Statistical optimization for subspace-based damage quantification

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ABSTRACT: The purpose of model updating is to minimize the misfit between the structural response measurements and an assumed numerical model. In the context of damage quantification, this misfit is characterized by some features computed from the response data measured on the faulty structure, and its Finite Element (FE) model in the healthy condition. The FE model is parameterized so that the estimated features are related to the physical parameters of the model. Therein, the parameterization size may be large. As a consequence of low instrumentation, different parameters can have a similar effect on the estimated features, resulting in non uniqueness of the updating problem solutions, even taking into account the inherent uncertainty errors, originating both from the model and the measurements. In this paper a model updating-based damage quantification strategy is proposed. It involves the minimization of two Hankel matrices, one computed from the data and another from the optimized model. The difference between those two matrices is studied, in particular in the practical case where the ambient excitation is unknown. It yields a statistical residual, whose deviations from zero can be evaluated through a statistical test. The resulting optimization is based on the Generalized Likelihood Ratio test as an objective function and uses its 95 per cent quantile as a measure of closeness for a stopping criterion for the optimization. Moreover, the large size of the finite element model to optimize compared to the low instrumentation has to be taken into account by clustering the parameter space. This clustering is proceeded using the well known stochastic subspace-based damage localisation method. The proposed framework is validated on simulations of a simple mechanical system.

KEY WORDS: Model updating, Damage quantification, Uncertainty quantification, subspace methods.

1 INTRODUCTION

The damage diagnosis problem can be divided into damage detection, localization, quantification and lifetime prediction [1]. Whereas the detection of damages from vibration measurements is well established e.g. in [2–7], the localization and quantification of damages is more complex, and requires additional physical information on the examined structure [8–11]. In this respect, few complete frameworks for damage identification exist, e.g. [12–14]. Some methods consider a specific structural type in their theory, e.g. beams [13], while other methods incorporate the required physical information from a FE model, which allows application to more complex structures. In this context, the sensitivities of a damage feature computed from the structural responses can be used to infer FE model parameter changes in [12], under the assumption of small damage extents. Contrary to these approaches, FE model optimization-based methods are usually not limited by the structural type or by the damage extent. Therein model updating is a generic term encompassing a family of many methods [15,16], that are often applied in the damage quantification context [17–20]. However, model updating methods are often poorly conditioned due to the possibly large FE parameter space in comparison to relatively few parameters that can be extracted from data, and are prone to statistical uncertainty errors of the estimated features. The objective of this paper is to develop a cascade framework where the damage is first localized and then quantified on the subset of the damaged parameter space, and which explicitly considers data-based uncertainties in the model optimization. An objective function is proposed based on the statistical hypothesis test of a residual composed from the Hankel matrices of output covariances of the monitored system compared to a numerical counterpart. The proposed residual unites simplicity of the damage feature, robustness towards changing ambient excitation conditions by an appropriate normalization, and a sound statistical evaluation within the local approach framework.

The paper is organized as follows. The considered model optimization problem, the background on system modeling is stated in Section 2. The objective function based on Hankel matrix difference is defined in Section 3. The damage quantification framework based on the damage localization-based parameter clustering and the stochastic optimization scheme is presented in Section 4. The application of the proposed framework on a numerical simulation of a simple mechanical system is enclosed in Section 5.

2 BACKGROUND

The overall goal of this paper is the estimation of the structural parameter change between a healthy reference state and the current damaged state. Let $\theta \in \Theta \subset \mathbb{R}^p$ be the parameter vector that contains the damage-sensitive parameters of the structural elements of interest for the considered problem within a bounded parameter space Θ. The parametrization is user-defined and adapted to the specific monitoring problem at hand. It is assumed that the parameter vector θ_0 in the healthy state of the structure is known, and let θ_* be the parameter vector in the damaged state. It is the goal to obtain an estimate $\hat{\theta}$ of θ_* from vibration measurements in the damaged states for estimating the damage extent $\delta = \theta_* - \theta_0$. Finding the optimal $\hat{\theta}$ can be formulated as an optimization problem, where an objective function $F(\theta)$, also called cost function, is minimized over the parameter space Θ.

The objective function is designed to represent the discrepancy between $\hat{v}$, the estimate of a feature vector computed from measurement data recorded under the (unknown) system parameter θ_*, and its counterpart $v(\theta)$ computed from a parametric model. The optimal solution for θ is then obtained as

$$\hat{\theta} = \underset{\theta\in\Theta}{\arg\min}\ F(\theta). \tag{1}$$

A classic feature vector $\hat{v}$ for the design of the objective function $F(\theta)$ in (1) is based on the modal parameters of the monitored system [21], and their statistical uncertainties obtained using e.g. [22–24]. In this work an objective function is proposed based on the statistical hypothesis test of a residual composed from the Hankel matrices of the monitored system compared to a numerical counterpart. An immediate advantage is to avoid the computation and the correspondence between the numerical modes and the estimated ones.

A decision whether the current parameter of the numerical model θ corresponds to the unknown parameter θ_* needs to take into account the uncertainty information of the estimated feature and its sensitivity towards the considered parameterization. An approach to assess this equivalence is to define a residual whose properties can be evaluated properly as the number of samples goes to infinity. For this, two hypotheses are defined

$$\begin{aligned} &\mathrm{H}_0 : \theta = \theta_* \quad \text{(model matched)}, \\ &\mathrm{H}_1 : \theta = \theta_* + \kappa/\sqrt{N} \quad \text{(model mismatched)}, \end{aligned} \tag{2}$$

where $\kappa = \sqrt{N}(\theta - \theta_*)$ is a change vector between the assumed system parameter θ and the the sought parameter θ_* . Notice that the hypotheses H_0 and H_1 are reversed from the classical damage detection setup in [8], where H_0 represented the healthy reference state, whereas here it represents the damaged state, which is now a reference for the model updating.

2.1 System and models

Assume that the dynamics of the monitored system can be modeled as linear time-invariant (LTI) with d degrees of freedom (DOF), which are described by the differential equation of motion

$$\mathcal{M}^\theta \ddot{q}(t) + \mathcal{D}^\theta \dot{q}(t) + \mathcal{K}^\theta q(t) = u(t) \tag{3}$$

where t denotes the continuous time, and the matrices $\mathcal{M}^\theta$, $\mathcal{D}^\theta$ and $\mathcal{K}^\theta \in \mathbb{R}^{d\times d}$ denote the mass, damping and stiffness matrices, respectively, which depend on the system parameter θ. The vectors $q(t) \in \mathbb{R}^d$ and $u(t) \in \mathbb{R}^d$ denote the continuous-time displacements and the unknown external forces, respectively. Observed at r sensor positions, e.g. by acceleration, velocity or displacement sensors at discrete time instants $t = k\tau$ (with sampling rate $1/\tau$), system (3) can be transformed into the discrete-time state-space model [25]

$$\begin{cases} x_{k+1}^\theta = A^\theta x_k^\theta + w_k \\ y_k^\theta = C^\theta x_k^\theta + v_k \end{cases} \tag{4}$$

where $x_k^\theta \in \mathbb{R}^n$ are the states, $y_k^\theta \in \mathbb{R}^r$ are the outputs, vectors w_k and v_k denote the white process and output noise respectively, $A^\theta \in \mathbb{R}^{n\times n}$, $C^\theta \in \mathbb{R}^{r\times n}$ are the state and observation matrices, and $n = 2m$ is the model order. The process noise v_k is assumed to be a stationary process with zero mean and covariance matrix $Q = \mathrm{E}(v_k v_k^T)$, w_k denotes the zero-mean output noise with covariance matrix $R = \mathrm{E}(w_k w_k^T)$, and the covariance between v_k and w_k is $S = \mathrm{E}(v_k w_k^T)$, where $\mathrm{E}(\cdot)$ denotes the expectation operator.

For simplicity, the $(\cdot)^\theta$ notation is dropped in the remainder of this paragraph. Let $\mathcal{R}_i = \mathrm{E}(y_k y_{k-i}^T) = CA^{i-1}G$ be the theoretical output covariances of the measurements, where $G = \mathrm{E}(x_{k+1} y_k^T) = A\Sigma^s C^T + S$ and $\Sigma^s = \mathrm{E}(x_k x_k^T) = A\Sigma^s A^T + Q$. The collection of $\mathcal{R}_i$ can be stacked to form a block Hankel matrix

$$\mathcal{H} = \begin{bmatrix} \mathcal{R}_1 & \mathcal{R}_2 & \dots & \mathcal{R}_q \\ \mathcal{R}_2 & \mathcal{R}_3 & \dots & \mathcal{R}_{q+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{R}_{p+1} & \mathcal{R}_{p+2} & \dots & \mathcal{R}_{p+q} \end{bmatrix} \in \mathbb{R}^{(p+1)r\times qr}, \tag{5}$$

where p and q are chosen such that $\min(pr, qr) \geq n$ with often $p+1 = q$. Matrix $\mathcal{H}$ enjoys the factorization property

$$\mathcal{H} = \mathcal{O}(C, A)\ \mathcal{C}(A, G), \tag{6}$$

where the observability and controllability matrices $\mathcal{O}(C, A)$ and $\mathcal{C}(A, G)$ are defined as

$$\mathcal{O}(C, A) = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^p \end{bmatrix},\ \mathcal{C}(A, G) = \begin{bmatrix} G & AG & \dots & A^{q-1}G \end{bmatrix}, \tag{7}$$

where (A, C) can be easily computed directly from $\mathcal{M}$, $\mathcal{D}$, $\mathcal{K}$ [25], and G is obtained based on the chosen noise properties after [26].

Consistent estimates $\hat{\mathcal{H}}$ can be obtained from the output covariances of the measurements $\{y_k\}_{k=1,\dots,N+p+q}$ e.g. by

$$\hat{\mathcal{H}} = \mathcal{Y}^+ \mathcal{Y}^{-T}, \tag{8}$$

where the data Hankel matrices $\mathcal{Y}^+$ and $\mathcal{Y}^-$ contain future and past time horizons

$$\mathcal{Y}^+ = \frac{1}{\sqrt{N}} \begin{bmatrix} y_{q+1} & y_{q+2} & \cdots & y_{N+q} \\ y_{q+2} & y_{q+3} & \cdots & y_{N+q+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{p+q+1} & y_{p+q+2} & \cdots & y_{p+q+N} \end{bmatrix},$$

$$\mathcal{Y}^- = \frac{1}{\sqrt{N}} \begin{bmatrix} y_q & y_{q+1} & \cdots & y_{N+q-1} \\ y_{q-1} & y_q & \cdots & y_{N+q-2} \\ \vdots & \vdots & \ddots & \vdots \\ y_1 & y_2 & \dots & y_N \end{bmatrix}.$$

The estimates of $\mathcal{O}(C, A)$ and $\mathcal{C}(A, G)$ are classically obtained from a singular value decomposition (SVD) of $\hat{\mathcal{H}}$ thanks to the factorization property (6).

2.2 Hankel matrix normalization scheme

Model-based Hankel matrix parametrized with θ, and Hankel matrix computed from data generated under parameter θ_* can be compared by a simple objective function for a model optimization-based damage quantification. Hankel matrices, however, are not only dependent on the dynamics of the system, but also on the unknown covariance of the noise processes Q. Then, since the model-based Hankel matrix can not be defined under the same Q as its data-based equivalent, their comparison in search of θ is meaningless. To overcome this problem, a proper normalization of the model-based Hankel matrix is recalled after [27], where this normalization was introduced to compensate change in the noise properties between different data sets. Here, since there is only one data set, the matrix Q is not changing but is still unknown, and the same normalisation is useful.

Let $\mathcal{H}^{\theta_*}_{\text{data}}$ and $\mathcal{H}^{\theta_*}_{\text{model}}$ be two exact Hankel matrices of rank n for a system in the state θ_*, subjected to process noise with different covariances Q_{data} and Q_{model}. A SVD of the juxtaposed matrices $\mathcal{H}_{\text{data}}$ and $\mathcal{H}_{\text{model}}$ writes

$$\left[\mathcal{H}^{\theta_*}_{\text{data}}\ \mathcal{H}^{\theta_*}_{\text{model}}\right] = \left[U_{\text{s}}\ U_{\text{ker}}\right] \begin{bmatrix} D_{\text{s}} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{\text{s}}^T \\ V_{\text{ker}}^T \end{bmatrix}, \tag{9}$$

where $\text{rank}\left(\left[\mathcal{H}^{\theta_*}_{\text{data}}\ \mathcal{H}^{\theta_*}_{\text{model}}\right]\right) = n$, $U_{\text{s}} \in \mathbb{R}^{(p+1)r\times n}$ contains the left singular vectors, $D_{\text{s}} \in \mathbb{R}^{n\times n}$ contains the non-zero singular values and $V_{\text{s}} \in \mathbb{R}^{2qr\times n}$ contains the right singular vectors, which are split into $V_{\text{s}}^T = \left[V_{\text{s,data}}^T\ V_{\text{s,model}}^T\right]$ corresponding to $\mathcal{H}^{\theta_*}_{\text{data}}$ and $\mathcal{H}^{\theta_*}_{\text{model}}$ respectively. Now define

$$\mathcal{Z}_{\text{data}} = D_{\text{s}} V_{\text{s,data}}^T, \quad \mathcal{Z}_{\text{model}} = D_{\text{s}} V_{\text{s,model}}^T, \tag{10}$$

where both $\mathcal{Z}_{\text{data}}$ and $\mathcal{Z}_{\text{model}}$ are full row rank. The exact Hankel matrices share the same image in the reference state

$$\left[\mathcal{H}^{\theta_*}_{\text{data}}\ \mathcal{H}^{\theta_*}_{\text{model}}\right] = U_{\text{s}}\left[\mathcal{Z}_{\text{data}}\ \mathcal{Z}_{\text{model}}\right]. \tag{11}$$

To compare $\mathcal{H}^{\theta_*}_{\text{data}}$ with $\mathcal{H}^{\theta_*}_{\text{model}}$ an appropriate normalization is given by

$$\overline{\mathcal{H}}_{\text{model}} = \mathcal{H}^{\theta_*}_{\text{model}} \mathcal{Z}^{\dagger}_{\text{model}} \mathcal{Z}_{\text{data}}, \tag{12}$$

where $\overline{\mathcal{H}}_{\text{model}}$ now shares the same $\mathcal{C}(A, G)$ as $\mathcal{H}^{\theta_*}_{\text{data}}$.

2.3 Hankel matrix difference-based residual for damage diagnosis

Let $\mathcal{H}^{\theta}_{\text{model}}$ be the model-based Hankel matrix generated under some chosen process noise covariance Q_{model}. Next, let $\hat{\mathcal{H}}^{\theta_*}_{\text{data}}$ be obtained from a data set of length N generated under a process noise covariance Q_{data}, with Q_{data} different from Q_{model}. Then, after (11) it yields

$$\mathcal{H}^{\theta}_{\text{model}} \mathcal{Z}^{\dagger}_{\text{model}} \mathcal{Z}_{\text{data}} - \mathcal{H}^{\theta_*}_{\text{data}} = 0 \quad \text{iff} \quad \theta = \theta_*, \tag{13}$$

and

$$\mathcal{H}^{\theta}_{\text{model}} \mathcal{Z}^{\dagger}_{\text{model}} \mathcal{Z}_{\text{data}} - \mathcal{H}^{\theta_*}_{\text{data}} \neq 0 \quad \text{iff} \quad \theta \neq \theta_*, \tag{14}$$

where $\mathcal{Z}_{\text{data}}$ and $\mathcal{Z}_{\text{model}}$ are obtained from an SVD of $\left[\mathcal{H}^{\theta_*}_{\text{data}}\ \ \mathcal{H}^{\theta}_{\text{model}}\right]$ as in (9) but truncated at order n.

Based on both (13) and (14), the change detection residual is defined as

$$\hat{\zeta}^{\theta} \stackrel{\text{def}}{=} \sqrt{N}\text{vec}(\mathcal{H}^{\theta}_{\text{model}} \hat{\mathcal{Z}}^{\dagger}_{\text{model}} \hat{\mathcal{Z}}_{\text{data}} - \hat{\mathcal{H}}^{\theta_*}_{\text{data}}), \tag{15}$$

where $\hat{\mathcal{Z}}_{\text{data}}$ and $\hat{\mathcal{Z}}_{\text{model}}$ are defined from the following SVD and partitioned at order n

$$\left[\hat{\mathcal{H}}^{\theta_*}_{\text{data}}\ \mathcal{H}^{\theta}_{\text{model}}\right] = \left[\hat{U}_{\text{s}}\ \hat{U}_{\text{ker}}\right] \begin{bmatrix} \hat{D}_{\text{s}} & 0 \\ 0 & \hat{D}_{\text{ker}} \end{bmatrix} \begin{bmatrix} \hat{V}_{\text{s,data}}^T & \hat{V}_{\text{s,model}}^T \\ \hat{V}_{\text{ker,data}}^T & \hat{V}_{\text{ker,model}}^T \end{bmatrix},$$

as $\hat{\mathcal{Z}}_{\text{data}} = \hat{D}_{\text{s}} \hat{V}_{\text{s,data}}^T$, $\hat{\mathcal{Z}}_{\text{model}} = \hat{D}_{\text{s}} \hat{V}_{\text{s,model}}^T$.

Then, (15) is zero if and only if the parameters θ and θ_* correspond, up to some statistical considerations that will be discussed in the next section.

Alternatively or concurrently to the difference of Hankel matrices, other metrics can be used, based on modal parameters or the kernel of the Hankel matrix used for damage localization in [8]. The present metric has the merits to relate to the classical Manhalobis distance [27].

3 OBJECTIVE FUNCTION BASED ON HANKEL MATRIX DIFFERENCE

In this section an objective function for model optimization is expressed by a difference between two Hankel matrices. The statistical properties of the residual defined in Section 2.2 are derived and a hypothesis test for data-model correspondence is established. Then, an objective function for optimization of the numerical model is constructed. Finally, a criterion for assessing the minima of the objective function is derived based on the quantile of the developed statistical distribution.

3.1 Residual distribution

To characterize the distribution of the residual (15) the asymptotic local approach for change detection [28] is used. Firstly, thanks to the Central Limit Theorem (CLT), the local approach ensures that $\hat{\mathcal{H}}^{\theta}_{\text{data}}$ is asymptotically Gaussian, and it holds

$$\sqrt{N}\,\text{vec}(\hat{\mathcal{H}}^{\theta_*}_{\text{data}} - \mathcal{H}^{\theta_*}_{\text{data}}) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Sigma_{\text{data}}), \tag{16}$$

where Σ_{data} is the asymptotic reference Hankel matrix covariance. Note that an estimate of Σ_{data} can be easily obtained from the sample covariance as in e.g. [29].

Then, it can be proved that the residual in (15) is asymptotically Gaussian with the following properties

$$\text{H}_0 : \hat{\zeta}^{\theta_*} \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Sigma_{\zeta}), \tag{17}$$

$$\text{H}_1 : \hat{\zeta}^{\theta} \xrightarrow{\mathcal{L}} \mathcal{N}(\mathcal{J}^{\zeta}_{\theta_*}\kappa, \Sigma_{\zeta})\ , \tag{18}$$

where

$$\Sigma_{\zeta} = \mathcal{J}^{\zeta}_{\mathcal{H}_{\text{data}}} \Sigma_{\text{data}} (\mathcal{J}^{\zeta}_{\mathcal{H}_{\text{data}}})^T, \tag{19}$$

and $\mathcal{J}^{\zeta}_{\mathcal{H}_{\text{data}}} = I_{(p+1)r} \otimes U_{\text{ker}} U_{\text{ker}}^T$. The sensitivity of the residual with respect to the chosen parameterization $\mathcal{J}^{\zeta}_{\theta_*}$ is defined as

$$\mathcal{J}^{\zeta}_{\theta_*} = \left(\left(\mathcal{Z}^{\dagger}_{\text{data}} \mathcal{Z}_{\text{data}}\right)^T \otimes U_{\text{ker}} U_{\text{ker}}^T \right) \mathcal{J}^{\mathcal{H}_{\text{data}}}_{\theta_*}, \tag{20}$$

where $\mathcal{J}^{\mathcal{H}_{\text{data}}}_{\theta_*} = \partial\text{vec}\,(\mathcal{H}_{\text{data}})/\partial\theta\ (\theta_*)$.

Notice that the sensitivity matrix $\mathcal{J}^{\mathcal{H}_{\text{data}}}_{\theta_*}$ is not computed from the unknown model θ_*, but from the data alone. In that sense, the sensitivities are expressed with respect to a data-driven parameterization, which consists in the modal parameters. As such, a consistent estimate of $\mathcal{J}^{\mathcal{H}_{\text{data}}}_{\theta_*}$ can be computed using the modal parameter estimates, as in [30].

3.2 Objective function for model optimization

Let $\widehat{\mathcal{J}}$ and $\widehat{\Sigma}$ respectively be consistent estimates of $\mathcal{J}^{\zeta}_{\theta_*}$ and Σ_{ζ}. Then a Generalized Likelihood Ratio (GLR) test to decide between H_0 and H_1 writes as

$$\mathrm{GLR}^{\theta} = (\hat{\zeta}^{\theta})^T\, \widehat{\Sigma}^{-1}\, \widehat{\mathcal{J}} \left(\widehat{\mathcal{J}}^T\, \widehat{\Sigma}^{-1}\, \widehat{\mathcal{J}}\right)^{-1} \widehat{\mathcal{J}}^T\, \widehat{\Sigma}^{-1} \hat{\zeta}^{\theta} \,. \quad (21)$$

Assuming Σ_{ζ} to be invertible, under H_0, GLR^{θ} follows a χ^2 distribution with $\mathrm{rank}(\mathcal{J}^{\zeta}_{\theta_*})$ degrees of freedom.

The test value from (21) can be directly used in the objective function for the model optimization as

$$F(\theta) = \mathrm{GLR}^{\theta}. \quad (22)$$

The quantile q_{χ^2} of the underlying χ^2 distribution, satisfying $\int_0^{q_{\chi^2}} f_{\chi^2}(x)dx = \gamma$, where γ is a desired confidence level, can be used to define an acceptance region

$$\Theta = \{\theta : \mathrm{GLR}^{\theta} \leq q_{\chi^2}\}, \quad (23)$$

which comprises all parameter vectors θ that yield $\mathcal{H}^{\theta}_{\mathrm{model}}$ to comply with the estimated reference $\hat{\mathcal{H}}^{\theta_*}_{\mathrm{data}}$ under the statistical distribution of the GLR. Thus, Θ comprises the statistically acceptable solutions for θ_* with regards to the considered models, and a stopping criterion of the optimization search can be formulated for $\theta \in \Theta$.

Notice that the stopping criterion and the objective function do not involve computation or matching of the numerical modes for any model. Only the modes identified from the data set are needed in the computation of $\widehat{\mathcal{J}}$. It yields a very direct and simple computational scheme for optimization.

4 DAMAGE QUANTIFICATION STRATEGY

In the previous section, an objective function has been elaborated whose set of minima corresponds to models that are statistically compliant with the Hankel matrix computed from measurements. As such, looking for the minima of this function will yield the sought models. This, however, is not trivial.

For example, some optimization strategies might be stuck in local minima due to the rugged nature of the proposed objective function. Moreover, the lack of identifiability for parameters of large FE models requires clustering in order to estimate the change over subsets of similar parameters, and consequently to quantify the possible damage. In this paper, it is proposed to chain a damage localization approach with a model optimization procedure to minimize the designed objective function (22) over the subset of parameters that have been recognized as damaged by the localization approach. In this way, the search domain for the optimization approach can be reduced efficiently. The resultant damage quantification framework is summarized in Figure 1.

For damage localization, the subspace-based damage localization approach [8] is chosen, and the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [31] is adapted as the optimization approach. Both methods are outlined in the following subsections. Similarly, other localization methods and other optimization methods can be chosen for this framework. The damage quantification procedure comprising the aforementioned strategies is summarized in Algorithm 1.

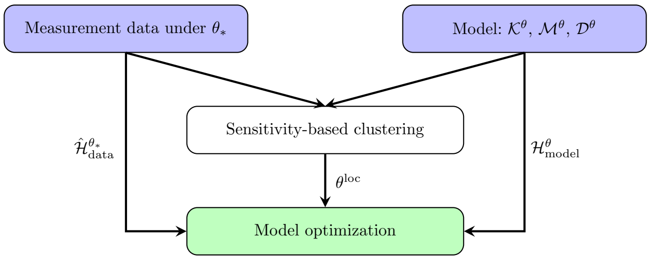

Figure 1. Damage quantification flow chart

4.1 Damage localization and statistical sensitivity-based clustering

Due to a possibly large FE model-based parametrization θ on one side and limited measurements regarding the number of sensors on the other side, the sensitivity of the optimized feature vector with respect to some components of θ may be equal or be very close. Thus, such parameter components are indistinguishable, and clustering of the parameters is performed to estimate their total change over some localized subset.

For this purpose the statistical sensitivity clustering, which is a part of the subspace-based damage localization [30,8,32] is used. The output of such a procedure is a subset of the parameter vector θ, denoted by θ^{loc}, which contains all parameters of the clusters that are recognized as damaged. Hence, it can be considered that the remaining parameters in θ are unchanged. Subsequently, the damage extent can be quantified by updating only the parameter subset θ^{loc}. Thus, linking the damage localization with an adequate model optimization method should lead to a damage quantification scheme, where the dimension of the search domain is reduced by the localization approach. Clustering the parameter space is essential for large parameterization. However, this will not be detailed in the considered numerical applications for proof of concept where all elements are well separated statistically.

4.2 Model optimization with modified CMA-ES

Starting with the initial value $\theta = \theta_{\mathrm{init}}$ corresponding to the model in the reference state, the CMA algorithm consists in generating λ model candidates θ^g_j, $j = 1, \ldots, \lambda$, in each population g, by sampling a multivariate Gaussian distribution. The model candidates are only modified on the parameter subset θ^{loc} that is given by the localization approach. The sampling is carried out on the considered parameter subset for the subsequent population $g+1$ as

$$(\theta^{\mathrm{loc}}_j)^{g+1} = m^g + \varepsilon_j, \quad \varepsilon_j \sim \sigma^g \mathcal{N}\left(0, C^g\right), \quad (24)$$

where $j = 1, \ldots, \lambda$ and m^g is a weighted mean of the model candidates $(\theta^{\mathrm{loc}}_j)^g$ in the parent generation. Then, the parameter subset of the full parameter vector θ^{g+1}_j is updated with $(\theta^{\mathrm{loc}}_j)^{g+1}$. Equation (24) represents a mutation and recombination into offsprings, for which the

CMA-ES algorithm adapts the parameters C^g and σ^g in each generation. The covariance matrix C^g of the added Gaussian noise represents the amplitude for the sampling to occur, and the scaling factor σ^g determines the range of the considered mutation. Consequently, the optimization continues and the best parent solutions replace the offspring until it converges to a solution. For CMA-ES, the covariance matrix C^g is incrementally updated with rank-one matrices representing the direction between the best parent solutions at two consecutive generations, such that the likelihood of previously successful search steps is increased [31].

For the convergence to a solution, a stopping criterion is included in the algorithm that is adapted to the acceptance region (23). Since the acceptance region consists of all models that are equally optimal in the statistical sense, the algorithm can stop once a number t_{opt} of population model candidates are inside the region, where $t_{\text{opt}} \geq 1$. A higher value leads to more confidence in the set of retained solutions. Once inside the acceptance region, there is no need to further minimize the objective function, avoiding unnecessary additional computations.

Then the final solution is computed as the mean of the selected model candidates $\theta^g_{j_k} \in \overline{\Theta}$ in the last population g that are lying in the acceptance region, with

$$\theta_{\text{sol}} = \tfrac{1}{t_{\text{opt}}} \sum_{k=1}^{t_{\text{opt}}} \theta^g_{j_k} \tag{25}$$

Finally, the change in the parameter vector is evaluated for damage quantification. Since the changes in the parameter components of each cluster are indistinguishable as described in the previous section, only the global change for each cluster can be evaluated, while the separate evaluation of changes of different parameters within the same cluster is impossible. Then, the change in any damaged cluster c can be evaluated as

$$\hat{\delta}_c = \sum_{k=1}^{p_c} \theta_{\text{sol}}^{i_c(k)} - \theta_{\text{init}}^{i_c(k)} \tag{26}$$

where $i_c(1), \ldots, i_c(p_c)$ are the indices of the components of parameter vector θ that correspond to cluster c, and p_c is the number of elements in c.

5 NUMERICAL APPLICATION

In this section, the proposed damage quantification framework is applied in a numerical experiment on a 6 DOF mechanical chain-like system that for any consistent set of units is modeled with spring stiffness $k_1 = k_3 = k_5 = 100$ and $k_2 = k_4 = k_6 = 200$, mass $m_i = 1/20$ and a proportional damping matrix such that all modes have a damping ratio of 3%. The chain is illustrated in Figure 2. The system is excited by a white noise acting at all DOFs whose covariance yields $Q = bb^T$ where $b \in \mathbb{R}^{6\times 6}$ is a matrix whose entries are drawn from a standard normal distribution. The structural accelerations are simulated at DOFs 1, 3 and 5 at a sampling frequency of 50 Hz, and white measurement noise with 5% of the standard deviation of the output is added to each response measurement. Each measurement comprises $N = 200{,}000$ samples.

Algorithm 1: Model optimization

Input : output measurements $\{y_k\}_{k=1,\ldots,N+p+q}$;
physical parameter θ_{init};
parameterized model $\mathcal{K}^\theta$, $\mathcal{M}^\theta$, $\mathcal{D}^\theta$;
optimization parameters from Table 1 ;

Output: estimated parameter value θ_{sol} and
estimated total change $\hat{\delta}$ for each cluster ;

1 compute $\hat{\mathcal{H}}^{\theta_*}_{\text{data}}$ from (8) and the estimate of Σ_{data} ;
2 create $\mathcal{K}^{\theta_{\text{init}}}$, $\mathcal{M}^{\theta_{\text{init}}}$, $\mathcal{C}^{\theta_{\text{init}}}$ and compute $\mathcal{H}^{\theta_{\text{init}}}_{\text{model}}$ from (6) and (7) ;
3 compute the residual $\hat{\zeta}^\theta$ (15), the estimate of Σ_ζ (19) and $\mathcal{J}^\zeta_{\theta_*}$ (20) ;
4 evaluate $F(\theta_{\text{init}})$ in (22) ;
5 **repeat**
6 get λ model candidates $\theta_1, \ldots, \theta_\lambda$ in (24) ;
7 repeat steps **2**, **3** and **4** with $\theta_1, \ldots, \theta_\lambda$;
8 update CMA-ES parameters after [31] ;
9 count the number t_Θ of model candidates θ_j with $\theta_j \in \overline{\Theta}$ in the acceptance region, see (23) ;
10 **until** $t_\Theta \geq t_{\text{opt}}$;
11 compute θ_{sol} in (25) as the mean of all model candidates in $\overline{\Theta}$;
12 compute the change in the parameter value $\hat{\delta}_c$ for each damaged cluster c in (26)

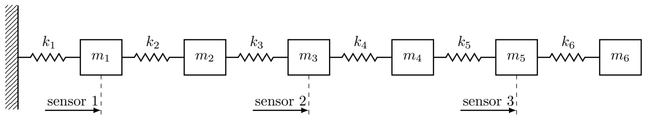

Figure 2. 6 DOF chain system sketch

In total 1000 data sets for each case of Q are realized. In each data set the damage is modeled as a stiffness reduction of the 4-th spring by 5%. Hereafter let θ be the collection of the element stiffness k_{1-6}. Albeit the damages considered herein relate to the stiffness changes, the proposed approach is general and can be formulated to quantify changes in any parameter of the numerical model for which the modal parameter sensitivities are nonzero. For damage quantification the procedure outlined in Algorithm 1 is used.

Due to the simplicity of the examined system, each parameter is reported in a separate cluster, indicating that the sensitivities of the estimated residual w.r.t. the considered system parameters are distinguishable. Moreover, a large probability for damage in the 4-th spring is reported by the subspace-based damage localization method, correctly mapping the location of damage. Since the parameterization is small enough, the full parameter set can be used for the damage quantification with Algorithm 1. In practice, however, the optimization would be only run on the detected clusters. The parameters used to initialize the optimization algorithm are depicted in Table 1.

Firstly, the proposed objective function is examined. The function $F(\theta)$ in (22) maps the p-dimensional parameter space to a multidimensional hyperplane whose shape can easily be illustrated for $p = 2$. Consider the estimate of $\mathcal{H}^{\theta_*}_{\text{data}}$ computed from the available data, and $\mathcal{H}^\theta_{\text{model}}$

Table 1. CMA-ES optimization parameters

σ_{init}	θ^i_{init}	$\theta^i_{\text{min-max}}$	λ	t_{opt}
60	150	75-225	1000	60% λ

parametrized with spring stiffnesses k_j. The corresponding function $F(\theta)$ is displayed for the parameter pair (k_3, k_4) in Figure 3 and (k_4, k_5) in Figure 4. Note that the remaining spring stiffnesses are $(k_1 = 100, k_2 = 200, k_6 = 200)$.

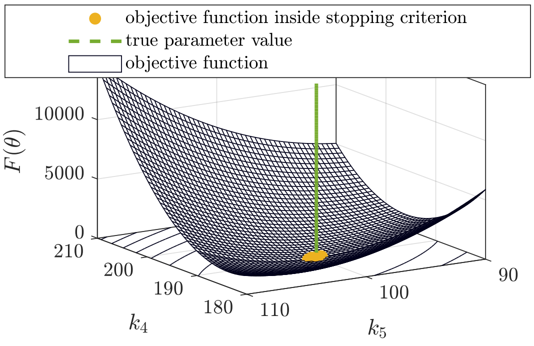

Figure 3. Objective function $F(\theta)$ for the parameter pair k_3 and k_4.

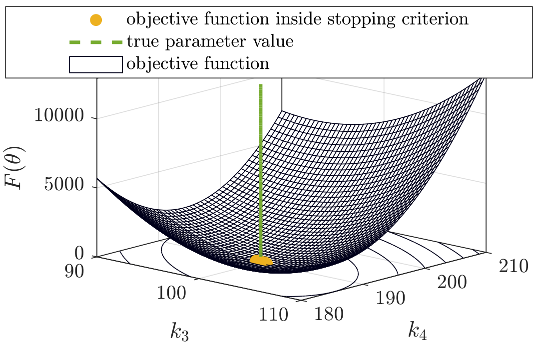

Figure 4. Objective function $F(\theta)$ for the parameter pair k_4 and k_5.

It can be viewed that $F(\theta)$ is convex in the considered parameter region. The global minimum of $F(\theta)$ is well-aligned with the value $F(\theta_*)$ at the exact parameter pair $(100, 190)$ for the parameters k_3 and k_4 and $(190, 100)$ for the parameters k_4 and k_5. A group of parameters around the true value satisfies the uncertainty-based stopping criterion. This is a consequence of the statistical uncertainty introduced by estimating $\mathcal{H}^{\theta_*}_{\text{data}}$ in $F(\theta)$.

Secondly, the optimization results based on a single measurement set are analyzed. Each optimization population contains λ parameters, from which t_{opt} parameters are required to be within the acceptance region for the algorithm to stop. As such, the estimated change can be examined for every parameter set in each iteration of the algorithm, illustrating its performance. This is depicted in Figure 5.

Figure 5 shows the iterations of the estimated change $\hat{\delta}_{k_3}$ and $\hat{\delta}_{k_4}$ until the last iteration. Notice that within the last iteration not all the parameters are compliant with the stopping criterion. Those parameters are characterized with a higher objective function value, which indicates that at least one considered model is not corresponding to

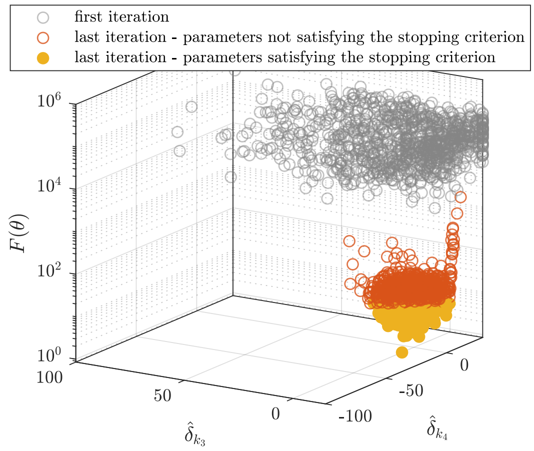

Figure 5. CMA-ES optimization of the change in parameters k_3 and k_4.

measurements. The model optimization takes 12 iterations to converge to a feasible solution, which together with the acceptance region is visualized in Figure 6.

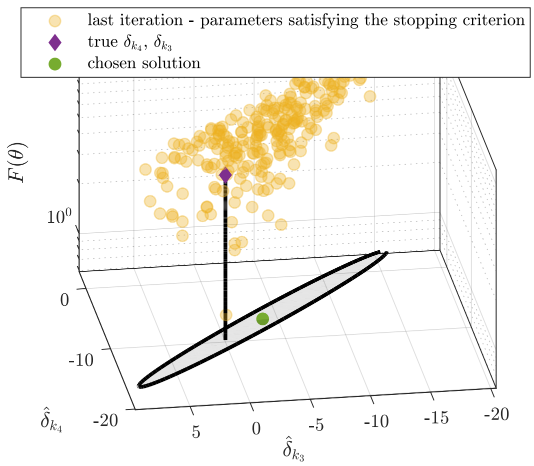

Figure 6. Zoom on the changes in k_3 and k_4 corresponding to the acceptance region in the last iteration.

A zoom on $\hat{\delta}_{k_3}$ and $\hat{\delta}_{k_4}$ illustrated in Figure 6, corresponds to the parameters complying with the stopping criterion. The estimates are spread around the exact value, whereas the mean solution is close to the exact value.

Next, consider estimating $\hat{\delta}_k$ for each of the 1000 measurement realizations. Each data set yields different estimates $\hat{\mathcal{H}}^{\theta_*}_{\text{data}}$ governed by the same statistical distribution. Consequently, the variability of each $\hat{\delta}_k$ stems from the statistical uncertainty of $\hat{\mathcal{H}}^{\theta_*}_{\text{data}}$. Thus computing $\hat{\delta}_k$ from independent data sets should yield a histogram whose variance corresponds to the variance of the estimate of δ_k. The histograms of $\hat{\delta}_k$ for each system parameter are plotted in Figures 7-9.

It can be viewed that for each parameter case the mean value of $\hat{\delta}_k$ from all histograms aligns with the true change value, and the 5% damage in k_4 is correctly quantified.

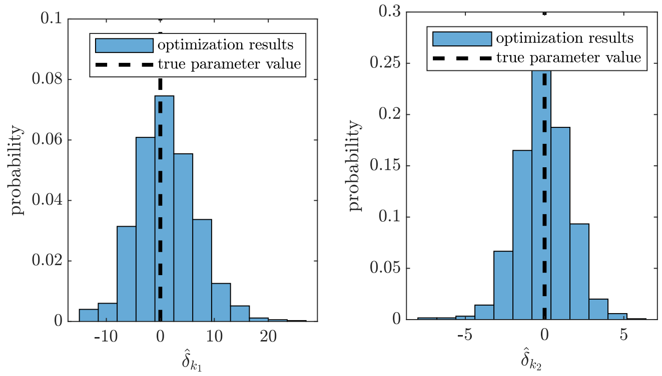

Figure 7. CMA-ES optimization of the change in parameters k_1 and k_2.

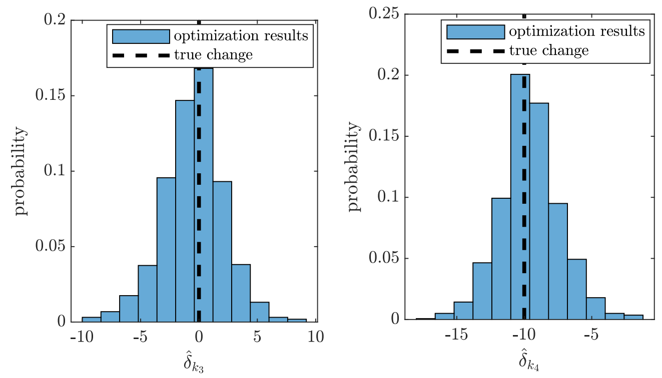

Figure 8. CMA-ES optimization of the change in parameters k_3 and k_4.

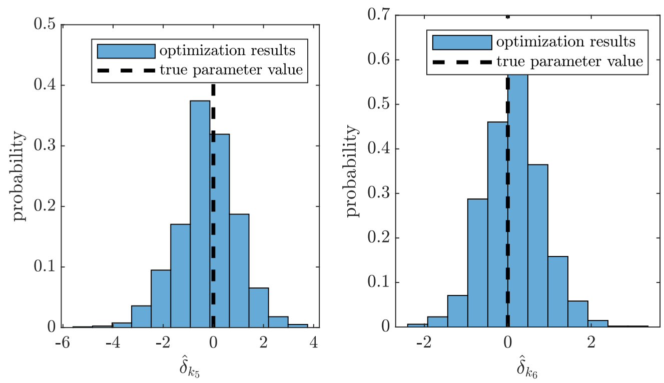

Figure 9. CMA-ES optimization of the change in parameters k_5 and k_6.

6 CONCLUSIONS

In this paper, a damage quantification strategy has been developed that uses the difference between a data-driven and a model-based Hankel matrices. A stopping criterion based on the quantile of the statistical law of a Mahalanobis metric based on these two matrices has been used in the CMA-ES-based optimization. Preliminary localization results have clustered the parameter space and shown what clusters were potentially damaged. It has then been shown that the damage extent is clearly identified for damaged elements in different clusters. To show the impact of clustering and how damage can not be identified and separated inside one cluster, further studies on large finite element models have to be conducted.

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