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On Resolvable Mixed Path Designs

JEAN-CLAUDE BERMOND, KATHERINE HEINRICH AND MIN-LI YU

For $k \geq 2$ the complete multigraph λK_n has a factorization into $s + t$ spanning subgraphs ($st \neq 0$), s of which are 1-factors and t of which are the union of n/k vertex-disjoint paths of length $k - 1$ iff $n \equiv 0 \pmod{2}$, $n \equiv 0 \pmod{k}$ and $ks + 2t(k - 1) = \lambda k(n - 1)$.

1. INTRODUCTION

Let G be a multigraph. An H -factor in G is a spanning subgraph of G , each component of which is isomorphic to the graph H . If the edges of G can be partitioned into H -factors we say that G has an H -factorization.

There has been considerable study of H -factorizations of λK_n (the complete multigraph on n vertices in which each edge has multiplicity λ). For example, when $H = K_2$ an H -factor is a 1-factor and λK_n has a 1-factorization iff n is even. When $H = C_k$, the cycle of length k , $k \geq 3$, it has been shown (see [1] and the references therein) that K_n has a C_k -factorization iff $n \equiv 0 \pmod{k}$ and n is odd. (This is often referred to as the Oberwolfach problem.) If $H = P_k$, the path of length $k - 1$, it was shown ([3, 5]) that for $k \geq 2$ a P_k -factorization of λK_n exists iff $n \equiv 0 \pmod{k}$ and $\lambda k(n - 1) \equiv 0 \pmod{2(k - 1)}$.

In this paper we are interested in the construction of factorizations of λK_n in which the factors are of two types. Denoting by $G(m)$ the union of m G -factors, a $(G(s), H(t))$ -factorization of λK_n is a factorization in which s of the factors are G -factors and t are H -factors. The question of partitioning the edges of λK_n into two types of factors is not new. The generalized Oberwolfach problem (first posed in [6]) asks for a $(P_2(1), C_k(t))$ -factorization of λK_n . It is known (see [1, 2, 4]) that such a factorization exists iff n is even and $n \equiv 0 \pmod{k}$, except possibly when $k \equiv 3 \pmod{12}$ and $n = 4k$. From this we can easily derive necessary and sufficient conditions for the existence of a $(P_2(s), C_k(t))$ -factorization of K_n if k is even. When k is odd the problem is more difficult. The only result in this case is that of Rees [7] who gives necessary and sufficient conditions for the existence of a $(P_2(s), C_3(t))$ -factorization of K_n .

We will give necessary and sufficient conditions for the existence of a $(P_2(s), P_k(t))$ -factorization of λK_n . In view of earlier comments we will assume that $st \neq 0$ and that $k \geq 3$. The goal of this paper is to prove the following result. (Observe that simple counting, first on vertices and second on edges, yields the necessary conditions of the theorem.)

THEOREM 1.1. *For $k \geq 2$ the complete multigraph λK_n has a factorization into $s + t$ spanning subgraphs ($st \neq 0$), s of which are 1-factors and t of which are P_k -factors (that is, a $(P_2(s), P_k(t))$ -factorization) iff $n \equiv 0 \pmod{2}$, $n \equiv 0 \pmod{k}$ and $ks + 2t(k - 1) = \lambda k(n - 1)$.*

2. BUILDING BLOCKS

The proof of Theorem 1.1 depends on several relatively simple constructions which we present in the following lemmas. The proofs of these lemmas use ideas similar to those used in the construction of a P_k -factorization of λK_n , as described in [3].

We remark that in the remainder of the paper all arithmetic calculation is either modulo k on the residues $1, 2, \dots, k$, or modulo $2k$ on the residues $1, 2, \dots, 2k$, as will be clear from context. Throughout we assume that $k \geq 3$.

LEMMA 2.1. For even k :

- (a) K_k has a P_k -factorization; and
- (b) $K_{k,k} - P_2(1)$ has a P_k -factorization.

PROOF. (a) This is well known. If $V(K_k) = \{1, 2, \dots, k\}$ a P_k -factorization of K_k is given by the paths $P(i) = [i, 1+i, k-1+i, 2+i, k-2+i, \dots, k/2+2+i, k/2-1+i, k/2+1+i, k/2+i]$, $1 \leq i \leq k/2$.

(b) Let $P = [1, 2, \dots, k]$ be one of the paths in a P_k -factorization of K_k . In $K_{k,k}$ P defines the P_k -factor $\{[x_1, y_2, x_3, \dots, x_{k-1}, y_k], [y_1, x_2, y_3, \dots, y_{k-1}, x_k]\}$. Repeating for each path in the P_k -factorization of K_k we obtain a P_k -factorization of $K_{k,k} - P_2(1)$, where $P_2(1) = \{\{x_1, y_1\}, \{x_2, y_2\}, \dots, \{x_k, y_k\}\}$. \square

LEMMA 2.2. For odd k :

- (a) $K_{2k} - P_2(1)$ has a P_k -factorization; and
- (b) $K_{2k,2k} - C_4(1)$ has a P_k -factorization.

PROOF. (a) Consider the P_{2k} -factorization of K_{2k} as given in the proof of Lemma 2.1(a). Each $P(i) = [i, 1+i, 2k-1+i, 2+i, 2k-2+i, \dots, k+2+i, k-1+i, k+1+i, k+i]$, $1 \leq i \leq k$, is the union of two paths of length k and the edge $\{(3k+1)/2+i, (k+1)/2+i\}$. Observe that these k edges are in fact the edges of a 1-factor in K_{2k} . Deleting them from the paths yields a P_k -factorization of $K_{2k} - P_2(1)$.

(b) This follows from (a) in the same way as Lemma 2.1(b) followed from Lemma 2.1(a). \square

DEFINITION 2.3. It is not difficult to see that in Lemma 2.2(b) we can permute the vertices of $K_{2k,2k}$ in such a way that if the vertex bipartition is (A, B) , where $A = \{a_1, \dots, a_{2k}\}$ and $B = \{b_1, \dots, b_{2k}\}$, then $C_4(1)$ consists of the k 4-cycles $C(i) = (a_{2i+1}, b_{2i+3}, a_{2i+2}, b_{2i+4})$, $1 \leq i \leq k$. We define T to be the graph obtained by identifying the vertices a_i and b_i , $1 \leq i \leq 2k$. If the vertices of T are labelled $V(T) = \{1, 2, \dots, 2k\}$, then T is the union of the four 1-factors:

$$F_1 = \{\{2i, 2i+1\}: 1 \leq i \leq k\},$$

$$F_2 = \{1, 3\} \cup \{4, 6\} \cup \{\{2i+3, 2i+6\}: 1 \leq i \leq k-2\},$$

$$F_3 = \{1, 4\} \cup \{\{2i+4, 2i+6\}, \{2i+1, 2i+3\}: i \in \{1, 3, \dots, k-2\}\}$$

and

$$F_4 = \{3, 6\} \cup \{\{2i+6, 2i+8\}, \{2i+3, 2i+5\}: i \in \{1, 3, \dots, k-2\}\}.$$

We now present the main lemma.

LEMMA 2.4. Let H be a multipartite graph with $V(H) = \bigcup_{i=1}^u V_i$ so that for $1 \leq i < j \leq u$ the bipartite subgraph on vertex set $V_i \cup V_j$ with bipartition (V_i, V_j) , is $\tau(\{i, j\})$ -regular, where τ is a mapping from the set $\{\{i, j\}: 1 \leq i \neq j \leq u\}$ to the non-negative integers. Let $G(H)$ be a graph with $V(G(H)) = \{1, 2, \dots, u\}$ in which the edge $\{i, j\}$ has multiplicity $\tau(\{i, j\})$. Then H has a $(P_2(s), P_k(t))$ -factorization if $G(H)$ does.

PROOF. Suppose that $G(H)$ has a $(P_2(s), P_k(t))$ -factorization with path-factors P^1, P^2, \dots, P^t and 1-factors F^1, F^2, \dots, F^s . To each edge $pq = \{p, q\} \in E(P^i)$, associate a 1-factor F_{pq}^i from the $\tau(\{p, q\})$ -regular bipartite subgraph with vertex-set $V_p \cup V_q$. Similarly, to each edge $xy = \{x, y\} \in E(F^j)$, associate a 1-factor F_{xy}^j from the $\tau(\{x, y\})$ -regular bipartite subgraph with vertex-set $V_x \cup V_y$. Do this so that the 1-factors associated with a given edge form a 1-factorization of that bipartite graph. Clearly $\bigcup_{pq \in E(P^i)} F_{pq}^i$ is a P_k -factor and $\bigcup_{xy \in E(F^j)} F_{xy}^j$ is a 1-factor. \square

COROLLARY 2.5. *Let H be as in Lemma 2.4. If $G(H)$ is one of (a) a cycle of length k , k even, in which edges alternatively have multiplicities $k/2 - 1$ and $k/2$, or (b) the graph T (see Definition 2.3) in which each edge has multiplicity $(k - 1)/2$, then H has a P_k -factorization.*

PROOF. We need only show that the graphs $G(H)$ have P_k -factorizations. In case (a) let $V(G(H)) = \{1, 2, \dots, k\}$ and the cycle be $(1, 2, \dots, k)$ in which the edges $\{1, 2\}, \{3, 4\}, \dots, \{k-1, k\}$ have multiplicity $k/2 - 1$ and the edges $\{2, 3\}, \{4, 5\}, \dots, \{k-2, k-1\}, \{k, 1\}$ have multiplicity $k/2$. Select the paths $Q(i) = [2i, 2i+1, \dots, 2i-1]$, $1 \leq i \leq k/2$. In case (b) let $V(G(H)) = \{1, 2, \dots, 2k\}$ and T be as described in Definition 2.3. First select the P_k -factors:

$$P(i) = \{[2i+1, 2i+4, 2i+5, 2i+8, 2i+9, \dots, 2i+2k-5, 2i+2k-2, 2i+2k-1], \\ [2i+2, 2i+3, 2i+6, 2i+7, 2i+10, \dots, 2i+2k-4, 2i+2k-3, 2i+2k]\}, \\ 1 \leq i \leq (k-1)/2,$$

and

$$R(i) = \{[2i+k, 2i+k+2, 2i+k+4, \dots, 2i+k-4, 2i+k-2], \\ [2i+k+1, 2i+k+3, 2i+k+5, \dots, 2i+k-3, 2i+k-1]\}, \quad 1 \leq i \leq (k-1)/2.$$

The edges remaining form the last P_k -factor which is

$$\{[1, 4, 5, 8, 9, \dots, k-4, k-1, k, k+2, k+4, \dots, 2k-1], \\ [2, 3, 6, 7, 10, \dots, k-3, k-2, k+1, k+3, k+5, \dots, 2k]\}$$

if $k \equiv 1 \pmod{4}$, and

$$\{[1, 4, 5, 8, 9, \dots, k-3, k-2, k+1, k+3, \dots, 2k], \\ [2, 3, 6, 7, 10, \dots, k-4, k-1, k, k+2, \dots, 2k-1]\}$$

if $k \equiv 3 \pmod{4}$. \square

3. THEOREM 1.1: PROOF OF SUFFICIENCY

Proof of sufficiency when k is even. From the first two necessary conditions we know that $n = kr$, and from the condition $ks + 2(k-1)t = \lambda k(n-1)$ we obtain $s \equiv \lambda(r-1) \pmod{k-1}$.

Let $V(\lambda K_n) = \{(i, j) : 1 \leq i \leq r, 1 \leq j \leq k\} = \bigcup_{i=1}^r H_i = \bigcup_{j=1}^k V_j$, where $H_i = \{(i, j) : 1 \leq j \leq k\}$ and $V_j = \{(i, j) : 1 \leq i \leq r\}$. Now λK_n is obtained from λK_r with vertex set $\{1, 2, \dots, r\}$ by replacing each vertex i by a copy of λK_k on the vertex set H_i and each edge $\{i, j\}$ by a copy of $K_{k,k}$ on the vertex set $H_i \cup H_j$ with bipartition (H_i, H_j) .

To each edge $\{i, j\}$ of λK_r associate in λK_n a 1-factor F_{ij} of $\lambda K_{k,k}$ with vertex bipartition (H_i, H_j) . Let R be the subgraph of λK_n consisting of the union of these 1-factors. Each vertex in R has degree $\lambda(r-1)$. First we will show that the 1-factors can be chosen so that R has a $(P_2(s_1), P_k(t_1))$ -factorization for $0 \leq s_1 \leq \lambda(r-1)$ and $s_1 \equiv \lambda(r-1) \pmod{k-1}$. Let $s_1 = \lambda(r-1) - q(k-1)$.

Let the 1-factor F_{ij} be either the 1-factor $C = \{(i, 2m-1), (j, 2m)\}, \{(i, 2m), (j, 2m-1)\}: 1 \leq m \leq k/2\}$ or the 1-factor $D = \{(i, 2m-1), (j, 2m-2)\}, \{(i, 2m-2), (j, 2m-1)\}: 1 \leq m \leq k/2\}$.

CLAIM. The 1-factors can be chosen in such a way that each vertex belongs to at least $q(k/2 - 1)$ 1-factors of type C and at least $q(k/2)$ of type D .

PROOF. If r is even take a 1-factorization of λK_r . To each edge of λK_r determined by $s_1 + q(k/2 - 1)$ of the 1-factors associate a type C 1-factor and to the remaining edges associate a type D 1-factor.

If r is odd, then $\lambda(r-1)$ is even and λK_r has a 2-factorization. If $s_1 + q(k/2 - 1)$ is even (and consequently so is $q(k/2)$), then to each edge of λK_r determined by $(s_1 + q(k/2 - 1))/2$ of the 2-factors associate a type C 1-factor and to the remaining edges associate a type D 1-factor. If $s_1 + q(k/2 - 1)$ is odd (and consequently so is $q(k/2)$), then to each edge of λK_r determined by $(s_1 - 1 + q(k/2 - 1))/2$ of the 2-factors associate a type C 1-factor and to the remaining edges associate a type D 1-factor. (Note that $s_1 \geq 1$ as if $s_1 = 0$, then $\lambda(r-1) = q(k-1)$, which is impossible as $k-1, q$ and r are all odd.) \square

It is not difficult to see that $G(R)$ consists of the edge-disjoint union of s_1 1-factors and q cycles of length k , in which alternate edges have multiplicities $k/2 - 1$ and $k/2$. By Lemma 2.4 and Corollary 2.5 the graph R has a $(P_2(s_1), P_k(t_1))$ -factorization.

Next we show that $\lambda K_n - R$ has a $(P_2(s_2), P_k(t_2))$ -factorization for any $s_2, 0 \leq s_2 \leq \lambda r(k-1)$ and $s_2 \equiv 0 \pmod{k-1}$.

If r is even λK_r admits a 1-factorization with 1-factors $F_1, F_2, \dots, F_{\lambda(r-1)}$ and to each 1-factor there corresponds in $\lambda K_n - R$ a $(K_{k,k} - P_2(1))$ -factor. Thus $\lambda K_n - R$ has a $((K_{k,k} - P_2(1))(\lambda(r-1)), K_k(\lambda))$ -factorization.

If r is odd λK_r admits a near 1-factorization (a near 1-factor is a matching which covers all but one vertex of the graph) with near 1-factors $M_1, M_2, \dots, M_{\lambda r}$ and each near 1-factor M_i corresponds in $\lambda K_n - R$ to a Q -factor, where Q consists of the vertex-disjoint union of $(r-1)/2$ copies of $K_{k,k} - P_2(1)$ and one copy of K_k . Thus $\lambda K_n - R$ has a Q -factorization.

Now we note that since k is even Lemma 2.1 assures us that each of $K_{k,k} - P_2(1), K_k$ and Q has a P_k -factorization consisting of $k/2$ P_k -factors. But these three graphs also have 1-factorizations made up of $k-1$ 1-factors. So in each of the $((K_{k,k} - P_2(1))(\lambda(r-1)), K_k(\lambda))$ -factorization of $\lambda K_n - R, r$ even, and the Q -factorization of $\lambda K_n - R, r$ odd, we replace $s_2/(k-1)$ of the factors by 1-factors and the remainder by P_k -factors.

The theorem then follows by letting $s_1 = s$ and $s_2 = 0$ if $s \leq \lambda(r-1)$, and $s_1 = \lambda(r-1)$ and $s_2 = s - \lambda(r-1)$ if $s \geq \lambda(r-1)$.

Proof of sufficiency when k is odd. From the first two necessary conditions we know that $n = 2kr$, and then from the condition $ks + 2(k-1)t = \lambda k(n-1)$ we obtain $s \equiv \lambda(2r-1) \pmod{2(k-1)}$. The construction to be presented is quite similar to that given when k is even.

Let $V(\lambda K_n) = \{(i, j): 1 \leq i \leq r, 1 \leq j \leq 2k\} = \bigcup_{i=1}^r H_i = \bigcup_{j=1}^{2k} V_j$, where $H_i = \{(i, j): 1 \leq j \leq 2k\}$ and $V_j = \{(i, j): 1 \leq i \leq r\}$. Now λK_n is obtained from λK_r with vertex set $\{1, 2, \dots, r\}$ by replacing each vertex i by a copy of λK_{2k} on the vertex set H_i and each edge $\{i, j\}$ by a copy of $K_{2k, 2k}$ on the vertex set $H_i \cup H_j$ with bipartition (H_i, H_j) .

To each edge $\{i, j\}$ of λK_r associate in λK_n a C_4 -factor C_{ij} of $K_{2k, 2k}$ with vertex bipartition (H_i, H_j) . To each vertex i of λK_r associate λ 1-factors, H_i^ε , $1 \leq \varepsilon \leq \lambda$, of the graph λK_{2k} with vertex set H_i . Let R be the $\lambda(2r-1)$ -regular subgraph of λK_n consisting of the union of these C_4 -factors and 1-factors. As in the previous case we begin by showing that these factors can be chosen so that R has a $(P_2(s_1), P_k(t_1))$ -factorization for $0 \leq s_1 \leq \lambda(2r-1)$ and $s_1 \equiv \lambda(2r-1) \pmod{2(k-1)}$. Let $s_1 = \lambda(2r-1) - 2q(k-1)$.

Suppose $\lambda(r-1)$ is even. Then λK_r has a 2-factorization. We arbitrarily direct the cycles in the 2-factorization, so yielding a directed λK_r in which each vertex has both in- and out-degree $\lambda(r-1)/2$. If the edge $\{i, j\}$ is directed from i to j (that is, it becomes the arc (i, j)), then let C_{ij} be as described in Definition 2.3 with $A = H_i$ and $B = H_j$. Thus $G(\bigcup_{i \neq j} C_{ij})$ consists of $\lambda(r-1)/2$ edge-disjoint subgraphs, each isomorphic to T . For each i choose the H_i^ε , $1 \leq \varepsilon \leq \lambda$, so that $\lfloor \lambda/4 \rfloor$ of them are $F_1^i = \{(i, 2j), (i, 2j+1)\}: 1 \leq j \leq k\}$, $\lfloor \lambda/4 \rfloor$ are $F_2^i = \{(i, 1), (i, 3)\}, \{(i, 4), (i, 6)\} \cup \{(i, 2j+3), (i, 2j+6)\}: 1 \leq j \leq k-2\}$, $\lfloor \lambda/4 \rfloor$ are $F_3^i = \{(i, 1), (i, 4)\} \cup \{(i, 2j+4), (i, 2j+6)\}, \{(i, 2j+1), (i, 2j+3)\}: j \in \{1, 3, \dots, k-2\}\}$, $\lfloor \lambda/4 \rfloor$ are $F_4^i = \{(i, 3), (i, 6)\} \cup \{(i, 2j+6), (i, 2j+8)\}, \{(i, 2j+3), (i, 2j+5)\}: j \in \{1, 3, \dots, k-2\}$ (recall Definition 2.3) and the remaining $\lambda' = \lambda - 4\lfloor \lambda/4 \rfloor$ are chosen arbitrarily. Thus $G(\bigcup_{i=1}^r \bigcup_{\varepsilon=1}^\lambda H_i^\varepsilon)$ consists of $\lfloor \lambda/4 \rfloor$ copies of T and λ' 1-factors and therefore $G(R)$ consists of $\lambda(r-1)/2 + \lfloor \lambda/4 \rfloor = \lfloor \lambda(2r-1)/4 \rfloor$ edge-disjoint copies of T and λ' 1-factors. Use Lemma 2.5(b) to determine a P_k -factorization of $(q(k-1)/2)T$. (Since $2q(k-1) < \lambda(2r-1)$ there are $q(k-1)/2$ copies of T available.) Each of the remaining copies of T in $G(R)$ has a 1-factorization. This now yields a $(P_2(s_1), P_k(t_1))$ -factorization of R .

We next consider the case when $\lambda(r-1)$ is odd (and hence λ is odd). In this case $\lambda K_r - P_2(1)$ has a 2-factorization. Proceed to define R as in the previous case using the 2-factorization of $\lambda K_r - P_2(1)$. To the remaining edges $\{i, j\}$ of λK_r (those of the deleted 1-factor F) associate the C_4 -factor $\{(i, 2p), (j, 2p), (i, 2p+1), (j, 2p+1)\}: 1 \leq p \leq k\}$. Again choose the λ 1-factors H_i^ε so that $G(\bigcup_{i=1}^r \bigcup_{\varepsilon=1}^\lambda H_i^\varepsilon)$ contains $\lfloor \lambda/4 \rfloor$ copies of T and λ' 1-factors. Furthermore, if $\lambda' = 3$, choose those λ' 1-factors to be F_2^i, F_3^i and F_4^i , as given previously. Let us now analyse the subgraph R . It consists of a 1-factor $F' = \{(i, 2p), (j, 2p)\}, \{(i, 2p+1), (j, 2p+1)\}: 1 \leq p \leq k, \{i, j\} \in F\}$ and a subgraph R' . If $\lambda' = 3$, $G(R')$ consists of $(\lambda(2r-1) - 1)/4$ edge-disjoint copies of T , and if $\lambda' = 1$, $G(R')$ consists of $(\lambda(2r-1) - 3)/4$ edge-disjoint copies of T and two 1-factors. In each case there are at least $q(k-1)/2$ copies of T available, and by Lemma 2.5(b) we have a P_k -factorization of $(q(k-1)/2)T$. Applying Lemma 2.4, $G(R')$, and therefore R' , has a $(P_2(s_1 - 1), P_k(t_1))$ -factorization. So R has a $(P_2(s_1), P_k(t_1))$ -factorization as required.

The final step, in which we show that $\lambda K_n - R$ has a $(P_2(s_2), P_k(t_2))$ -factorization for any s_2 , $0 \leq s_2 \leq 2\lambda r(k-1)$ and $s_2 \equiv 0 \pmod{2(k-1)}$, is quite straightforward.

If r is even we use a 1-factorization of K_r to obtain a $((K_{2k, 2k} - C_4(1))(\lambda(r-1)), (K_{2k} - P_2(1))(\lambda))$ -factorization of $\lambda K_n - R$, and if r is odd we use a near 1-factorization of K_r to obtain a Z -factorization of $\lambda K_n - R$, where Z is the vertex-disjoint union of $(r-1)/2$ copies of $K_{2k, 2k} - C_4(1)$ and one copy of $K_{2k} - P_2(1)$. By Lemma 2.2 the graphs $K_{2k, 2k} - C_4(1)$ and $K_{2k} - P_2(1)$ have P_k -factorizations, each with k P_k -factors. In addition, they both have 1-factorizations with $2k-2$ 1-factors. So on $s_2/2(k-1)$ occasions we choose the 1-factorization, and on the remaining occasions the P_k -factorization.

The theorem is now completed by letting $s_1 = s$ and $s_2 = 0$ if $s \leq \lambda(2r-1)$, and $s_1 = \lambda(2r-1)$ and $s_2 = s - \lambda(2r-1)$ if $s \geq \lambda(2r-1)$.

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