

Supplemental Document

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INTRODUCTION

We group here some additional material not included in the main text of *Physical validation of simulators in Computer Graphics: A new framework dedicated to slender elastic structures and frictional contact*, refereed to as 'the Paper' in the following. The present document comprises six sections. In Section 1 we present convergence studies for the different codes used in the four protocols introduced in the Paper. In Section 2, we show some of the meshes used in the study. In Section 3, we present in detail our protocols for experimental validation. In Section 4 we present the numerical and experimental results for a setup derived from the **Lateral Buckling** test. In Section 5 we list all the numerical plots on the **Cantilever** test as some of these could not be included in the Paper, due to space limitation. Finally in Section 6 we give some additional information on the codes used in the present study.

1 CONVERGENCE STUDIES

We show in this section the output of some of the codes tested in the Paper, but with different number of elements for the spatial discretisation of the structures.

Please note that computing times are given here as mere indications and should not be seen as benchmarking.

1.1 Convergence results for the Cantilever test

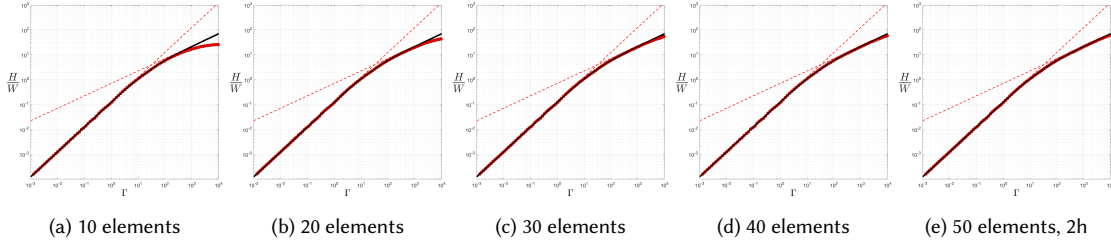


Fig. 1. Convergence results of SUPER-HELIX on the Cantilever test.

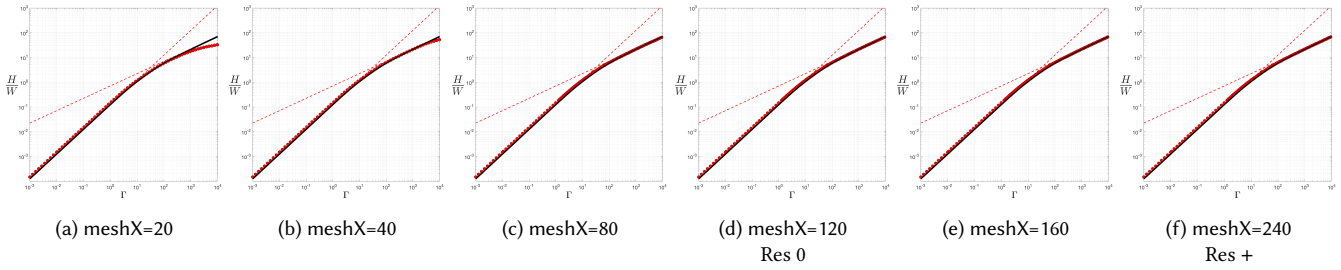


Fig. 2. Convergence results of FENICSHELL on the Cantilever test. The parameter meshX is the number of mesh points along L , the number mesh points along w being $\text{meshX} \times w/L$. The entire set of graphs was produced in about 1h.

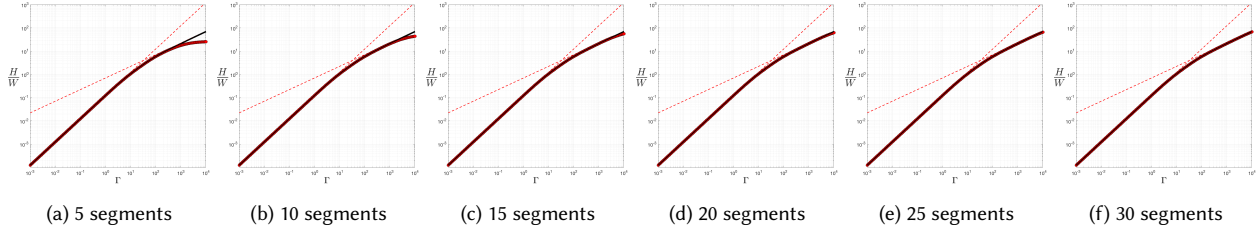


Fig. 3. Convergence results of SUPER-RIBBON on the Cantilever test. The entire set of graphs was produced in about 10 min.

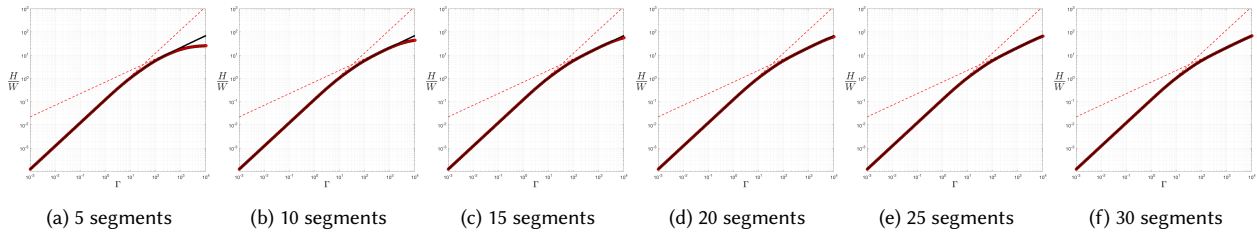


Fig. 4. Convergence results of SUPER-CLOTHOID on the Cantilever test. The entire set of graphs was produced in about 20 min.

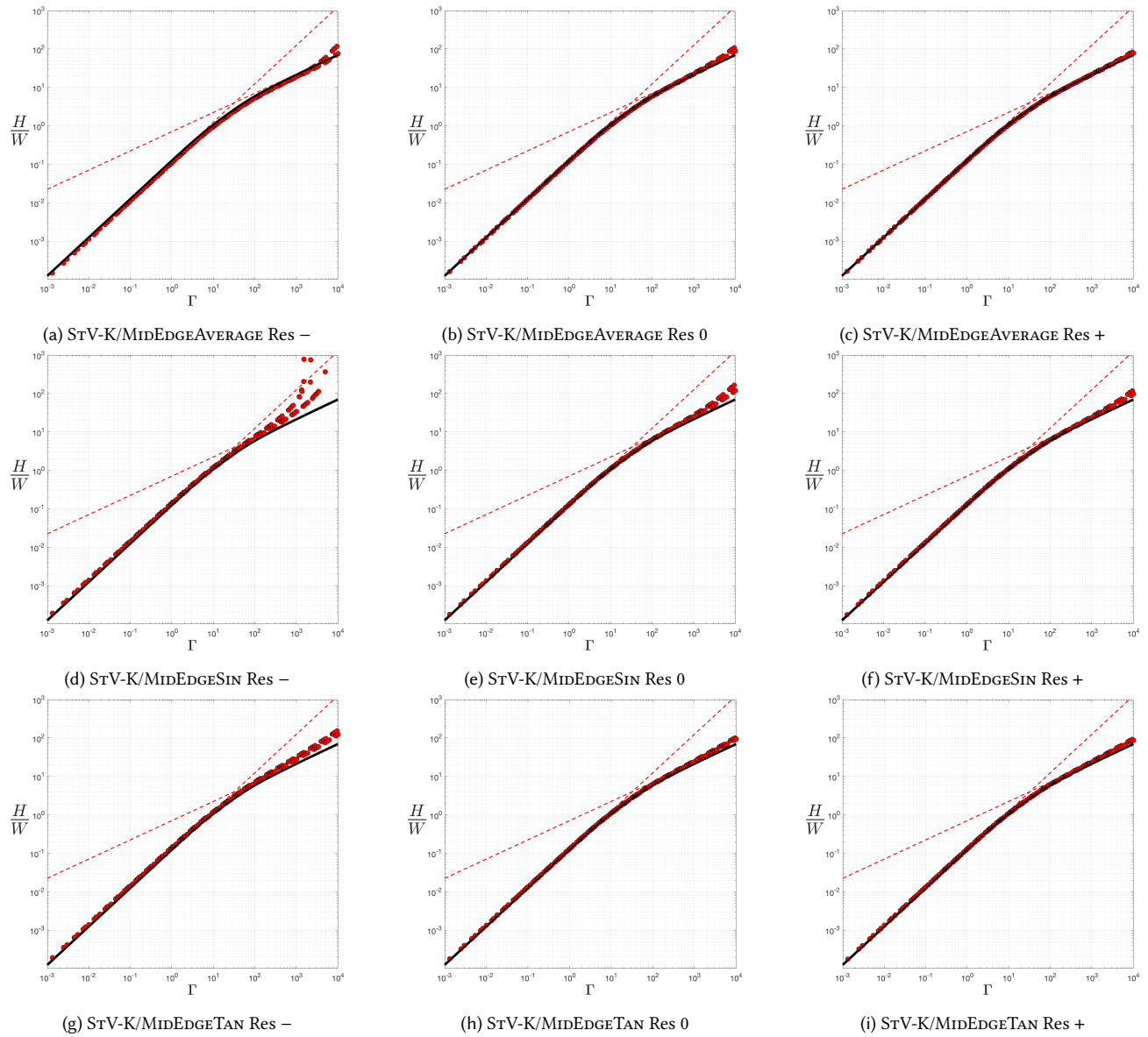


Fig. 5. Convergence results of LIBSHELL on the **Cantilever** test. Plots on the left column took about 10 min each, plots in the center column about 1 or 2h each, and plots in the right column about 8h each. For the **Cantilever** test, Res - has meshX=55, Res 0 has meshX=120, and Res + has meshX=240. The parameter meshX is the number of mesh points along L , the number mesh points along w being $\text{meshX} \times w/L$.

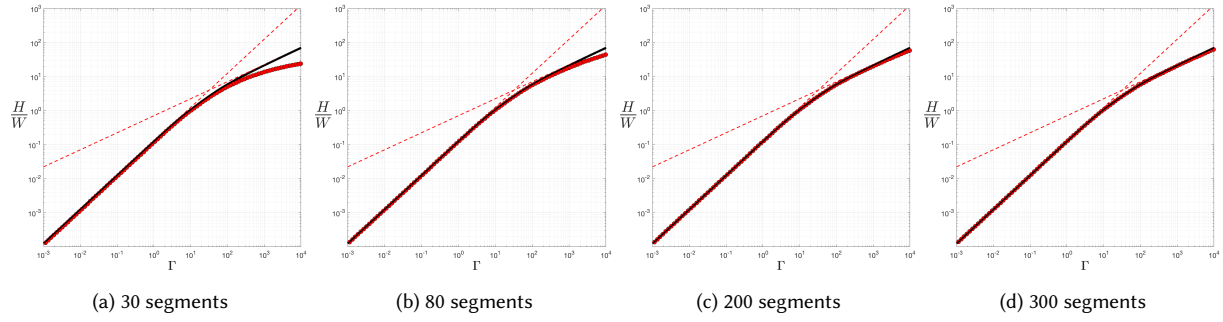


Fig. 6. Convergence results of DISCRETE ELASTIC ROD on the **Cantilever** test. The entire set of graphs was produced in about 3 days.

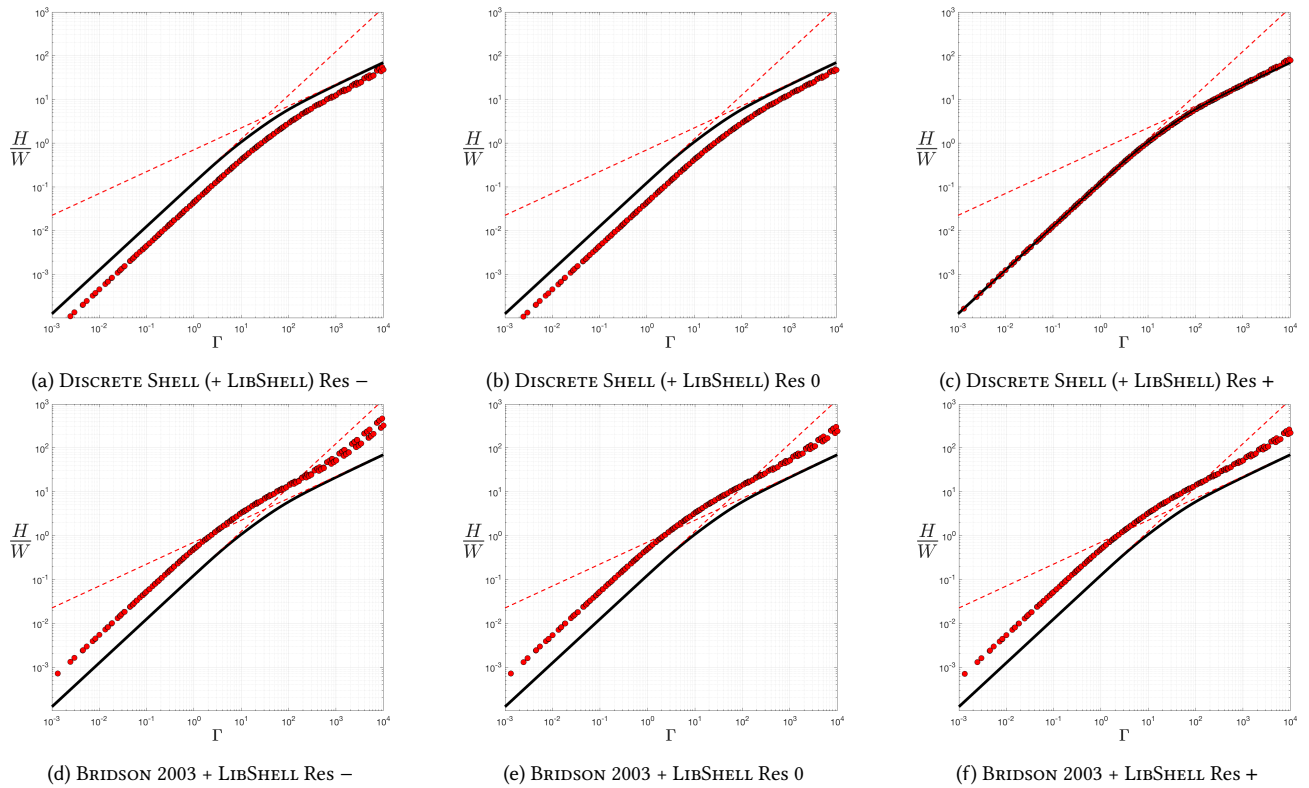


Fig. 7. Convergence results of the hinge models on the **Cantilever** test.

1.2 Convergence results for the **Bend-Twist** test

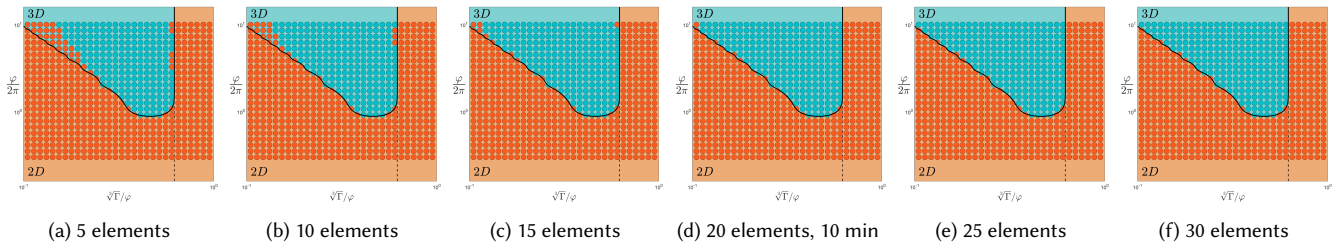


Fig. 8. Convergence results of SUPER-CLOTHOID on the **Bend-Twist** test. Approximate time for computing all these graphs: ~ 1h

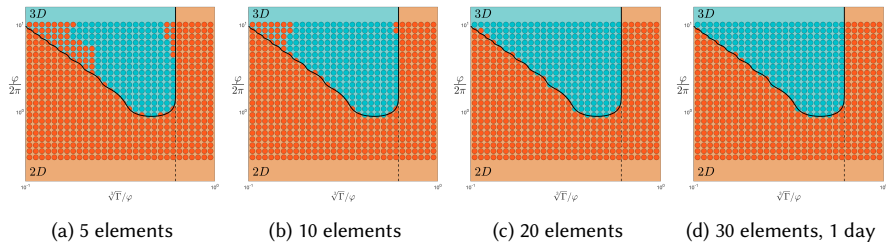


Fig. 9. Convergence results of SUPER-HELIX on the **Bend-Twist** test.

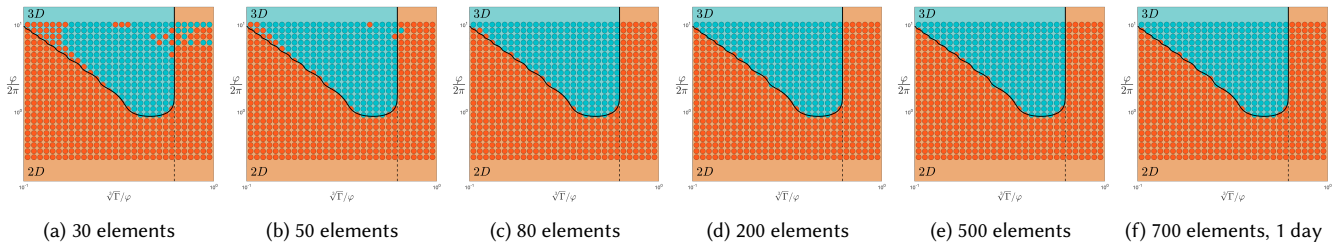


Fig. 10. Convergence results of DISCRETE ELASTIC ROD on the **Bend-Twist** test.

1.3 Convergence results for the **Lateral Buckling** test

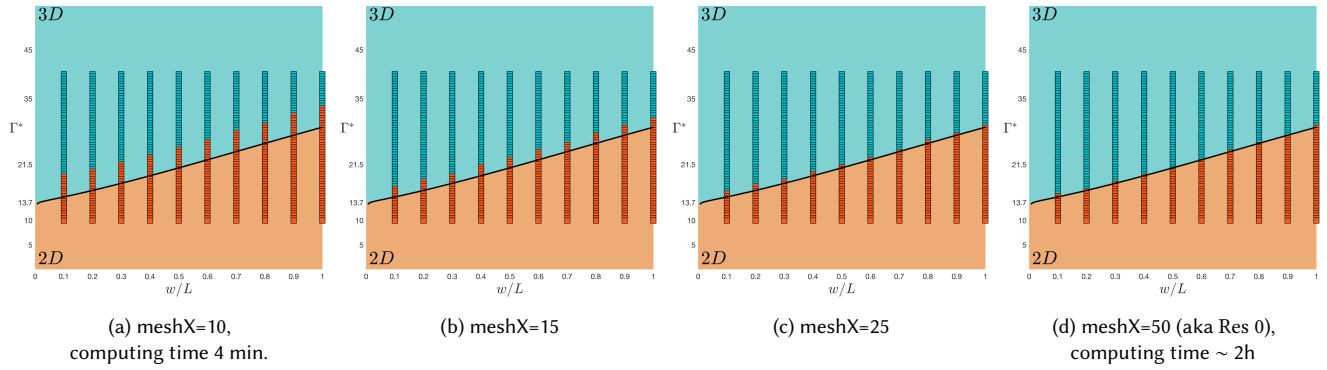


Fig. 11. Convergence results of FENICSSHELL on the **Lateral Buckling** test. The parameter meshX is the number of mesh points along L , the number mesh points along w being $\text{meshX} \times w/L$.

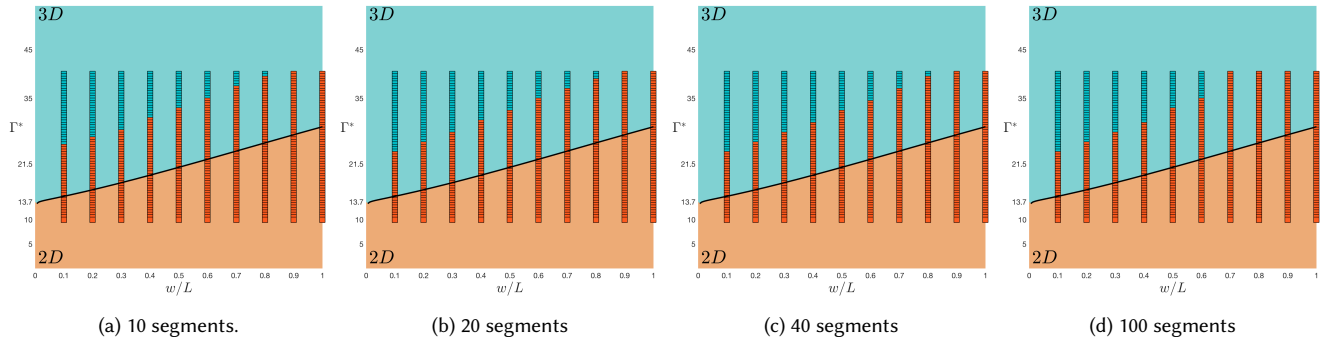


Fig. 12. Convergence results of WUNDERLICH CLAMPED on the **Lateral Buckling** test. This set of data took around 10 min to be generated.

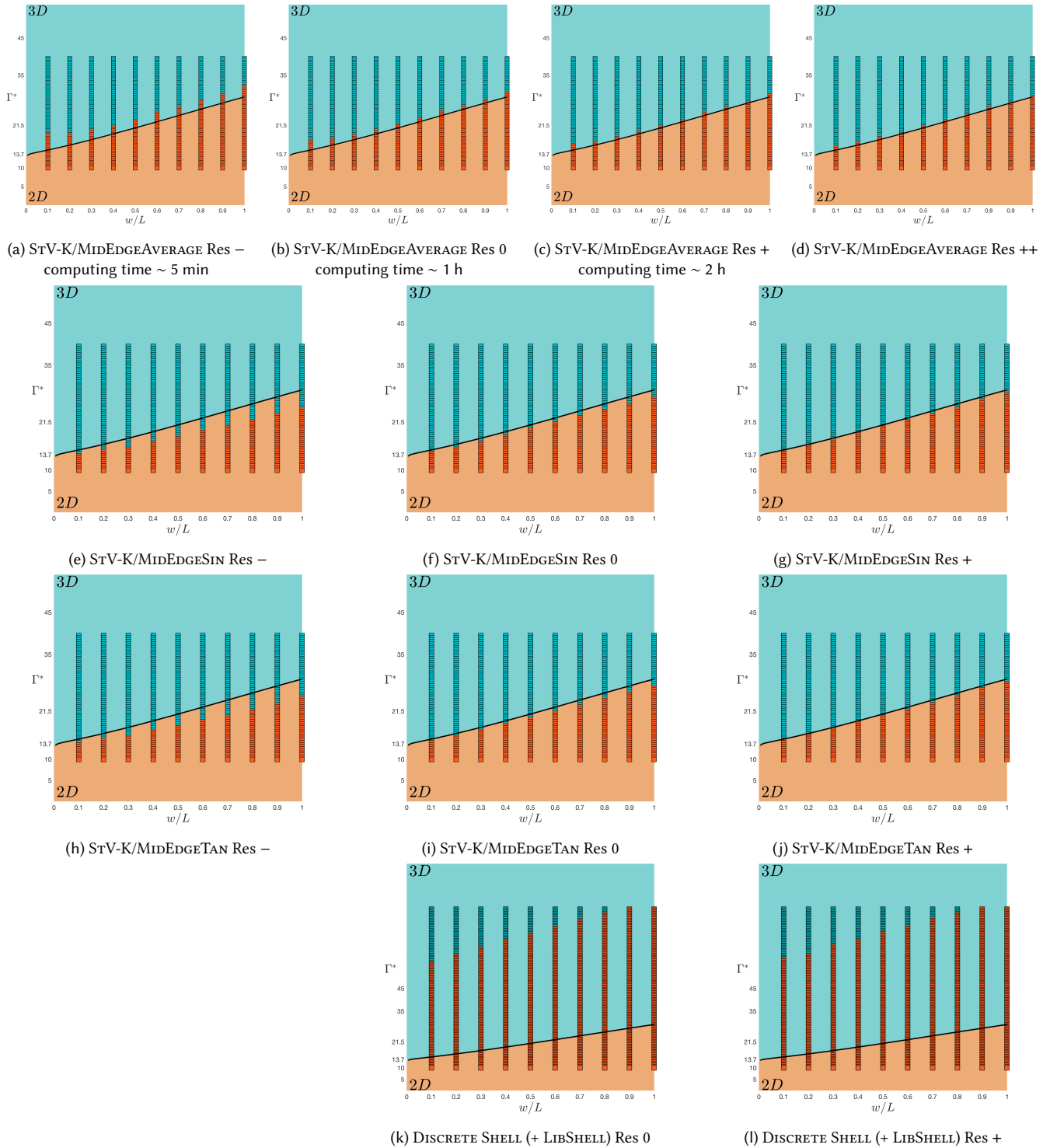


Fig. 13. Convergence results of LIBSHELL on the **Lateral Buckling** test. The parameter meshX is the number of mesh points along L , the number mesh points along w being $\text{meshX} \times w/L$. For the **Lateral Buckling** test, Res - has meshX=25, Res 0 has meshX=50, Res + has meshX=75, and Res ++ has meshX=100.

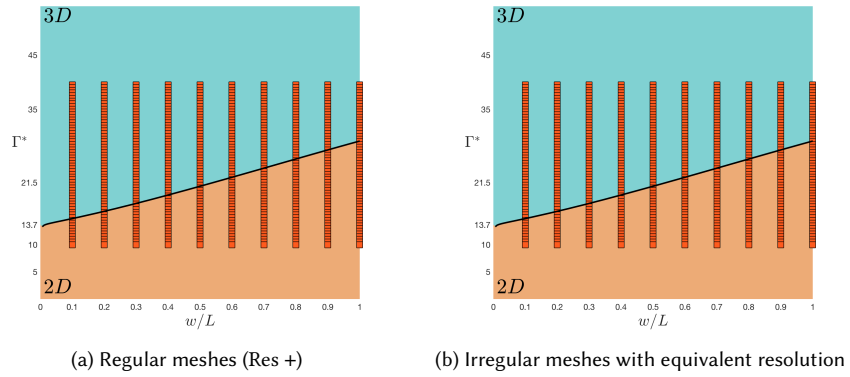


Fig. 14. Comparison of the results of the **Lateral Buckling** test on DISCRETE SHELL (+ LIBSHELL) with differently mesh inputs. We observed no notable difference between the regular and irregular mesh cases.

1.4 Convergence results for the **Stick-Slip** test

For ARGUS NON ADAPTIVE, BRIDSON-HARMON, and PROJECTIVE FRICTION, the fixed mesh had 350 vertices in total with 50 vertices along the length. The adaptive mesh for ARGUS was an empty mesh with just the corner points. All meshes were of length 0.2 m and width 0.01 m.

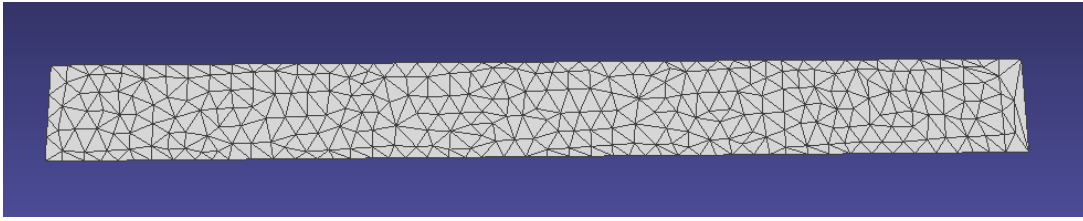


Fig. 15. Example mesh for the **Stick-Slip** test.

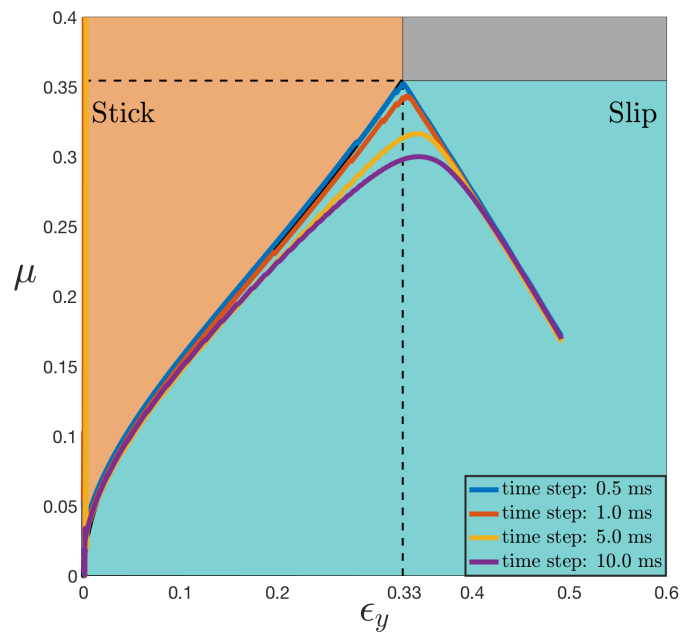


Fig. 16. Convergence study for the **Stick-Slip** test with the So-BOGUS (+ SUPER-HELIX 2D) code. We show $\mu = Q/P$ for different values of the time step. As the time step is decreased, the total number of steps is increased in order to keep a total simulated duration of 60 sec. The coefficients for viscosity and internal friction are set to 10.

2 SOME OF THE MESHES USED

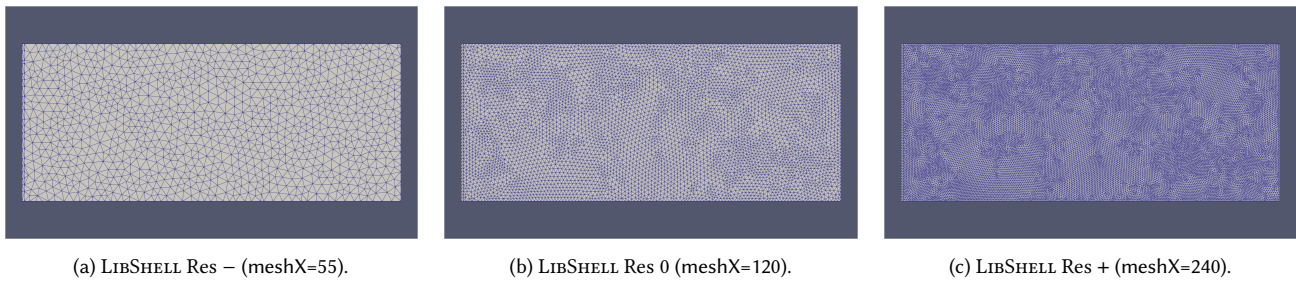


Fig. 17. Some of the meshes used on the **Cantilever** setup. The parameter meshX is the number of mesh points along L , the number mesh points along w being $\text{meshX} \times w/L$. Here $w/L = 0.4125$.

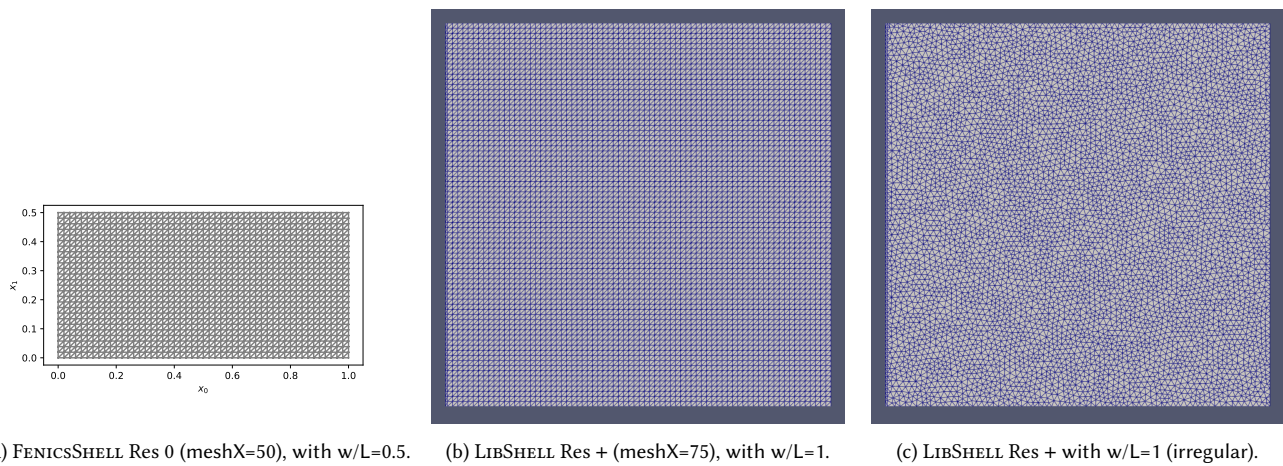


Fig. 18. Some of the meshes used on the **Lateral Buckling** setup. The parameter meshX is the number of mesh points along L , the number mesh points along w being $\text{meshX} \times w/L$.

3 EXPERIMENTAL PROTOCOLS

3.1 Materials

Our four tests require rods, plates and ribbons with controlled geometry and known mechanical properties. Ribbons and plates are made by cutting rectangular pieces, with the desired dimensions, out of large plastic sheets. The most relevant characteristic is the flatness of the sample. The PRO Shim Kit by RS Components (RS Stock No.:770-816) was found to be the best for our purposes, see Figure 19. This set of plastic sheets are made out of polyethylene terephthalate, PET, which has a Young modulus in the order of 5 GPa and they come in thickness ranging from 50 to 500 μm . These values are only estimations and for our tests we characterize our samples mechanically and geometrically by ourselves.



Fig. 19. Samples of PET sheets used for our experimental tests.

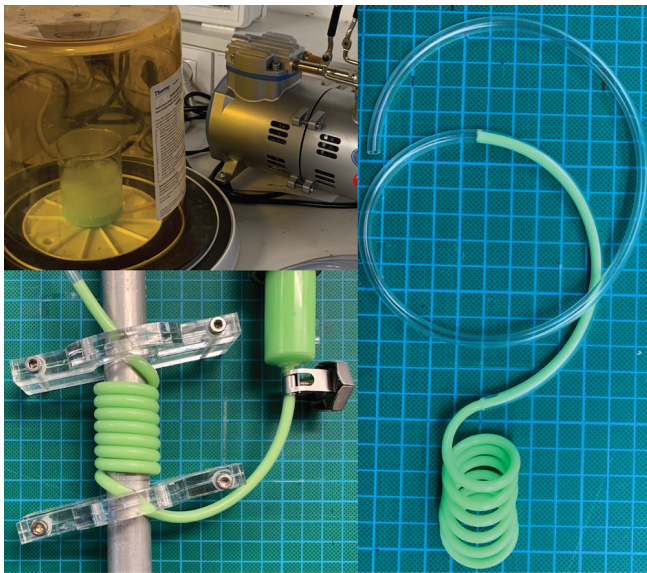


Fig. 20. **Fabrication process of an elastomeric model rod.** *Top left:* Degassing in a vacuum chamber. *Bottom left:* Polymer injection on coiled PVC tube. *Right:* Final shape of the rod after pulling out a large portion of rod from the PVC tube.

Curly and flat rods are fabricated using vinyl polysiloxane commercialised by Zhermack™. For our rods to have a Young's modulus in the order of 1 MPa, we use Elite32™. This product comes with a base and a curing catalyser that have to be scrupulously mixed in a 1:1 proportion. The mixing process produces accumulation of air in the mixture, therefore to achieve the expected elastic behaviour one needs to remove it while the mixture is fluid. To this end, we

degas the mixture by reducing pressure in a vacuum chamber for 2 minutes (see top left image in Figure 20). We observe that in this time most of the trapped air has been removed and the mixture remains fluid.

By means of a syringe, we inject the polymer into a PVC tube (see bottom left image in Figure 20). The final natural geometry of the moulded rod will be determined by the shape in which the PVC tube is arranged. Naturally curly rods are obtained by coiling the PVC tube around a metallic cylinder and holding it in a given configuration. For straight rods, we keep the PVC tube straight. Polymerisation takes about few hours, but to be on the safe side we usually let the polymer settle for 24 hours before removing it from the tube. Because silicon based polymers do not adhere to PVC, we simply pull the elastic rod out of the tube, see right image in Figure 20.

3.2 Experimental setups

The Cantilever setup, shown in Figure 25a, is a simple arrangement of a clamp held by two pillars where rods and ribbons are clamped tightly. The length of the sample is measured when it is placed on the clamp and we are careful to orthogonally align the sample to the clamp. To measure the deformation, we fix a graduated rail that allows vertical displacements of an horizontal metallic bar. For each sample length, we let the sample deflect and reach its equilibrium shape. We move the horizontal bar to the same height as the free end and measure H on the graduated vertical rail. Finally, W is measured with a ruler along the horizontal bar from the rail to the sample free-end. In Figure 21, we show the results for the measured aspect ratio as a function of the length for samples with different thickness, widths and made of different materials (VPS, Acetate and PET).

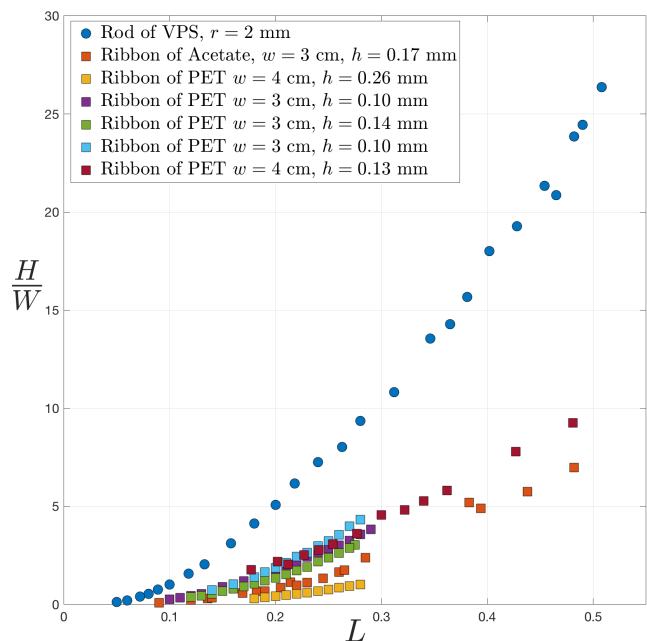


Fig. 21. Raw data **Cantilever test**

The **Bend-Twist** setup, shown in Figure 25b, is built only for illustration purposes since we rely on the experimental validation and the large data set presented in [Miller et al. 2014]. A rod of a given length is clamped in such a way that its tangent coincides with the gravity. Depending on the rod's geometry and its material properties, one observes that the deformed shape either remains in a plane or goes 3D, the **Bend-Twist** transition separating these two behaviours.



Fig. 22. **Bend-Twist** transition. (Left) 3D rod, with $L = 9.6$ cm. (Right) 2D rod, with $L = 9$ cm.

For this test, we fabricate our own helical rod as described in section 3.1. The moulding cylinder had a diameter of 3 cm. The coiled tube, where VPS was injected, had an interior diameter of 3.6 mm and a wall thickness of 0.75 mm. From these values, we estimate the radius and the pitch of the helical shape to be $R = 1.75$ cm and $p = 5.1$ mm, equivalent to a curvature $\kappa = 57.02 \text{ m}^{-1}$ and a torsion $\tau = 2.65 \text{ m}^{-1}$. This small intrinsic torsion is an imperfection in our setup, but the 2D/3D transition is evident as you increase the length of the rod. We show this transition in Figure 22, which happens between $L = 9$ cm and $L = 9.6$ cm.

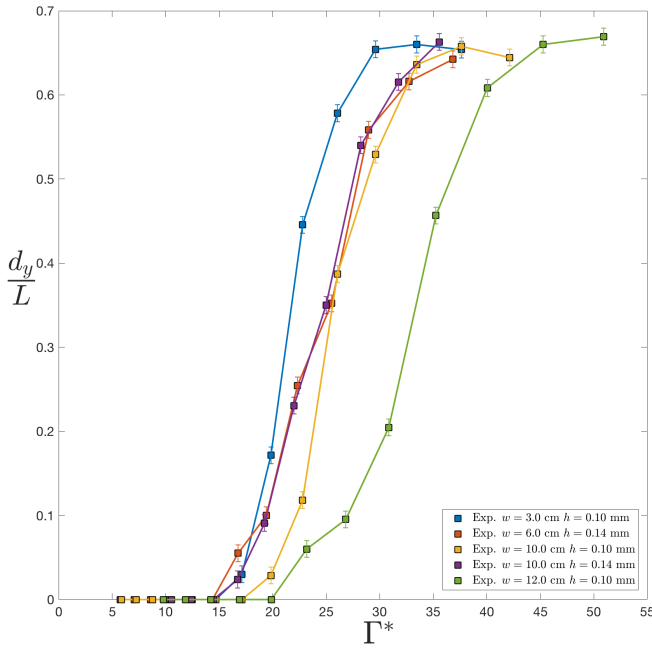


Fig. 23. Raw data **Lateral Buckling** test

The **Lateral Buckling** setup is shown in Figure 25c. This experiment is complex because the plate undergoes a large 3D deformation. To avoid time consuming digital reconstructions, we measure directly the lateral displacement of the centerline. To this end, we find the clamp projection to a position on the floor of the experimental setup. Once the plate has settled on its equilibrium shape, we gently

approach a square cross-section beam to project to the floor the position of the centerline at the free end. Finally, we measure with two orthogonal rulers the lateral displacement. The procedure is repeated for different free lengths L to sample Γ^* values. In Figure 23, we present our measurements for plates of polyethylene terephthalate (PET) of different widths w and thicknesses h .

The **Stick-Slip** setup, shown in Figure 25d, is a complex experimental setup, where we quasi-statically impose the vertical displacement of the clamp whilst measuring the lateral and normal forces. We use two calibrated Futek load cells and the force signal is digitalised with a National Instrument DAQ USB-6002. The vertical displacement is characterised by small steps of 2 mm followed by an in-position force measurement. The whole experiment is composed of a descending phase to the maximum displacement that we set to $\Delta_y = 0.33L$ and an ascending phase returning to the initial configuration to make sure the sample was not deformed plastically. However the experiment is stopped prematurely if the sample slides. Each vertical displacement is captured by a camera, and the slippage process is also captured in a video. We present in Figure 24 our force measurements for different samples with large enough friction coefficients so that sliding did not occur in the $\Delta_y \in [0, 0.33]L$ range. Each force measurement is an average of 2000 points acquired during 2 seconds at 1 kHz while the clamp remains at a given position.

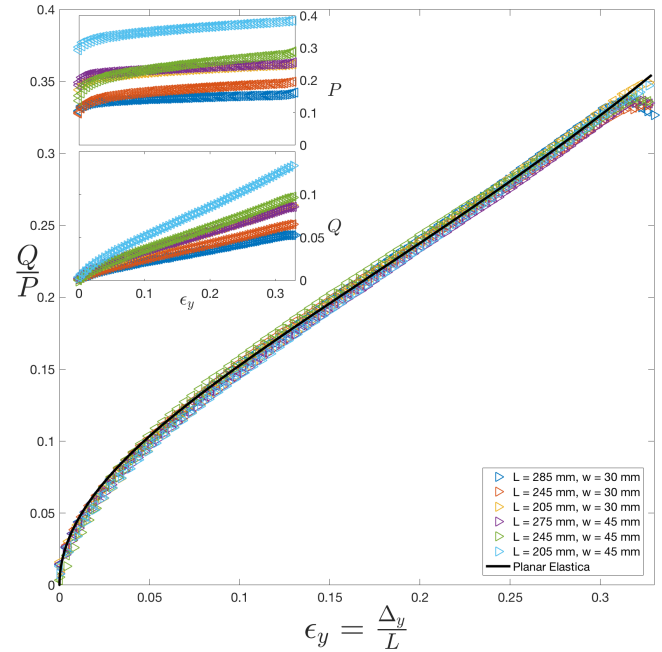


Fig. 24. Raw data **Stick-Slip** test

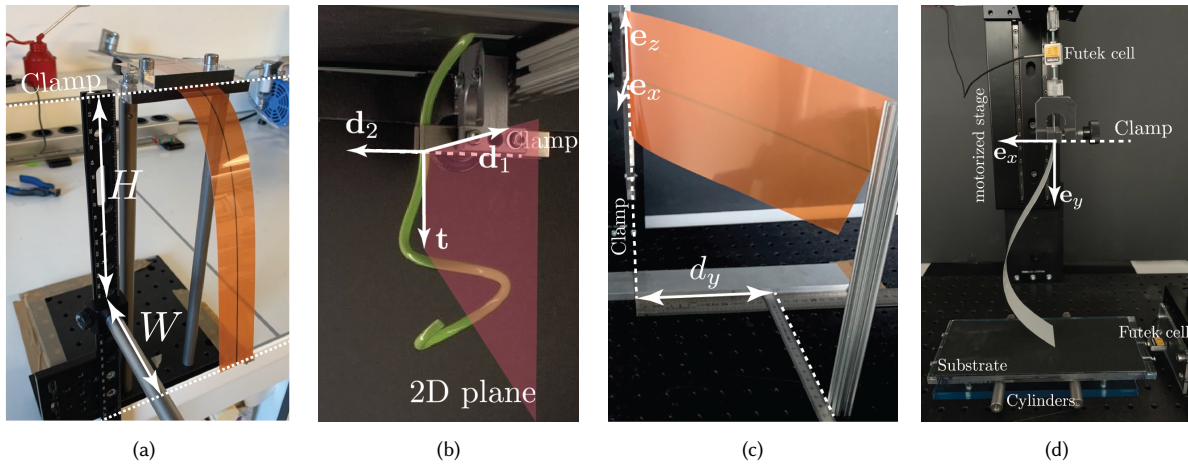


Fig. 25. Our experimental setups for the four validation tests **Cantilever** (a), **Bend-Twist** (b), **Lateral Buckling** (c), and **Stick-Slip** (d).

4 ROTATION EXPERIMENT BEHIND THE LATERAL BUCKLING TEST

4.1 Description

We present here a modification of the **Lateral Buckling** test. The clamped orientation is now tilted by an angle α from the vertical direction. The case $\alpha = 0$ corresponds to the classic **Lateral Buckling** geometry, and the case $\alpha = \pi/2$ corresponding to the **Cantilever** geometry. We let the plate hang and sag under its own weight and record the lateral displacement as α is varied from $\pi/2$ down to 0.

We have performed experiments with the same Poly-Styrene plates as in the main text, thickness $h = 0.10$ mm, density $\rho = 1410$ kg/m³, Young's modulus $E/(1 - \nu^2) \approx 7.75$ GPa, and Poisson's ratio $\nu = 0.35$. The width is here fixed to $w = 10$ cm, and three different lengths have been used $L = 17, 23,$ and 27 cm, corresponding to $\Gamma^* \approx 10.52, 26.05,$ and 42.14 respectively.

In addition we have simulated this new setup with the following codes: SUPER-RIBBON-clamped, LIBSHELL (STV-K/MIDEDGE/TAN), ©ABAQUS, and FENICSSHELL.

4.2 Results and analysis

The classic **Lateral Buckling** test was a bifurcation test with a binary output: either 2D or 3D solution. Here in contrast we record the continuous value of the lateral displacement as α is varied. We see that for the case $L = 17$ cm the lateral displacement is zero when $\alpha = 0$, i.e. this case lies in the 2D region of the phase diagram (Figure 4 of the Paper), far from the threshold curve. For this case all numerical codes agree fairly well with experiments. The same agreement is found for the case $L = 27$ cm where the lateral displacement at $\alpha = 0$ is clearly non zero, as this case is deep in the 3D region of the phase diagram. The case $L = 23$ cm, which is nearer to the threshold curve, is more demanding and we see that the agreement between experiments and numerics is here not uniform. Indeed, for large α all curves agree very well, but when α becomes small and approaches zero, the curves show a divergence. Here the lateral displacement quickly drops to a non-zero value, as $L = 23$ cm is indeed in the 3D region of the phase diagram, but the different axis intercepts do not agree, the experimental one being much larger than the numerical ones. This mismatch is typical of pitchfork bifurcations which are altered by imperfections, present in experiments and not taken into account in the tested numerical codes.

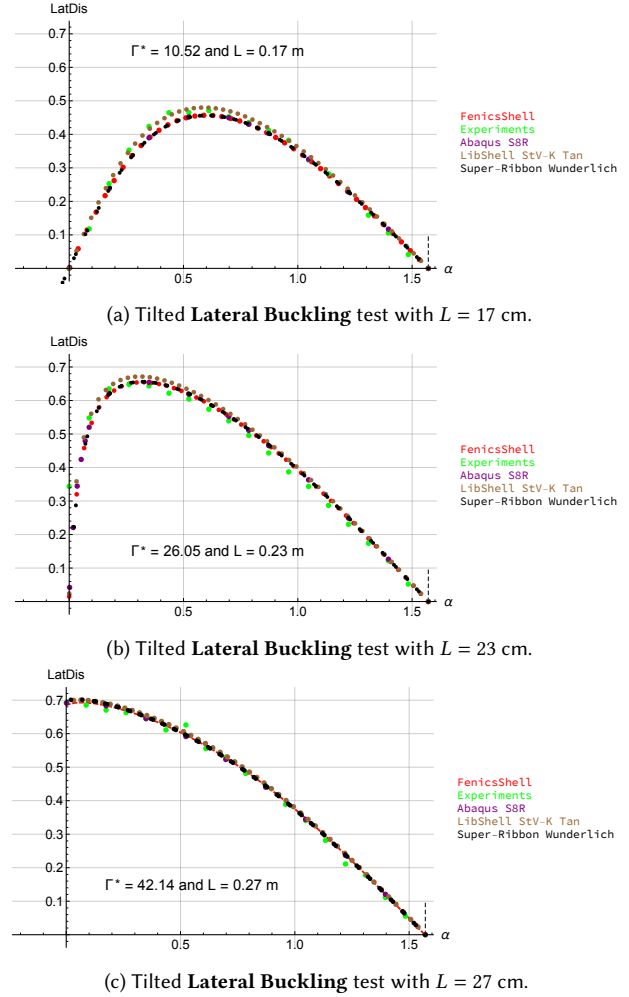


Fig. 26. Rotations experiments for the **Lateral Buckling** test.

5 FULL RESULTS ON THE CANTILEVER TEST

Here we added a **OK** score for variants that almost pass the test. This intermediary score does not need to be used in the main paper since for each test, we always obtained either one clear success or only clear failures among the different variants of the same code.



Fig. 27. Full results on the **Cantilever** test. NH stands for Neo-Hookean.

6 SOME MORE INFORMATION ON THE CODES USED

For each of the different codes used in our four tests, we list in Table 1 some aspects that were used. Please note that we do not list *general* features of the codes, but the actual setups that we used in present study.

Table 1. Codes configurations used in the *present* study. *some of the plots of Figure 27 were generated with a fixed mesh, see details in the captions of Figure 27.

Tested code	parameter dimensions	static / dynamic	adaptative grid or mesh
Rod			
DISCRETE ELASTIC ROD [Bergou et al. 2010]	dim	dyn	no
SUPER-HELIX [Bertails et al. 2006]	dim-less	dyn	no
SUPER-CLOTHOID [Casati and Bertails-Descoubes 2013]	dim-less	stat	no
Ribbon			
SUPER-RIBBON [Charrondière et al. 2020].	dim-less	stat	no
Plate			
LIBSHELL [Chen et al. 2018]	dim	stat	no
DISCRETE SHELL (+ LIBSHELL) [Grinspun et al. 2003]	dim	dyn	no
ARCSIM [Narain et al. 2012]	dim	dyn	yes*
DISCRETE SHELL + ARCSIM (tentative fix of ARCSIM)	dim	dyn	yes*
PROJECTIVE DYNAMICS [Bouaziz et al. 2014]	dim	dyn	no
Contact & friction			
VISCOUS FRICTION (+ SUPER-HELIX 2D)	dim	dyn	no
So-BOGUS [Daviet et al. 2011] (+ SUPER-HELIX 2D)	dim	dyn	no
ARGUS (\approx ARCSIM + So-BOGUS) [Li et al. 2018]	dim	dyn	yes
ARGUS NON ADAPTIVE (fix of ARGUS)	dim	dyn	no
BRIDSON-HARMON [Bridson et al. 2002; Harmon et al. 2008] (+ ARCSIM)	dim	dyn	no
PROJECTIVE FRICTION [Ly et al. 2020]	dim	dyn	no
Reference codes in Mechanical Engineering			
FENICSSHELL [Hale et al. 2018]	dim-less	stat	no
©ABAQUS	dim	stat & dyn	yes

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