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# Crosstalk Mitigation in Long-reach Multicore Fiber Communication Systems using RKHS based Nonlinear Equalization

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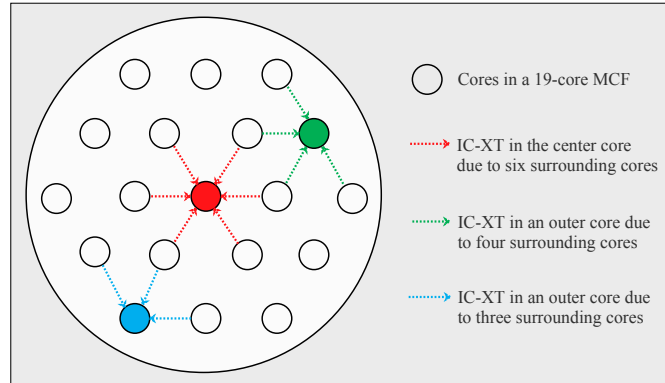
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**Abstract.** The transmission reach of multi-core fiber (MCF) communication systems is severely affected by inter-core crosstalk (IC-XT), which limits its application for long-reach core optical network. One of the major factors limiting the transmission reach of MCF is nonlinear IC-XT interference, which makes the overall system nonlinear, thereby resulting in a poor bit error rate (BER) performance. Conventional Volterra series based nonlinear equalizer are computationally complex, and impaired by modeling error due to the truncation of polynomial kernel. In this paper, for the first time, we propose multivariate kernel least mean square (KLMS) based adaptive nonlinear equalizer for mitigating IC-XT impairments in MCF communication systems. The proposed scheme is inspired from reproducing kernel Hilbert space (RKHS) based machine learning algorithms. Simulations are performed for different multi-core structures, fiber lengths, and modulation schemes, which show that the proposed KLMS algorithm exhibit superior BER performance over the existing Volterra series equalizer.

**Keywords:** Multicore fiber, crosstalk, adaptive, nonlinear, equalizer, reproducing kernel Hilbert space.

## 1 Introduction

Telecommunication traffic generated by mobile and fixedline users is aggregated through metro and access networks into the core optical fiber communication networks. Currently, optical fibers carry about 99% of global Internet traffic [1]. Cisco forecasted a compound annual growth rate of 26% in global IP traffic from 2017 to 2022 [2]. Currently deployed single-core fibers (SCFs) will not be able to support the continuously increasing bandwidth demands in future. Thus space division multiplexing (SDM) technology has been proposed to further increase the optical fiber capacity in spatial dimension using multicore fibers (MCFs) and/or multimode fibers (MMFs) [3–6]. This work is focussed on MCF optical communication systems.



**Fig. 1.** IC-XT in different cores in a 19-core MCF (the value of  $h_{i,j}$  given by (1) in channel matrix  $\mathbf{H}$  depends on the number of surrounding cores shown above, and other MCF parameters specified in (2), causing non-linear IC-XT interference)

MCFs can increase the capacity of optical communication systems, however, the transmission reach in such systems is severely affected by inter-core crosstalk (IC-XT) [3–6]. The IC-XT increases with increase in fiber length. Hence, it is difficult to perform all-optical transmission in core networks, where transmission reach of the order of thousands of kilometers is required. With the ever-increasing traffic demands, the amount of traffic in the core optical networks aggregated through metro/access networks will increase. Thus, to support the increasing bandwidth demands using MCFs in long-reach core networks, use of digital signal processing (DSP) at the receiver end is unavoidable [3].

IC-XT in an MCF depends on the core arrangement, number of cores, and fiber length along with other specific parameters related to fiber manufacturing [3–6]. Thus, IC-XT for different cores in an MCF is different, and is dependent of the number of surrounding cores. For instance, in a 19-core MCF shown in Fig. 1, the cores shown in red, green, and blue color will be affected by IC-XT due to six, four, and three number of cores, respectively. Moreover, the IC-XT between any two cores in an MCF is nonlinearly related to the core pitch, bending radius, coupling coefficient, propagation constant, and the fiber length [3–6]. This results in nonlinear IC-XT interference, which increases with fiber length, thereby resulting in a poor bit error rate (BER) performance in long-reach MCF optical communication systems.

*Related Works:* The existing approaches to deal with IC-XT are mainly based on methods which ensure that the spectrum is allocated in a way that the transmitted signals do not exceed maximum allowable threshold XT levels [5,6]. These approaches result in inefficient spectrum utilization, and spectrum fragmentation since a significant amount of spectrum in different cores is left unused to ensure threshold XT levels [6]. In addition to this, the transmission reach possible through these approaches ranges from hundreds to few thousand kilometers, depending on the modulation scheme used. For instance, using worst-case

XT (WC-XT) method in a 7-core hexagonal MCF structure, maximum possible transmission reach using binary phase shift keying (BPSK), quadrature phase shift keying (QPSK), and 16-quadrature amplitude modulation (QAM) is 1118 km, 397 km, and 89 km, respectively [5]. Thus, these existing approaches are not applicable for long-reach core optical networks, where shortest routes amongst different node pairs exceed these reach values. For example, in UBN24 network topology, the maximum length of shortest paths among node pairs is 6650 km [7] (i.e., between the nodes at San Diego and Ithaca [8]).

To enable communication through MCFs in high-capacity core networks such that long-reach using higher modulation formats can be obtained, an effective nonlinear equalization mechanism is required at the receiver for mitigating nonlinear IC-XT impairments. Volterra series based nonlinear equalizers have been proposed in the literature for compensating nonlinear distortions [9,10], however, they are suitable only for short reach optical transmissions [9]. Furthermore, Volterra series based approaches are computationally complex, and impaired by modeling error due to abrupt truncation of Taylor's series till second/third order terms [11,12].

Conventional linear adaptive filtering algorithms [13] like least mean square (LMS) algorithm, recursive least squares (RLS) algorithm, and their variants deliver sub-optimal performance for nonlinear systems [14]. Recently, reproducing kernel Hilbert space (RKHS) based machine learning algorithms have shown promise in the area of nonlinear channel equalization/estimation, system identification, time series prediction, wind forecasting, and pattern classification. Hence, linear adaptive filters have been combined with the RKHS framework to yield a class of kernel adaptive filters (KAFs) [15] like kernel LMS (KLMS) [14] and kernel RLS (KRLS) algorithms [16].

In this paper, we propose RKHS inspired KLMS based nonlinear equalizer for compensating the nonlinear distortions introduced by nonlinear IC-XT. Proposed RKHS based equalizer have the following advantages over the existing polynomial (like Volterra) series based nonlinear equalization; First, it transforms a non-convex optimization problem in lower dimensional input space to a convex optimization problem in RKHS, which guarantees the global optimal Wiener solution [17]. Secondly, the proposed equalizer has the ability to model any arbitrary nonlinearity without its *a priori* knowledge by using reproducing kernel. This property of the proposed equalizer stems from Riesz representation theorem [15]. Third, the proposed equalizer does not suffer from modeling error, which is not the case with Volterra equalizers. Fourth, the proposed equalizers are computationally efficient, which can be achieved by using sparsification techniques like the novelty criterion (NC), the coherence criterion, the quantization criterion, surprise criterion, and fixed budget models [15]. In this work, we incorporate NC in the proposed equalizer to yield sparse KLMS-NC algorithm, which makes the proposed equalizer suitable for practical deployments.

The organization of the rest of this paper is as follows: Section II gives brief overview of the system model considered in this work. Proposed RKHS based

nonlinear equalizer is detailed in Section III. Simulation results are provided in Section IV to illustrate the performance of the proposed algorithm. Finally, Section V concludes the paper.

*Notations:* Following terminology is used throughout the paper:  $\mathbb{R}$  and  $\mathbb{C}$  denote field of real and complex numbers respectively. Matrices are denoted by capital bold-face letters like  $\mathbf{H}$  with dimensions  $\mathbb{R}^{m \times n}$ , where  $m$  and  $n$  indicate the row and column dimension, respectively. Vectors at  $k^{\text{th}}$  time instant are denoted by small bold-face letters like  $\mathbf{x}_k$  with dimensions  $\mathbb{C}^n$  while scalars are represented by simple lower-case letters.  $\mathbf{I}_N$  denotes a  $N \times N$  identity matrix, and  $\mathbf{O}$  denotes null matrix.  $(\cdot)^H$  denotes Hermitian transpose of  $(\cdot)$ . The operator  $\mathbb{E}\{\cdot\}$  denotes statistical expectation. Inner product in RKHS  $\mathcal{H}$  is represented by  $\langle \cdot, \cdot \rangle_{\mathcal{H}}$ , and  $\|\cdot\|_{\mathcal{H}}$  denotes norm in RKHS  $\mathcal{H}$ . Gradient of  $x$  with respect to  $y$  is denoted by  $\nabla_y(x)$ .

## 2 System model

In this Section, system model considered in this work is described. Data bits are first modulated by using QPSK/QAM schemes to generate modulated symbols. Let  $\mathbf{x}_k = [x^{(1)}, x^{(2)}, \dots, x^{(l)}, \dots, x^{(N_c)}]^T \in \mathbb{C}^{N_c}$  denotes the modulated input symbol vector at  $k^{\text{th}}$  time instant, where  $x^{(l)}$  denotes the signal corresponding to  $l^{\text{th}}$  core of MCF, and  $N_c$  indicates number of cores in MCF. Input symbols  $\mathbf{x}_k$  are then transmitted through multi-core optical fiber. The cross-talk channel matrix  $\mathbf{H}$  for MCF is modeled as [18]

$$\mathbf{H} \in \mathbb{R}^{N_c \times N_c} = \begin{bmatrix} h_1 & h_{1,2} & \cdots & h_{1,N_c} \\ h_{2,1} & \ddots & & h_{2,N_c} \\ \vdots & & \ddots & \vdots \\ h_{N_c,1} & h_{N_c,2} & \cdots & h_{N_c} \end{bmatrix},$$

where  $h_{i,j}$  represents the cross-talk between  $i^{\text{th}}$  and  $j^{\text{th}}$  core. Based on coupled-power theory,  $h_{i,j}$  is expressed as [3, 6]

$$\begin{aligned} h_{i,j} &= \frac{1 - e^{-2 \cdot L \cdot r_{i,j}}}{1 + e^{-2 \cdot L \cdot r_{i,j}}} \quad \forall i \neq j, \\ h_i &= 1 - \sum_{i \neq j} h_{i,j} \end{aligned} \quad (1)$$

where  $L$  is the length of optical fiber, and  $r_{i,j}$  represents the mean XT increase per unit length of the fiber, which is given by

$$r_{i,j} = \frac{2 \cdot \kappa_{i,j}^2 \cdot R}{\beta \cdot \Lambda}, \quad (2)$$

where  $\kappa_{i,j}$ ,  $R$ ,  $\beta$ , and  $\Lambda$  are MCF parameters called coupling coefficient between  $i^{\text{th}}$  and  $j^{\text{th}}$  core, bending radius, propagation constant, and the core pitch, respectively. The received signal  $\mathbf{y}_k = [y^{(1)}, y^{(2)}, \dots, y^{(l)}, \dots, y^{(N_c)}]^T \in \mathbb{C}^{N_c}$  (where

$y^{(l)}$  denotes the received signal corresponding to  $l^{\text{th}}$  core of MCF) at  $k^{\text{th}}$  time instant can be written as

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{n}_k, \quad (3)$$

where  $\mathbf{n}_k = [n^{(1)}, n^{(2)}, \dots, n^{(l)}, \dots, n^{(N_c)}]^T \in \mathbb{C}^{N_c} \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_{N_c})$  is independent and identically distributed (i.i.d) complex additive white Gaussian noise (AWGN) vector with mean 0 and co-variance matrix  $\sigma_n^2 \mathbf{I}_{N_c}$ ,  $n^{(l)}$  denotes the noise in  $l^{\text{th}}$  core of MCF. (3) can be decomposed as follows:

$$\begin{aligned} y^{(1)} &= h_{1,1}.x^{(1)} + \underbrace{(h_{1,2}.x^{(2)} + \dots + h_{1,N_c}.x^{(N_c)})}_{\text{IC-XT interference}} + n^{(1)} \\ &\vdots \\ y^{(N_c)} &= h_{N_c,N_c}.x^{(N_c)} + \underbrace{(h_{N_c,1}.x^{(1)} + \dots + h_{N_c,N_c-1}.x^{(N_c-1)})}_{\text{IC-XT interference}} + n^{(N_c)} \end{aligned} \quad (4)$$

The terms inside the parenthesis in (4) represents the additive IC-XT interference. The nonlinear expression for cross-talk  $h_{i,j}$  in (1) makes the mapping between received symbol  $\mathbf{y}_k$  and transmitted symbol  $\mathbf{x}_k$  nonlinear, which results in nonlinear distortions at the receiver. To circumvent, received symbol  $\mathbf{y}_k$  is given as an input to the proposed RKHS based nonlinear equalizer detailed in Section III.

### 3 Proposed RKHS Based Nonlinear Equalizer

In this Section, RKHS based nonlinear equalizer is proposed for mitigating IC-XT interference in MCFs. Further, NC based sparsification and computational complexity is discussed in the next subsections. As inferred from (1) and (3), received symbols  $\mathbf{y}_k$  are impaired by nonlinear IC-XT interference, therefore, received observations  $\mathbf{y}_k$  are mapped from finite dimensional input space to the high dimensional feature space such as RKHS, denoted by  $\mathcal{H}$  by using a nonlinear transformation  $\Psi(\cdot)$  [15]. Mapping to RKHS is essential because a nonlinear problem in original input space is converted to a linear problem in high dimensional RKHS (from Cover's theorem). Let  $\mathbf{\Omega}_k$  denote the implicit weight matrix in RKHS at  $k^{\text{th}}$  time instant. Using training observations  $\{\mathbf{x}_k, \psi(\mathbf{y}_k)\}_{k=1}^{N_{\text{tr}}}$  (where  $N_{\text{tr}}$  is the number of training observations), the empirical loss function for multivariate KLMS algorithm<sup>3</sup> is formulated according to minimum mean square error (MMSE) criterion in RKHS as [14, 15]

$$\mathcal{L}_{\text{KLMS}}(\mathbf{\Omega}_k) = \min_{\mathbf{\Omega}_k} \mathbb{E} \left\{ \frac{1}{2} \|\mathbf{x}_k - \mathbf{\Omega}_k \Psi(\mathbf{y}_k)\|_{\mathcal{H}}^2 \right\} \quad (5)$$

where  $\mathbf{e}_k = \mathbf{x}_k - \mathbf{\Omega}_k \Psi(\mathbf{y}_k)$  is the prediction error vector in RKHS. Weight update equation for the proposed multivariate KLMS based equalizer can be written by

<sup>3</sup> Multivariate KLMS algorithm is the generalization of classical univariate KLMS algorithm [14] for vector-valued desired data [17].

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**Algorithm 1** Proposed nonlinear equalizer in RKHS for IC-XT mitigation in MCF

---

**Input:** Received observations  $\mathbf{y}_k$   
**Output:** Equalized symbols  $\hat{\mathbf{x}}_k$   
**% Initialization:**  
Choose  $k = 1$ , maximum iterations: MAXITER,  $\eta, \sigma, \delta_o, \delta_e, \mathbf{e}_1 \leftarrow \mathbf{s}_1, \mathcal{D}[1] \leftarrow \{\mathbf{y}_1\}$ ,  
and  $\mathcal{D}[1] \leftarrow \{\mathbf{e}_1\}$   
**% Computation:**  
**while**  $k \leq \text{MAXITER}$  **do**  
  **% Computation of estimated output**  
 $\hat{\mathbf{x}}_k = \mu_{\text{KLMS}} \sum_{j=1}^{|\mathcal{D}_k|} \mathcal{D}_k^j \mathcal{K}_{G,\sigma}(\mathcal{D}_k^j, \mathbf{y}_k)$   
  **% Computation of error vector**  
 $\mathbf{e}_k = \mathbf{x}_k - \hat{\mathbf{x}}_k$   
  **% Sparsification by NC**  
  **if**  $\min_{\forall j} \|\mathbf{y}_k - \mathcal{D}_k^j\| \geq \delta_o$  and  $\|\mathbf{e}_k\| \geq \delta_e$  **then**  
     $\mathcal{D}_{k+1} = \{\mathcal{D}_k \cup \mathbf{y}_k\}$   
     $\mathcal{D}_{k+1} = \{\mathcal{D}_k \cup \mathbf{e}_k\}$   
  **else**  
     $\mathcal{D}_{k+1} = \{\mathcal{D}_k\}$   
     $\mathcal{D}_{k+1} = \{\mathcal{D}_k\}$   
  **end if**  
   $k = k + 1$   
**end while**

---

applying stochastic gradient descent algorithm as follows [19]

$$\boldsymbol{\Omega}_{k+1} = \boldsymbol{\Omega}_k - \mu_{\text{KLMS}} \nabla_{\boldsymbol{\Omega}_k} (\mathcal{L}_{\text{KLMS}}(\boldsymbol{\Omega}_k)) = \boldsymbol{\Omega}_k + \mu_{\text{KLMS}} \mathbf{e}_k \Psi^H(\mathbf{y}_k), \quad (6)$$

where  $\mu_{\text{KLMS}}$  is the step-size for KLMS algorithm. The recursive weight update equation can be written as (assuming  $\boldsymbol{\Omega}_k = \mathbf{O}$ )

$$\boldsymbol{\Omega}_{k+1} = \mu_{\text{KLMS}} \sum_{i=1}^k \mathbf{e}_i \Psi^H(\mathbf{y}_i), \quad (7)$$

Estimated output of the proposed KLMS based equalizer can be written as [17]

$$\hat{\mathbf{x}}_k = \boldsymbol{\Omega}_k \Psi(\mathbf{y}_k) = \mu_{\text{KLMS}} \sum_{i=1}^{k-1} \mathbf{e}_i \langle \Psi(\mathbf{y}_i), \Psi(\mathbf{y}_k) \rangle_{\mathcal{H}} \quad (8)$$

Inner product in (8) is difficult to compute as  $\Psi(\cdot)$  is not known a priori. However, according to Mercer's theorem, inner product in RKHS can be evaluated by using Mercer kernel as follows [15]

$$\mathcal{K}(\mathbf{y}_i, \mathbf{y}_k) = \langle \Psi(\mathbf{y}_i), \Psi(\mathbf{y}_k) \rangle_{\mathcal{H}} \quad (9)$$

Invoking (9) in (8) we get

$$\hat{\mathbf{x}}_k = \mu_{\text{KLMS}} \sum_{i=1}^{k-1} \mathbf{e}_i \mathcal{K}_{\sigma,G}(\mathbf{y}_i, \mathbf{y}_k), \quad (10)$$

where  $\mathcal{H}_{\sigma, \mathbb{C}}(\mathbf{y}_i, \mathbf{y}_k) = \exp\left(-\frac{\sum_q (\mathbf{y}_i^q - \mathbf{y}_k^{q*})^2}{2\sigma^2}\right)$  is the complex Gaussian kernel [15] with  $\sigma$  as the kernel width, and superscript  $q$  denotes  $q^{th}$  entry of the vector. The main advantage of the proposed nonlinear equalizer in RKHS is that it does not require weights  $\mathbf{\Omega}_k$  and nonlinear function  $\Psi(\cdot)$  to be known a priori as long as the kernel function  $\mathcal{H}(\cdot, \cdot)$  is specified.

### 3.1 NC based sparsification

The main issue with the proposed KLMS based nonlinear equalizer in RKHS is that its computational complexity linearly increases with each observation as  $O(k)$ , which hinders the practical viability of the proposed algorithm. However, all the observations are not required for training and equalization. Hence, a dictionary based sparsification technique based on Platt's novelty criterion (NC) [15] is incorporated in the proposed KLMS based equalizer to yield sparse KLMS-NC algorithm [15]. We introduce the notion of dictionary of observations (denoted by  $\mathcal{D}_k$ ) which keeps track of all the input regressors used for training the system, and dictionary of error terms (denoted by  $\mathcal{E}_k$ ). Initially, dictionary starts from the first observation as  $\mathcal{D}_k = \{\mathbf{y}_1\}$  and  $\mathcal{E}_k = \{\mathbf{e}_1\}$ , and the incoming observations are selectively added to the dictionary according to the following two rules of Platt's NC as: (1) First, the Euclidean distance of the incoming observation is computed with the existing dictionary as  $\min_{\forall j} \|\mathbf{r}_k - \mathcal{D}_k^j\|_2 \quad \forall j = 1, 2, \dots, S$ , where  $S$  denotes number of observations in the current dictionary  $\mathcal{D}_k$ . If it is less than some pre-defined threshold  $\delta_o$  then the current incoming observation is discarded and not added to the dictionary. However, if  $\min_{\forall j} \|\mathbf{r}_k - \mathcal{D}_k^j\|_2 \geq \delta_o$  then second rule is checked; (2) If  $\|\mathbf{e}_k\| \geq \delta_e$  then the current observation is added to the dictionary as  $\mathcal{D}_{k+1} = \{\mathcal{D}_k, \mathbf{y}_k\}$  and size of the dictionary gets increased by one. Otherwise, dictionary remains unchanged as  $\mathcal{D}_{k+1} = \{\mathcal{D}_k\}$ . Proposed algorithm is detailed in **Algorithm 1**.

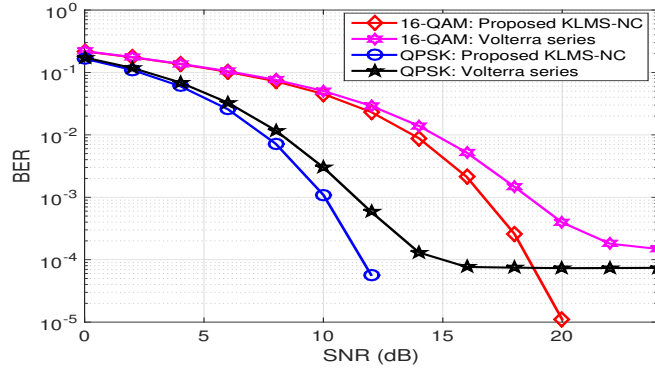
### 3.2 Computational complexity

Computational complexity of the Volterra series equalizer is  $O(M^P)$  [9], where  $M$  is the number of filter taps used for  $P^{th}$  order Volterra filter. Computational complexity of the Volterra equalizer increases with the increase in order  $P$ . After incorporating NC based sparsification, the computational complexity of the proposed KLMS-NC algorithm get reduced from  $O(k)$  to  $O(|\mathcal{D}_k|)$  [20, 21], where  $|\mathcal{D}_k|$  indicates cardinality of the dictionary (dictionary-size), i.e., the number of observations present in the dictionary at convergence.

## 4 Simulations

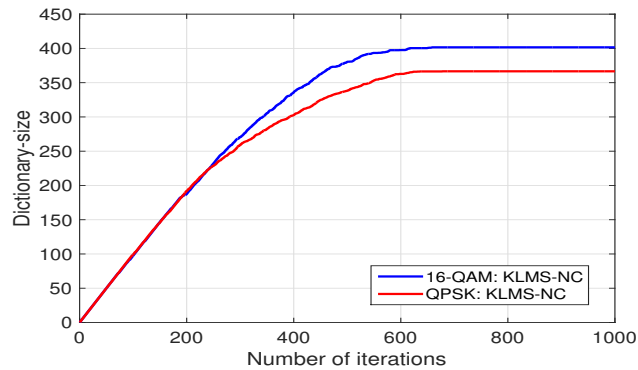
In this Section, simulations are presented to demonstrate the performance of the proposed algorithm. QPSK and 16-QAM modulation schemes are considered.





**Fig. 2.** SNR vs BER performance for Volterra series and the proposed KLMS algorithm for 12-core MCF using QPSK and 16-QAM modulation schemes

An ensemble of  $10^4$  symbols are considered over 100 independent Monte-carlo trials. Two different core structures, i.e., 12-core dual ring structure (DRS)-MCF structure, and 19-core hexagonal MCF structure are considered with the following parameters:  $\kappa = 3.5 \times 10^{-3}$ ,  $R = 80$  mm,  $\beta = 4 \times 10^6$ , and  $\Lambda$  is chosen as  $40 \mu\text{m}$ , and  $35 \mu\text{m}$  for 12-core, and 19-core MCF, respectively [3]. Proposed KLMS based nonlinear equalizer is compared with the conventional Volterra series based nonlinear equalizer. Volterra series equalizer with 65 first order, and 45 second order filter taps are chosen. Out of  $10^4$  symbols, 1000 symbols are used for training. Kernel width  $\sigma = 5$  is set by Silverman's rule [15], step-size  $\mu_{\text{KLMS}} = 0.45$  ( $0 < \mu_{\text{KLMS}} < 2$  as given in [15, 22]), observation-quantization constant  $\delta_o = 10^{-3}$ , and error-quantization constant  $\delta_e = 0.1$  is set according to the rules given in [15].



**Fig. 3.** Evolution of dictionary-size for the proposed KLMS algorithm for 12-core MCF using QPSK and 16-QAM modulation schemes

**Table 1.** Comparison of computational complexity for Volterra series, KLMS, and KLMS-NC based equalizers for 12-core MCF using 16-QAM

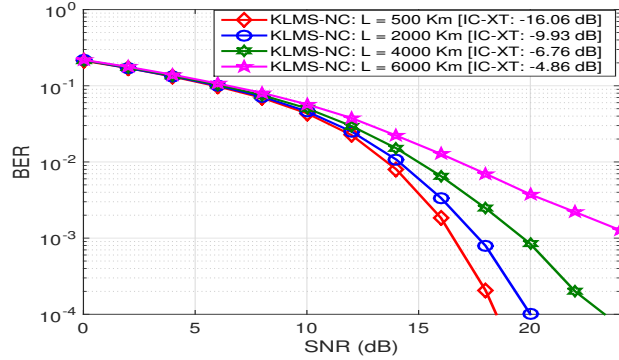
Equalization algorithm	Number of multiplications	Complexity order
Volterra series	2025	$O(M^P)$
KLMS	1000	$O(k)$
KLMS-NC	400	$O( \mathcal{D}_k )$

First, the SNR vs BER performance, and evolution of dictionary-size for 12-core MCF with fiber length 1000 km, are shown in Fig. 2, and Fig. 3, respectively, using QPSK and 16-QAM modulation schemes. It can be observed from Fig. 2 that the proposed KLMS equalizer delivers 3-5 dB gain at an approximate BER of  $10^{-4}$  over the conventional Volterra series equalizer for QPSK and 16-QAM constellations. It can be also observed from Fig. 2 that the Volterra series equalizer has a BER error floor in high SNR regime due to the truncation error. Further, it is observed from Fig. 3 that, out of 1000 observations, proposed algorithm requires only 360 observations for QPSK, and 400 observations for 16-QAM, which highlights the computational simplicity of the proposed algorithm. Computational complexity details are given in Table I.

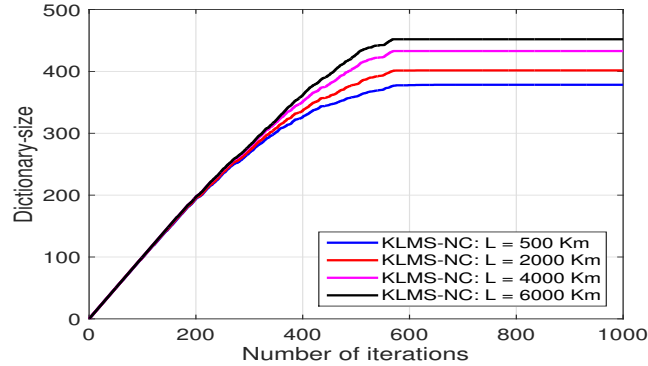
Next, the SNR vs BER performance is evaluated by varying the length of the fiber in Fig. 4 for 12-core MCF using 16-QAM modulation scheme. The mean IC-XT values corresponding to different fiber lengths are shown in Fig. 4, which exceed the threshold IC-XT for 16-QAM modulation scheme [5]. It can be observed from Fig. 4 that the BER performance degrades with increase in fiber length since IC-XT increases with increase in fiber length. However, using the proposed nonlinear equalizer, longer reach using higher modulation schemes (i.e., 16-QAM) can be achieved, which demonstrates the practical viability of the proposed equalizer for core optical communication networks. Further, dictionary-size performance for the proposed algorithm is analyzed in Fig. 5 for 12-core MCF using 16-QAM modulation scheme. As inferred from Fig. 5, the dictionary-size also increases with the increase in fiber length, i.e., more number of regressors/observations are required for equalization of increased IC-XT levels.

To study the variation of nonlinear IC-XT on different core arrangements, the BER performance of the proposed algorithm is compared for 12-core DRS-MCF and 19-core hexagonal MCF structures in Fig. 6 using 16-QAM. It can be observed from Fig. 6 that the BER for 19-core MCF is higher than that for the 12-core MCF. This is due to the difference in core arrangement in the 12-core DRS-MCF, and 19-core hexagonal MCF structure. In a 12-core DRS-MCF, the maximum number of surrounding cores is 4 (for internal ring), whereas, in a 19-core MCF, the maximum number of surrounding cores is 6 (for central and inner hexagon cores). This increase in the number of surrounding cores leads to increase in the number of non-zero elements  $h_{i,j} \forall i \neq j$  in the channel matrix  $\mathbf{H}$ . In addition to this, due to smaller core pitch in 19-core hexagonal MCF,

the value of  $h_{i,j} \forall i \neq j$  increases, thereby resulting in increased nonlinear IC-XT interference, and hence increased BER in the 19-core MCF as compared to the 12-core MCF.



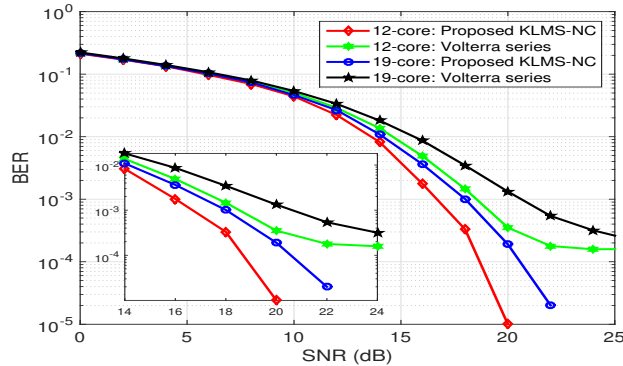
**Fig. 4.** SNR vs BER performance for the proposed KLMS algorithm for 12-core MCF using 16-QAM for different optical fiber lengths



**Fig. 5.** Evolution of dictionary-size for the proposed KLMS algorithm for different lengths of 12-core MCF using 16-QAM

## 5 Conclusion

In this paper, the problem of nonlinear IC-XT interference in long-reach MCF optical communication systems is addressed. KLMS based supervised machine learning algorithm in RKHS is proposed for mitigating IC-XT impairments in



**Fig. 6.** SNR vs BER performance for the proposed KLMS algorithm for 12-core and 19-core MCF using 16-QAM

MCF. Further, novelty criterion based sparsification is incorporated in the proposed equalizer to reduce its computational complexity. Simulations are performed for 12-core DRS-MCF, and 19-core hexagonal MCF structures, which shows the nonlinear IC-XT variation with core arrangement. The proposed algorithm is evaluated by varying modulation schemes, and fiber length. Simulation results indicate that the proposed KLMS based nonlinear equalizer outperforms the existing Volterra series based nonlinear equalizer. This indicates that the proposed equalizer is a viable approach for mitigating nonlinear IC-XT impairments in long-reach MCF optical communication systems.

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