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Abstract

In this paper we address the problem of state observation of linear time-varying (LTV) systems with delayed measurements, which has attracted the attention of many researchers—see [9] and references therein. We show that, the parameter estimation-based observer (PEBO) design proposed in [4,5] provides a very simple solution to the problem with reduced prior knowledge. Moreover, when PEBO is combined with the dynamic regressor extension and mixing (DREM) estimation technique [1,6], the estimated state converges in fixed-time with extremely weak excitation assumptions.

Key words: Linear time-varying systems; state observer; delay systems; parameter estimation.

1 Introduction and Problem Formulation

It is common in control applications that the measuring devices introduce time-varying delays. This fact complicates the problem of estimating the state of a dynamical system since the output signal will be subject to delays, a problem that has been studied by many authors. For linear time invariant (LTI) systems, such a difficulty is well understood and the convergence of the observer is usually verified checking the feasibility of a linear matrix inequality [2]. On the other hand, the problem for LTV systems is widely open—see the literature review and the list of references in the recent papers [7,9].

We consider in this paper an LTV system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(\varphi(t))x(\varphi(t)), \qquad (1)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^q$, $\varphi(t)$ is a continuous *known* nonnegative delay function that codifies the measurement delay, that is

$$\varphi(t) := t - d(t),$$

where $d(t)$ is the time-varying delay verifying

$$0 \le d(t) \le d_M, \qquad (2)$$

and the state initial condition $x(0) = x_0 \in \mathbb{R}^n$.

All system matrices are continuous and *known* with $C(\varphi(t))$ bounded and $A(t)$, $B(t)$ verifying the following.

Assumption 1 The state transition matrix of the homogeneous part of the system (1), denoted $\Phi(t,\tau)$, verifies

$$\|\Phi(t,\tau)\| \le c_1, \ \forall t \ge \tau.$$

Assumption 2

$$\int_{\tau}^{t} \|\Phi(t,\tau)B(\tau)\|d\tau \le c_2, \ \forall t \ge \tau.$$

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Remark 1 Assumption 1 is equivalent to *uniform stability* of the homogeneous part of the system (1) [8, Theorem 6.4], while Assumption 2 is a necessary and sufficient condition for *bounded-input-bounded-state* stability of (1) [8, Theorem 12.2].

In this paper we design an observer

$$\dot{\chi}(t) = F(\chi(t), u(t), y(t))$$
$$\hat{x}(t) = H(\chi(t), u(t), y(t))$$

with $\chi(t) \in \mathbb{R}^{n_\chi}$ such that all signals are bounded and *fixed convergence time* (FCT) of the estimated state is ensured, that is,

$$\hat{x}(t) = x(t), \ \forall t \geq t_c, \tag{3}$$

for some $t_c \in (0, \infty)$ and for all $x_0 \in \mathbb{R}^n, \chi(0) \in \mathbb{R}^{n_\chi}$ and all continuous bounded $u(t)$.

2 Main result

To streamline the presentation of our observer we recall from [3] the following.

Definition 1 A bounded signal $\Delta(t) \in \mathbb{R}$ is an interval exciting (IE) if there exists a time $t_{\text{IE}} \in (0, \infty)$ such that

$$\int_0^{t_{\text{IE}}} \Delta^2(s) ds \geq \rho, \tag{4}$$

for some $\rho > 0$.

Proposition 1 Consider the system (1) satisfying Assumptions 1 and 2 and $d(t)$ verifying (2) with the generalized PEBO+DREM

$$\dot{\xi}(t) = A(t)\xi(t) + B(t)u(t) \tag{5a}$$
$$\dot{\Phi}_A(t) = A(t)\Phi_A(t), \ \Phi_A(0) = I_n \tag{5b}$$
$$\dot{Y}(t) = -\lambda Y(t) + \lambda \Psi^\top(t)z(t) \tag{5c}$$
$$\dot{\Omega}(t) = -\lambda\Omega(t) + \lambda\Psi^\top(t)\Psi(t) \tag{5d}$$

with $\lambda > 0$ and the gradient parameter estimator [1]

$$\dot{\hat{\theta}}(t) = -\gamma\Delta(t)[\Delta(t)\hat{\theta}(t) - \mathcal{Y}(t)] \tag{6}$$

with $\gamma > 0$, and the definitions

$$\Psi(t) := C(\varphi(t))\Phi_A(\varphi(t)) \tag{7a}$$
$$z(t) := C(\varphi(t))\xi(\varphi(t)) - y(t) \tag{7b}$$
$$\mathcal{Y}(t) := \text{adj}\{\Omega(t)\}Y(t) \tag{7c}$$
$$\Delta(t) := \det\{\Omega(t)\}, \tag{7d}$$

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[1] See [10, Chapter 2] for the definition of a gradient estimator.

where $\text{adj}\{\cdot\}$ is the adjugate matrix. Define the state estimate as

$$\hat{x}(t) = \xi(t) - \Phi_A(t)\frac{1}{1 - w_c(t)}[\hat{\theta}(t) - w_c(t)\hat{\theta}(0)], \tag{8}$$

with

$$\dot{w}(t) = -\gamma\Delta^2(t)w(t), \ w(0) = 1, \tag{9}$$

and $w_c(t)$ defined via the clipping function

$$w_c(t) = \begin{cases} w(t) & \text{if } w(t) \leq 1 - \mu \\ 1 - \mu & \text{if } w(t) > 1 - \mu, \end{cases}, \tag{10}$$

where $\mu \in (0, 1)$ is a designer chosen parameter. Assume $\Delta(t)$ verifies (4) with

$$\rho = -\frac{1}{\gamma}\ln(1 - \mu). \tag{11}$$

Then, (3) is ensured for some $t_c \geq t_{\text{IE}}$, and all signals remain bounded.

PROOF. First, we prove the signal boundedness claim. Due to Assumptions 1 and 2 and boundedness of $u(t)$, $\xi(t)$ and the fundamental matrix $\Phi_A(t)$ are bounded. This, together with boundedness of $C(\varphi(t))$, guarantees that $z(t)$ and $\Psi(t)$ are bounded. Finally, positivity of λ ensures that $Y(t)$ and $\Omega(t)$ are also bounded.

Second, we apply the PEBO technique to derive a (vector) linear regression equation (LRE). For this purpose, define the error signal $e(t) := \xi(t) - x(t)$, which satisfies

$$\dot{e}(t) = A(t)e(t),$$

hence

$$e(t) = \Phi_A(t)\theta,$$

with $\theta := e(0)$. Consequently,

$$x(t) = \xi(t) - \Phi_A(t)\theta.$$

The output of the system (1) then satisfies

$$y(t) = C(\varphi(t))\left[\xi(\varphi(t)) - \Phi_A(\varphi(t))\theta\right].$$

From which we get a LRE, that allows us to identify θ, as

$$z(t) = \Psi(t)\theta, \tag{12}$$

where we used the definitions in (7a) and (7b). Notice that these signals are well defined because the assumption (2) ensures that $\varphi(t)$ is lower bounded.

Third, we apply the DREM procedure to (12) to generate n scalar LREs. Towards this end, we consider the

following chain of implications

$$(12) \Rightarrow \Psi^\top(t)z(t) = \Psi^\top(t)\Psi(t)\theta \quad (\Leftarrow \Psi^\top(t)\times)$$
$$\Rightarrow Y(t) = \Omega(t)\theta \left(\Leftarrow \frac{\lambda}{\mathbf{p}+\lambda}[\cdot] \text{ and } (5c),(5d)\right)$$
$$\Rightarrow \mathcal{Y}(t) = \Delta(t)\theta, \ (\Leftarrow \mathrm{adj}\{\Omega\} \times \text{ and } (7c),(7d)),$$

with $\mathbf{p} = \frac{d}{dt}$, where we have used the fact that for any, *possibly singular*, $n \times n$ matrix L we have $\mathrm{adj}\{L\}L = \det\{L\}I_n$ in the last line.

The last step is the analysis of the gradient estimator (6) with the FCT observer (8)-(10). Replacing the latter identity in (6) yields the error equation

$$\dot{\tilde{\theta}}(t) = -\gamma\Delta^2(t)\tilde{\theta}(t),$$

where $\tilde{\theta}(t) := \hat{\theta}(t) - \theta$. The solution of this equation is given by

$$\tilde{\theta}(t) = e^{-\gamma\int_0^t \Delta^2(s)ds}\tilde{\theta}(0).$$

Notice that the solution of (9) is

$$w(t) = e^{-\gamma\int_0^t \Delta^2(s)ds},$$

hence we have that

$$\tilde{\theta}(t) = w(t)\tilde{\theta}(0).$$

Clearly, this is equivalent to

$$[1 - w(t)]\theta = \hat{\theta}(t) - w(t)\hat{\theta}(0).$$

On the other hand, if $\Delta(t)$ is IE with ρ satisfying (11), we have that there exists a $t_c \geq t_0$ such that

$$w(t) = w_c(t) < 1, \ \forall t \geq t_c.$$

Consequently, we conclude that

$$\frac{1}{1 - w_c(t)}[\hat{\theta}(t) - w_c(t)\hat{\theta}(0)] = \theta, \ \forall t \geq t_c.$$

Replacing this identity in (8) completes the proof.

Remark 2 Note that the upper bound on the delay implicitly impacts the value of t_c given in (4), thus, it predefines the fixed time of convergence.

Remark 3 Another, more complex, solution to this problem that requires strict positivity of the delay $d(t)$ and the *knowledge* of its derivative is reported in [9] under the classical assumption of existence of an exponentially stable Luenberger observer for the LTV system (1) with $\varphi(t) = t$, *i.e.* [9, Assumption 2]. The estimator is based on—the now classical—PDE representation of the delay, with an observer designed for the coupled LTV-PDE system.

Remark 4 In [7] a Kalman-Bucy-like observer with fractional powers is proposed, which ensures that the state estimation enters a *residual set* in a finite time provided the pair $(C(t), A(t))$ is uniformly completely observable. As it is well known [8], this assumption is a *sufficient* condition for the verification of [9, Assumption 2]. It is obvious that both assumptions are *strictly stronger* than our IE condition.

3 Simulation Results

Consider the LTV system (1) with $m = q = 1, n = 2$ and

$$A = \begin{bmatrix} 0 & 1 \\ -\sin^2(t) & 0 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

For simulations consider the initial conditions $x(0) = \mathrm{col}(1, 2)$ and zero for all the other states.

We consider three cases:

C1 $\varphi(t) = t$ (Fig. 1 and Fig. 2);
C2 $\varphi(t) = \varphi(t-\tau), \tau = 1+0.9\sin(t)$ (Fig. 3 and Fig. 4);
C3 $\varphi(t) = \varphi(t-\tau), \tau = 0.1+\cos^2(3t)$ (Fig. 5 and Fig. 6).
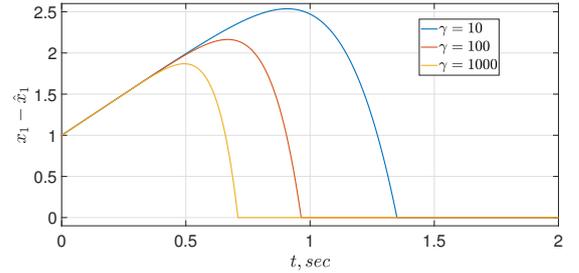

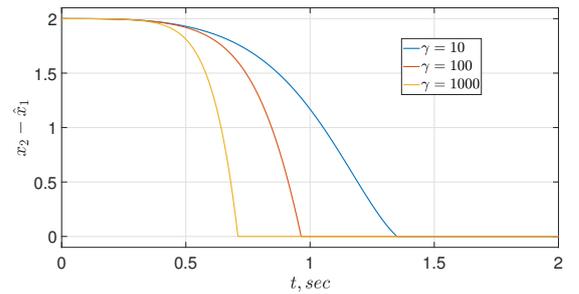Fig. 1. Error transients $x_1(t) - \hat{x}_1(t)$ for different γ and case **C1**

Fig. 2. Error transients $x_2(t) - \hat{x}_2(t)$ for different γ and case **C1**

4 Conclusions and Future Research

A state observer for LTV systems with delayed measurements of the form (1) has been proposed. The main feature of the observer is that, thanks to the use of DREM,
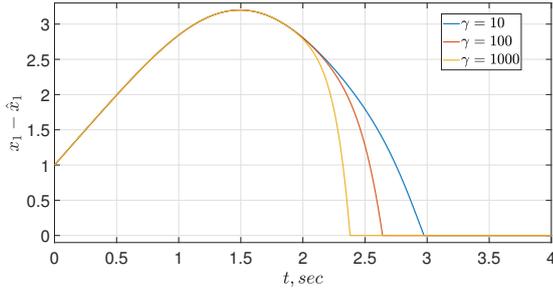
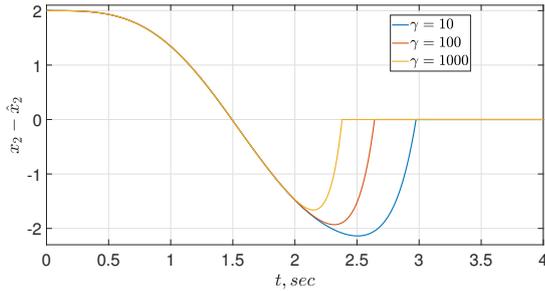Fig. 3. Error transients $x_1(t) - \hat{x}_1(t)$ for different γ and case **C2**

Fig. 4. Error transients $x_2(t) - \hat{x}_2(t)$ for diffrerent γ and case **C2**
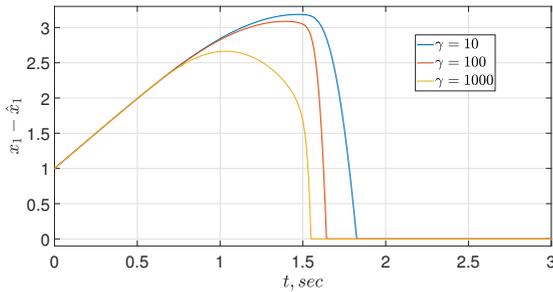

Fig. 5. Error transients $x_1(t) - \hat{x}_1(t)$ for diffrerent γ and case **C3**
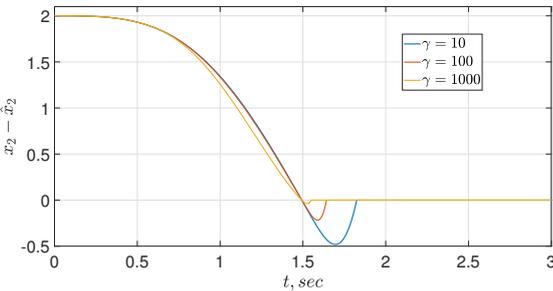

Fig. 6. Error transients $x_2(t) - \hat{x}_2(t)$ for diffrerent γ and case **C3**

it ensures convergence in fixed time under the extremely weak assumption of IE. Following the PEBO approach, the observation of the state is carried out via the estima-

tion of some suitable initial conditions and, in contrast to [9], we do not require any kind of "delay compensation". To the best of our knowledge this is the strongest result available for this problem.

Current research is under way to relax the strict assumption of exact knowledge of the system dynamics and the time delay function.

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