Damage Localization in Mechanical Systems by Lasso Regression

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Abstract: Early signs of mechanical characteristic changes are essential for structural health monitoring (SHM). Due to the complexity of civil, mechanical or aeronautical structures, SHM is often faced with high dimensional mechanical characteristics together with limited sensor instrumentation. In this paper, Lasso regression is applied to address this complexity issue, based on its ability for solving large regression problems. The mechanical vibration model is first appropriately transformed into a linear regression model, with its parameters corresponding to small changes in the monitored mechanical characteristics, then these parameters are estimated from mechanical sensor signals under the assumption that most of the parameters are zeros. The performance of the proposed method is illustrated with a simulated truss structure.

Keywords: Fault isolation, mechanical system, damage localization, vibrations, regression, Lasso

1. INTRODUCTION

Vibration analysis plays an important role in structural health monitoring (SHM), notably for civil, mechanical and aeronautical structures (Farrar and Worden, 2007; Brownjohn, 2007). Structural *damages* cause changes in the characteristics of the associated mechanical system. Early signs of these changes are essential for deciding appropriate operations in order to prevent serious damages possibly implying dramatic consequences, such as complete structural failures. Vibration analysis is a powerful tool to detect and to localize such early signs. In this context, the tasks of *damage detection and localization* correspond to *fault detection and isolation* investigated in the literature on fault diagnosis.

There exist purely data-based methods for mechanical structure monitoring (Carden and Fanning, 2004), in particular for *damage detection*, see e.g. (Basseville et al., 2000; Balmes et al., 2006; Fassois and Sakellariou, 2009; Ramos et al., 2010; Viefhues et al., 2018). Such methods have the advantage of being easily applicable to structures sufficiently equipped with sensors collecting data, but they provide limited information about the physical origin of detected structural changes. In contrast, model-based methods are more powerful in terms of damage localization and diagnosis by associating physical model properties to sensor measurements (Teughels and De Roeck, 2005; Döhler et al., 2016), but more design efforts are required for the application of such methods (Allahdadian et al., 2019). The method proposed in this paper falls into the second category.

In a large and complex mechanical structure, damages may occur at many different places. In terms of a finite element (FE) model, damage localization consists in isolating the parameters of elements affected by damages, typically among *a large number of elements* involved in the FE model (Brownjohn et al., 2001; Simoen et al., 2015). This complexity, due to a large number of elements subject to damages, should be taken into account in the design of damage localization methods. Furthermore, random estimation uncertainties due to modeling errors and noisy measurements from limited sensors impair the analysis for damage localization in a large parameter space.

In this paper, Lasso regression (Tibshirani, 1996) will be applied to mechanical structure damage localization. Under the assumption that only a small number of parameters are non zeros (sparseness assumption), Lasso (least absolute shrinkage and selection operator) is able to address regression problems involving a large number of parameters based on fewer data than those required by conventional regression methods or by statistical tests. In the context of damage localization, damages are represented by deviations of model parameters from their nominal values. Lasso will be applied to the estimation of such parameter deviations, by assuming that only a small number of parameters would deviate from their nominal values. As a matter of fact, it is unlikely that many different components of a structure are damaged at the same time. Though damages may happen at many different places in a mechanical structure, only a few of them would occur simultaneously.

However, like other methods for sparse regression, Lasso is usually applied to problems formulated in the form of *linear regressions*, whereas mechanical structures are dynamic systems described with state-space models involving internal states. This formalism incompatibility represents a major difficulty for the application of sparse regression techniques to mechanical structure damage localization. A key step of the method proposed in this paper consists of establishing a link between these two formalisms, for efficient application of Lasso regression to damage localization.

It is assumed in this paper that a FE model of the monitored mechanical structure is available, in order to localize damages in terms of changes in its parameters. A discrete time *state-space model* will also be established in order to process sensor data sampled at discrete time instants. Inspired by the results reported in (Zhang and Basseville, 2014; Döhler et al., 2015; Döhler et al., 2017), this state-space model will be transformed in several steps in order to obtain a linear regression relating the mechanical sensor data to the monitored changes in the parameters of the FE model.

2. DIFFERENTIAL AND STATE-SPACE MODELS

Based on Newton's laws of motion and on spatial discretization, the vibration behavior of a mechanical structure subject to unknown ambient excitations is generally described by a vectorial ordinary differential equation (ODE)

$$\mathcal{M}\ddot{\mathcal{X}}(t) + \mathcal{C}\dot{\mathcal{X}}(t) + \mathcal{K}\mathcal{X}(t) = f(t) \tag{1}$$

where t denotes the time; $\mathcal{M}, \mathcal{C}, \mathcal{K} \in \mathbb{R}^{m\times m}$ are mass, damping, and stiffness matrices, respectively; $\mathcal{X}(t) \in \mathbb{R}^m$ is the displacement vector of m degrees of freedom of the structure; and $f(t) \in \mathbb{R}^m$ is some external unmeasured force, treated as random disturbances. Matrices $\mathcal{M}, \mathcal{C}, \mathcal{K}$ usually result from a spatial discretization of the structure by a finite element model, which is supposed to be known in the nominal state.

If the mechanical system described by the ODE equation (1) is equipped with accelerometers, they are installed at particular positions of the structure, delivering acceleration signals corresponding to $L_a\ddot{\mathcal{X}}(t)$, with a matrix L_a selecting some components of the acceleration vector $\ddot{\mathcal{X}}(t)$. Similarly, when velocity and displacement sensors are available, they deliver respectively velocity and displacement signals corresponding to $L_v\dot{\mathcal{X}}(t)$ and $L_d\mathcal{X}(t)$ with two selection matrices L_v and L_d. Typically the number of available sensors is much smaller than the dimension of $\mathcal{X}(t)$. This lack of sensors represents a major difficulty for mechanical structure SHM.

As sensor signals are sampled at discrete time instants ,say $t_k = k\tau$ (with $k = 0, 1, 2, \ldots$ and some sampling period τ), it is more convenient to process the data with a discrete time model. Discretizing the ODE (1) in time then results in a discrete-time state space system model (Juang, 1994):

$$\begin{cases} z_{k+1} = Fz_k + w_k \\ y_k = Hz_k + v_k \end{cases} \tag{2}$$

where the state vector $z_k = \left[\mathcal{X}(k\tau)^T \ \dot{\mathcal{X}}(k\tau)^T\right]^T \in \mathbb{R}^n$ with $n = 2m$, the measured output vector $y_k \in \mathbb{R}^r$ is filled with r sensor signals, and the system matrices

$$F = \exp(F^c\tau) \in \mathbb{R}^{n\times n}, \quad F^c = \begin{bmatrix} 0 & I \\ -\mathcal{M}^{-1}\mathcal{K} & -\mathcal{M}^{-1}\mathcal{C} \end{bmatrix}, \tag{3}$$

$$H = \begin{bmatrix} L_d - L_a\mathcal{M}^{-1}\mathcal{K} & L_v - L_a\mathcal{M}^{-1}\mathcal{C} \end{bmatrix} \in \mathbb{R}^{r\times n}, \tag{4}$$

with selection matrices L_d, L_v, L_a related to the positions of displacement, velocity or acceleration sensors, respectively. The state noise w_k and output noise v_k are unmeasured and assumed to be Gaussian, zero-mean and white.

In this paper, damage localization will be focused on the stiffness properties of the ODE model (1), corresponding to changes related to the parameters of structural elements. The stiffness matrix $\mathcal{K}$ is a function of these parameters. For the purpose of localizing damages in the structural elements, the stiffness matrix $\mathcal{K}$ can be parametrized by a vector of *independent parameters* η corresponding to the structural elements with $\mathcal{K} = \mathcal{K}(\eta)$. Changes in the physical parameter η provoke changes in the eigenstructure of system (1), and consequently of system (2). The related eigenstructure parameter vector θ is defined in the following.

The eigenvalues $\mu_i \in \mathbb{C}$ and eigenvectors $\phi_i \in \mathbb{C}^m$ of system (1) are $2m$ pairs of scalar values and vectors satisfying

$$(\mathcal{M}\mu_i^2 + \mathcal{C}\mu_i + \mathcal{K})\phi_i = 0, \quad i = 1, 2, \ldots, 2m.$$

They are related to the eigenvalues λ_i and eigenvectors ψ_i of the matrix F in (2), which satisfy

$$F\psi_i = \lambda_i\psi_i, \quad i = 1, 2, \ldots, n = 2m, \tag{5}$$

through

$$\lambda_i = \mathrm{e}^{\mu_i\tau}, \quad \psi_i = \begin{bmatrix} \phi_i \\ \mu_i\phi_i \end{bmatrix}. \tag{6}$$

Assume that the eigenstructure of the considered system contains only complex modes. Then, the eigenvalues μ_i consist of m complex conjugate pairs, so do the eigenvalues λ_i. Let

$$\mu \triangleq [\mu_1, \mu_2, \ldots, \mu_m]^T \in \mathbb{C}^m, \quad \lambda \triangleq [\lambda_1, \lambda_2, \ldots, \lambda_m]^T \in \mathbb{C}^m$$

be the vectors containing m of the $2m$ eigenvalues, one out of each of the m complex conjugate pairs, and

$$\phi \triangleq [\phi_1, \phi_2, \ldots, \phi_m] \in \mathbb{C}^{m\times m}$$

be composed of the corresponding eigenvectors. The eigenvalues contained in μ and eigenvectors in ϕ could be used to parametrize the state-space model (2), but they have the drawback of being complex numbers, whereas the variables and matrices involved in (2) are all of real values. To avoid complex valued parametrization, the eigenvalues and eigenvectors (μ_i, ϕ_i) are then replaced by the equivalent real eigenstructure parameter vector $\theta \in \mathbb{R}^{2m+2m^2}$ defined as

$$\theta \triangleq \begin{bmatrix} \Re(\mu) \\ \Im(\mu) \\ \mathrm{vec}(\Re(\phi)) \\ \mathrm{vec}(\Im(\phi)) \end{bmatrix} \tag{7}$$

where $\Re$ and $\Im$ denote respectively the real part and the imaginary part of a complex vector or matrix.

Assume that the matrix F in (5) is diagonalizable, then

$$F = T(\theta)A(\theta)T^{-1}(\theta) \tag{8}$$

with real matrices

$$A(\theta) = \begin{bmatrix} \mathrm{diag}(\Re(\lambda)) & \mathrm{diag}(\Im(\lambda)) \\ -\mathrm{diag}(\Im(\lambda)) & \mathrm{diag}(\Re(\lambda)) \end{bmatrix}, \tag{9}$$

$$T(\theta) = \begin{bmatrix} \Re(\phi) & \Im(\phi) \\ \Re(\phi\,\mathrm{diag}(\mu)) & \Im(\phi\,\mathrm{diag}(\mu)) \end{bmatrix}. \tag{10}$$

Similarly, following from (3), (4) and (8), the matrix H is then rewritten as

$$H = [L_d \ L_v] + [0_{r,m} \ L_a]\, T(\theta)A_c(\theta)T^{-1}(\theta) \tag{11}$$

where

$$A_c(\theta) = \begin{bmatrix} \text{diag}(\Re(\mu)) & \text{diag}(\Im(\mu)) \\ -\text{diag}(\Im(\mu)) & \text{diag}(\Re(\mu)) \end{bmatrix}.$$

With the parametrization of F and H by θ expressed in (8) and (11), the state-space model (2) is rewritten as

$$z_{k+1} = T(\theta)A(\theta)T^{-1}(\theta)z_k + w_k \tag{12a}$$

$$y_k = \left([L_d\ L_v] + [0_{r,m}\ L_a]\, T(\theta)A_c(\theta)T^{-1}(\theta)\right) z_k + v_k. \tag{12b}$$

It is assumed that the nominal values of the mechanical system matrices $\mathcal{M}$, $\mathcal{C}$ and $\mathcal{K}$ are available, typically based on a finite element model of the monitored structure. The nominal value θ^0 of the parameter vector θ is then accordingly deduced.

In the following sections, a perturbation analysis will be performed so that small changes in θ will appear as additive terms in the state-space model. Then, a link to the physical parameterization η will be made, and damage localization will be made in terms of changes in η.

3. PERTURBATION ANALYSIS

In the state-space model (2), parametrized as in (12), the unknown changes in parameter vector θ appear in the system matrices $F(\theta)$ and $H(\theta)$, which are in product with the unknown state vector z_k. In order to efficiently apply Lasso regression for damage localization, the state-space model will be transformed through a perturbation analysis based on a small parameter change assumption, so that the changes in parameter vector θ appear as additive terms.

Let θ^0 be the nominal value of θ, and assume a small change

$$\theta = \theta^0 + \varepsilon\theta^1. \tag{13}$$

Then accordingly

$$A(\theta) = A(\theta^0) + \varepsilon A(\theta^1) \triangleq A^0 + \varepsilon A^1, \tag{14a}$$

$$A_c(\theta) = A_c(\theta^0) + \varepsilon A_c(\theta^1) \triangleq A_c^0 + \varepsilon A_c^1, \tag{14b}$$

$$T(\theta) = T(\theta^0) + \varepsilon T(\theta^1) \triangleq T^0 + \varepsilon T^1. \tag{14c}$$

Let z_k^0 be the state trajectory estimate assuming $\theta = \theta^0$ that can be obtained from a Kalman filter based on the nominal system (2) with $F = F(\theta^0)$ and $H = H(\theta^0)$, and assume that

$$z_k = z_k^0 + \varepsilon z_k^1. \tag{15}$$

Substitute (14) into (12) while neglecting high order terms in ε, the state-space model then becomes

$$z_{k+1} \approx F^0 z_k + \Psi_k^\theta \tilde{\theta} + w_k \tag{16a}$$

$$y_k \approx H^0 z_k + \Phi_k^\theta \tilde{\theta} + v_k \tag{16b}$$

where $F^0 = T^0 A^0 (T^0)^{-1}$, the matrices $\Psi_k^\theta \in \mathbb{R}^{n\times q}$ and $\Phi_k^\theta \in \mathbb{R}^{r\times q}$ with $q = 2m + 2m^2$ are filled with known signals, as respectively detailed in Appendix A and Appendix B. The matrices Ψ_k^θ and Φ_k^θ are fully computed from known signals, hence the parameter change $\tilde{\theta}$ is additive in (16).

3.1 Link to physical parametrization

In this paper, damage localization will be focused on changes in the stiffness matrix $\mathcal{K} = \mathcal{K}(\eta)$, while the mass and damping matrices $\mathcal{M}$ and $\mathcal{C}$ are assumed remaining unchanged. This is a typical case in structural health monitoring applications, and it can be easily generalized to changes in the other matrices. The physical parameter vector $\eta = [\eta_1, \eta_2, ..., \eta_p]^T \in \mathbb{R}^p$ constitutes an independent parametrization, which is linked to the structural type and geometry. Usually, damage is related to changes in few components of η, which affect the entire parameter vector θ. As in (13), the parameter η is decomposed into a nominal value and a small change by

$$\eta = \eta^0 + \varepsilon\eta^1,$$

and $\tilde{\eta} = \varepsilon\eta^1$.

The parameter vector θ in (7) and the physical parameter vector η are related through (Heylen et al., 1998)

$$\frac{\partial\mu_i}{\partial\eta_k} = -\frac{1}{a_i}\phi_i^T \frac{\partial\mathcal{K}(\eta)}{\partial\eta_k}\phi_i, \tag{17}$$

$$\frac{\partial\phi_i}{\partial\eta_k} = \sum_{l=1,l\neq i}^{m} \frac{1}{a_l}\frac{1}{\mu_l - \mu_i}\phi_l^T \frac{\partial\mathcal{K}(\eta)}{\partial\eta_k}\phi_i\phi_l + \sum_{l=1}^{m} \frac{1}{a_l^*}\frac{1}{\mu_l^* - \mu_i}\phi_l^H \frac{\partial\mathcal{K}(\eta)}{\partial\eta_k}\phi_i\phi_l^* \tag{18}$$

where

$$a_i = 2\mu_i\phi_i^T\mathcal{M}\phi_i + \phi_i^T\mathcal{C}\phi_i,$$

"$*$" denotes the complex conjugate and "^{H}" the conjugate transpose. Assembling the real and imaginary parts of (17) and (18) for $i = 1, \ldots, m$ in the rows and for $k = 1, \ldots, p$ in the columns leads to the Jacobian matrix

$$\mathcal{J}_{\theta,\eta} = \frac{\partial\theta}{\partial\eta},$$

which is computed at θ^0 and η^0, and thus

$$\tilde{\theta} \approx \mathcal{J}_{\theta,\eta}\tilde{\eta}.$$

Then, following from (16a) and (16b), the change detection problem in the physical parameter vector η is transformed to the detection of *additive* changes in the system

$$z_{k+1} \approx F^0 z_k + \Psi_k^\eta \tilde{\eta} + w_k \tag{19a}$$

$$y_k \approx H^0 z_k + \Phi_k^\eta \tilde{\eta} + v_k, \tag{19b}$$

with

$$\Psi_k^\eta = \Psi_k^\theta\, \mathcal{J}_{\theta,\eta} \tag{20}$$

$$\Phi_k^\eta = \Phi_k^\theta\, \mathcal{J}_{\theta,\eta}. \tag{21}$$

4. DAMAGE LOCALIZATION BY LASSO REGRESSION

If the vectors of displacement $\mathcal{X}$, velocity $\dot{\mathcal{X}}$ and acceleration $\ddot{\mathcal{X}}$ were fully available directly from sensor signals, then it would be easy to apply Lasso to a sampled version of the ODE model (1). However, in practice, only a small number of selected components of these vectors are measured by sensors. For this reason, the state-space model (19) subject to additive parameter changes will be transformed into an equivalent linear regression model, through an advanced residual analysis of the Kalman filter, so that Lasso regression can be easily applied.

Design a Kalman filter for the *nominal system*, which is described by the state-space model (19), but with $\tilde{\eta} = 0$. This Kalman filter is implemented with the system matrices F^0 and H^0 in (19) and the noise covariance matrices

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} = \mathrm{E}\left(\begin{bmatrix} w_k \\ v_k \end{bmatrix} \begin{bmatrix} w_k^T & v_k^T \end{bmatrix} \right).$$

Let K be the steady state Kalman gain and the innovation covariance matrix

$$\Sigma = H^0 P (H^0)^T + R, \tag{22}$$

where P is the solution of the algebraic Riccati equation associated to the Kalman filter.

Apply this Kalman filter to the *possibly faulty system* (19). Denote the one step ahead prediction $z_k^0 = z_{k|k-1}(\eta^0)$ and the innovation sequence

$$\zeta_k \triangleq y_k - F^0 z_k^0.$$

If actually $\tilde{\eta} = 0$, then it is well known that ζ_k is a centered Gaussian white noise, otherwise the innovation sequence is biased and satisfies the linear regression equation (Zhang and Basseville, 2014):

$$\zeta_k \approx (H^0 \Gamma_k + \Phi_k^\eta)\tilde{\eta} + e_k \tag{23}$$

where Γ_k is recursively computed as

$$\Gamma_{k+1} = F^0 (I_n - K H^0)\Gamma_k + \Psi_k^\eta - F^0 K \Phi_k^\eta \tag{24}$$

$$\Gamma_0 = 0, \tag{25}$$

and e_k is a white Gaussian noise of zero mean with covariance Σ as expressed in (22).

In the linear regression equation (23), apart from the unknown parameter increment $\tilde{\eta}$ and the white Gaussian noise e_k, the other involved variables are either known or can be recursively computed: ζ_k is the innovation sequence of the Kalman filter, H^0 is known, Γ_k is computed with (24), and Φ_k^η is given by (21) (See Appendices A and B for the details about Ψ_k^θ and Φ_k^θ).

With the linear Gaussian model (23), damage detection then consists in testing $\tilde{\eta} \neq 0$ against $\tilde{\eta} = 0$. This hypothesis testing problem can be solved by the generalized likelihood ratio (GLR) test. See (Basseville and Nikiforov, 1993) and also (Zhang and Basseville, 2014). In what follows, it is assumed that the application of the GLR test has already detected some damages. Then Lasso regression will be applied to localize the damages.

Given sensor data collected for $k = 1, 2, \ldots, N$, build a matrix $\boldsymbol{X}$ and a vector $\boldsymbol{y}$ as

$$\boldsymbol{X} = \begin{bmatrix} H^0\Gamma_1 + \Phi_1^\eta \\ H^0\Gamma_2 + \Phi_2^\eta \\ \vdots \\ H^0\Gamma_N + \Phi_N^\eta \end{bmatrix}, \quad \boldsymbol{y} = \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_N \end{bmatrix}. \tag{26}$$

Then, according to (23),

$$\boldsymbol{y} = \boldsymbol{X}\tilde{\eta} + \boldsymbol{e} \tag{27}$$

with the error vector

$$\boldsymbol{e} = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix}. \tag{28}$$

Sparse estimation algorithms can be applied to efficiently solve equation (27) for $\tilde{\eta}$ under the assumption that $\tilde{\eta}$ has few non-zero components. In particular, Lasso (Tibshirani, 1996) consists in solving a regularized least squares problem:

$$\hat{\tilde{\eta}} = \arg\min_{\tilde{\eta}} \left(\frac{1}{2N} \|\boldsymbol{X}\tilde{\eta} - \boldsymbol{y}\|_2^2 + \lambda \|\tilde{\eta}\|_1 \right) \tag{29}$$

where $\|\cdot\|_1$ and $\|\cdot\|_2$ denote respectively the l_1 norm and l_2 norm, and $\lambda > 0$ is a weighting coefficient.

The term $\lambda\|\tilde{\eta}\|_1$ controls the number of non-zero components in the Lasso estimate $\hat{\tilde{\eta}}$. A tricky point is how to choose the value of λ in practice. The number of non-zero components in the Lasso solution $\hat{\tilde{\eta}}$ decreases when λ increases, and there is a wide range of λ in which $\hat{\tilde{\eta}}$ keeps the same number of non-zero components. The value of λ can be chosen at the middle of this range, by numerically exploring different values of λ. A numerical example will be given in the next section. Damages are then localized in terms of non-zero components of $\hat{\tilde{\eta}}$.

5. NUMERICAL EXAMPLE

In this example, the proposed method is applied to damage localization for a simulated truss structure with 25 elements of equal stiffness properties, as illustrated in Fig. 1. Each truss element has a Young's modulus of $E =$ 14.4 GPa and a cross-sectional area of $A = (10/144)$ m^2. Each non-diagonal truss element has a length of $L = 10$ m. The parameter vector $\eta = [\eta_1, ..., \eta_{25}]^T$ is defined as the axial stiffness of each of the elements. Damage is simulated by decreasing the stiffness of elements 3 and 7 by 20% each. The structure is excited by Gaussian white noise forces acting at the six positions indicated by arrows in Fig. 1. Six acceleration sensors are attached to these positions in vertical direction, as symbolized by the blue nodes in the same figure. The standard deviation of the measurement noises is equal to 5% of the standard deviation of the sensor signals. Sensor data records are generated with different data lengths N (the number of time instants at which sensor data are sampled) at a sampling frequency of 100 Hz.

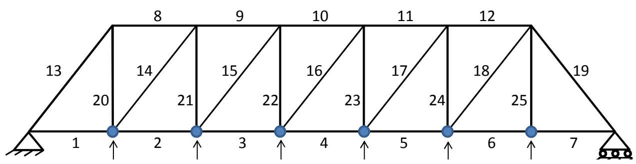

Fig. 1. Truss structure comprising 25 elements, with six sensors.

To apply Lasso to damage localization, as formulated in (29), it is necessary to choose the value of the weighting coefficient λ. The number of non-zero components in the Lasso estimate $\hat{\tilde{\eta}}$ decreases when λ increases. An example is given in Fig. 2 for Lasso estimates based on $N = 2000$ data points. There is a wide range where the number of non-zero components, represented by red circles, remains 2. The value of λ is automatically selected at the middle of the widest range in which the number of non-zero components is constant.

Due to random uncertainties, sometimes damage localization yields wrong results. The rate of successful localization increases with the data sample length N. Based on 1000 random realizations simulated with each of the

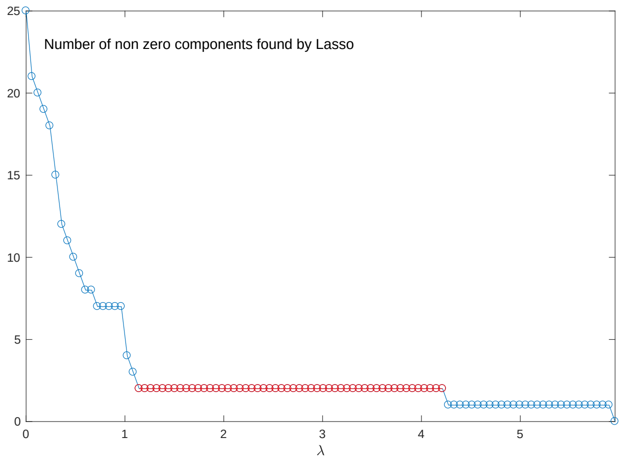

Fig. 2. Example of number of non-zero components of the Lasso estimate $\hat{\tilde{\eta}}$ per λ. This number remains 2 in the range represented by red circles.

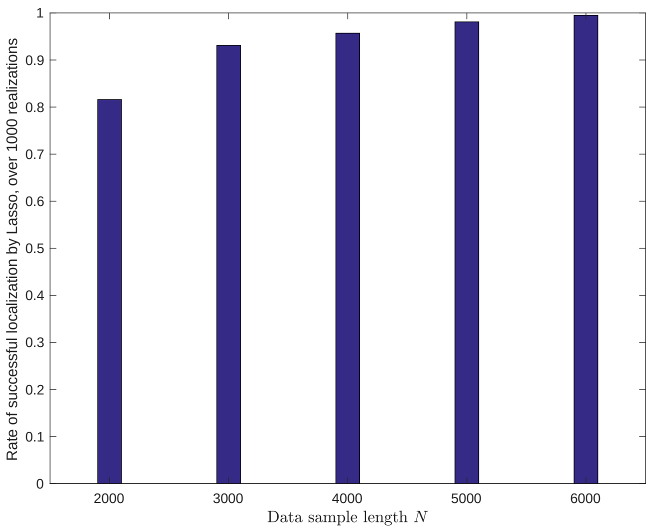

Fig. 3. Rate of successful damage localization by Lasso evaluated over 1000 random realizations simulated with each of the data sample lengths $N = 2000, 3000, 4000, 5000, 6000$.

data sample lengths $N = 2000, 3000, 4000, 5000, 6000$, the rates of successful localization are shown in Fig. 3. In these results, localization is successful in 81.6% of the simulated realizations with $N = 2000$, and 99.6% with $N = 6000$. For the simulations with $N = 10000$, the simulated damages have been correctly localized in all the 1000 random realizations.

6. CONCLUSIONS

In this paper, the ability of Lasso regression for addressing the complexity issue in SHM is illustrated through its application to a simulated truss structure. Due to limited available sensors subject to random measurement noises, traditional methods require the processing of large data sets in order to localize damages corresponding to changes in some mechanical characteristic parameters within a large dimensional parameter space. The reported results show that Lasso can correctly localize damages by processing moderately large data sets. This preliminary study will be completed by more realistic simulations and laboratory experiments in future works.

Appendix A. STATE EQUATION PERTURBATION

Consider the above perturbation for the state equation (12a). By using the approximation

$$(T^0 + \varepsilon T^1)^{-1} \approx (T^0)^{-1} - \varepsilon (T^0)^{-1} T^1 (T^0)^{-1},$$

consider the matrix $T(\theta)A(\theta)T^{-1}(\theta)$ in (12) as a perturbed matrix $T^0 A^0 (T^0)^{-1}$, then

$$\begin{aligned} T(\theta)A(\theta)T^{-1}(\theta) &= (T^0 + \varepsilon T^1)(A^0 + \varepsilon A^1)(T^0 + \varepsilon T^1)^{-1} \\ &\approx T^0 A^0 (T^0)^{-1} + \varepsilon \Big(T^1 A^0 (T^0)^{-1} + \\ &\quad T^0 A^1 (T^0)^{-1} - T^0 A^0 (T^0)^{-1} T^1 (T^0)^{-1} \Big) \end{aligned}$$

where the terms involving $\varepsilon^2, \varepsilon^3$ have been omitted. Then, using (15),

$$\begin{aligned} T(\theta)A(\theta)T^{-1}(\theta) z_k &\approx T^0 A^0 (T^0)^{-1} z_k + \varepsilon \Big(T^1 A^0 (T^0)^{-1} \\ &+ T^0 A^1 (T^0)^{-1} - T^0 A^0 (T^0)^{-1} T^1 (T^0)^{-1} \Big) z_k^0. \end{aligned} \tag{A.1}$$

In the above parenthesis in product with z_k^0, the matrices T^0, A^0 are independent of θ, A^1 and T^1 are linearly parametrized by θ^1, see (14). Let the small change in θ (see (13)) be denoted by

$$\tilde{\theta} = \varepsilon \theta^1.$$

The following lemma has been shown in (Döhler et al., 2017).

Lemma 1. It holds

$$\begin{aligned} \varepsilon \left(T^1 A^0 (T^0)^{-1} + T^0 A^1 (T^0)^{-1} \right. \\ \left. - T^0 A^0 (T^0)^{-1} T^1 (T^0)^{-1} \right) z_k^0 \approx \Psi_k^\theta \tilde{\theta}, \end{aligned} \tag{A.2}$$

where matrix $\Psi_k^\theta \triangleq \Psi_k^{(1)} + \Psi_k^{(2)} + \Psi_k^{(3)}$ is filled with known signals, as follows:

- $\Psi_k^{(1)} \triangleq \begin{bmatrix} 0 & 0 & \bar{l}_k^T \otimes I_m & \underline{l}_k^T \otimes I_m \\ \Psi_k^{(1,1)} & \Psi_k^{(1,2)} & \Psi_k^{(1,3)} & \Psi_k^{(1,4)} \end{bmatrix}$, where
$$\begin{aligned} \Psi_k^{(1,1)} &= \Re(\phi^0)\overline{L}_k + \Im(\phi^0)\underline{L}_k \\ \Psi_k^{(1,2)} &= -\Im(\phi^0)\overline{L}_k + \Re(\phi^0)\underline{L}_k \\ \Psi_k^{(1,3)} &= (\overline{L}_k \Re(\mu^0) + \underline{L}_k \Im(\mu^0))^T \otimes I_m \\ \Psi_k^{(1,4)} &= (-\overline{L}_k \Im(\mu^0) + \underline{L}_k \Re(\mu^0))^T \otimes I_m, \end{aligned}$$
with
$$\begin{bmatrix} \bar{l}_k \\ \underline{l}_k \end{bmatrix} \triangleq A^0 (T^0)^{-1} z_k^0, \; \overline{L}_k \triangleq \mathrm{diag}(\bar{l}_k), \; \underline{L}_k \triangleq \mathrm{diag}(\underline{l}_k),$$
where $\bar{l}_k, \underline{l}_k \in \mathbb{R}^m$.
- $\Psi_k^{(2)} \triangleq -T^0 A^0 (T^0)^{-1} P_k$, where matrix P_k is defined analogously to $\Psi_k^{(1)}$ when replacing $\bar{l}_k$, $\underline{l}_k$, $\overline{L}_k$ and $\underline{L}_k$ by $\bar{h}_k$, $\underline{h}_k$, $\overline{H}_k$ and $\underline{H}_k$, respectively, where
$$\begin{bmatrix} \bar{h}_k \\ \underline{h}_k \end{bmatrix} \triangleq (T^0)^{-1} z_k^0, \; \overline{H}_k \triangleq \mathrm{diag}(\bar{h}_k), \; \underline{H}_k \triangleq \mathrm{diag}(\underline{h}_k).$$
- $\Psi_k^{(3)} \triangleq [Q_k \;\; 0_{n,2m^2}]$, where
$$Q_k = \tau T^0 \begin{bmatrix} \overline{H}_k & \underline{H}_k \\ \underline{H}_k & -\overline{H}_k \end{bmatrix} \begin{bmatrix} \mathrm{diag}(\Re(\lambda^0)) & -\mathrm{diag}(\Im(\lambda^0)) \\ \mathrm{diag}(\Im(\lambda^0)) & \mathrm{diag}(\Re(\lambda^0)) \end{bmatrix}.$$

Appendix B. OUTPUT EQUATION PERTURBATION

Analogously to (A.1), a perturbation in the output equation (12b) yields

$$\big([L_d\ L_v] + [0_{r,m}\ L_a]\, T(\theta)A_c(\theta)T^{-1}(\theta)\big)z_k \approx H^0 z_k + \varepsilon\,[0_{r,m}\ L_a]\Big(T^1 A_c^0 (T^0)^{-1} + T^0 A_c^1 (T^0)^{-1} - T^0 A_c^0 (T^0)^{-1} T^1 (T^0)^{-1}\Big) z_k^0,$$

where

$$H^0 = [L_d\ L_v] + [0_{r,m}\ L_a]\, T^0 A_c^0 (T^0)^{-1}.$$

Similarly as in the previous section, the terms A_c^1 and T^1 in the above parenthesis are linearly parametrized by θ^1, as shown in (Döhler et al., 2017):

Lemma 2. It holds

$$\varepsilon\,[0_{r,m}\ L_a]\Big(T^1 A_c^0 (T^0)^{-1} + T^0 A_c^1 (T^0)^{-1} - T^0 A_c^0 (T^0)^{-1} T^1 (T^0)^{-1}\Big) z_k^0 \approx \Phi_k^\theta \tilde{\theta}, \quad \text{(B.1)}$$

where matrix $\Phi_k^\theta \triangleq L_a(\Phi_k^{(1)} + \Phi_k^{(2)} + \Phi_k^{(3)})$ is filled with known signals, as follows:

- $\Phi_k^{(1)} \triangleq \Big[\Phi_k^{(1,1)}\ \Phi_k^{(1,2)}\ \Phi_k^{(1,3)}\ \Phi_k^{(1,4)}\Big]$, where
$$\Phi_k^{(1,1)} = \Re(\phi^0)\overline{D}_k + \Im(\phi^0)\underline{D}_k$$
$$\Phi_k^{(1,2)} = -\Im(\phi^0)\overline{D}_k + \Re(\phi^0)\underline{D}_k$$
$$\Phi_k^{(1,3)} = (\overline{D}_k\Re(\mu^0) + \underline{D}_k\Im(\mu^0))^T \otimes I_m$$
$$\Phi_k^{(1,4)} = (-\overline{D}_k\Im(\mu^0) + \underline{D}_k\Re(\mu^0))^T \otimes I_m,$$
with
$$\begin{bmatrix}\overline{d}_k\\ \underline{d}_k\end{bmatrix} \triangleq A_c^0 (T^0)^{-1} z_k^0,\ \overline{D}_k \triangleq \mathrm{diag}(\overline{d}_k),\ \underline{D}_k \triangleq \mathrm{diag}(\underline{d}_k),$$
and $\overline{d}_k, \underline{d}_k \in \mathbb{R}^m$.
- $\Phi_k^{(2)} \triangleq -[0_{m,m}\ I_m]\, T^0 A_c^0 (T^0)^{-1} P_k$
- $\Phi_k^{(3)} \triangleq [0_{m,m}\ I_m]\, T^0 \begin{bmatrix}\overline{H}_k & \underline{H}_k & 0_{m,2m^2}\\ \underline{H}_k & -\overline{H}_k & 0_{m,2m^2}\end{bmatrix}$

Note that $\Phi_k^\theta = 0$ in the case where only displacements or velocities are measured, excluding accelerations.

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