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Learning the sampling density in 2D SPARKLING MRI acquisition for optimized image reconstruction

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Abstract—The SPARKLING algorithm was originally developed for accelerated 2D magnetic resonance imaging (MRI) in the compressed sensing (CS) context. It yields non-Cartesian sampling trajectories that jointly fulfill a target sampling density while each individual trajectory complies with MR hardware constraints. However, the two main limitations of SPARKLING are first that the optimal target sampling density is unknown and thus a user-defined parameter and second that this sampling pattern generation remains disconnected from MR image reconstruction thus from the optimization of image quality. Recently, data-driven learning schemes such as LOUPE have been proposed to learn a discrete sampling pattern, by jointly optimizing the whole pipeline from data acquisition to image reconstruction. In this work, we merge these methods with a state-of-the-art deep neural network for image reconstruction, called XPDNET, to learn the optimal target sampling density. Next, this density is used as input parameter to SPARKLING to obtain 20x accelerated non-Cartesian trajectories. These trajectories are tested on retrospective compressed sensing (CS) studies and show superior performance in terms of image quality with both deep learning (DL) and conventional CS reconstruction schemes.

Index Terms—non-Cartesian trajectories, MRI, Compressed Sensing, reconstruction networks

I. INTRODUCTION

Compressed sensing (CS) in MRI [1] has led to a large reduction in scan time while maintaining a reasonable reconstructed MR image quality. Practically, CS is implemented by undersampling pseudo-randomly the k-space according to a variable sampling density [2]–[5]. The sampling pattern may be composed of multiple individual Cartesian lines (Cartesian Sampling), in which case variable density sampling (VDS) is implemented only along the phase encoding dimension. To go to higher reduction in scan times, non-Cartesian sampling is really helpful as it permits the implementation of 2D VDS with the help of non-Cartesian trajectories, such as radial spokes [6] and spiral interleaves [7]). Although radial and spiral sampling are widespread, they are not really optimal as radial spokes don't cover the k-space perfectly and spiral interleaves do not exactly match a prescribed sampling density. Hence, severe artifacts impede image quality during CS reconstruction.

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The Spreading Projection Algorithm for Rapid K-space sampling, or SPARKLING [8] has been introduced as an iterative scheme that optimizes for each k-space trajectory to be compliant with MRI hardware constraints (particularly maximum gradient and slew rate constraints), while ensuring that the overall sampling pattern obtained with all the trajectories follows a target sampling density. Further, the algorithm ensures that optimized k-space sampling pattern does not have any local clusters, leading to locally uniform sampling patterns. This algorithm was extended to 3D [9], [10] and showed superior performance in both terms of a peaky point spread function and image quality.

However, a major drawback of SPARKLING algorithm is the need to setup a target sampling density as an input to the algorithm. In our earlier studies, we relied on heuristic methods to set this sampling density. The latter was parameterized to be radially decaying and its optimal parameters (decay, cutoff) were grid searched during retrospective reconstruction studies in which image quality was maximized as a function of optimized trajectories for varied target densities. However, this approach is too computationally expensive. Also, with a parameterized target density, the search space is too constrained, preventing us to obtain organ, imaging-contrast or orientation-specific sampling schemes. One way to tackle this problem is by learning the target sampling density using data-driven approaches.

In [11], the authors proposed a naive approach to choose the target sampling density by averaging the power spectra of multiple MR images in a dataset. This method results in sampling densities that enforce denser sampling in the low frequencies. In [11] the authors showed that this approach outperforms standard VDS and remains robust to variability in anatomy and orientation. However, this method focuses purely on the MRI dataset and is agnostic to the reconstruction technique. All MRI reconstruction algorithms enforce a prior (like sparsity in the wavelet or image gradient domain). Recent DL reconstruction algorithms [12]–[14] have learned more complex priors based on the organ or contrast-specific dataset. The target sampling density can be more efficient if it takes these priors into account and enforce denser samples in regions where the degree of uncertainty associated with such priors for reconstruction is higher.

More recently, methods like [15], [16] learn the sampling pattern for MRI in a data-driven manner while optimizing for image quality at the reconstruction stage. In the deep learning setting, LOUPE [16] jointly optimizes the sampling density and the weights of a U-net architecture for image reconstruction. However, these studies are limited to Cartesian sampling. Most appealing contributions [17], [18] tend to directly learn the trajectories in a data-driven manner under MR hardware constraints. However, these optimization methods have a large number of trainable parameters that makes them stuck in sub-optimal local minimizers resulting in final trajectories that just coincide with perturbed initialization.

In this work, we use the target densities obtained by LOUPE as an input to the SPARKLING algorithm to generate 2D SPARKLING non-Cartesian trajectories. We carry out retrospective studies and compare them with densities obtained from average power spectra over the fastMRI dataset. We perform image reconstruction using both CS technique and the newly developed NC-XPDNET [19] which is a density compensated unrolled Neural network for non-Cartesian MRI reconstruction. We conclude that the proposed solution (LOUPE+2D SPARKLING) outperforms other VDS approaches in terms of image quality.

II. MATERIALS AND METHODS

Here, we detail the methods used to optimize for sampling density and thereby how the latter is injected as an input to SPARKLING algorithm to generate non-Cartesian trajectories. We later study the performance of the corresponding sampling schemes on retrospective MR image reconstruction studies.

A. 2D Non-Cartesian Trajectories

Throughout this work, we follow the formulation we developed in [10], for the case of 2D imaging. Let the MR image size be $N \times N$, over a field of view $\mathcal{F} \times \mathcal{F}$. Then the 2D k-space of the image is defined in $[-K_{\max}, K_{\max}]^2$, with $K_{\max} = \frac{N}{2\mathcal{F}}$. In all our trajectories, we kept $N = 320$ and $\mathcal{F} = 0.23$ m. For the sake of simplicity, let us normalize the k-space to $\Omega = [-1, 1]^2$. We are optimizing for the 2D k-space sampling pattern \mathbf{K} which is composed of several shots N_c , $\mathbf{K} = (\mathbf{k}_i)_{i=1}^{N_c}$. Each 2D shot $\mathbf{k}_i(t) = (k_{i,x}(t), k_{i,y}(t))$ is controlled by the magnetic field gradients $\mathbf{G}_i(t) = (G_{i,x}(t), G_{i,y}(t))$ as follows: $\mathbf{k}_i(t) = \frac{\gamma}{2\pi} \int_0^t \mathbf{G}_i(\tau) d\tau$ with γ the gyro-magnetic ratio ($\gamma = 42.57\text{MHz/T}$ for proton imaging). Each shot is sampled at the pace of gradient raster time Δt , throughout the readout time T_{obs} , resulting in $N_s = \lfloor \frac{T_{\text{obs}}}{\Delta t} \rfloor$ samples per shot. The k-space data from the scanner is sampled at dwell time δt , which in practice is a fraction of Δt . Thus the total received k-space samples are of the form $\mathcal{K} \in \mathbb{C}^{N_c \times N_s \times \frac{\Delta t}{\delta t}}$. In our studies, we used dwell time ($\delta t = 2\mu\text{s}$) and gradient raster time ($\Delta t = 10\mu\text{s}$), thereby having 5 times more k-space sample points than the measurements defined by the gradient wave forms.

The MR hardware constraints of maximum gradient strength ($G_{\max} = 40$ mT/m) and slew rate ($S_{\max} = 180$ T/m/s) results in a constrained trajectory with limited speed (α) and

acceleration (β). Note that the speed constraint also handles the Nyquist sampling criterion (see [20]). We define this constraint set as $\mathcal{Q}_{\alpha,\beta}^{N_c}$, see [10].

B. SPARKLING Algorithm

Let the target sampling distribution be $\rho : \Omega \rightarrow \mathbb{R}$, with $\rho(x) \geq 0$ for all x and $\int \rho(x) dx = 1$. Given ρ , the SPARKLING algorithm optimizes for the k-space sampling pattern \mathbf{K} such that the actual sampling distribution is closest to ρ , while being locally uniform. Although theoretically SPARKLING takes a continuous distribution ρ as input parameter, in practice, we discretize the distribution to obtain $\rho \in \mathbb{R}^{N \times N}$. Further, the algorithm ensures that the each k-space shot $\mathbf{k}_i(t)$ in optimal $\hat{\mathbf{K}}$ lies in $\mathcal{Q}_{\alpha,\beta}^{N_c}$. We can now summarize the SPARKLING algorithm as follows:

$$\hat{\mathbf{K}} = \mathcal{S}(\rho, \mathcal{Q}_{\alpha,\beta}^{N_c}, \mathbf{K}_0) \quad (1)$$

with \mathbf{K}_0 being the initialization. The detailed algorithm is presented in [10]. Hereafter, we discuss different gridded distributions ρ that were obtained for our study.

C. Target sampling density learning

In this work, we broadly use four methods for estimating or learning target sampling densities. All these methods are data-driven and we relied on the fastMRI dataset [21] to compute them. Let $\{\mathbf{x}_j \in \mathbb{R}^{N \times N}\}_{j=1}^n$ denote a brain MR image from the fastMRI dataset, where j is the scan number and n is the total number of images (for simplicity, we used magnitude-only images). Let $\{\mathbf{v}_j \in \mathbb{C}^{N \times N}\}_{j=1}^n$ correspond to its respective discrete k-space on a grid (Fourier spectrum) obtained by a fast Fourier transform.

1) *VDS-based (ρ_{vds}):* The first method we employed to obtain a density is based on naive VDS. For this, we parameterized the density as radially decaying with cutoff C and decay D as described in [10]:

$$\rho_{\text{vds}}^{C,D}(x) = \begin{cases} \kappa & |x| < C \\ \kappa \left(\frac{C}{|x|}\right)^D & |x| > C \end{cases} \quad (2)$$

In our experiments, we grid searched for optimal parameters heuristically and used $C = 25\%$ and $D = 2$ as the best density.

2) *Spectrum-based ($\rho_{\text{sb}}, \rho_{\text{lsb}}$):* Next we obtained sampling density based on [11] which involves averaging the spectra of brain images from the fastMRI dataset. Let \mathbf{v}_{avg} correspond to the average of all the spectra \mathbf{v}_j in the dataset. With this, we can normalize the spectrum to obtain a distribution ρ_{sb} on the grid $N \times N$:

$$\rho_{\text{sb}}(p, q) = \frac{\mathbf{v}_{\text{avg}}(p, q) - \min(\mathbf{v}_{\text{avg}})}{\sum_{p,q} [\mathbf{v}_{\text{avg}}(p, q) - \min(\mathbf{v}_{\text{avg}})]}. \quad (3)$$

Further, we observed that the spectra have very large magnitudes at lower frequencies as compared with higher frequencies. In an effort to flatten the distribution so that we may give more importance to both frequencies, we relied on average *log-spectrum* of the fastMRI images and obtained the distribution ρ_{lsb} by using Eq. (3).

3) *LOUPE-based* (ρ_{lb}): As the spectrum-based methods are agnostic to image reconstruction, to fill this gap we used the Cartesian acquisition model from LOUPE [16]. LOUPE is actually a DL-based optimization scheme that learns a Cartesian under-sampling pattern for a prescribed sparsity level γ , which provides the percentage of discarded measurements as compared to a full sampling. Hence, γ is defined as the inverse of under-sampling factor R . In the case of non-Cartesian sampling, this factor R reads:

$$R = \frac{N \times N}{N_c \times N_s \times \frac{\Delta t}{\delta t}} = \frac{1}{\gamma} \quad (4)$$

In practice, we used $R = 2.5$ so that $\gamma = 0.4$. Using LOUPE, we can learn a gridded sampling density ρ_{lb} by jointly optimizing the acquisition and reconstruction frameworks in the Cartesian domain. In [16], the authors used conventional U-Net [22] for carrying out reconstructions. In contrast here, we integrate LOUPE's acquisition network with a modular cross-domain neural network called XPDNET [13], [23] which stood second in the 2020 fastMRI brain reconstruction challenge. Hence, we jointly optimize for the sampling distribution ρ_{lb} and the reconstruction network. In regards to the LOUPE model, we initialize the *sigmoid sample slope* $s = 20$ and trained this network for 100 epochs over all the training set ($n = 4469$ MR images) and probed for the target sampling densities. We ensured that there was no leaking of the k-space data into the reconstruction network by checking the resulting binary sampling masks (see [16]).

D. Retrospective Studies

With different target sampling distributions as input, we carried out an extensive retrospective study on 50 slices points from the validation data of FastMRI dataset. The k-space data was obtained by applying a forward NUFFT operator (F) to the input multi-coil brain images. We carried out reconstructions using two different methods:

1) *CS reconstruction*: First we used the synthesis formulation of self-calibrating CS image reconstruction [24] by solving for the wavelet coefficients \mathbf{z} as follows:

$$\hat{\mathbf{z}} = \underset{\mathbf{z} \in \mathbb{C}^{N \times N}}{\operatorname{argmin}} \frac{1}{2} \sum_{\ell=1}^L \|F_{\Omega} \mathbf{S}_{\ell} \Psi^* \mathbf{z} - \mathbf{y}_{\ell}\|_2^2 + \lambda \|\mathbf{z}\|_1 \quad (5)$$

where the L is the number of coils. Here the data consistency is enforced with SENSE operators $(F_{\Omega} \mathbf{S}_{\ell})_{\ell}$, where F_{Ω} is the NUFFT masked to Ω and \mathbf{S}_{ℓ} is sensitivity map for ℓ^{th} coil estimated by density compensated adjoint of the 20% of acquired k-space center (see details in [24]). $\lambda > 0$ is the regularization parameter for ℓ_1 -sparsity which was promoted in the wavelet domain Ψ . For our reconstructions, we used Symlet 8 wavelet with 4 scales for Ψ . The regularization parameter λ was grid searched between $(10^{-4}, 10^0)$ while maximizing for the reconstruction quality using structural similarity index (SSIM) in retrospective reconstruction. In order to accelerate convergence, we preconditioned the k-space using density compensation. The compensation weights were

estimated with 10 iterations of method as described in [25]. Final MR images were reconstructed as follows: $\hat{\mathbf{x}} = \Psi \hat{\mathbf{z}}$.

2) *DL reconstruction network (NC-XPDNET)*: For an extension into DL-based reconstruction, we used NC-XPDNET [19], which is a non-Cartesian extension of the XPDNET used in training for learning the sampling pattern. Particularly, the model we used was a density compensated unrolled non-Cartesian reconstruction network, which was trained on the multi-coil knee dataset with radial trajectories. While re-training the model on brain data with the new SPARKLING trajectories would provide improved results, the image quality we obtained on reconstructed images was already promising for most of our studies due to the robustness of this model to organ and trajectory discrepancy.

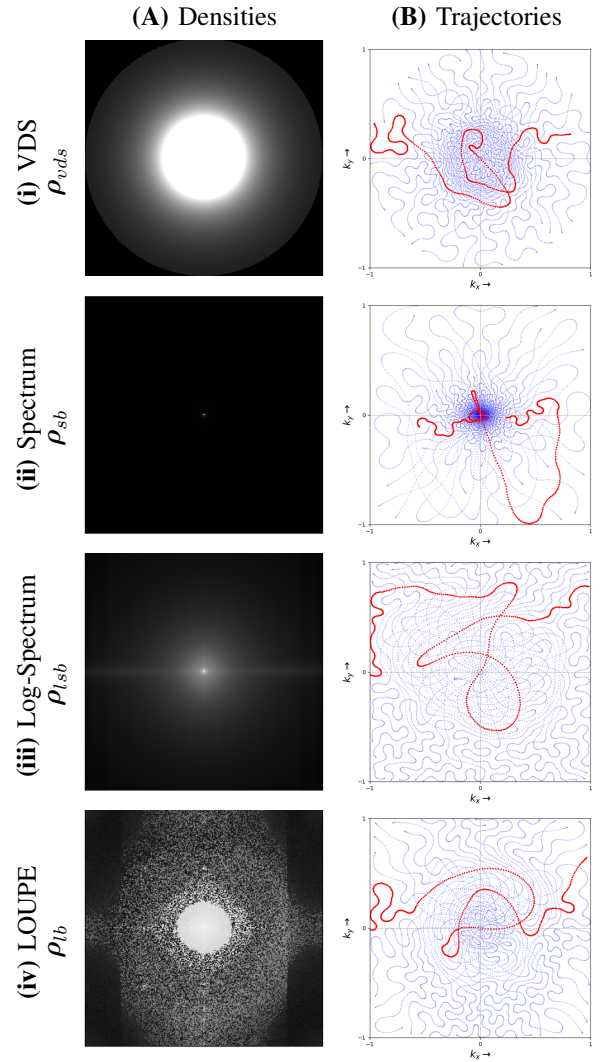


Fig. 1. (A): The target sampling densities obtained for T₁-weighted images with: (i) VDS (ρ_{vds}), a radially decaying parameterized density, with C=25% and D=2 in [10]; (ii) Average spectrum (ρ_{sb}) over the dataset based on [11]; (iii) Average logarithm of the spectrum (ρ_{lsb}) over the dataset, to flatten the density in (ii); (iv) LOUPE (ρ_{lb}) [16] coupled with XPDNET [23] reconstruction. (B): Corresponding k-space trajectories generated with $N_c = 16$ ($R = 2.5$), $N_s = 512$, $G_{\max} = 40$ mT/m and $S_{\max} = 180$ T/m/s. For illustration purpose, a single shot is colored in red.

III. RESULTS

In this section we briefly present the densities and trajectories for various methods of estimating the target sampling densities as described in Sec. II-C. Then we briefly go through the retrospective reconstruction results that we obtained.

A. Densities and trajectories

The varied target sampling densities and their respective SPARKLING trajectories are presented in Fig. 1. We see that the direct spectrum-based density ρ_{sb} is extremely dense at the center of k-space, leading to really dense sampling here in the respective trajectories. The *log-spectrum* method does indeed flatten out the density ρ_{lsb} , allowing for trajectories to explore more into higher frequencies. Finally, the LOUPE based density does oversample the center of k-space resulting in a scheme very similar to variable density sampling. However, the density ρ_{lb} from LOUPE is more grainy since the learning of this density happens on a Cartesian grid.

B. Retrospective image reconstruction studies

1) *Quantitative results:* Next, we carried out retrospective studies on 50 slices of the validation data (two imaging contrasts, namely T_1 and T_2) in fastMRI dataset for all the above trajectories generated. We computed the SSIM and peak signal-to-noise ratio (PSNR) metrics on the reconstructed MR images with a mask on the brain in order to assess image quality. The results are presented in Tab. I.

TABLE I

RETROSPECTIVE STUDY ON DIFFERENT TRAJECTORIES FOR $R = 2.5$ ON 50 SLICES. THE SSIM AND PSNR SCORES ARE INDICATED AND THE BEST CASES ACROSS ROWS ARE MARKED IN BOLD FONT.

Target Density	CS-Based Recon				NC-XPDNET			
	T_1		T_2		T_1		T_2	
	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR
ρ_{vds}	0.936	31.824	0.915	29.652	0.909	32.653	0.925	30.437
ρ_{sb}	0.920	28.850	0.919	28.351	0.937	31.554	0.929	29.860
ρ_{lsb}	0.902	29.838	0.908	28.322	0.907	30.931	0.919	29.772
ρ_{lb}	0.938	32.667	0.915	30.247	0.920	34.240	0.929	31.588

Firstly, we note that all methods perform pretty decently as long as the sampling density has been optimized. However, we see that the SPARKLING trajectories with ρ_{lb} densities consistently perform well throughout with SSIMs always close to 0.92. Also, this method has highest PSNR among all the methods. This confirms our hypothesis that a method which is both data and reconstruction aware would perform well in learning the target densities. With respect to SSIM, ρ_{sb} is sometimes slightly better than ρ_{lb} . Another aspect is that for T_1 -weighted imaging, CS based reconstruction outperforms NC-XPDNET reconstruction. A likely explanation for this is that the network was trained with the knee fastMRI dataset and under-sampled radial trajectories. Therefore we could expect improved results if NC-XPDNET was trained with brain data and SPARKLING trajectories, however this has not been done so far given the required time for the training stage (approx. 3 days). Finally, we noticed that ρ_{vds} performs similarly to

ρ_{lb} with respect to SSIM in most of cases. This might be due to the properties of k-space content in brain imaging, which is radially symmetric. Hence optimizing for a radially decaying density gives similar performances to LOUPE-based methods. However, we need to note that this optimization of parameterized density is very computationally intensive involving trajectory generation and retrospective reconstruction to understand which parameter affects the most image quality.

2) *Qualitative results:* For visual inspection, we present the results of image reconstruction from data undersampled using SPARKLING trajectories generated for various target densities in Fig. 2 (T_1 -weighted images) and Fig. 3 (T_2 -weighted images). For the sake of space, we only report the best reconstruction results, namely CS-based for T_1 -w and NC-XPDNET-based for T_2 -w contrasts. For T_1 -weighted contrast, ρ_{vds} and ρ_{lb} give best while blurry results. The blurriness could be due to the fact that the grid search for the value of λ was carried out while maximizing for SSIM. It is actually known that the SSIM metric is relatively insensitive to the degree of blurriness. Importantly, we can see in Fig. 2 that the density ρ_{lb} better retains the contrast with respect to the reference image. On the contrary, for T_2 -weighted contrast, ρ_{lb} outperforms the other densities as reflected both visually in Fig. 3 and quantitatively (see metrics in Tab. I).

IV. CONCLUSIONS

In this study, we addressed the main drawback of SPARKLING algorithm, namely the need for a good target sampling density as an input parameter. We setup four different methods to generate optimized target sampling densities and design SPARKLING trajectories accordingly. We showed that the LOUPE-based approach is the most promising as it provides consistent results across contrasts. However, a current limitation of this work is that there remains some split between the acquisition and reconstruction models in a fully non-Cartesian setting. Under the current study, the sampling density was jointly optimized with a Cartesian DL reconstruction network. Then, non-Cartesian SPARKLING trajectories were generated and retrospective validation was performed using a non-Cartesian DL network. There is thus still a gap between the training and validation stage in this pipeline. However, in spite of this limitation, we obtained promising results in this study. In terms of perspective, we plan to work on a joint network between NC-XPDNET and SPARKLING to efficiently learn the k-space trajectories in a data-driven manner, under the MR Hardware constraints.

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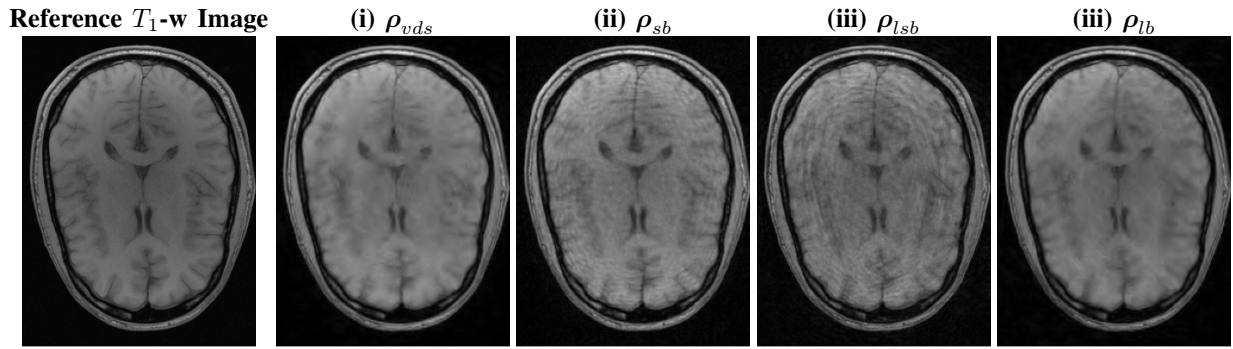


Fig. 2. CS-based image reconstruction for retrospective T_1 -w imaging with slice 6 in *file_brain_AXT1_201_6002725.h5* from validation data in fast MRI dataset for different target sampling densities.

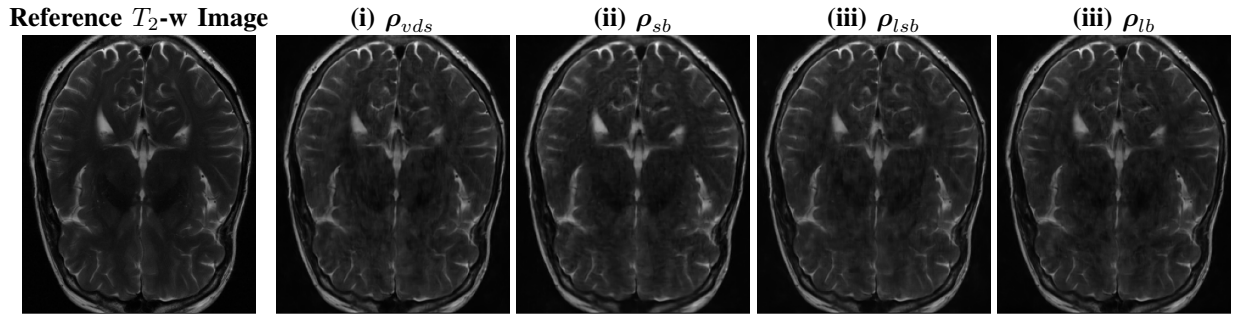


Fig. 3. NC-XPDNET-based image reconstruction for retrospective T_2 -w imaging with slice 5 in *file_brain_AXT2_200_2000019.h5* from validation data in fast MRI dataset for different target sampling densities.

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