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# Simultaneous Information and Energy Transmission with Finite Constellations

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**Abstract:** In this report, the fundamental limits on the rates at which information and energy can be simultaneously transmitted over an additive white Gaussian noise channel are studied under the following assumptions: (a) the channel is memoryless; (b) the number of channel input symbols (constellation size) and block length are finite; and (c) the decoding error probability (DEP) and the energy outage probability (EOP) are bounded away from zero. In particular, it is shown that the limits on the maximum information and energy transmission rates; and the minimum DEP and EOP, are essentially set by the type induced by the code used to perform the transmission. That is, the empirical frequency with which each channel input symbol appears in the codewords. Using this observation, guidelines for optimal constellation design for simultaneous energy and information transmission are presented.

**Key-words:** Simultaneous information and energy transmission, finite constellations

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## Transmission simultanée d'information et d'énergie avec des constellations finies

**Résumé :** Dans ce rapport, les limites fondamentales des débits auxquelles l'information et l'énergie peuvent être transmises simultanément sur un canal de bruit gaussien blanc additif sont étudiées sous les hypothèses suivantes: (a) le canal est sans mémoire; (b) le nombre de symboles d'entrée du canal (taille de la constellation) et la durée de la transmission sont finis; et (c) la probabilité d'erreur de décodage (DEP) et la probabilité de coupure d'énergie (EOP) sont limitées par des bornes inférieures strictement positives. En particulier, il est montré que les limites sur les débits maximaux de transmission d'information et d'énergie et les DEP et EOP minimaux, sont essentiellement fixés par le *type* induit par le code utilisé pour effectuer la transmission. C'est-à-dire la fréquence empirique à laquelle chaque symbole d'entrée de canal apparaît dans les mots de code. En utilisant cette observation, des recommandations pour la conception optimale des constellations pour la transmission simultanée d'information et d'énergie sont présentées.

**Mots-clés :** Transmission simultanée d'information et d'énergie, constellations finies

## 1 Introduction

Nikola Tesla suggested that radio frequency signals can be used to simultaneously transmit both information and energy in 1914 [1]. About two centuries later, simultaneous information and energy transmission (SIET), also known as simultaneous wireless information and power transfer (SWIPT), is one of the technologies that might be implemented in 6G communications systems in the near future, c.f., [2] and [3]. In the following, for the sake of correctness, when radio signals are involved, the denomination “energy transmission” is preferred against “power transfer”. Thus, the acronym SIET is adopted instead of SWIPT, in the remaining of this report.

SIET implies a fundamental trade-off between the amount of energy and information that can be simultaneously transmitted by a signal. This has been the subject of intense research, c.f., [4–7] and [8]. In [4], a capacity-energy function is defined in order to determine the fundamental limit on the information transmission rate subject to the fact that the average energy at the channel output is not smaller than a given threshold. Therein, the underlying assumption is that the communication duration in channel uses is infinitely long. This guarantees that the decoding error probability (DEP) and the energy outage probability (EOP) can be made arbitrarily close to zero, and thus, the focus is only on the information transmission rate and the energy transmission rate. This analysis has been extended to multi-user channels. In this case, the notion of information-energy capacity region generalizes to the set of all information and energy rate tuples that can be simultaneously achieved in the asymptotic block length regime [5]. For instance, the information-energy capacity region of the Gaussian multiple access channel is characterized in [9], whereas the information-energy capacity region of the Gaussian interference channel is approximated in [10].

A first attempt to study the fundamental limits of SIET under the assumption of finite transmission duration with finite constellation sizes is presented in [6]. Therein, the study is restricted to discrete memoryless channels and show that finite transmission duration implies DEPs and EOPs that are bounded away from zero.

This work contributes in this direction and considers the problem of SIET in additive white Gaussian noise channels considering finite transmission duration and finite constellation sizes. The main results in this work highlight the intuition that codes that uniformly use all channel input symbols are associated with high information rates, whereas, codes that exclusively use the channel input symbols that carry the largest amount of energy are associated with high energy rates. More specifically, a characterization of the maximum information and energy rates and minimum DEP and EOP that can be simultaneously achieved is formulated in terms of the type the code induces on the set of channel input symbols. In this work a type is understood in the sense of the empirical frequency with which each channel input symbol appears in the codewords [11].

## 2 Notation

The natural, real and complex numbers are denoted by  $\mathbb{N}$ ,  $\mathbb{R}$  and  $\mathbb{C}$ , respectively. In particular,  $0 \notin \mathbb{N}$ . Random variables and random vectors are denoted by uppercase letters and uppercase bold letters. Scalars and vectors are denoted by lowercase letters and lowercase bold letters. The real and imaginary parts of a complex number  $c \in \mathbb{C}$  are denoted by  $\Re(c)$  and  $\Im(c)$ . The Kullback-Leibler (KL) divergence between two measures  $P$  and  $Q$ , with  $P$  absolutely continuous with  $Q$ , is denoted by  $D(P||Q) = \int \log \frac{dP}{dQ} dP$ , where  $\frac{dP}{dQ}$  denotes the Radon-Nykodim derivate of  $P$  with respect to  $Q$ .

### 3 System Model

Consider a communication system formed by a transmitter, an information receiver (IR), and an energy harvester (EH). The objective of the transmitter is to simultaneously send information to the IR at a rate of  $R$  bits per second; and energy to the EH at a rate of  $B$  Joules per second over an additive white Gaussian noise (AWGN) channel. That is, given a channel input  $\mathbf{x} = (x_1, x_2, \dots, x_n)^\top \in \mathbb{C}^n$ , with  $n \in \mathbb{N}$ , the output of the channel are the random vectors

$$\mathbf{Y} = \mathbf{x} + \mathbf{N}_1, \text{ and} \quad (1a)$$

$$\mathbf{Z} = \mathbf{x} + \mathbf{N}_2, \quad (1b)$$

where  $n$  is the duration of the transmission in channel uses; and the vectors  $\mathbf{Y} = (Y_1, Y_2, \dots, Y_n)^\top \in \mathbb{C}^n$  and  $\mathbf{Z} = (Z_1, Z_2, \dots, Z_n)^\top \in \mathbb{C}^n$  are the inputs of the IR and the EH, respectively. The components of the random vectors  $\mathbf{N}_1 = (N_{1,1}, N_{1,2}, \dots, N_{1,n})^\top \in \mathbb{C}^n$  and  $\mathbf{N}_2 = (N_{2,1}, N_{2,2}, \dots, N_{2,n})^\top \in \mathbb{C}^n$  are independent and identically distributed. More specifically, for all  $(i, j) \in \{1, 2\} \times \{1, 2, \dots, n\}$ ,  $N_{i,j}$  is a complex circularly symmetric Gaussian random variable whose real and imaginary parts have zero means and variances  $\frac{1}{2}\sigma^2$ .

That is, for all  $\mathbf{y} = (y_1, y_2, \dots, y_n)^\top \in \mathbb{C}^n$ , for all  $\mathbf{z} = (z_1, z_2, \dots, z_n)^\top \in \mathbb{C}^n$ , and for all  $\mathbf{x} = (x_1, x_2, \dots, x_n)^\top \in \mathbb{C}^n$ , it holds that the joint probability density function of the channel outputs  $(\mathbf{Y}, \mathbf{Z})$  satisfies  $f_{\mathbf{Y}\mathbf{Z}|\mathbf{X}}(\mathbf{y}, \mathbf{z}|\mathbf{x}) = f_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}|\mathbf{x})f_{\mathbf{Z}|\mathbf{X}}(\mathbf{z}|\mathbf{x})$ , where

$$f_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}|\mathbf{x}) = \prod_{t=1}^n f_{Y|X}(y_t|x_t) \text{ and} \quad (2)$$

$$f_{\mathbf{Z}|\mathbf{X}}(\mathbf{z}|\mathbf{x}) = \prod_{t=1}^n f_{Z|X}(z_t|x_t), \quad (3)$$

and for all  $t \in \{1, 2, \dots, n\}$ ,

$$f_{Y|X}(y_t|x_t) = \frac{1}{\pi\sigma^2} \exp\left(-\frac{(\Re(y_t) - \Re(x_t))^2 + (\Im(y_t) - \Im(x_t))^2}{\sigma^2}\right), \quad (4a)$$

$$f_{Z|X}(z_t|x_t) = \frac{1}{\pi\sigma^2} \exp\left(-\frac{(\Re(z_t) - \Re(x_t))^2 + (\Im(z_t) - \Im(x_t))^2}{\sigma^2}\right). \quad (4b)$$

Within this framework, two tasks must be accomplished: information transmission and energy transmission.

#### 3.1 Information Transmission

Assume that the information transmission takes place using a modulation scheme that uses  $L$  symbols. That is, there is a set

$$\mathcal{X} \triangleq \{x^{(1)}, x^{(2)}, \dots, x^{(L)}\} \subset \mathbb{C} \quad (5)$$

that contains all possible channel input symbols, and

$$L \triangleq |\mathcal{X}|. \quad (6)$$

Let  $M$  be the number of the message indices to be transmitted within  $n$  channel uses. That is,

$$M \leq 2^{n \lceil \log_2 L \rceil}. \quad (7)$$

To reliably transmit a message index, the transmitter uses an  $(n, M)$ -code defined as follows.

**Definition 1.**  $(n, M)$ -code: An  $(n, M)$ -code for the random transformation in (1) is a system:

$$\{(\mathbf{u}(1), \mathcal{D}_1), (\mathbf{u}(2), \mathcal{D}_2), \dots, (\mathbf{u}(M), \mathcal{D}_M)\}, \quad (8)$$

where, for all  $(i, j) \in \{1, 2, \dots, M\}^2, i \neq j$ ,

$$\mathbf{u}(i) = (u_1(i), u_2(i), \dots, u_n(i)) \in \mathcal{X}^n, \quad (9a)$$

$$\mathcal{D}_i \cap \mathcal{D}_j = \phi, \quad (9b)$$

$$\bigcup_{i=1}^M \mathcal{D}_i \subseteq \mathbb{C}^n, \text{ and} \quad (9c)$$

$$|u_t(i)| \leq P. \quad (9d)$$

Assume that the transmitter uses the  $(n, M)$ -code

$$\mathcal{C} \triangleq \{(\mathbf{u}(1), \mathcal{D}_1), (\mathbf{u}(2), \mathcal{D}_2), \dots, (\mathbf{u}(M), \mathcal{D}_M)\}, \quad (10)$$

that satisfies (9). The information rate of any  $(n, M)$ -code is given by

$$R = \frac{\log_2 M}{n} \quad (11)$$

in bits per channel use. To transmit the message index  $i$ , with  $i \in \{1, 2, \dots, M\}$ , the transmitter uses the codeword  $\mathbf{u}(i) = (u_1(i), u_2(i), \dots, u_n(i))$ . That is, at channel use  $t$ , with  $t \in \{1, 2, \dots, n\}$ , the transmitter inputs the symbol  $u_t(i)$  into the channel. At the end of  $n$  channel uses, the IR observes a realization of the random vector  $\mathbf{Y} = (Y_1, Y_2, \dots, Y_n)^T$  in (1a). The IR decides that message index  $j$ , with  $j \in \{1, 2, \dots, M\}$ , was transmitted, if the following event takes place:

$$\mathbf{Y} \in \mathcal{D}_j, \quad (12)$$

with  $\mathcal{D}_j$  in (10). That is, the set  $\mathcal{D}_j \in \mathbb{C}^n$  is the region of correct detection for message index  $j$ . Therefore, the DEP associated with the transmission of message index  $i$  is

$$\gamma_i(\mathcal{C}) \triangleq 1 - \int_{\mathcal{D}_i} f_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}|\mathbf{u}(i))d\mathbf{y}, \quad (13)$$

and the average DEP is

$$\gamma(\mathcal{C}) \triangleq \frac{1}{M} \sum_{i=1}^M \gamma_i(\mathcal{C}). \quad (14)$$

Using this notation, Definition 1 can be refined as follows.

**Definition 2**  $((n, M, \epsilon)$ -codes). An  $(n, M)$ -code for the random transformation in (1), denoted by  $\mathcal{C}$ , is said to be an  $(n, M, \epsilon)$ -code if

$$\gamma(\mathcal{C}) < \epsilon. \quad (15)$$

### 3.2 Energy Transmission

Let  $g: \mathbb{C} \rightarrow [0, +\infty]$  be a positive function such that given a channel output  $z \in \mathbb{C}$ , the value  $g(z)$  is the energy harvested from such channel output. The energy transmission task must ensure that a minimum average energy  $B$  is harvested at the EH at the end of  $n$  channel uses. Let



$\bar{g} : \mathbb{C}^n \rightarrow [0, +\infty]$  be a positive function such that given  $n$  channel outputs  $\mathbf{z} = (z_1, z_2, \dots, z_n)$ , the average energy is

$$\bar{g}(\mathbf{z}) = \frac{1}{n} \sum_{t=1}^n g(z_t). \quad (16)$$

Assume that the transmitter uses the code  $\mathcal{C}$  in (10). Then, the EOP associated with the transmission of message index  $i$ , with  $i \in \{1, 2, \dots, M\}$ , is

$$\theta_i(\mathcal{C}, B) \triangleq \Pr[\bar{g}(\mathbf{Z}) < B | \mathbf{X} = \mathbf{u}(i)], \quad (17)$$

where the probability is with respect to the probability density function  $f_{\mathbf{Z}|\mathbf{X}}$  in (3); and the average EOP is given by

$$\theta(\mathcal{C}, B) \triangleq \frac{1}{M} \sum_{i=1}^M \theta_i(\mathcal{C}, B). \quad (18)$$

This leads to the following refinement of Definition 2.

**Definition 3** ( $(n, M, \epsilon, B, \delta)$ -code). *An  $(n, M, \epsilon)$ -code for the random transformation in (1), denoted by  $\mathcal{C}$ , is said to be an  $(n, M, \epsilon, B, \delta)$ -code if*

$$\theta(\mathcal{C}, B) < \delta. \quad (19)$$

## 4 Main Results

The results in this section are presented in terms of the types induced by the codewords of a given code. Given an  $(n, M, \epsilon, B, \delta)$ -code denoted by  $\mathcal{C}$  of the form in (10), the type induced by the codeword  $\mathbf{u}(i)$ , with  $i \in \{1, 2, \dots, M\}$ , is a probability mass function (pmf) whose support is  $\mathcal{X}$  in (5). Such pmf is denoted by  $P_{\mathbf{u}(i)}$  and for all  $\ell \in \{1, 2, \dots, L\}$ ,

$$P_{\mathbf{u}(i)}(x^{(\ell)}) \triangleq \frac{1}{n} \sum_{t=1}^n \mathbb{1}_{\{u_t(i)=x^{(\ell)}\}}. \quad (20)$$

The type induced by all the codewords in  $\mathcal{C}$  is also a pmf on the set  $\mathcal{X}$  in (5). Such pmf is denoted by  $P_{\mathcal{C}}$  and for all  $\ell \in \{1, 2, \dots, L\}$ ,

$$P_{\mathcal{C}}(x^{(\ell)}) \triangleq \frac{1}{M} \sum_{i=1}^M P_{\mathbf{u}(i)}(x^{(\ell)}). \quad (21)$$

Using this notation, the main results are essentially upper bounds on the information rate  $R$  in (11) and energy rate  $B$  in (17); as well as, lower bounds on the average DEP  $\epsilon$  and average EOP  $\delta$  for all possible  $(n, M, \epsilon, B, \delta)$ -codes. These bounds are provided assuming that the  $L$  channel input symbols in  $\mathcal{X}$  and the block length  $n$  are finite.

### 4.1 Energy Transmission Rate and Average EOP

The following lemma introduces an upper bound on the energy transmission rate that holds for all possible  $(n, M, \epsilon, B, \delta)$ -codes.

**Lemma 1.** *Given an  $(n, M, \epsilon, B, \delta)$ -code denoted by  $\mathcal{C}$  for the random transformation in (1) of the form in (10), the energy transmission rate  $B$  satisfies,*

$$B \leq \frac{1}{1-\delta} \sum_{\ell=1}^L P_{\mathcal{C}}(x^{(\ell)}) \mathbb{E} [g(x^{(\ell)} + W)], \quad (22)$$

where  $P_{\mathcal{C}}$  is defined in (21) and the expectation is with respect to  $W$ , which is a complex circularly symmetric Gaussian random variable whose real and imaginary parts have zero means and variances  $\frac{1}{2}\sigma^2$ .

*Proof.* From (17), the following holds:

$$\theta_i(\mathcal{C}, B) = \Pr \left[ \frac{1}{n} \sum_{t=1}^n g(u_t(i) + W) < B \right] \quad (23)$$

$$= \Pr \left[ \frac{1}{n} \sum_{\ell=1}^L n P_{\mathbf{u}(i)}(x^{(\ell)}) g(x^{(\ell)} + W) < B \right] \quad (24)$$

$$= \Pr \left[ \sum_{\ell=1}^L P_{\mathbf{u}(i)}(x^{(\ell)}) g(x^{(\ell)} + W) < B \right], \quad (25)$$

where the probability is with respect to the random variable  $W$  defined in Lemma 1. Plugging (25) in (18) yields,

$$\theta(\mathcal{C}, B) = \frac{1}{M} \sum_{i=1}^M \Pr \left[ \sum_{\ell=1}^L P_{\mathbf{u}(i)}(x^{(\ell)}) g(x^{(\ell)} + W) < B \right] \quad (26)$$

$$= \frac{1}{M} \sum_{i=1}^M \left( 1 - \Pr \left[ \sum_{\ell=1}^L P_{\mathbf{u}(i)}(x^{(\ell)}) g(x^{(\ell)} + W) \geq B \right] \right) \quad (27)$$

$$\geq \frac{1}{M} \sum_{i=1}^M \left( 1 - \frac{1}{B} \mathbb{E} \left[ \sum_{\ell=1}^L P_{\mathbf{u}(i)}(x^{(\ell)}) g(x^{(\ell)} + W) \right] \right) \quad (28)$$

$$= 1 - \frac{1}{BM} \sum_{i=1}^M \sum_{\ell=1}^L P_{\mathbf{u}(i)}(x^{(\ell)}) \mathbb{E} [g(x^{(\ell)} + W)] \quad (29)$$

$$= 1 - \frac{1}{B} \sum_{\ell=1}^L P_{\mathcal{C}}(x^{(\ell)}) \mathbb{E} [g(x^{(\ell)} + W)], \quad (30)$$

where the probability in (26) is with respect to the random variable  $W$ ; the inequality in (28) follows from Markov's inequality and the expectation is with respect to the random variable  $W$ ; and the equality in (30) follows from (21). Finally, from (19) and (30),

$$\delta \geq 1 - \frac{1}{B} \sum_{\ell=1}^L P_{\mathcal{C}}(x^{(\ell)}) \mathbb{E} [g(x^{(\ell)} + W)], \quad (31)$$

which implies,

$$B \leq \frac{1}{1-\delta} \sum_{\ell=1}^L P_{\mathcal{C}}(x^{(\ell)}) \mathbb{E} [g(x^{(\ell)} + W)], \quad (32)$$

and this completes the proof.  $\square$

An observation from Lemma 1 is that the limit on the energy rate  $B$  does not depend on the individual types  $P_{\mathbf{u}(1)}, P_{\mathbf{u}(2)}, \dots, P_{\mathbf{u}(M)}$  but on the type induced by all codewords, i.e.,  $P_{\mathcal{C}}$ . The inequality in (31) implies the following corollary.

**Corollary 1.** *Given an  $(n, M, \epsilon, B, \delta)$ -code denoted by  $\mathcal{C}$  for the random transformation in (1) of the form in (10), the average EOP  $\delta$  satisfies,*

$$\delta \geq \left( 1 - \frac{1}{B} \sum_{\ell=1}^L P_{\mathcal{C}}(x^{(\ell)}) \mathbb{E} [g(x^{(\ell)} + W)] \right)^+. \quad (33)$$

where  $P_{\mathcal{C}}$  is defined in (21) and the expectation is with respect to  $W$ , which is a complex circularly symmetric Gaussian random variable whose real and imaginary parts have zero means and variances  $\frac{1}{2}\sigma^2$ .

A particular class of channel input symbols is that of circular constellations.

**Definition 4** (Circular Constellation). *The set  $\mathcal{X}$  in (5) is said to form a circular constellation if*

$$\mathcal{X} = \{Ae^{i\frac{2\pi}{L}k} \subseteq \mathbb{C} : k \in \{0, 1, 2, \dots, (L-1)\}, A \in (0, P]\}, \quad (34)$$

where  $i$  is the imaginary unit.

The following corollary introduces an upper bound on the energy transmission rate of codes with circular constellations (Definition 4).

**Corollary 2.** *Given an  $(n, M, \epsilon, B, \delta)$ -code  $\mathcal{C}$  for the random transformation in (1) of the form in (10) with a circular constellation, the energy transmission rate  $B$  satisfies*

$$B \leq \frac{1}{1-\delta} \mathbb{E} [g(x + W)], \quad (35)$$

where  $x \in \mathcal{X}$ ,  $P_{\mathcal{C}}$  is defined in (21) and the expectation is with respect to  $W$ , which is a complex circularly symmetric Gaussian random variable whose real and imaginary parts have zero means and variances  $\frac{1}{2}\sigma^2$ .

Corollary 2 follows immediately from Lemma 1 given the fact that for all  $(x_1, x_2) \in \mathcal{X}^2$ , with  $\mathcal{X}$  in (34), it holds that  $\mathbb{E} [g(x_1 + W)] = \mathbb{E} [g(x_2 + W)]$ , where the random variable  $W$  is defined in Lemma 1.

## 4.2 Average Decoding Error Probability

The analysis of the information transmission rate and the average DEP of a given code  $\mathcal{C}$  of the form in (10) depends on the choice of the decoding sets  $\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_M$ . Without any loss of generality, assume that for all  $i \in \{1, 2, \dots, M\}$ , the decoding set  $\mathcal{D}_i$  is written in the form

$$\mathcal{D}_i = \mathcal{D}_{i,1} \times \mathcal{D}_{i,2} \times \dots \times \mathcal{D}_{i,n}, \quad (36)$$

where for all  $t \in \{1, 2, \dots, n\}$ , the set  $\mathcal{D}_{i,t}$  is a subset of  $\mathbb{C}$ .

The following lemma introduces a lower bound on the average DEP that holds for all possible  $(n, M, \epsilon, B, \delta)$ -codes.

**Lemma 2.** Given an  $(n, M, \epsilon, B, \delta)$ -code  $\mathcal{C}$  for the random transformation in (1) of the form in (10), the average DEP  $\epsilon$  satisfies,

$$\epsilon \geq 1 - \frac{1}{M} \sum_{i=1}^M \exp \left( -nH(P_{\mathbf{u}(i)}) - nD(P_{\mathbf{u}(i)}||Q) + n \log \sum_{j=1}^L \int_{\mathcal{E}_j} f_{Y|X}(y|x^{(j)}) dy \right), \quad (37)$$

where,  $P_{\mathbf{u}(i)}$  is the type defined in (20); the function  $Q$  is a pmf on  $\mathcal{X}$  in (5) such that for all  $i \in \{1, 2, \dots, L\}$ ,

$$Q(x^{(i)}) = \frac{\int_{\mathcal{E}_i} f_{Y|X}(y|x^{(i)}) dy}{\sum_{j=1}^L \int_{\mathcal{E}_j} f_{Y|X}(y|x^{(j)}) dy}; \quad (38)$$

and for all  $\ell \in \{1, 2, \dots, L\}$ , the set  $\mathcal{E}_\ell$  is

$$\mathcal{E}_\ell \triangleq \arg \min_{(i,t) \in \{1,2,\dots,M\} \times \{1,2,\dots,n\}} \int_{\mathcal{D}_{i,t}} f_{Y|X}(y|x^{(\ell)}) dy. \quad (39)$$

*Proof.* The DEP associated with message index  $i$ ,  $\gamma_i(\mathcal{C})$  in (13), satisfies

$$\gamma_i(\mathcal{C}) = 1 - \prod_{t=1}^n \int_{\mathcal{D}_{i,t}} f_{Y|X}(y|u_t(i)) dy \quad (40)$$

$$\geq 1 - \prod_{\ell=1}^L \left( \int_{\mathcal{E}_\ell} f_{Y|X}(y|x^{(\ell)}) dy \right)^{nP_{\mathbf{u}(i)}(x^{(\ell)})} \quad (41)$$

where  $P_{\mathbf{u}(i)}(x^{(\ell)})$  is defined in (20); the equality in (40) follows from (2) and (13); and the inequality in (41) follows from the definition of  $\mathcal{E}_\ell$  in (39). Hence, from (14) and (41), it follows that:

$$\gamma(\mathcal{C}) \geq \frac{1}{M} \sum_{i=1}^M \left( 1 - \prod_{\ell=1}^L \left( \int_{\mathcal{E}_\ell} f_{Y|X}(y|x^{(\ell)}) dy \right)^{nP_{\mathbf{u}(i)}(x^{(\ell)})} \right). \quad (42)$$

Evaluating the product in (42), the following holds:

$$\prod_{\ell=1}^L \left( \int_{\mathcal{E}_\ell} f_{Y|X}(y|x^{(\ell)}) dy \right)^{nP_{\mathbf{u}(i)}(x^{(\ell)})} = \exp \left( \log \prod_{\ell=1}^L \left( \int_{\mathcal{E}_\ell} f_{Y|X}(y|x^{(\ell)}) dy \right)^{nP_{\mathbf{u}(i)}(x^{(\ell)})} \right), \quad (43)$$

$$= \exp \left( \sum_{\ell=1}^L nP_{\mathbf{u}(i)}(x^{(\ell)}) \log \int_{\mathcal{E}_\ell} f_{Y|X}(y|x^{(\ell)}) dy \right), \quad (44)$$

$$= \exp \left( \sum_{\ell=1}^L nP_{\mathbf{u}(i)}(x^{(\ell)}) \left( \log \frac{\int_{\mathcal{E}_\ell} f_{Y|X}(y|x^{(\ell)}) dy}{\sum_{j=1}^L \left( \int_{\mathcal{E}_\ell} f_{Y|X}(y|x^{(j)}) dy \right)} \right) \right. \\ \left. + \log \sum_{j=1}^L \left( \int_{\mathcal{E}_\ell} f_{Y|X}(y|x^{(j)}) dy \right) + \log P_{\mathbf{u}(i)}(x^{(\ell)}) \right. \\ \left. - \log P_{\mathbf{u}(i)}(x^{(\ell)}) \right). \quad (45)$$

Plugging (38) in (45) yields,

$$\begin{aligned} \prod_{\ell=1}^L \left( \int_{\mathcal{E}_\ell} f_{Y|X}(y|x^{(\ell)}) dy \right)^{nP_{\mathbf{u}(i)}(x^{(\ell)})} &= \exp \left( \sum_{\ell=1}^L nP_{\mathbf{u}(i)}(x^{(\ell)}) \log \frac{Q(x^{(\ell)})}{P_{\mathbf{u}(i)}(x^{(\ell)})} \right. \\ &\quad \left. + nP_{\mathbf{u}(i)}(x^{(\ell)}) \log \left( \sum_{j=1}^L \int_{\mathcal{E}_\ell} f_{Y|X}(y|x^{(j)}) dy \right) \right. \\ &\quad \left. + nP_{\mathbf{u}(i)}(x^{(\ell)}) \log P_{\mathbf{u}(i)}(x^{(\ell)}) \right) \end{aligned} \quad (46)$$

$$\begin{aligned} &= \exp \left( -nH(P_{\mathbf{u}(i)}) - nD(P_{\mathbf{u}(i)}||Q) \right. \\ &\quad \left. + n \log \sum_{j=1}^L \int_{\mathcal{E}_\ell} f_{Y|X}(y|x^{(j)}) dy \right), \end{aligned} \quad (47)$$

and plugging (47) into (42) yields,

$$\gamma(\mathcal{C}) \geq 1 - \frac{1}{M} \sum_{i=1}^M \exp \left( -nH(P_{\mathbf{u}(i)}) - nD(P_{\mathbf{u}(i)}||Q) + n \log \sum_{j=1}^L \int_{\mathcal{E}_\ell} f_{Y|X}(y|x^{(j)}) dy \right). \quad (48)$$

Finally, from (15) and (47), it follows that

$$\epsilon > 1 - \frac{1}{M} \sum_{i=1}^M \exp \left( -nH(P_{\mathbf{u}(i)}) - nD(P_{\mathbf{u}(i)}||Q) + n \log \sum_{j=1}^L \int_{\mathcal{E}_\ell} f_{Y|X}(y|x^{(j)}) dy \right), \quad (49)$$

which completes the proof.  $\square$

A class of codes that is of particular interest in this study is that of homogeneous codes, which are defined hereunder.

**Definition 5** (Homogeneous Codes). *An  $(n, M, \epsilon, B, \delta)$ -code denoted by  $\mathcal{C}$  for the random transformation in (1) of the form in (10) is said to be homogeneous if for all  $i \in \{1, 2, \dots, M\}$  and for all  $\ell \in \{1, 2, \dots, L\}$ , it holds that*

$$P_{\mathbf{u}(i)}(x^{(\ell)}) = P_{\mathcal{C}}(x^{(\ell)}), \quad (50)$$

and for all  $(i, t) \in \{1, 2, \dots, M\} \times \{1, 2, \dots, n\}$  for which  $u_t(i) = x^{(\ell)}$ , for some  $\ell \in \{1, 2, \dots, L\}$ , it holds that,

$$\mathcal{D}_{i,t} = \mathcal{E}_\ell, \quad (51)$$

where,  $P_{\mathbf{u}(i)}$  and  $P_{\mathcal{C}}$  are the types defined in (20) and (21), respectively; the sets  $\mathcal{D}_{i,t}$  are defined in (36); and the sets  $\mathcal{E}_\ell$  are defined in (39).

Homogeneous codes are essentially  $(n, M, \epsilon, B, \delta)$ -codes that satisfy two conditions. First, a given channel input symbol is used the same number of times in all codewords; and second, every channel input symbol is decoded with the same decoding set independently of the codeword and/or the position in the codeword. Lemma 2 simplifies for the case of homogeneous codes (Definition 5) as follows.

**Corollary 3.** Given a homogeneous  $(n, M, \epsilon, B, \delta)$ -code denoted by  $\mathcal{C}$  for the random transformation in (1) of the form in (10), the DEP  $\epsilon$  satisfies that

$$\epsilon > 1 - \exp\left(-nH(P_{\mathcal{C}}) - nD(P_{\mathcal{C}}||Q) + n \log \sum_{j=1}^L \int_{\mathcal{E}_j} f_{Y|X}(y|x^{(j)}) dy\right), \quad (52)$$

where,  $P_{\mathcal{C}}$  is the type defined in (21); the pmf  $Q$  is defined in (38) and for all  $j \in \{1, 2, \dots, L\}$ , the sets  $\mathcal{E}_j$  are defined in (39).

Note that the right-hand side of (52) is minimized when the following condition is met,

$$P_{\mathcal{C}} = Q. \quad (53)$$

This observation leads to the following corollary.

**Corollary 4.** Given a homogeneous  $(n, M, \epsilon, B, \delta)$ -code denoted by  $\mathcal{C}$  for the random transformation in (1) of the form in (10), such that (53) is satisfied, then the DEP  $\epsilon$  satisfies that

$$\epsilon \geq 1 - \exp\left(-nH(Q) + n \log \left(\sum_{j=1}^L \int_{\mathcal{E}_j} f_{Y|X}(y|x^{(j)}) dy\right)\right), \quad (54)$$

where the pmf  $Q$  is defined in (38) and for all  $j \in \{1, 2, \dots, L\}$ , the sets  $\mathcal{E}_j$  are defined in (39).

Note that the equality in (53) reveals the optimal choice of  $P_{\mathcal{C}}$  with respect to the average DEP, which is using the channel input symbols in  $\mathcal{X}$  with a probability that is proportional to the probability of correct decoding.

### 4.3 Information Transmission Rate

A first upper bound on the information rate is obtained by upper bounding the number of codewords that a code might possess given the particular types  $P_{\mathbf{u}(1)}, P_{\mathbf{u}(2)}, \dots, P_{\mathbf{u}(n)}$ ; or the average type  $P_{\mathcal{C}}$  in (21). The following lemma introduces such a bound for the case of a homogeneous code.

**Lemma 3.** Given a homogeneous  $(n, M, \epsilon, B, \delta)$ -code denoted by  $\mathcal{C}$  for the random transformation in (1) of the form in (10), the information transmission rate  $R$  in (11) is such that

$$R \leq \frac{1}{n} \log_2 \left( \frac{n!}{\prod_{\ell=1}^L (nP_{\mathcal{C}}(x^{(\ell)}))!} \right) \leq \log_2 L, \quad (55)$$

where,  $P_{\mathcal{C}}$  is the type defined in (21).

*Proof.* The largest number of codewords of length  $n$  that can be formed with  $L$  different channel input symbols is  $L^n$ . In this case, the codewords do not exhibit the type  $P_{\mathcal{C}}$ . Hence, from (11), in the absence of a constraint on the type  $P_{\mathcal{C}}$ , the largest rate is  $\log_2 L$  bits per channel use. This justifies the inequality in the right-hand side of (55). Alternatively, given a code type  $P_{\mathcal{C}}$  that satisfies (50), the number of codewords that can be constructed is upper bounded by

$$\binom{n}{nP_{\mathcal{C}}(x^{(1)})} \binom{n - nP_{\mathcal{C}}(x^{(1)})}{nP_{\mathcal{C}}(x^{(2)})} \cdots \binom{n - \sum_{\ell=1}^{L-1} nP_{\mathcal{C}}(x^{(\ell)})}{nP_{\mathcal{C}}(x^{(L)})} = \frac{n!}{\prod_{\ell=1}^L (nP_{\mathcal{C}}(x^{(\ell)}))!}.$$

Therefore, the information rate  $R$  in (11) satisfies

$$R \leq \frac{1}{n} \log_2 \left( \frac{n!}{\prod_{\ell=1}^L (nP_{\mathcal{C}}(x^{(\ell)}))!} \right), \quad (56)$$

which completes the proof.  $\square$

Lemma 3 can be written in terms of the entropy of the type  $P_{\mathcal{C}}$  as shown by the following corollary.

**Corollary 5.** *Given a homogeneous  $(n, M, \epsilon, B, \delta)$ -code denoted by  $\mathcal{C}$  for the random transformation in (1) of the form in (10), the information transmission rate  $R$  in (11) is such that*

$$\begin{aligned} R \leq & H(P_{\mathcal{C}}) + \frac{1}{n^2} \left( \frac{1}{12} - \sum_{\ell=1}^L \frac{1}{12nP_{\mathcal{C}}(x^{(\ell)}) + 1} \right) \\ & + \frac{1}{n} \left( \log(\sqrt{2\pi}) - \sum_{\ell=1}^L \log \sqrt{2\pi P_{\mathcal{C}}(x^{(\ell)})} \right) - \frac{\log n}{n} \left( \frac{L-1}{2} \right), \end{aligned} \quad (57)$$

where,  $P_{\mathcal{C}}$  is the type defined in (21).

*Proof.* From (55), the following holds

$$R \leq \frac{1}{n} \log(n!) - \frac{1}{n} \sum_{\ell=1}^L \log((nP_{\mathcal{C}}(x^{(\ell)}))!), \quad (58)$$

and using the Stirling's approximation [12] on the factorial terms yields

$$(nP_{\mathcal{C}}(x^{(\ell)}))! \geq \sqrt{2\pi} (nP_{\mathcal{C}}(x^{(\ell)}))^{nP_{\mathcal{C}}(x^{(\ell)}) + \frac{1}{2}} \exp\left(-nP_{\mathcal{C}}(x^{(\ell)}) + \frac{1}{12nP_{\mathcal{C}}(x^{(\ell)}) + 1}\right), \text{ and } (59)$$

$$n! \leq \sqrt{2\pi} n^{n + \frac{1}{2}} \exp\left(-n + \frac{1}{12n}\right). \quad (60)$$

From (59) and (60), the following holds,

$$\begin{aligned} \log((nP_{\mathcal{C}}(x^{(\ell)}))!) & \geq \log(\sqrt{2\pi}) + \left(nP_{\mathcal{C}}(x^{(\ell)}) + \frac{1}{2}\right) \log(nP_{\mathcal{C}}(x^{(\ell)})) - nP_{\mathcal{C}}(x^{(\ell)}) \\ & \quad + \frac{1}{12nP_{\mathcal{C}}(x^{(\ell)}) + 1} \end{aligned} \quad (61)$$

$$\begin{aligned} & = \log(\sqrt{2\pi}) + nP_{\mathcal{C}}(x^{(\ell)}) \log(P_{\mathcal{C}}(x^{(\ell)})) + \frac{1}{2} \log(P_{\mathcal{C}}(x^{(\ell)})) \\ & \quad + \left(nP_{\mathcal{C}}(x^{(\ell)}) + \frac{1}{2}\right) \log(n) - nP_{\mathcal{C}}(x^{(\ell)}) + \frac{1}{12nP_{\mathcal{C}}(x^{(\ell)}) + 1} \text{ and } , \end{aligned} \quad (62)$$

$$\begin{aligned} \log(n!) & \leq \log(\sqrt{2\pi}) + \left(n + \frac{1}{2}\right) \log(n) - n + \frac{1}{12n} \\ & = n \log(n) - n + \frac{1}{12n} + \frac{1}{2} \log(2\pi n). \end{aligned} \quad (63)$$

The sum in (58) satisfies,

$$\begin{aligned} \sum_{\ell=1}^L \log \left( (nP_{\mathcal{C}}(x^{(\ell)}))! \right) &\geq L \log(\sqrt{2\pi}) - nH(P_{\mathcal{C}}) + \frac{1}{2} \sum_{\ell=1}^L \log(P_{\mathcal{C}}(x^{(\ell)})) + n \log(n) \\ &\quad + \frac{L}{2} \log(n) - n + \sum_{\ell=1}^L \frac{1}{12nP_{\mathcal{C}}(x^{(\ell)}) + 1}. \end{aligned} \quad (64)$$

Using (63) and (65) in (58) yields,

$$\begin{aligned} R &\leq \log(n) - 1 + \frac{1}{12n^2} + \frac{1}{2n} \log(2\pi n) - \frac{L}{n} \log(\sqrt{2\pi}) + H(P_{\mathcal{C}}) - \frac{1}{2n} \sum_{\ell=1}^L \log(P_{\mathcal{C}}(x^{(\ell)})) \\ &\quad - \log(n) - \frac{L}{2n} \log(n) + 1 - \frac{1}{n} \sum_{\ell=1}^L \frac{1}{12nP_{\mathcal{C}}(x^{(\ell)}) + 1} \end{aligned} \quad (65)$$

$$\leq H(P_{\mathcal{C}}) + \frac{1}{n^2} \left( \frac{1}{12} - \sum_{\ell=1}^L \frac{1}{12nP_{\mathcal{C}}(x^{(\ell)}) + 1} \right) + \frac{1}{2n} \left( \log(2\pi n) - \sum_{\ell=1}^L \log(2\pi n P_{\mathcal{C}}(x^{(\ell)})) \right) \quad (66)$$

$$\begin{aligned} &= H(P_{\mathcal{C}}) + \frac{1}{n^2} \left( \frac{1}{12} - \sum_{\ell=1}^L \frac{1}{12nP_{\mathcal{C}}(x^{(\ell)}) + 1} \right) + \frac{1}{n} \left( \log(\sqrt{2\pi}) - \sum_{\ell=1}^L \log \sqrt{2\pi P_{\mathcal{C}}(x^{(\ell)})} \right) \\ &\quad - \frac{\log n}{n} \left( \frac{L-1}{2} \right), \end{aligned} \quad (67)$$

which completes the proof.  $\square$

Note that all terms in (57), except the entropy  $H(P_{\mathcal{C}})$ , vanish with the block length  $n$ . This implies that the information rate is essentially constrained by the entropy of the channel input symbols. In particular, note that  $H(P_{\mathcal{C}}) \leq \log_2 L$ .

#### 4.4 Information and Energy Trade-Off

The results presented above lead to the following result for homogeneous codes.

**Theorem 1.** *Given a homogeneous  $(n, M, \epsilon, B, \delta)$ -code denoted by  $\mathcal{C}$  for the random transformation in (1) of the form in (10), the following holds*

$$\begin{aligned} R &\leq H(P_{\mathcal{C}}) + \frac{1}{n^2} \left( \frac{1}{12} - \sum_{\ell=1}^L \frac{1}{12nP_{\mathcal{C}}(x^{(\ell)}) + 1} \right) \\ &\quad + \frac{1}{n} \left( \log(\sqrt{2\pi}) - \sum_{\ell=1}^L \log \sqrt{2\pi P_{\mathcal{C}}(x^{(\ell)})} \right) - \frac{\log n}{n} \left( \frac{L-1}{2} \right); \end{aligned} \quad (68)$$

$$B \leq \frac{1}{1-\delta} \sum_{\ell=1}^L P_{\mathcal{C}}(x^{(\ell)}) \mathbb{E} \left[ g(x^{(\ell)} + W) \right]; \text{ and} \quad (69)$$

$$\epsilon > 1 - \exp \left( -nH(P_{\mathcal{C}}) - nD(P_{\mathcal{C}}||Q) + n \log \sum_{j=1}^L \int_{\mathcal{E}_j} f_{Y|X}(y|x^{(j)}) dy \right). \quad (70)$$

where,  $P_{\mathcal{C}}$  is the type defined in (21); the pmf  $Q$  is defined in (38); for all  $j \in \{1, 2, \dots, L\}$ , the sets  $\mathcal{E}_j$  are defined in (39); and the expectation in (69) is with respect to  $W$ , which is a complex



circularly symmetric Gaussian random variable whose real and imaginary parts have zero means and variances  $\frac{1}{2}\sigma^2$ .

*Proof.* The proof of (68) follows from Corollary 5; the proof of (69) follows from Lemma 1; and the proof of (70) follows from Corollary 3.  $\square$

Theorem 1 quantifies the trade-off between all parameters of a homogeneous  $(n, M, \epsilon, B, \delta)$ -code for the random transformation in (1). Let such a code be denoted by  $\mathcal{C}$  and of the form in (10). Hence, both the upper bound on the information rate in (68) and the upper bound on the energy rate in (69) depend on the type  $P_{\mathcal{C}}$ . The information rate is essentially upper bounded by the entropy of  $P_{\mathcal{C}}$ . Hence, codes whose codewords are such that every channel input symbol is used the same number of times are less constrained in terms of information rate. This is the case in which  $P_{\mathcal{C}}$  is a uniform distribution. Alternatively, using a uniform type  $P_{\mathcal{C}}$  might dramatically constrain the energy transmission rate. For instance, if the constellation is such that for at least one pair  $(x_1, x_2) \in \mathcal{X}^2$ , with  $\mathcal{X}$  in (34), it holds that  $\mathbb{E}[g(x_1 + W)] < \mathbb{E}[g(x_2 + W)]$  where the random variable  $W$  is defined in Lemma 1, then using the symbol  $x_1$  equally often than  $x_2$  certainly constraints the energy rate. Codes that might potentially exhibit the largest energy rates are those in which the symbols  $x$  that maximize  $\mathbb{E}[g(x + W)]$  are used more often. Clearly this deviates from the uniform distribution and thus, constraints the information rate.

Another interesting trade-off appears between the DEP and EOP. From Corollary 1, it follows that codes whose codewords contain mainly channel input symbols  $x$  that maximize  $\mathbb{E}[g(x + W)]$  can be more reliable from the perspective of energy transmission. Alternatively, from Corollary 3, it follows that codes that are more reliable in terms of information transmission are those whose codewords contain more channel input symbols with the smallest DEP. That is, when  $P_{\mathcal{C}}$  approaches the pmf  $Q$  in (38).

The following section provides an example in which these trade-offs are evidenced through a numerical analysis.

## 5 Examples

Consider a homogeneous  $(n, M, \epsilon, B, \delta)$ -code  $\mathcal{C}$  (Definition 5) for the random transformation in (1) of the form in (10) built upon a 16-QAM constellation. More specifically, the set of channel input symbols  $\mathcal{X}$  in (5) is:

$$\begin{aligned} \mathcal{X} = \{ & (5 + i5), (-5 + i5), (-5 - i5), (5 - i5), (15 + i5), \\ & (5 + i15), (-5 + i15), (-15 + i5), (-15 - i5), (-5 - i15), \\ & (5 - i15), (15 - i5), (15 + i15), (-15 + i15), (-15 - i15), \\ & (15 - i15) \}. \end{aligned} \tag{71}$$

The decoding sets  $\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_{16}$  in (51) are those obtained from the maximum-à-posteriori (MAP) estimator [13] given the type  $P_{\mathcal{C}}$  in (21). The block-length is fixed at  $n = 80$  channel uses and the noise variance  $\sigma^2 = 1$  in (4). The energy harvested at the EH is calculated taking into account the non-linearities at the receiver described in [14]. In particular, given type  $P_{\mathcal{C}}$ , the expression  $\sum_{\ell=1}^L P_{\mathcal{C}}(x^{(\ell)})\mathbb{E}[g(x^{(\ell)} + W)]$  in (69) is obtained by using [14, Proposition 1], which is denoted by  $P_{del}$  therein. The choice of parameters while using [14, Proposition 1] is  $k_2 = k_4 = 1$  and  $|h| = |\tilde{h}| = 1$ .

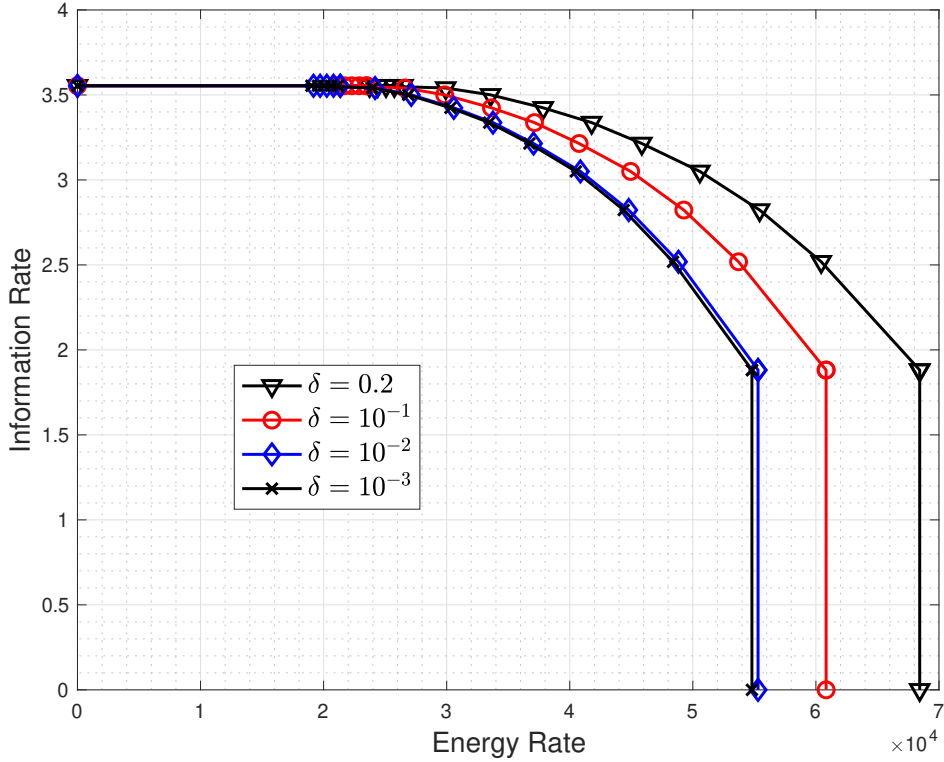


Figure 1: Information-energy rate capacity regions for a homogeneous code employing a 16-QAM constellation.

In Figure 1, given a type  $P_{\mathcal{E}}$ , the upper bound on the information transmission rate and the upper bound on the energy transmission rate in (68) and (69), respectively, are depicted. Each point in the curve corresponds to a different type  $P_{\mathcal{E}}$ . Therein, it becomes evident that smaller EOPs imply smaller information and energy transmission rates. The maximum information rate achievable with a 16-QAM constellation is four bits per channel use. Nonetheless, when codewords are forced to exhibit the same type (homogeneous code) and a particular DEP is targeted, the information transmission rate does not exceed 3.51 bits per channel use. This maximum holds when the energy transmission rate is smaller than  $2 \times 10^4$  energy units, which corresponds to the maximum energy that might be delivered by codes whose type  $P_{\mathcal{E}}$  approaches the uniform distribution on  $\mathcal{X}$ . Beyond this value, increasing the energy transmission rate implies deviating from the uniform distribution, which penalises the information transmission rate. The maximum energy rate implies types  $P_{\mathcal{E}}$  that concentrate on the symbols  $(15 + i15)$ ,  $(-15 + i15)$ ,  $(-15 - i15)$ ,  $(15 - i15)$ . A type  $P_{\mathcal{E}}$  that approaches the uniform distribution over these four channel inputs induces an upper bound on the information transmission rate of 1.9 bits per channel use. A type  $P_{\mathcal{E}}$  that concentrates on one of these channel input symbols induce a zero information rate. Figure 2 illustrates the upper bounds information and energy transmission rates in (68) and (69) as a function of the lower bound on the DEP in (70) when the EOP is kept constant at  $\delta = 10^{-4}$  and  $n = 80$ . More specifically, each point in the curves in Figure 2

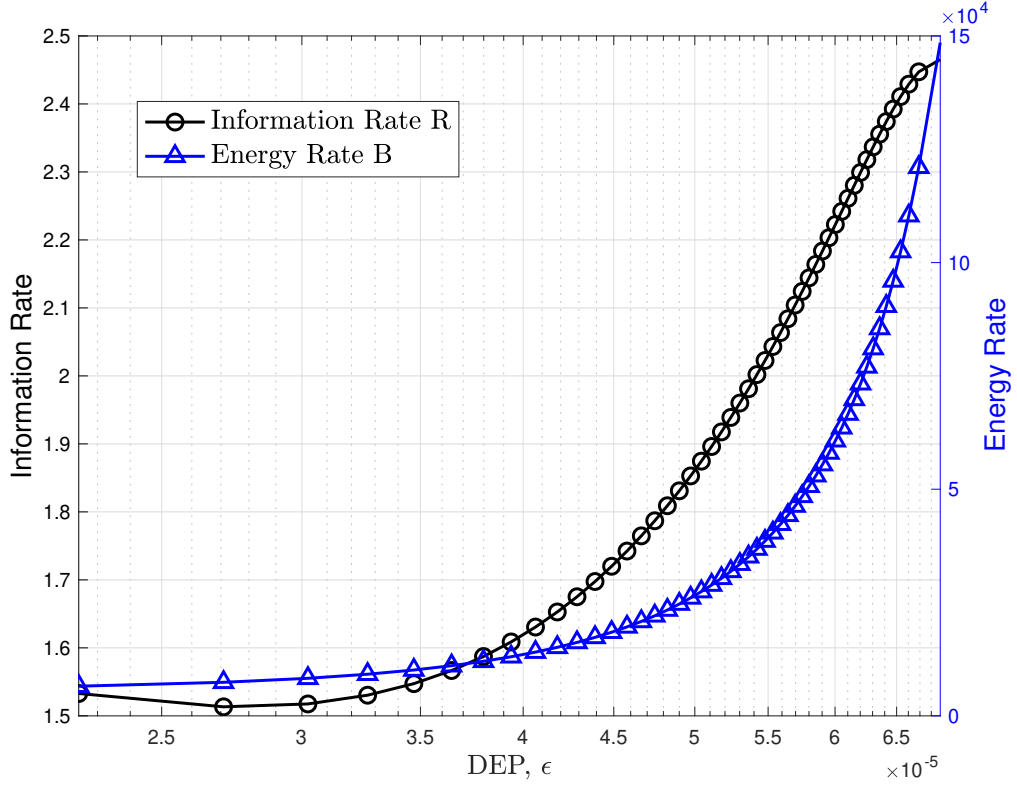


Figure 2: Information and energy rates as a function of the DEP.

for a given value of the DEP is associated with a type  $P_{\mathcal{E}}$ . This type is used to determine upper bounds on both the information and energy transmission rates, which leads to the curves in Figure 2.

For a higher DEP require types  $P_{\mathcal{E}}$  with less constraints, which increase the bounds on both information and energy rates.

## 6 Conclusions

In this report, the fundamental trade-offs between information and energy transmission rates have been studied in memoryless AWGN channels under the assumptions that the DEP and EOP are bounded away from zero; and the constellation size and the block length are finite. These trade-offs are essentially translated into constraints over the type that the code must induce, which implies constraints on how often each point in the constellation is used. Using this observation the fundamental limits on both information and energy transmission rates have been characterized for any homogeneous code leading to guidelines on optimal constellation designs for SIET.

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