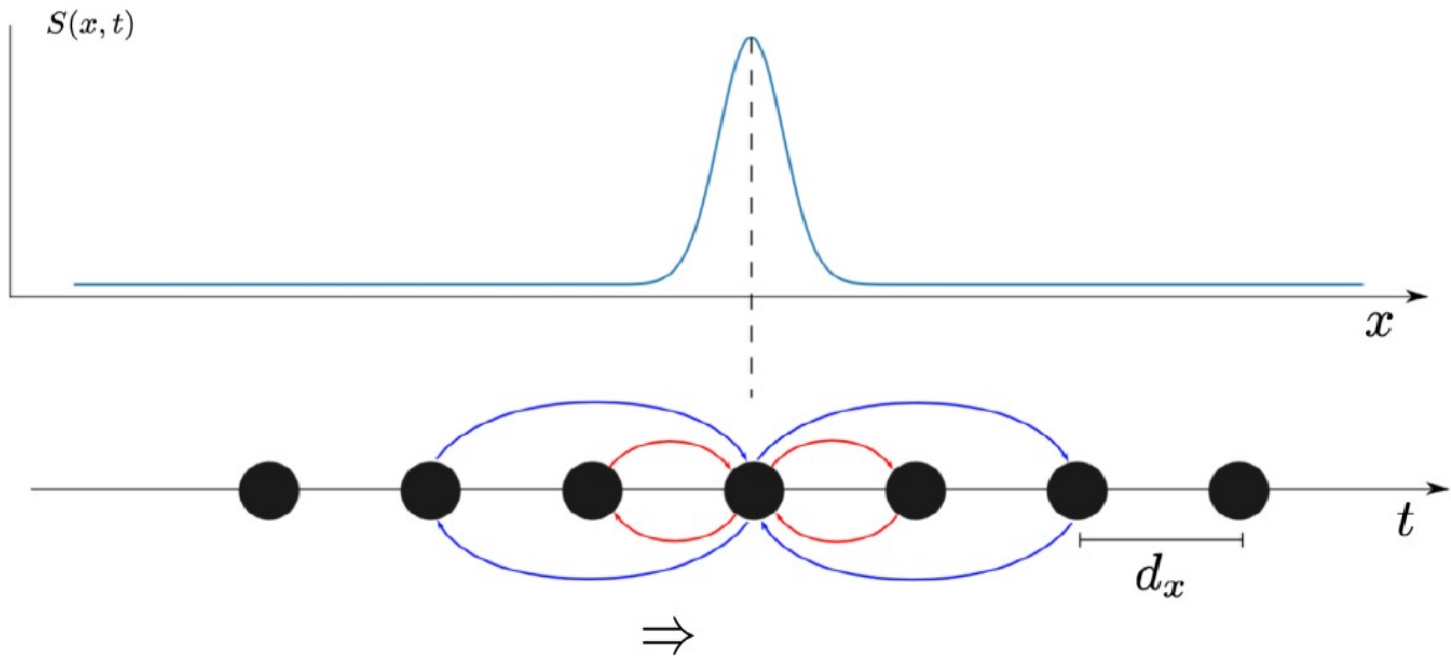
Spontaneous spiking activity



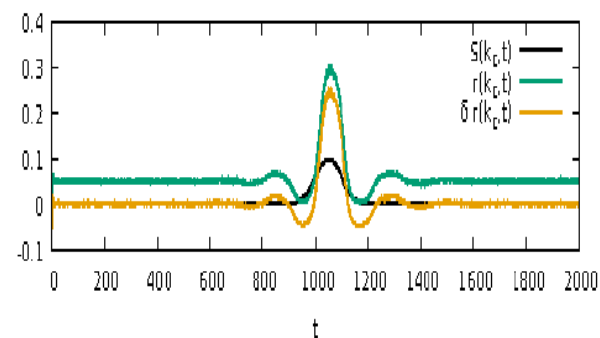
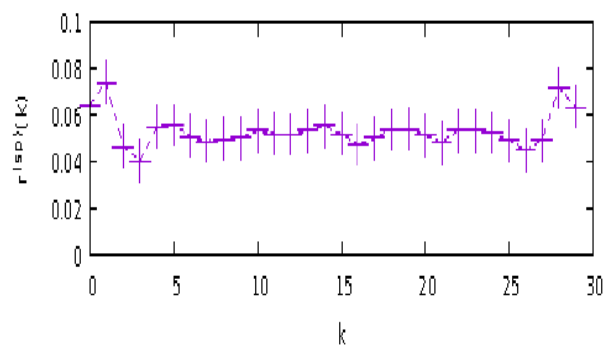
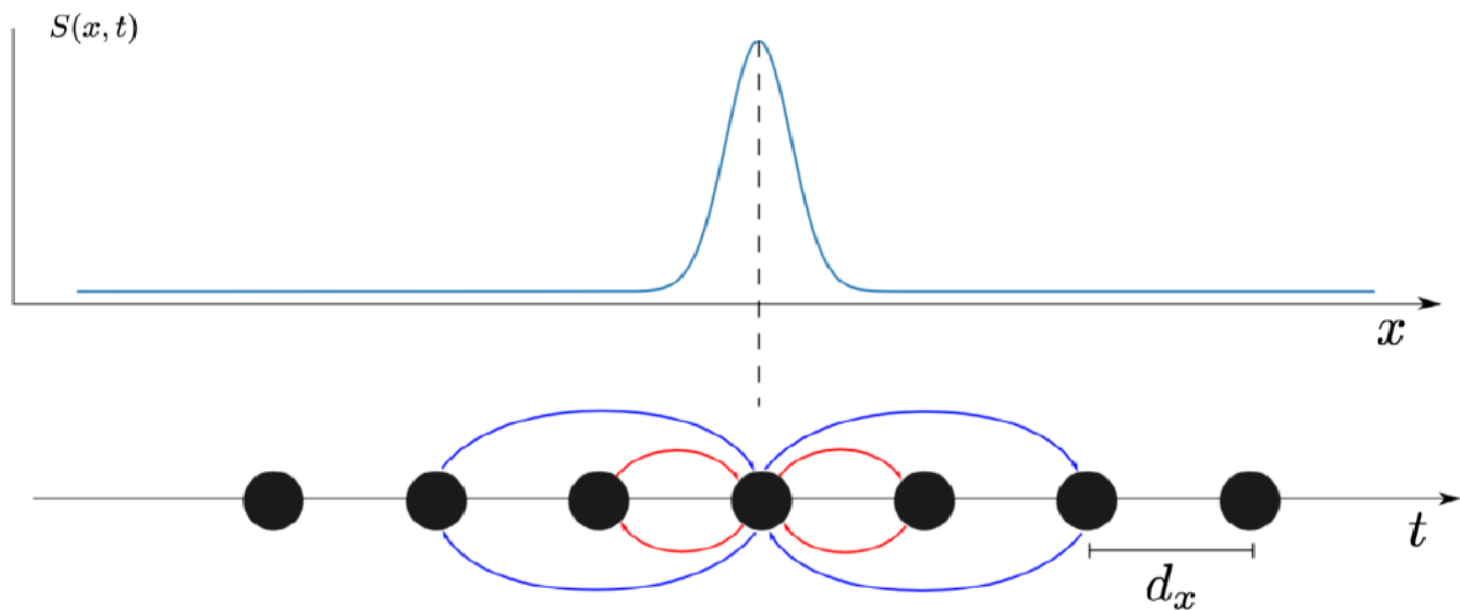
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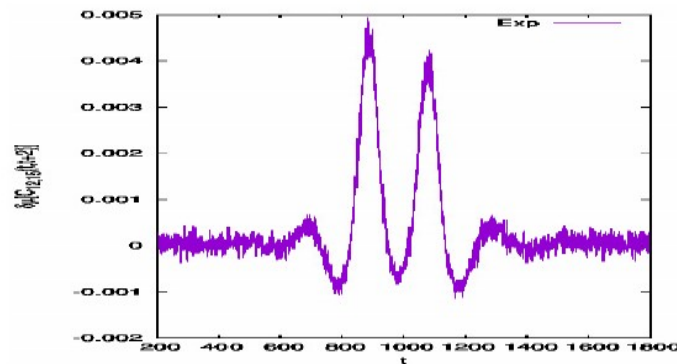
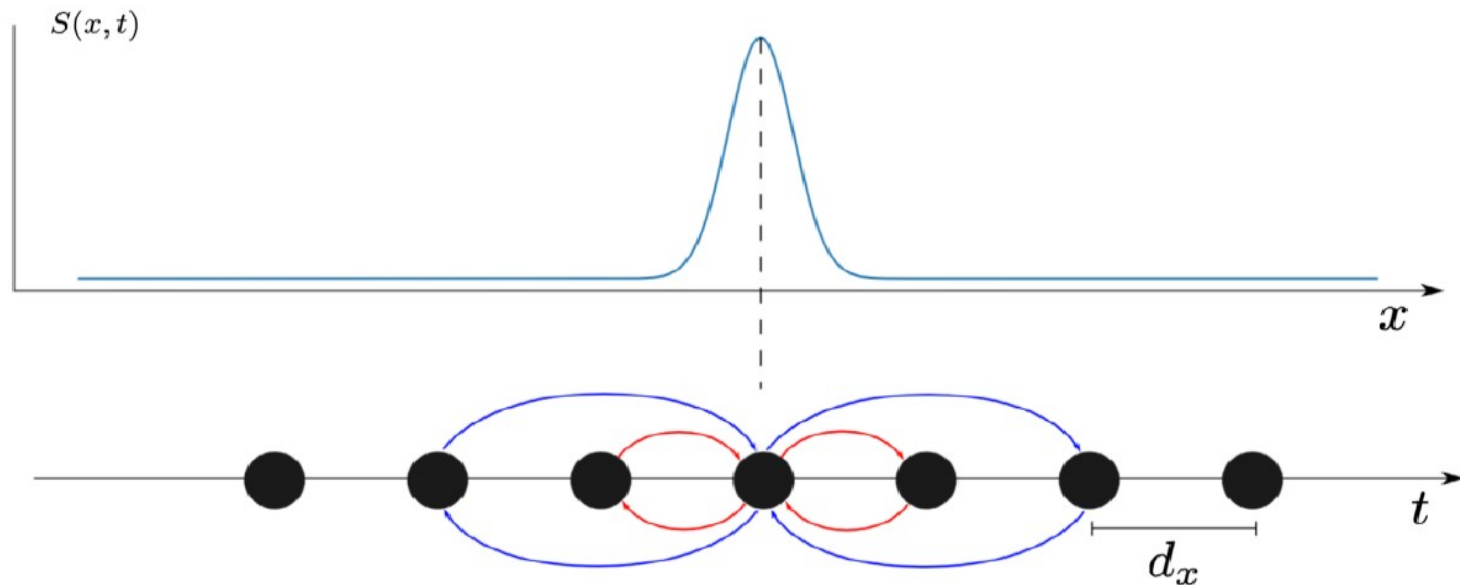


Spontaneous spiking activity



Sample averaging (non stationary)

Spontaneous spiking activity



Variation in the correlation $C_{12,15}(t, t+2)$

Correlation variations are triggered by the stimulus, but they are constrained by the network dynamics.

How to compute the spatio-temporal response of a neuronal network to a time dependent stimulus ?

Can we compute the time variation of spike-observables as a function of the stimulus and spontaneous dynamics ?

B. Cessac, I. Ampuero, R. Cofre, Linear response for spiking neuronal networks with unbounded memory. *Entropy*, MDPI, 2021, 23 (2), pp.155.

A conductance-based Integrate and Fire model

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t, \omega) V_k = i_k(t, \omega).$$

- Linear in V .
- Spike history-dependent.

Dynamics integration

$$\Gamma_k(t_1, t, \omega) = e^{-\frac{1}{C_k} \int_{t_1}^t V_{\tau_k}(t, \omega) g_k(u, \omega) du}$$

$$V_k(t, \omega) = \underbrace{V_k^{(sp)}(t, \omega) + V_k^{(S)}(t, \omega)}_{V_k^{(det)}(t, \omega)} + V_k^{(noise)}(t, \omega)$$

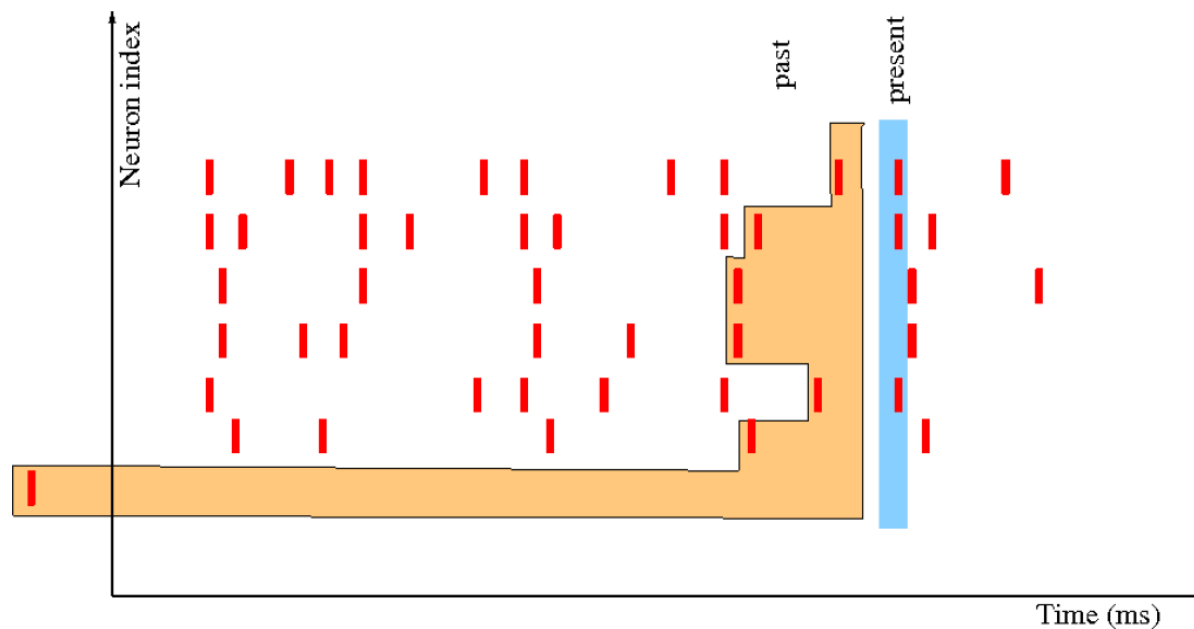
$$V_k^{(sp)}(t, \omega) = V_k^{(syn)}(t, \omega) + V_k^{(L)}(t, \omega),$$

$$V_k^{(syn)}(t, \omega) = \frac{1}{C_k} \sum_{j=1}^N W_{kj} \int_{\tau_k(t, \omega)}^t \Gamma_k(t_1, t, \omega) \alpha_{kj}(t_1, \omega) dt_1$$

A conductance-based Integrate and Fire model

Variable length Markov chain with unbounded memory

$$P_n [\omega(n) \mid \omega_{-\infty}^{n-1}] \equiv \Pi \left(\omega(n), \frac{\theta - V_k^{(det)}(n-1, \omega)}{\sigma_k(n-1, \omega)} \right)$$



Response to stimuli

B. Cessac, I. Ampuero, R. Cofre, Linear response for spiking neuronal networks with unbounded memory. Entropy, MDPI, 2021, 23 (2), pp.155.

GIF model without stimulus \Rightarrow *stationary Gibbs distribution*.

time-dependent stimulus $S(t) \Rightarrow$ *non stationary Gibbs distribution*.

How is the average of an observable $f(\omega, t)$ affected by the stimulus ?

If S is weak enough: $\delta\mu[f(t)] = [\kappa_f * S](t)$, (linear response).

$$\kappa_{k,f}(t - t_1) = \frac{1}{C_k} \sum_{r=-\infty}^{n=[t]} C^{(sp)} \left[f(t, \cdot), \frac{\mathcal{H}_k^{(1)}(r, \cdot)}{\sigma_k(r-1, \cdot)} \int_{\tau_k(r-1, \cdot)}^{r-1} S_k(t_1) \Gamma_k(t_1, r-1, \cdot) dt_1 \right]$$

Firing rates

Computations have been done using the INRIA Pranas software, B. Cessac et al, *Frontiers in Neuroinformatics*, Vol 11, page 49, (2017).

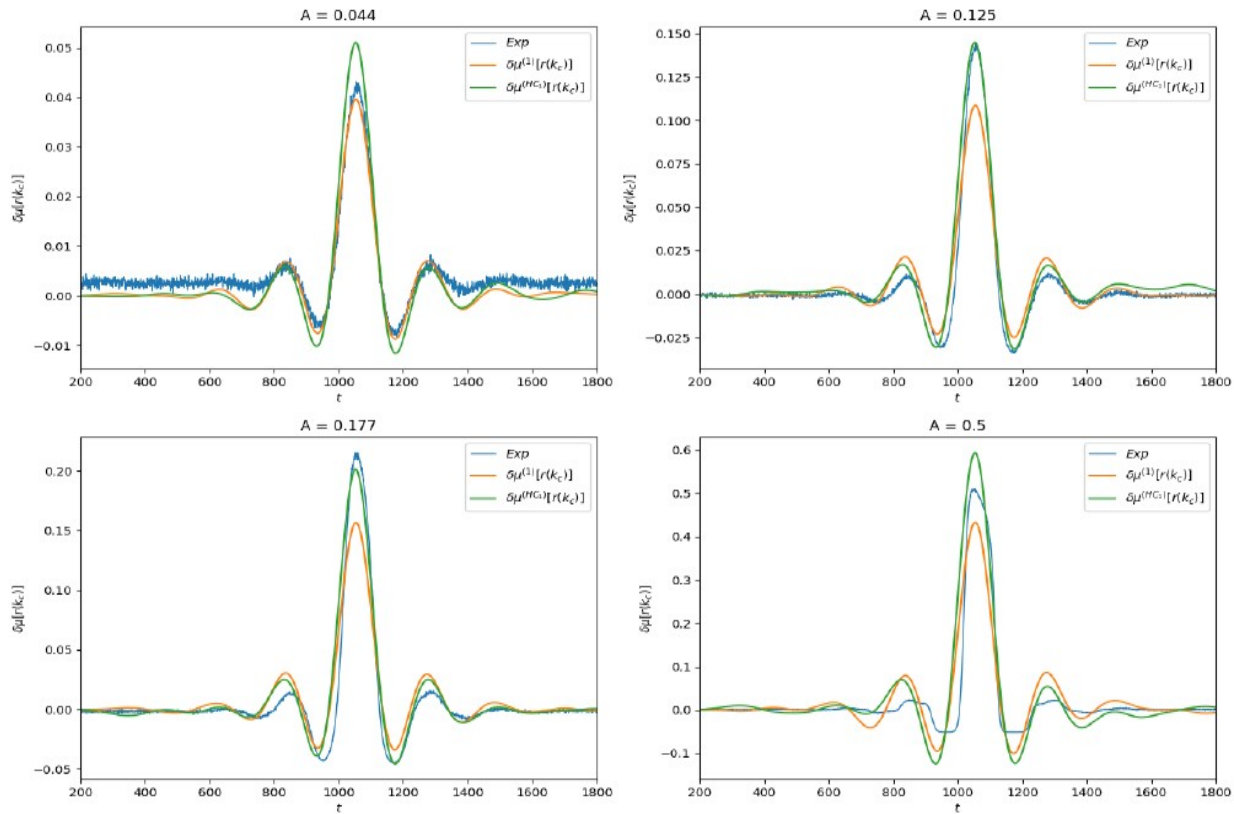


Figure: Linear response of $f(\omega, n) = \omega_{k_c}(n)$ for different values of stimulus amplitude A .

Higher order observable

Computations have been done using the INRIA Pranas software, B. Cessac et al, *Frontiers in Neuroinformatics*, Vol 11, page 49, (2017).

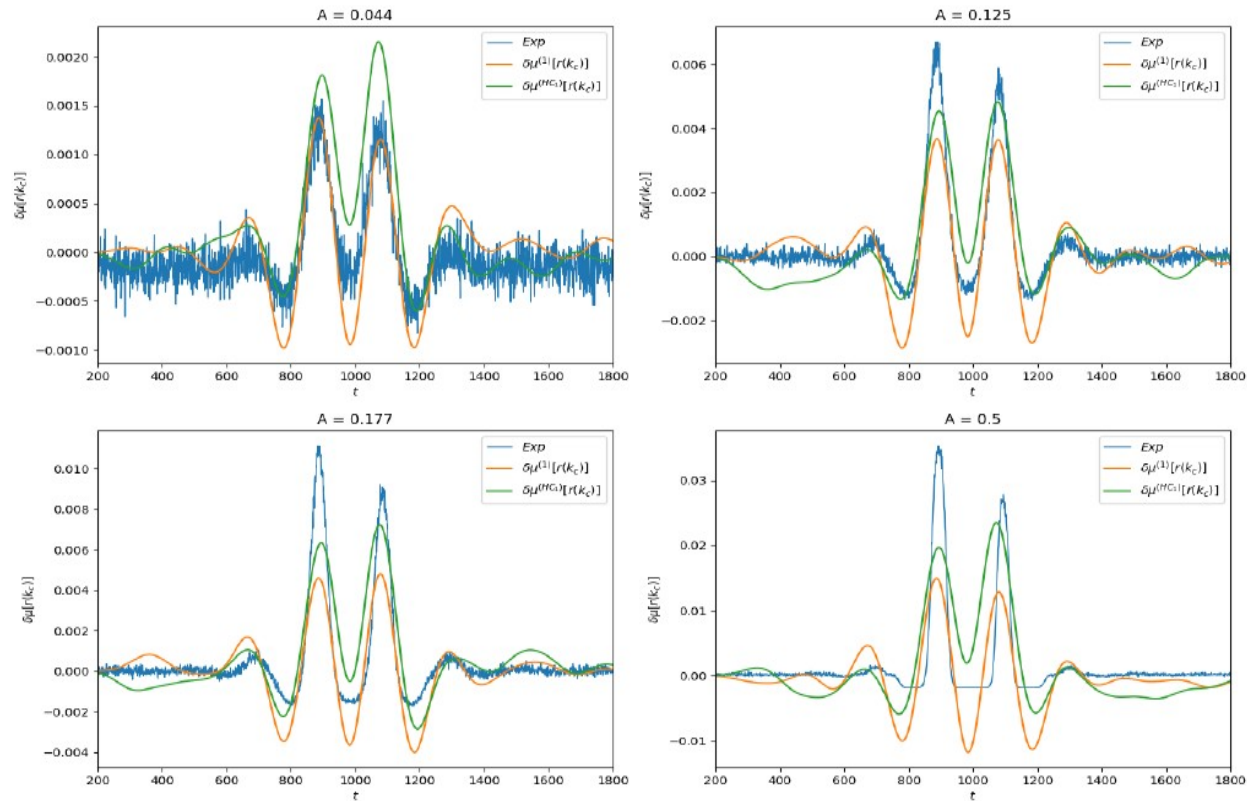


Figure: Linear response of the observable $f(n, \omega) = \omega_{k_c - 2}(n - 3)\omega_{k_c}(n)$.

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How could the visual cortex « decode » such signals ?
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Thanks 😊