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► **To cite this version:**

Long Li, Deremble Bruno, Noé Lahaye, Etienne Mémin. Stochastic modeling of the oceanic mesoscale eddies. STUOD 2021 - 6th Sandbox Stochastic Transport in Upper Ocean Dynamics Workshop, Feb 2021, London, United Kingdom. pp.1-24. hal-03140513

**HAL Id: hal-03140513**

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Submitted on 15 Feb 2021

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# Stochastic modeling of the oceanic mesoscale eddies

The 6th STUDO Sandbox Workshop — 12 February 2021

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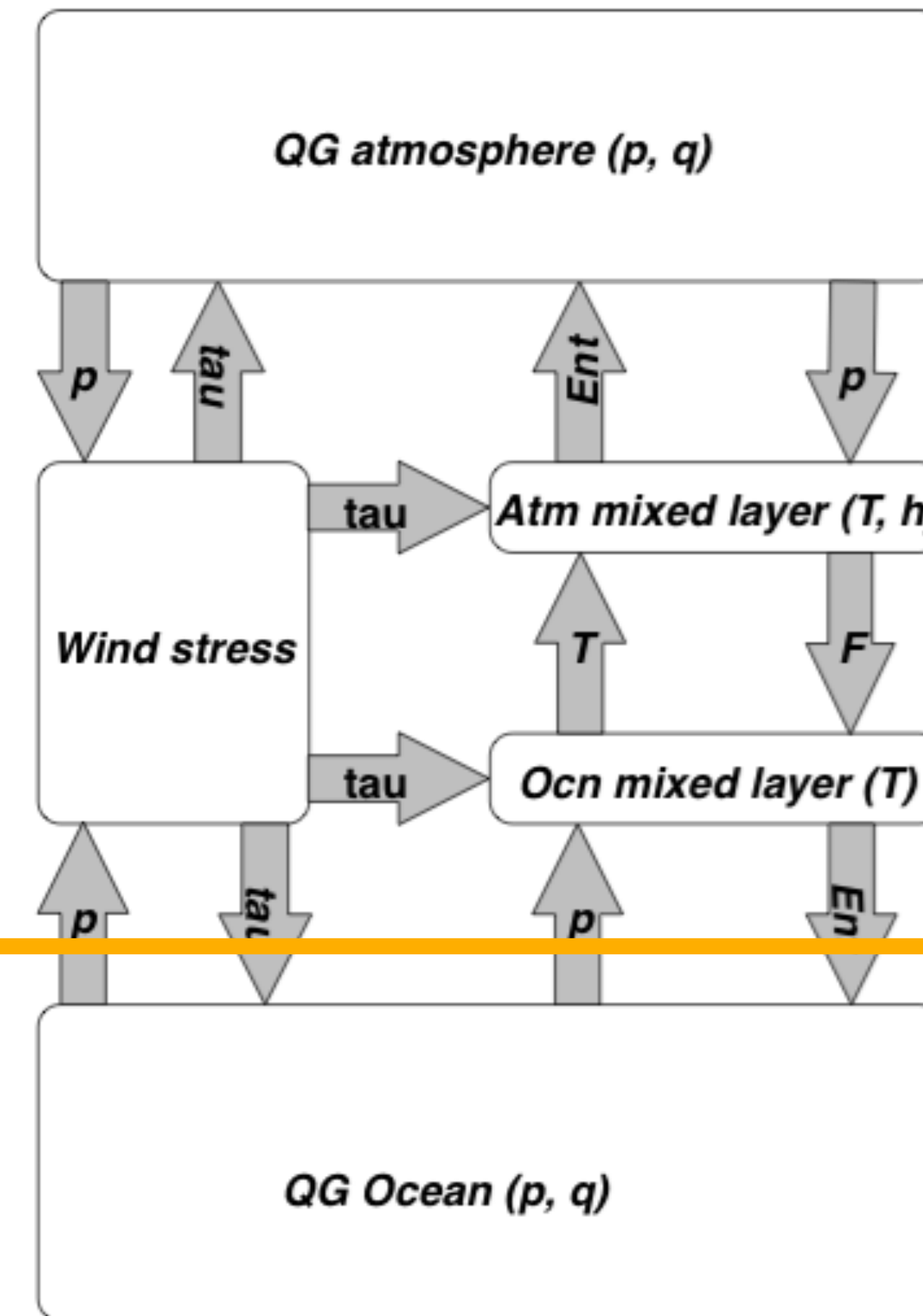
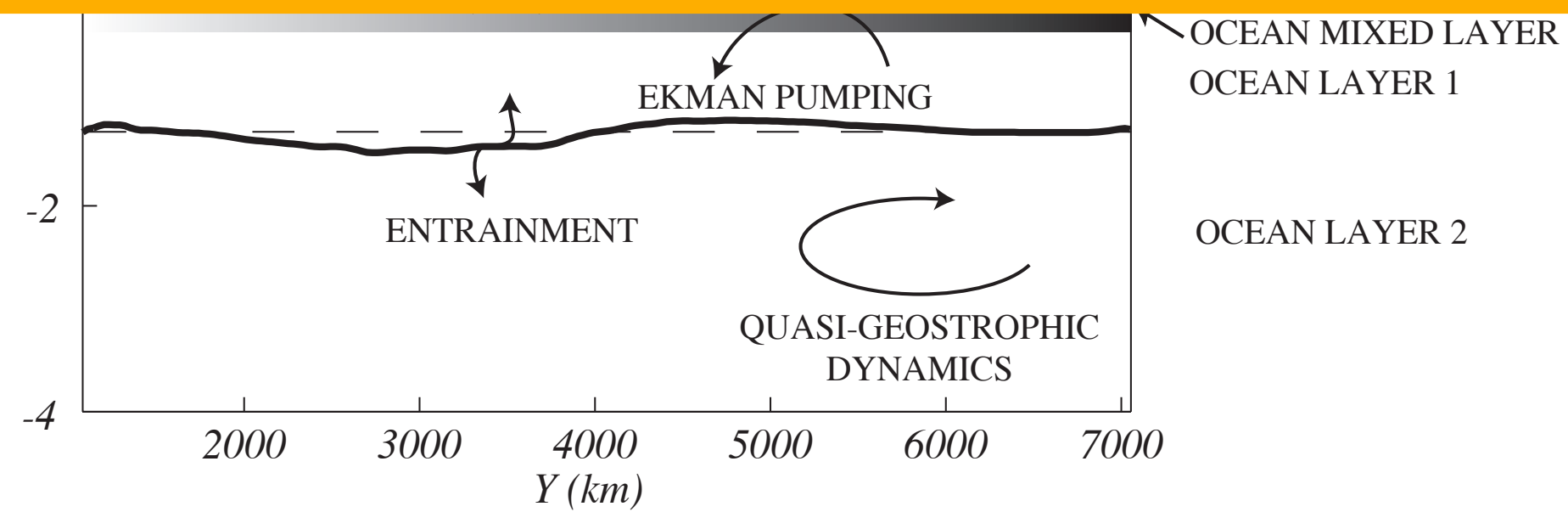
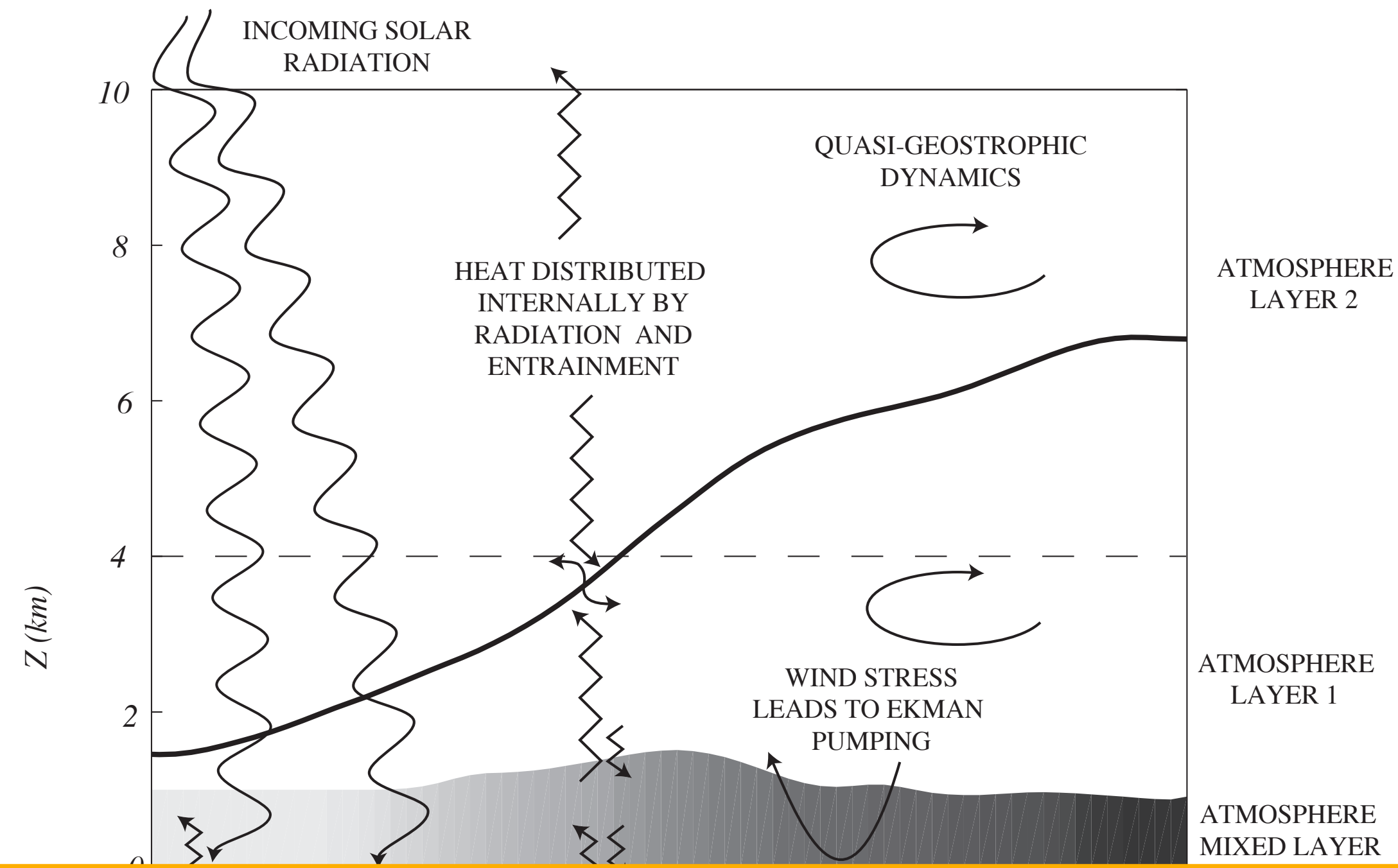


# Motivations

- I. Can LU model better represent the mesoscale eddies effect on the large-scale circulation?
- II. Can LU model improve the internal variability of coarse-resolution ocean models?



# Dynamical core



Quasi-Geostrophic Coupled Model (Q-GCM) [<http://q-gcm.org>]



# Deterministic QG equations

Exp.2

- Evolution of  $k$ -th layer potential vorticity (PV)

$$\partial_t q_k = - \nabla \cdot (\mathbf{u}_k q_k) + \frac{A_2}{f_0} \nabla^4 p_k - \frac{A_4}{f_0} \nabla^6 p_k - \frac{f_0}{H_k} (w_k - w_{k-1})$$

- Update  $k$ -th layer pressure, height and velocity

$$q_k = \frac{1}{f_0} \nabla^4 p_k + \beta(y - y_0) + \frac{f_0}{H_k} (\eta_k - \eta_{k-1})$$

$$\eta_k = \frac{p_{k+1} - p_k}{g'_k}, \quad g'_k = \frac{g(\rho_{k+1} - \rho_k)}{\rho_0}$$

$$\mathbf{u}_k = \frac{1}{f_0} \nabla^\perp p_k$$

Exp.1

- Evolution of mixed layer temperature (or SST)

$$\mathbf{u}_m = \mathbf{u}_1 + \frac{\boldsymbol{\tau}^\perp}{f_0 H_m}$$

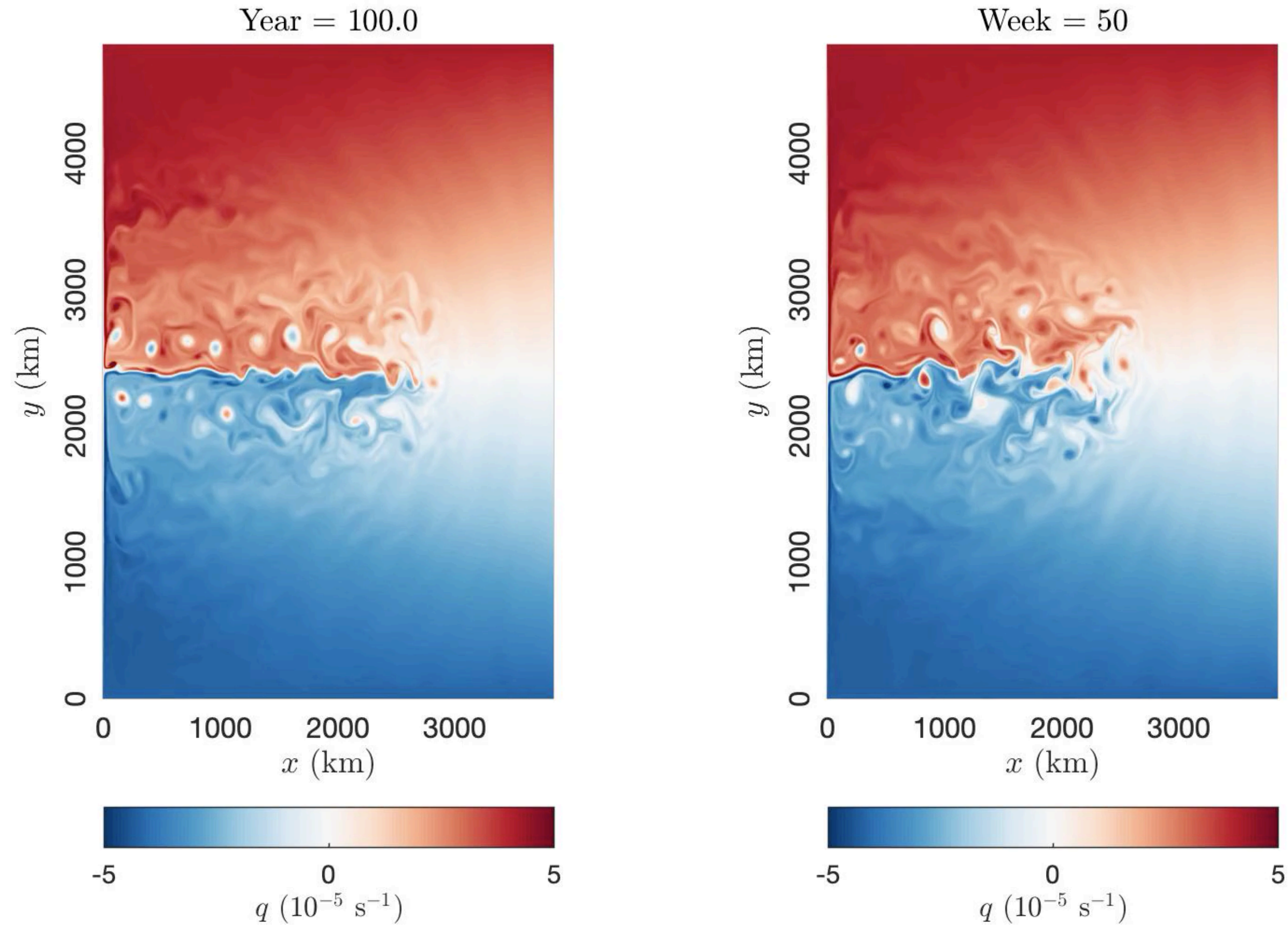
$$\partial_t T_m = - \nabla \cdot (\mathbf{u}_m T_m) + K_2 \nabla^2 T_m - K_4 \nabla^4 T_m + w_0 \frac{T_1 + T_m}{2H_m} - F_m$$

$$w_1 = - \frac{\Delta_m T}{2\Delta_1 T} w_0$$

$$\begin{bmatrix} w_0 = \frac{1}{f_0} \nabla \times \boldsymbol{\tau} \\ w_N = \frac{\delta_{ek}}{2f_0} \nabla^2 p_N \end{bmatrix}$$



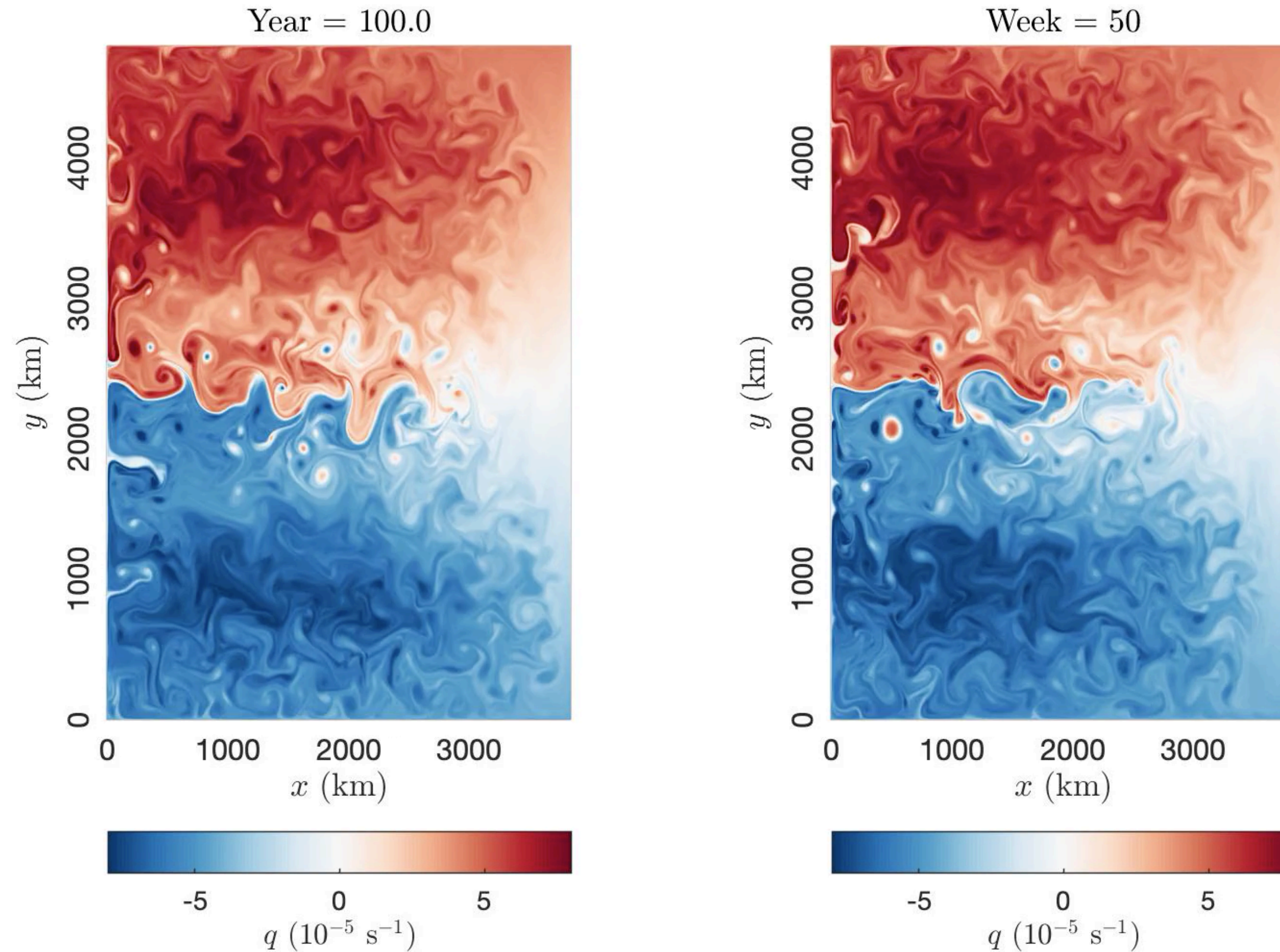
# Eddy-resolving simulations



**Upper-layer PV ( $\Delta x = 5$  km,  $L_d = [39, 22]$  km) of Exp.1 (without SST)**



# Eddy-resolving simulations



**Upper-layer PV ( $\Delta x = 5$  km,  $L_d = [39, 22]$  km) of Exp.2 (with SST)**



# Stochastic QG equations

- $k$ -th layer eddy velocity noise (m/s)  $\sigma_k \dot{B}_t$

- $k$ -th layer variance tensor (m<sup>2</sup>/s<sup>2</sup>)  $\frac{\mathbf{a}_k}{dt} = \mathbb{E} \left[ (\sigma_k \dot{B}_t - \mathbb{E}[\sigma_k \dot{B}_t]) (\sigma_k \dot{B}_t - \mathbb{E}[\sigma_k \dot{B}_t])^T \right]$

- Evolution of  $k$ -th layer PV  $\partial_t q_k = -\nabla \cdot (\mathbf{u}_k q_k) + \frac{A_2}{f_0} \nabla^4 p_k - \frac{A_4}{f_0} \nabla^6 p_k - \frac{f_0}{H_k} (w_k - w_{k-1}) + \nabla \cdot \mathbf{F}_k$

$$\mathbf{F}_k = \underbrace{-\left(\sigma_k \dot{B}_t - \frac{1}{2} \nabla \cdot \mathbf{a}_k\right) q_k}_{\text{Advection flux}} + \underbrace{\frac{1}{2} \mathbf{a}_k \nabla q_k}_{\text{Diffusion flux}} - \underbrace{\left(\mathbf{u}_k \cdot \nabla^\perp \sigma_k \dot{B}_t + \mathbf{a}_k \nabla f\right)}_{\text{Sources flux}} + \underbrace{\frac{1}{2} \sum_{i=1,2} \partial_{x_i}^\perp \mathbf{a}_k \nabla u_k^i}_{\text{Sinks flux}}$$

Advection flux

Diffusion flux

Sources flux

Sinks flux

- Evolution of SST  $\partial_t T_m = -\nabla \cdot (\mathbf{u}_m T_m) + K_2 \nabla^2 T_m - K_4 \nabla^4 T_m + w_0 \frac{T_1 + T_m}{2H_m} - F_m + \nabla \cdot \mathbf{F}_m$

$$\mathbf{F}_m = \underbrace{-\left(\sigma_1 \dot{B}_t - \frac{1}{2} \nabla \cdot \mathbf{a}_1\right) T_m}_{\text{Advection flux}} + \underbrace{\frac{1}{2} \mathbf{a}_1 \nabla T_m}_{\text{Diffusion flux}}$$



# Data-driven modeling

## (i) LU-POD

$$\mathbf{u}_k^{\text{HR}} - \overline{\mathbf{u}_k^{\text{HR}}} = \mathcal{F}\mathbf{u}_k^{\text{HR}} + (1 - \mathcal{F})\mathbf{u}_k^{\text{HR}} - \overline{\mathcal{F}\mathbf{u}_k^{\text{HR}} + (1 - \mathcal{F})\mathbf{u}_k^{\text{HR}}}$$

$$= (\mathbf{u}_k^{\text{LR}} - \overline{\mathbf{u}_k^{\text{LR}}}) + (1 - \mathcal{F})\mathbf{u}_k^{\text{HR}} - \overline{(1 - \mathcal{F})\mathbf{u}_k^{\text{HR}}}$$

POD

$$\{\phi_{k,n}\}_{n=1,\dots,N_t}$$

$$\mathbf{u}_k^* = -\mathcal{F}\overline{(1 - \mathcal{F})\mathbf{u}_k^{\text{HR}}}$$

$[\mathbf{u}_k^* \neq 0 \text{ if } \mathcal{F}^2 \neq \mathcal{F}]$

$\xi_n \sim \mathcal{N}(0, 1)$   
 $(M_0 < M_1 < N_t)$

$$\sigma_k \dot{\mathbf{B}}_t = \sum_{n=M_0}^{M_1} \phi_{k,n} \xi_n + \mathbf{u}_k^*$$

$$\frac{\mathbf{a}_k}{dt} = \sum_{n=M_0}^{M_1} \phi_{k,n} \phi_{k,n}^T$$



# Data-driven modeling

## (ii) LU-POD-P

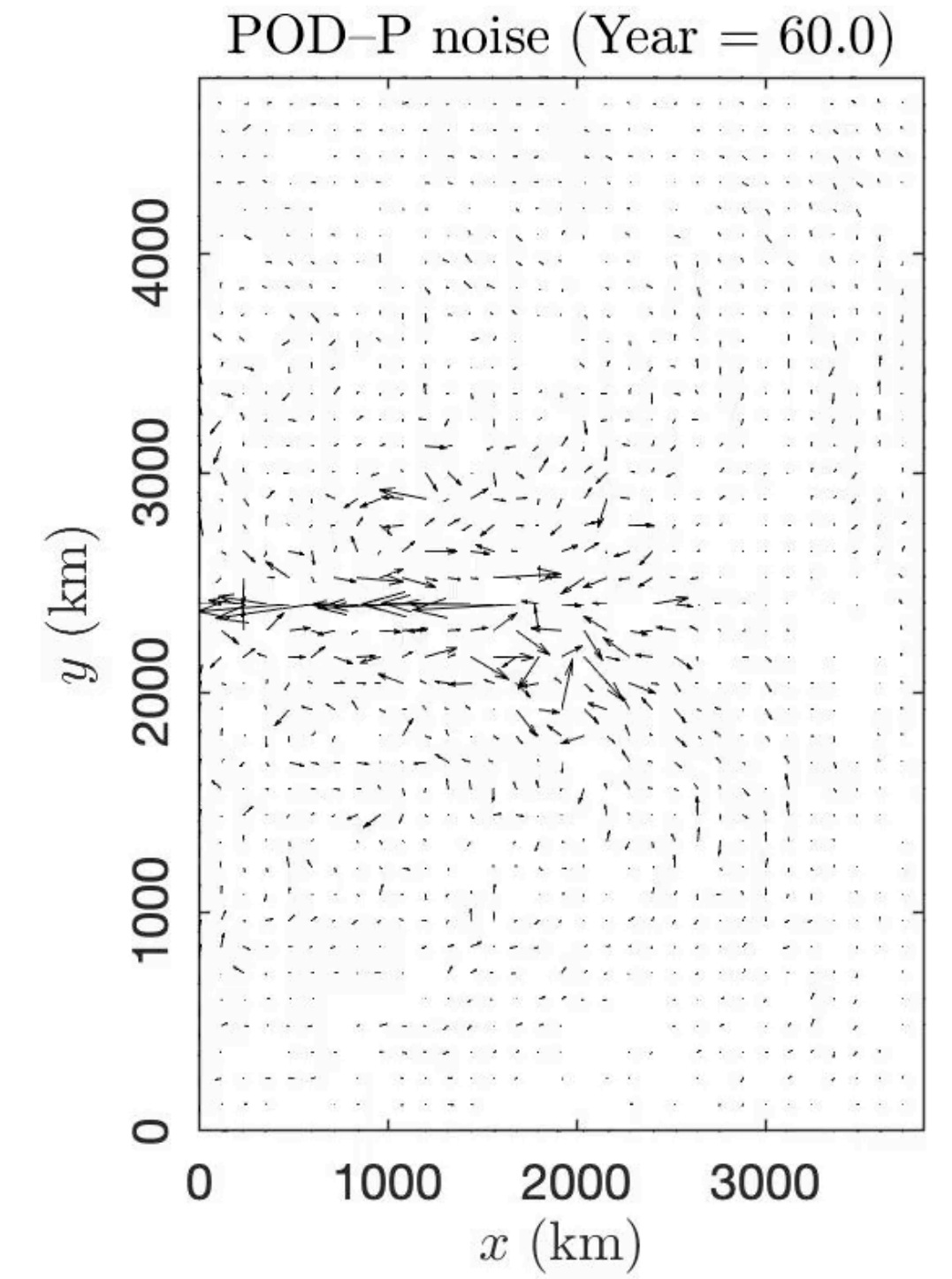
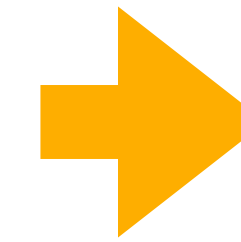
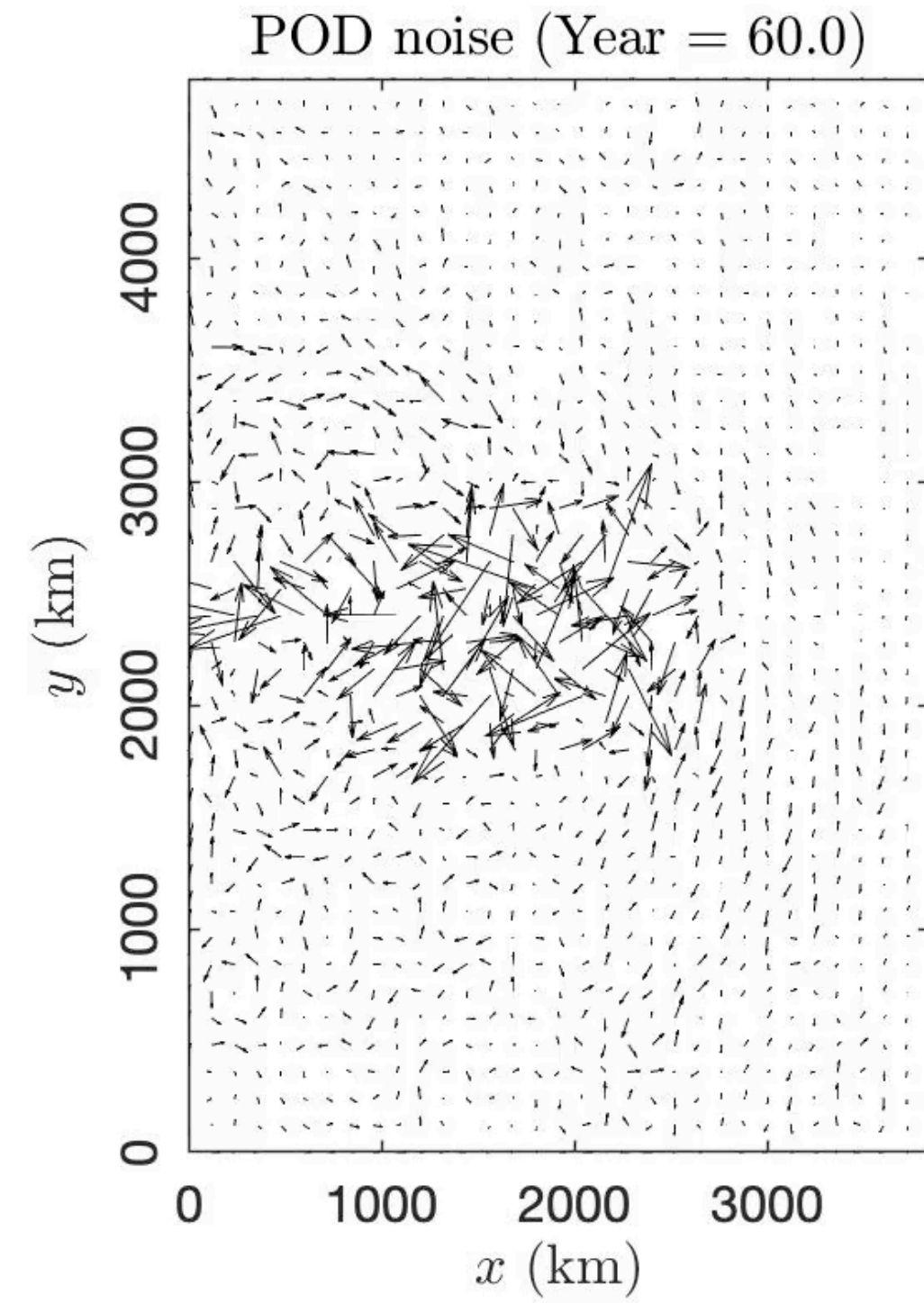
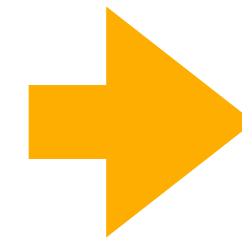
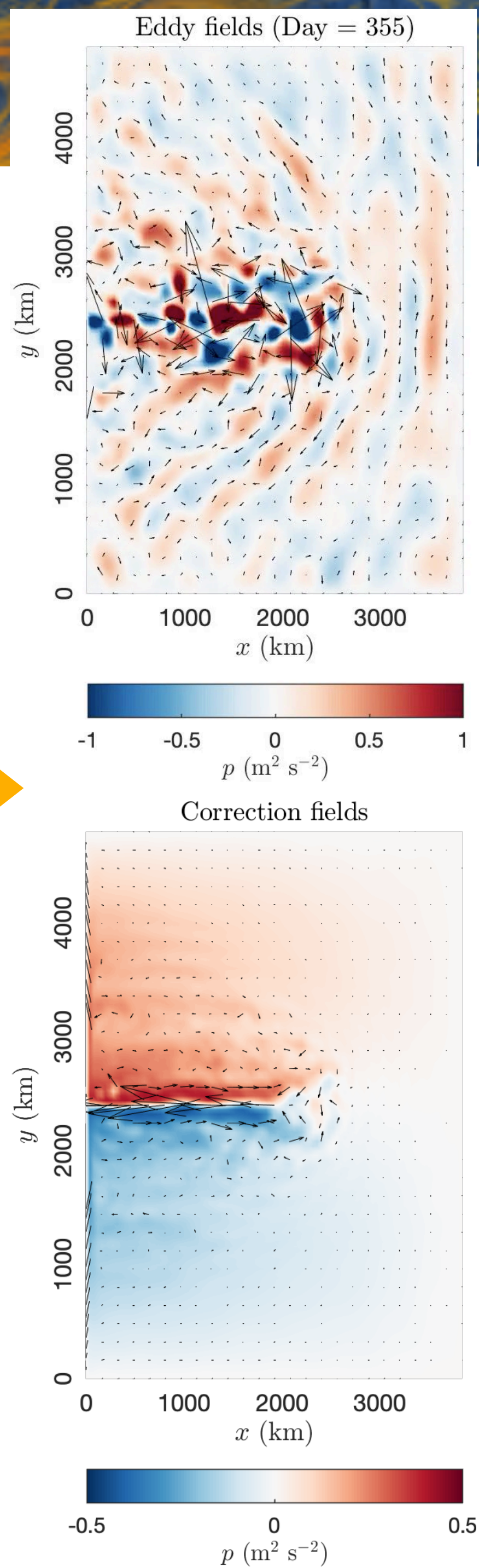
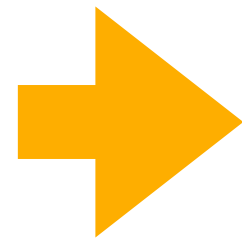
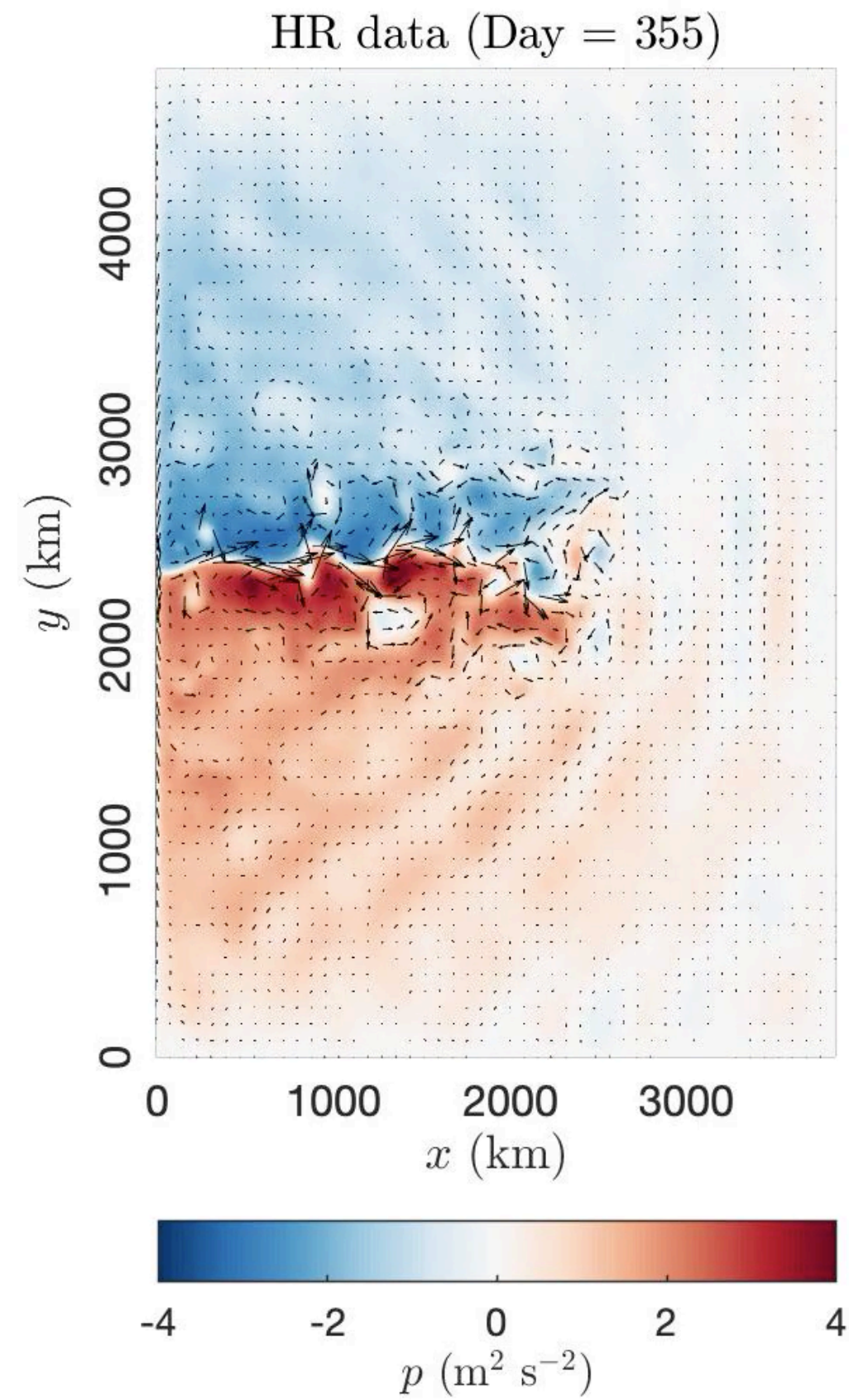
$$\mathbf{P}_k = \mathbf{I} - \frac{\nabla \theta_k (\nabla \theta_k)^T}{\|\nabla \theta_k\|^2}, \quad \theta_k = \frac{f_0}{H_k} (\eta_k - \eta_{k-1})$$

$$\tilde{\sigma}_k \dot{\mathbf{B}}_t = \mathbf{P}_k \boxed{\sigma_k \dot{\mathbf{B}}_t}, \quad \tilde{\mathbf{a}}_k = \mathbf{P}_k \mathbf{a}_k \mathbf{P}_k^T$$

LU-POD



# Illustration of noises

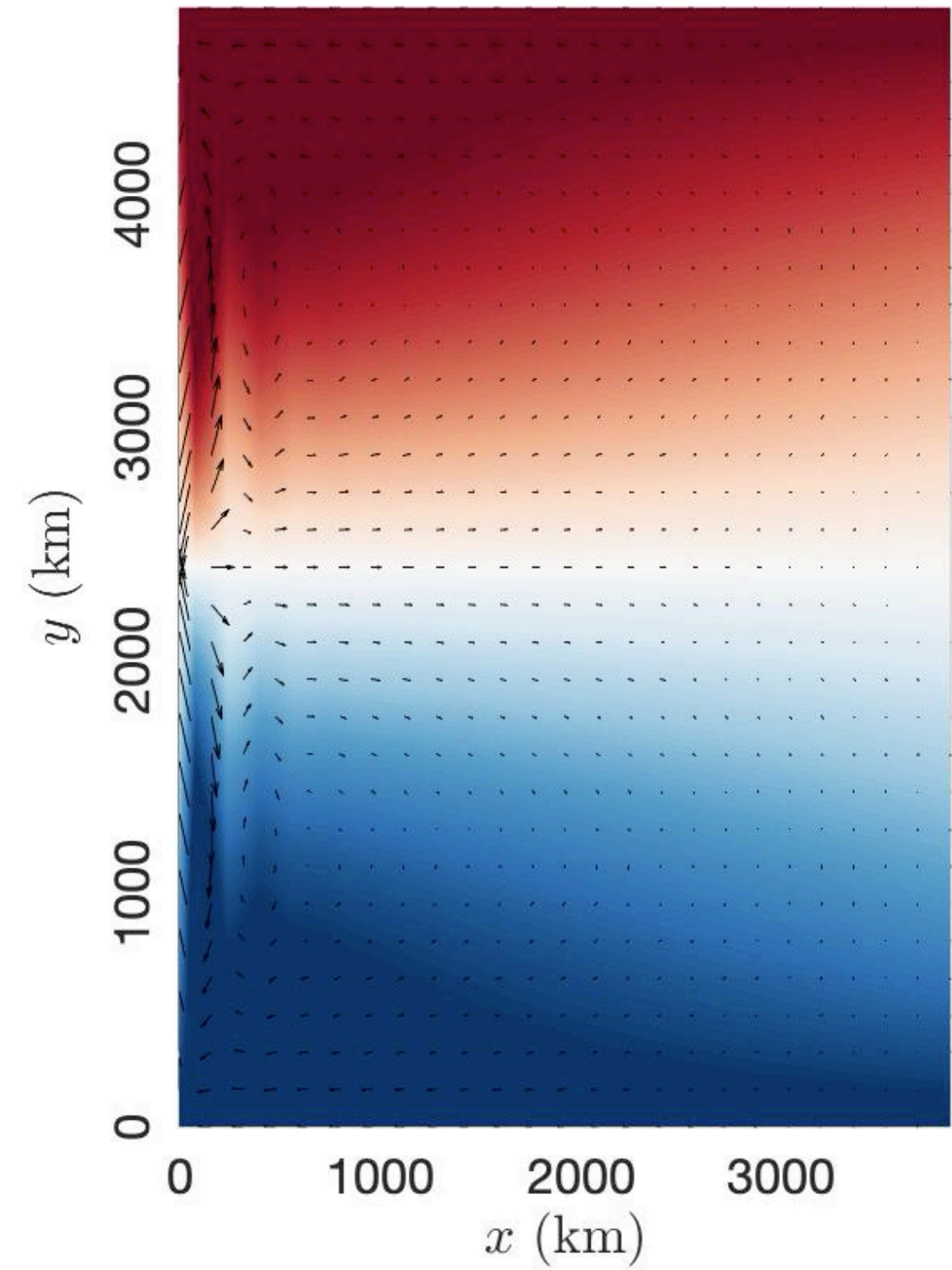




# Coarse-resolution simulations

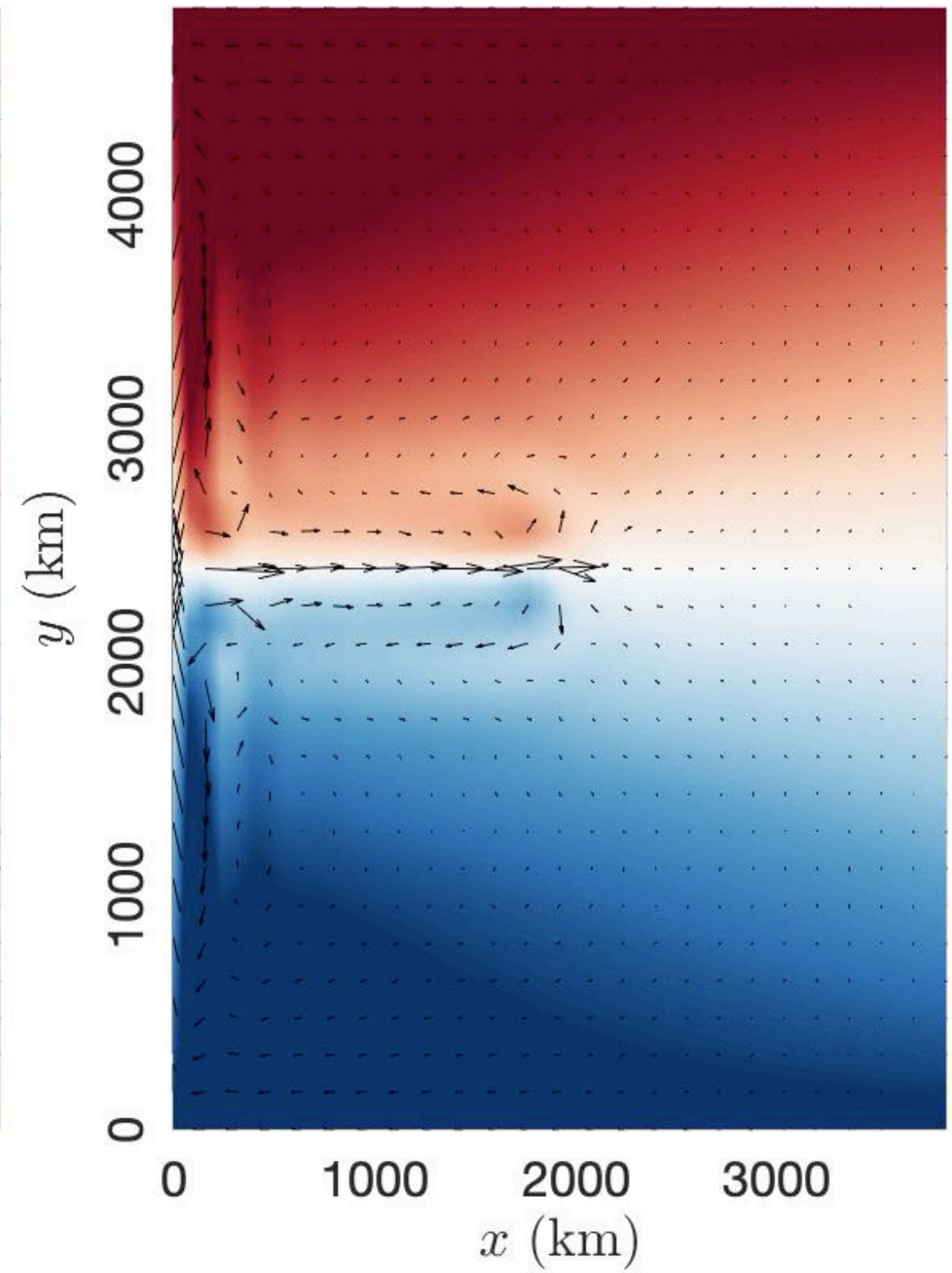
**LR**

Year = 100.0



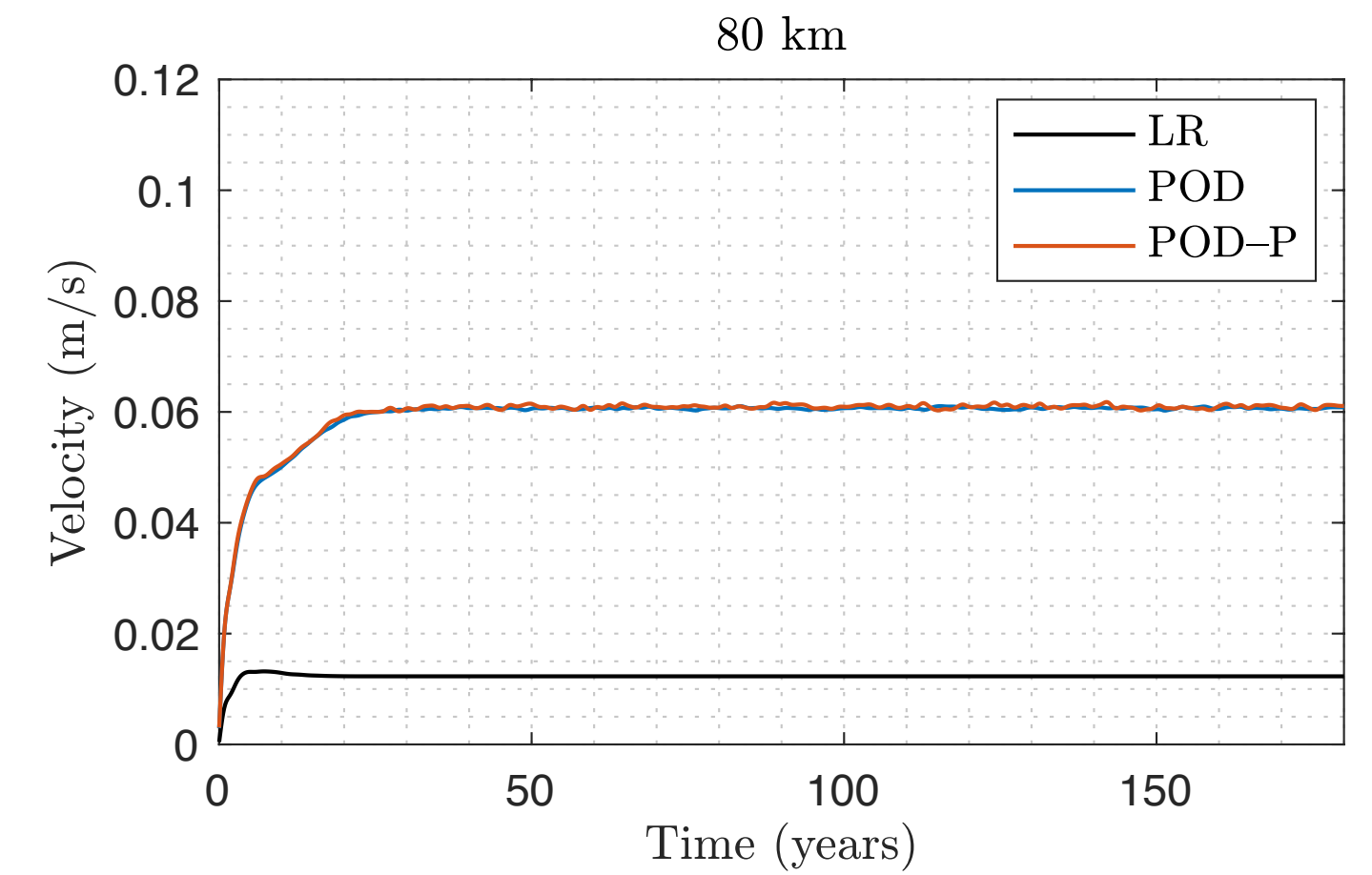
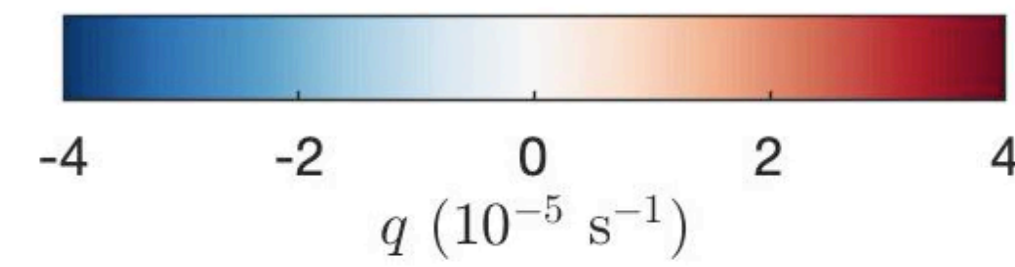
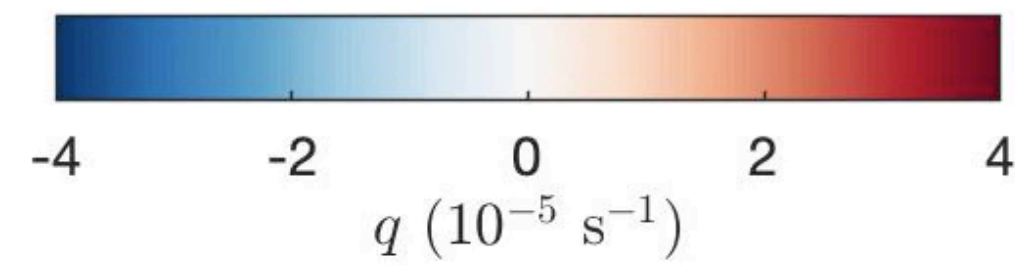
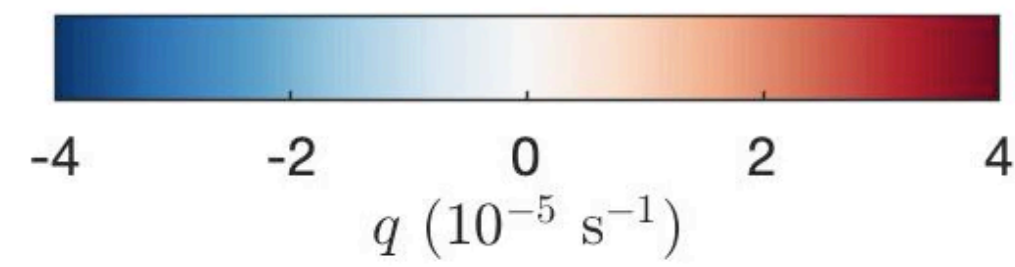
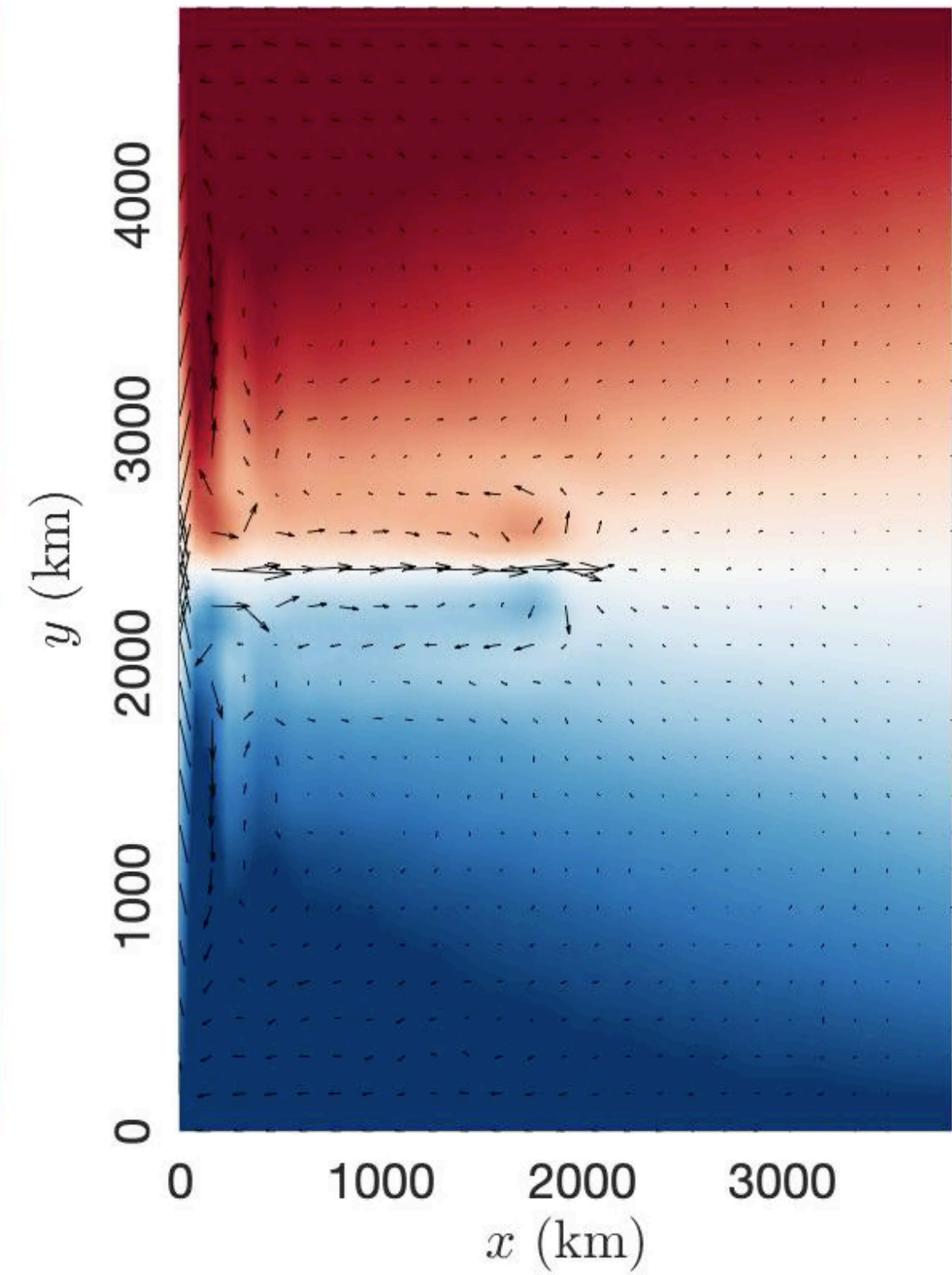
**LU-POD**

Year = 100.0



**LU-POD-P**

Year = 100.0

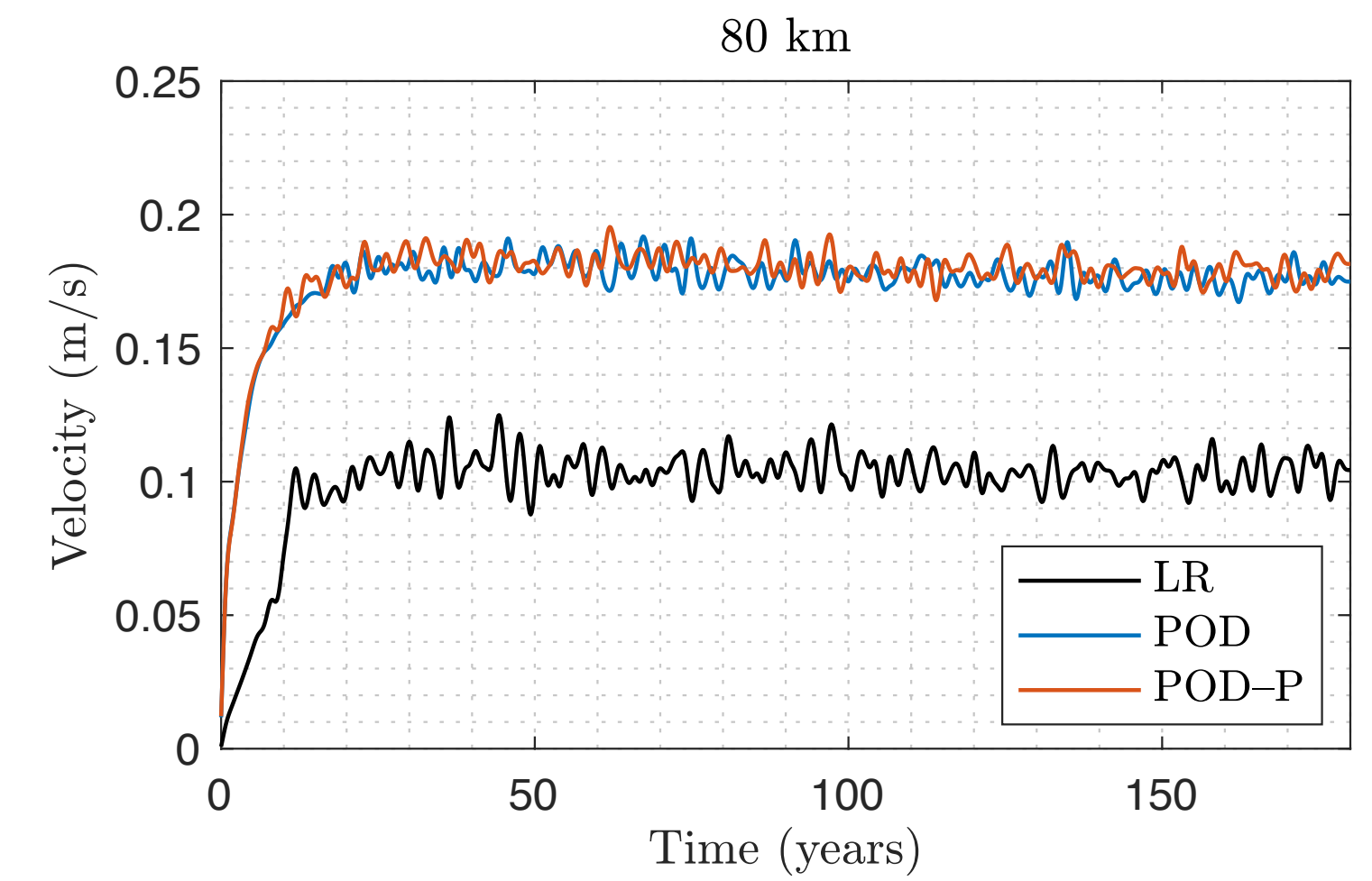
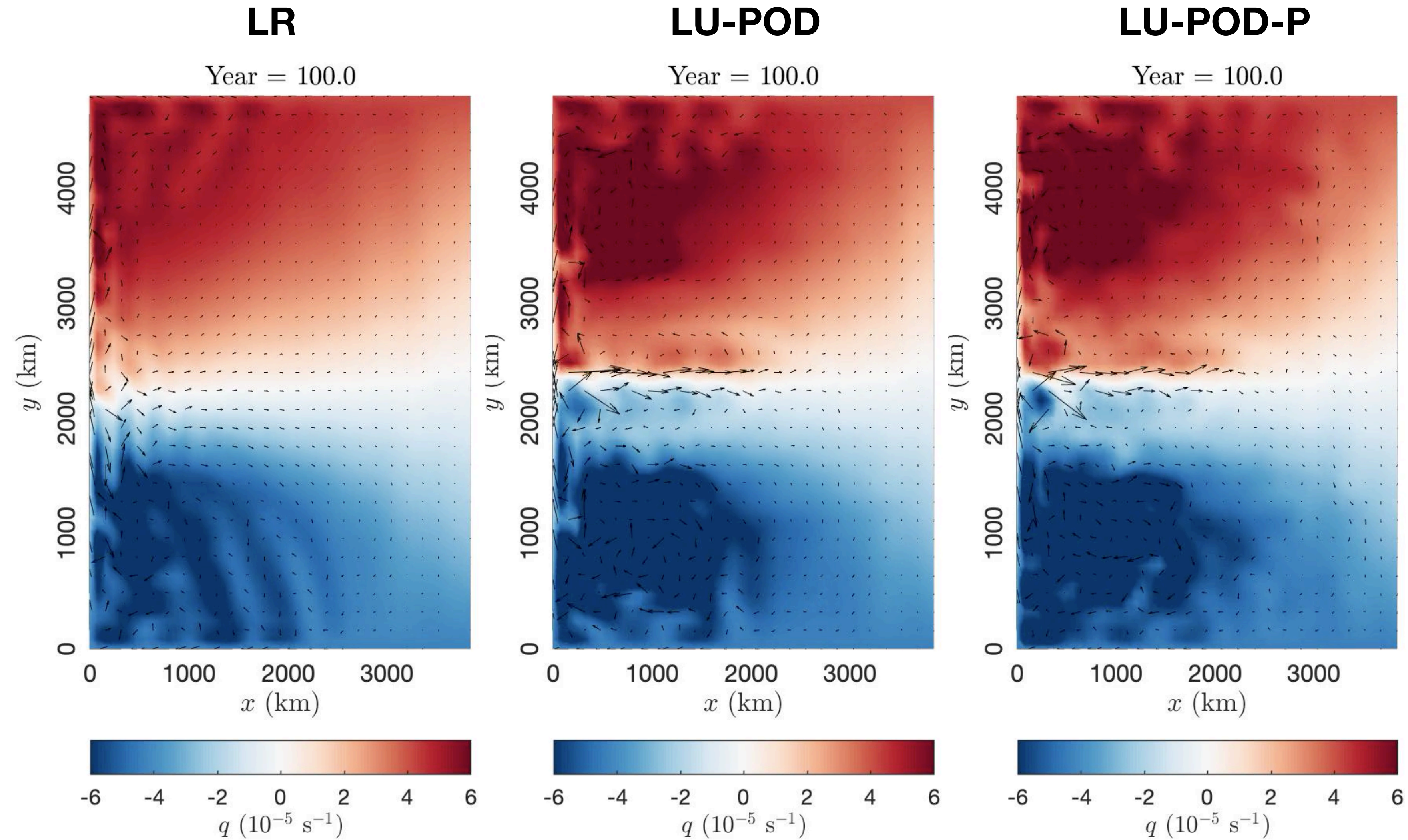


Comparison of jet velocity magnitude

**Upper-layer PV ( $\Delta x = 80$  km,  $L_d = [39, 22]$  km) of Exp.1 (without SST)**



# Coarse-resolution simulations



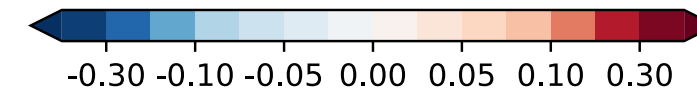
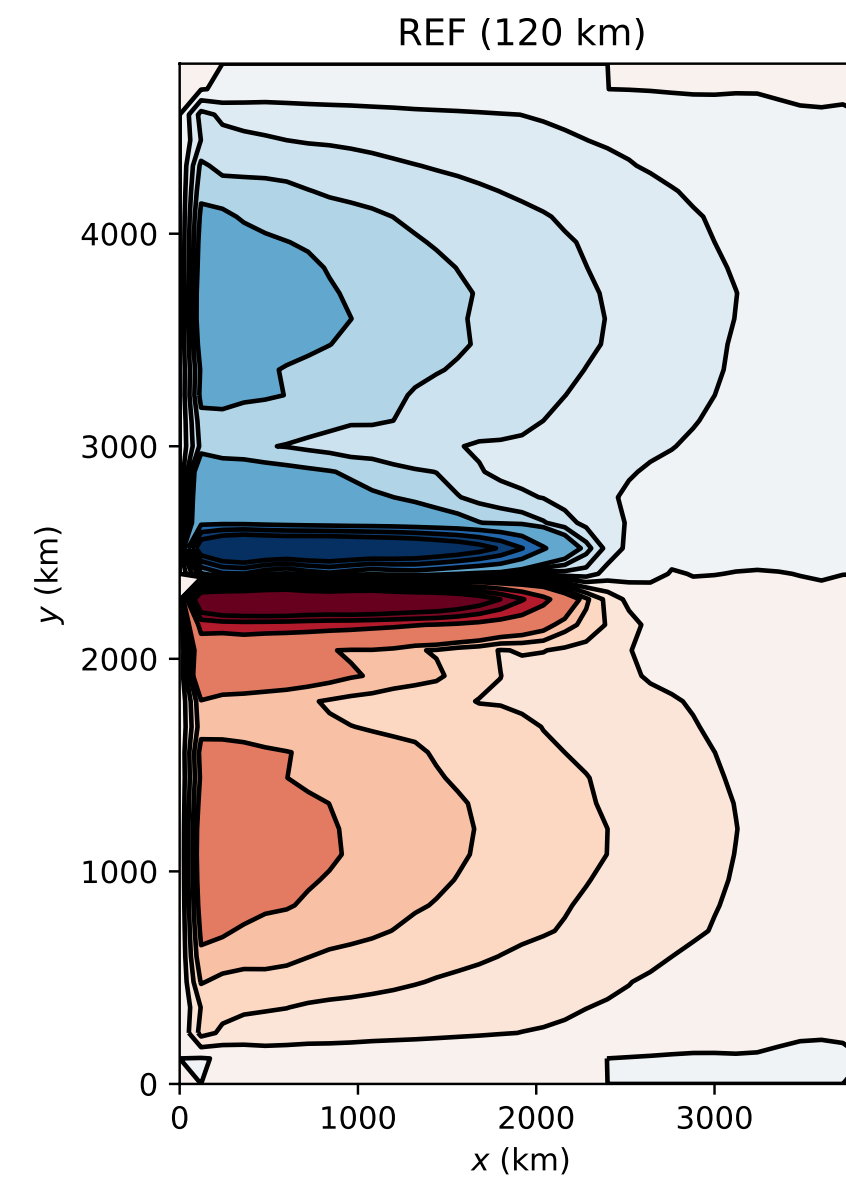
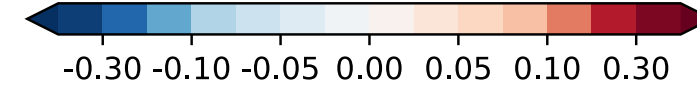
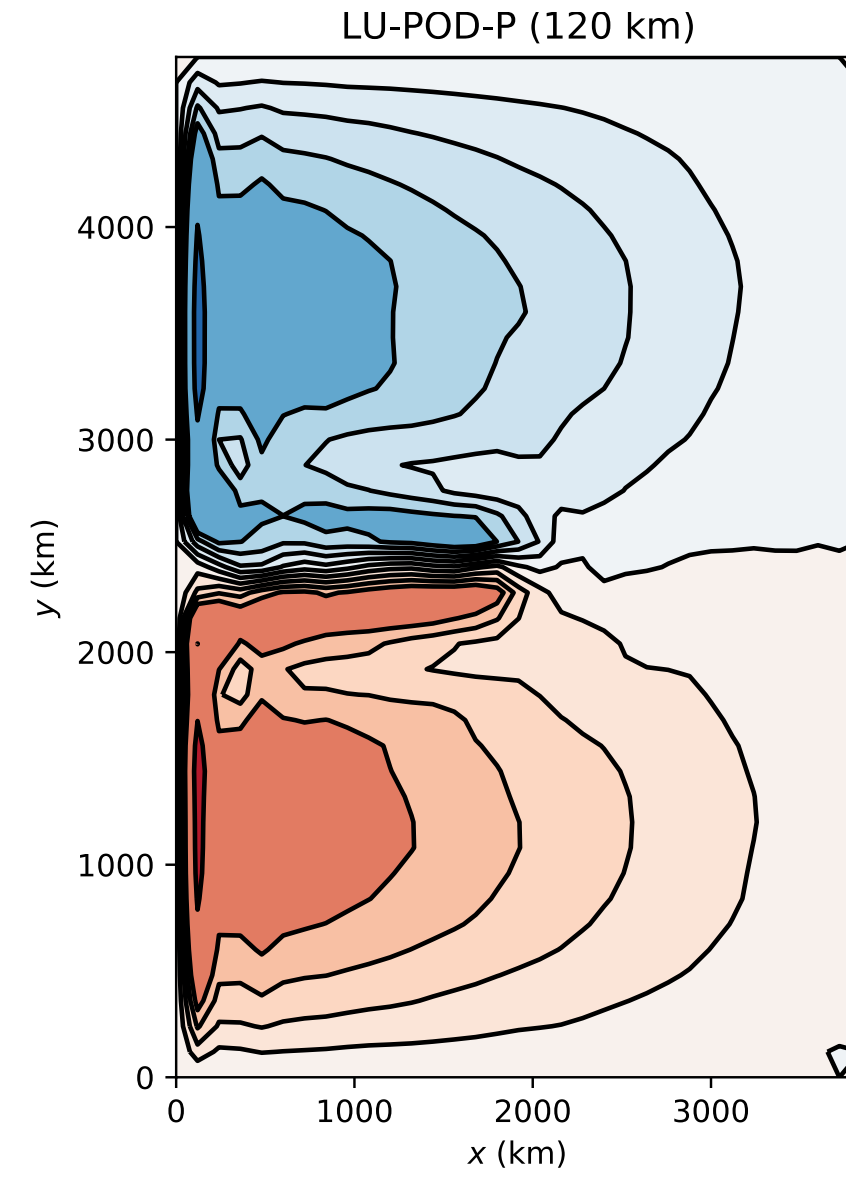
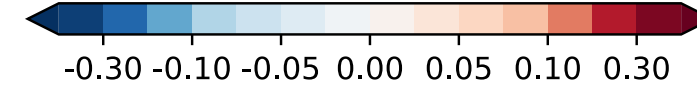
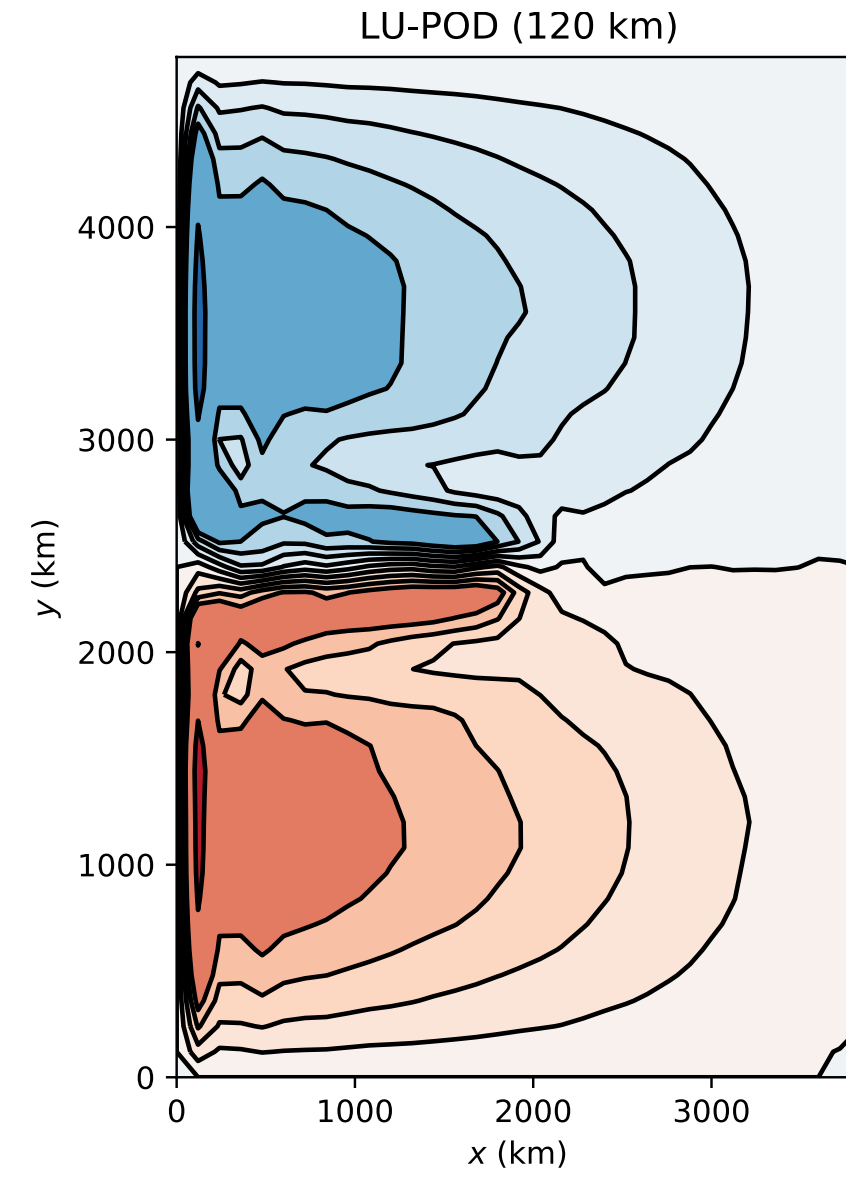
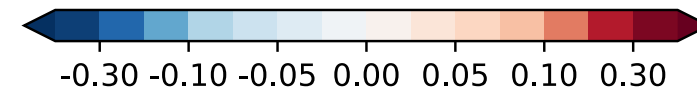
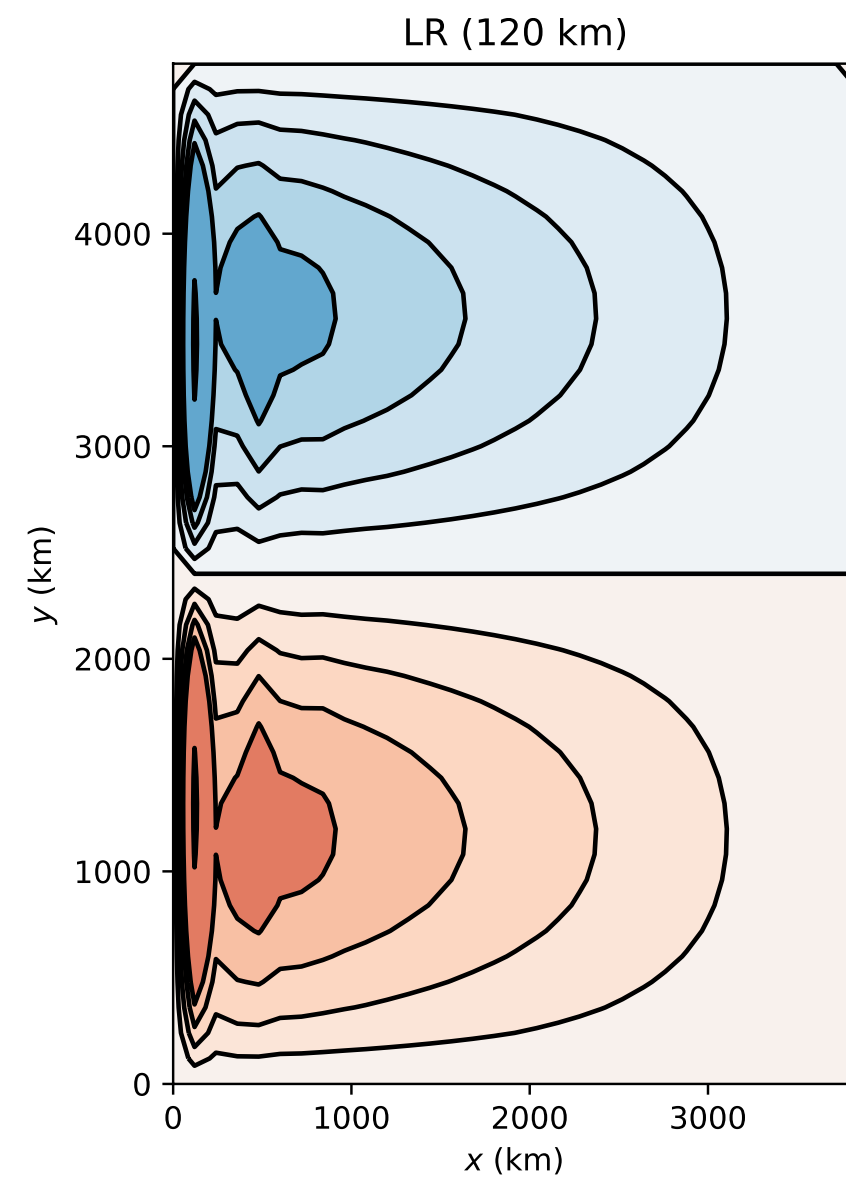
Comparison of jet velocity magnitude

**Upper-layer PV ( $\Delta x = 80 \text{ km}$ ,  $L_d = [39, 22] \text{ km}$ ) of Exp.2 (with SST)**

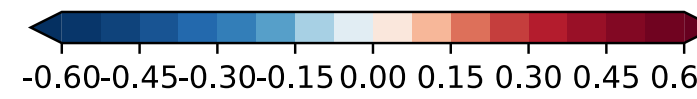
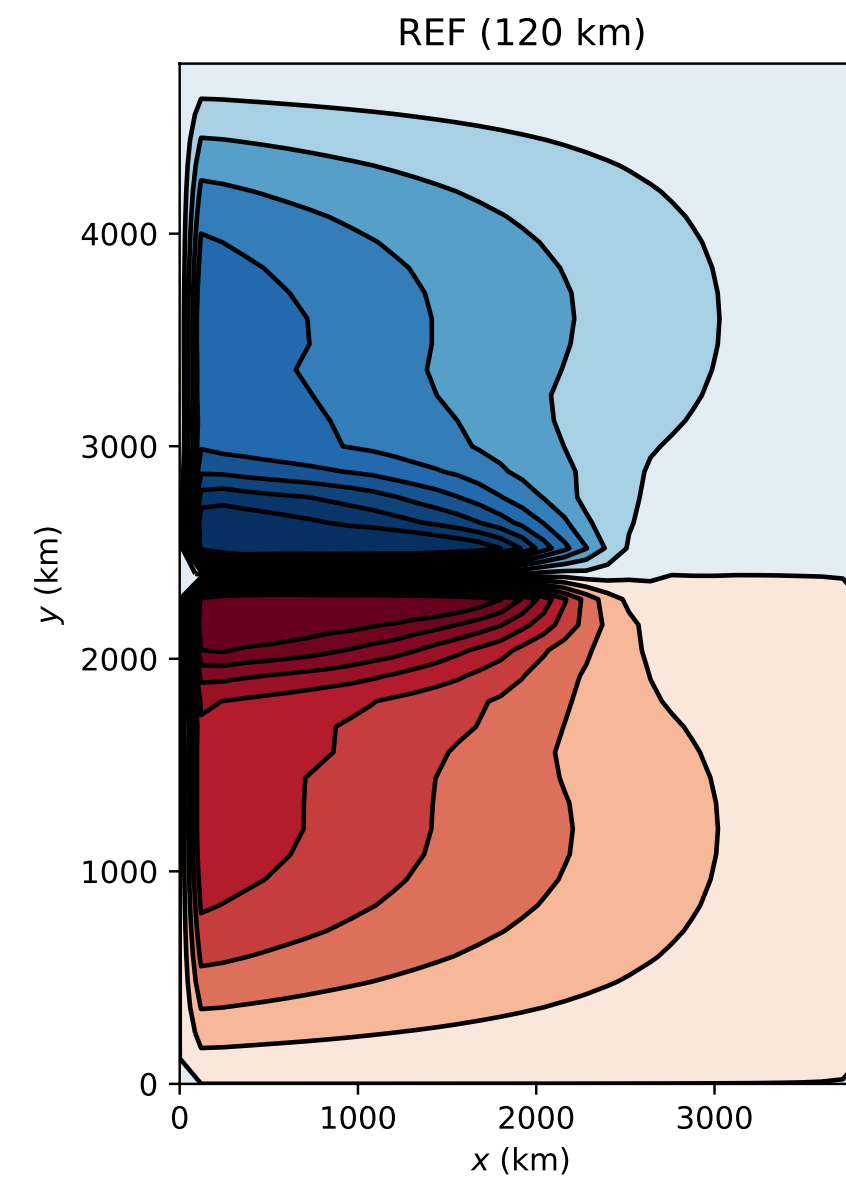
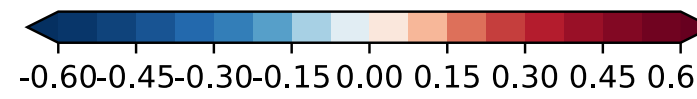
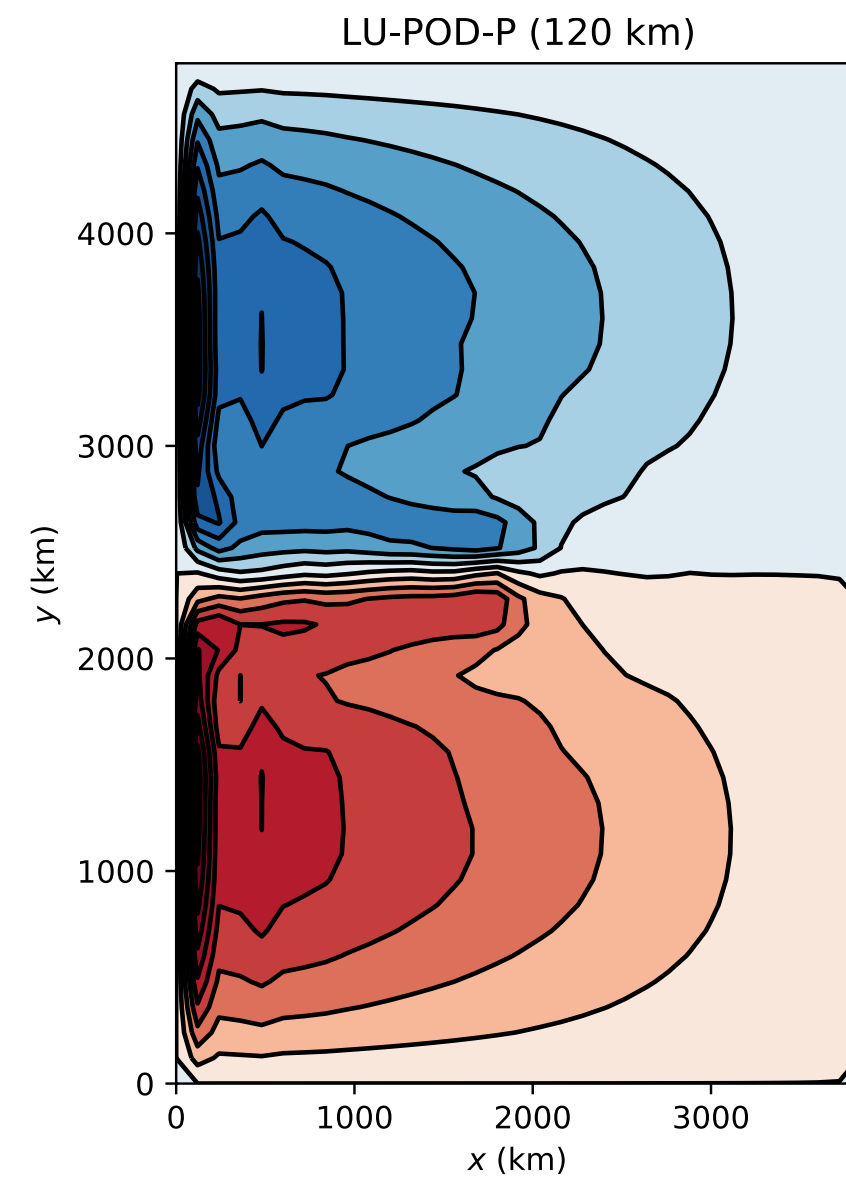
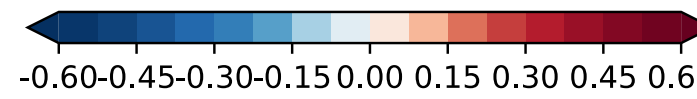
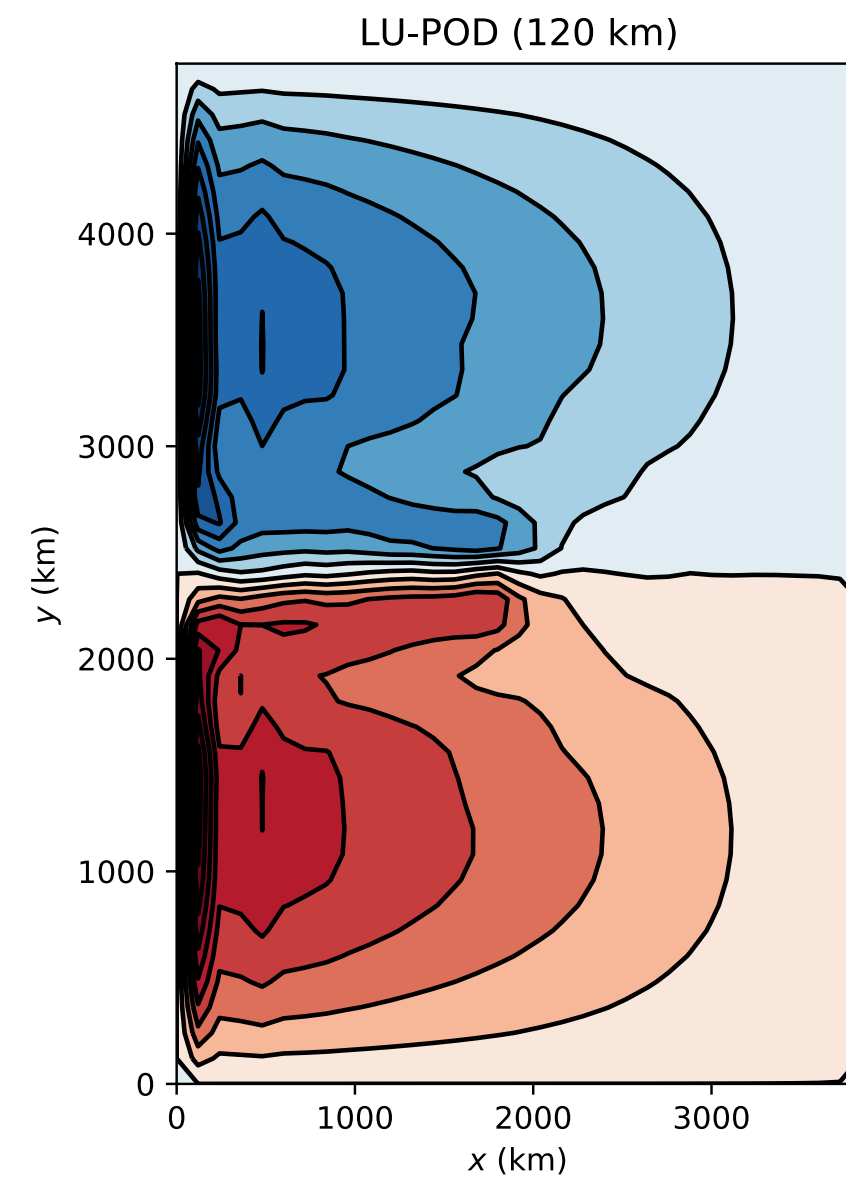
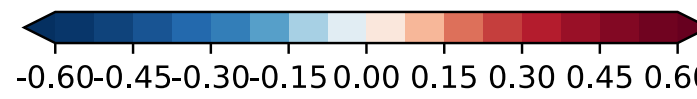
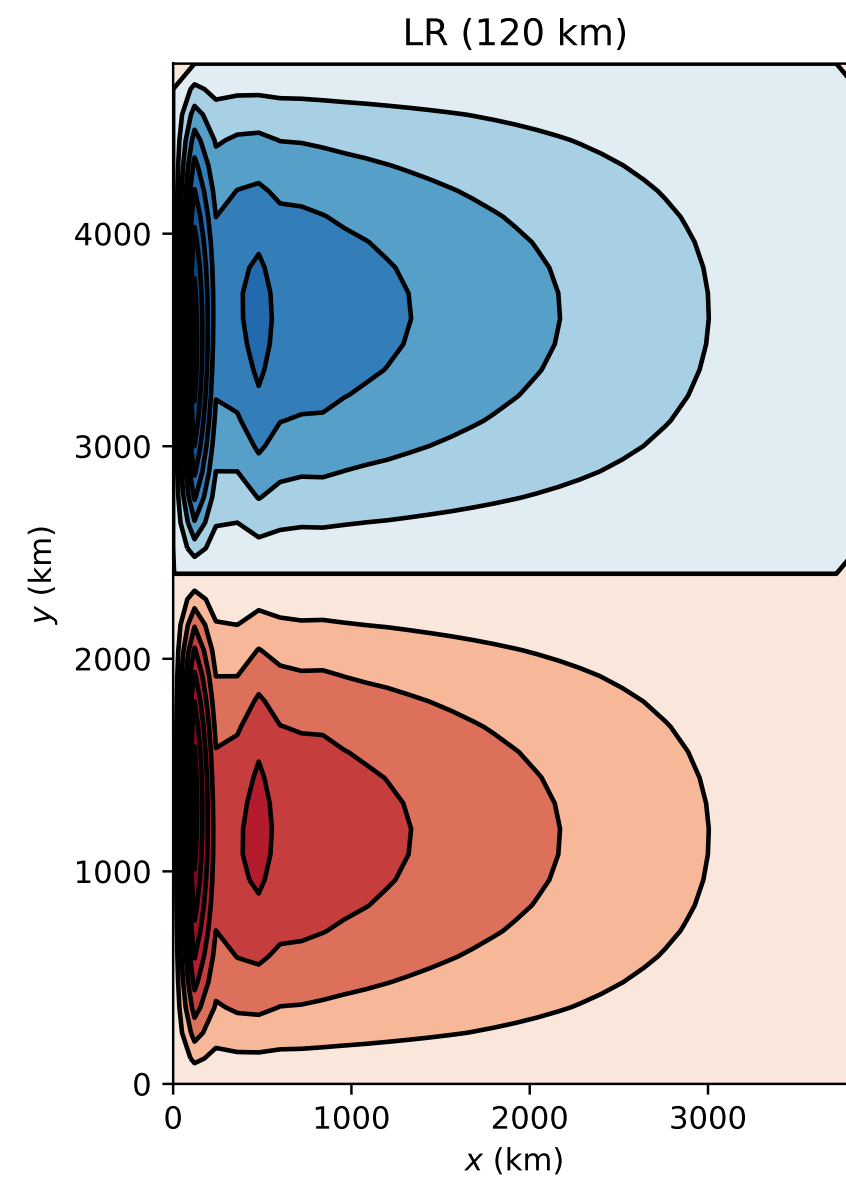


# Statistical prediction

**Barotropic mode**



**1st Baroclinic mode**

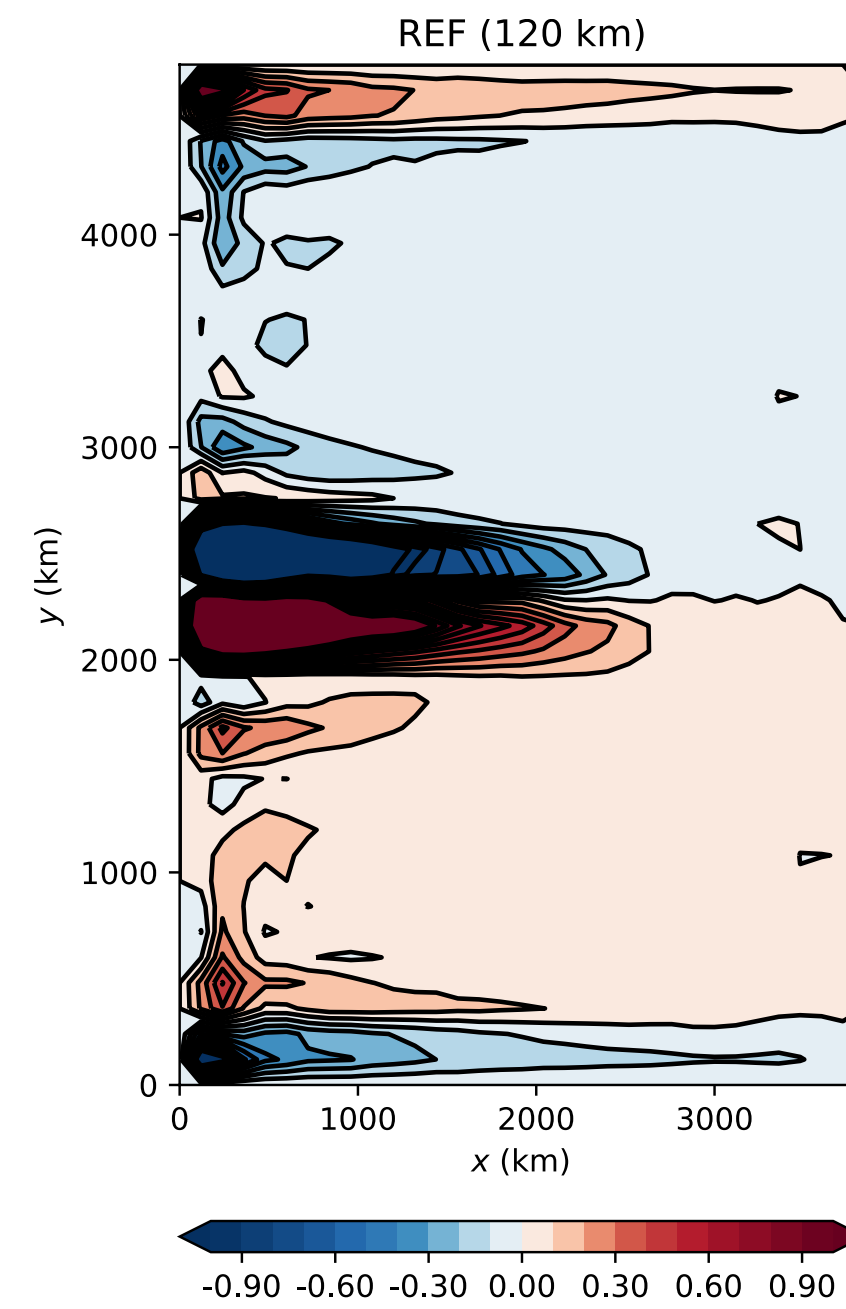
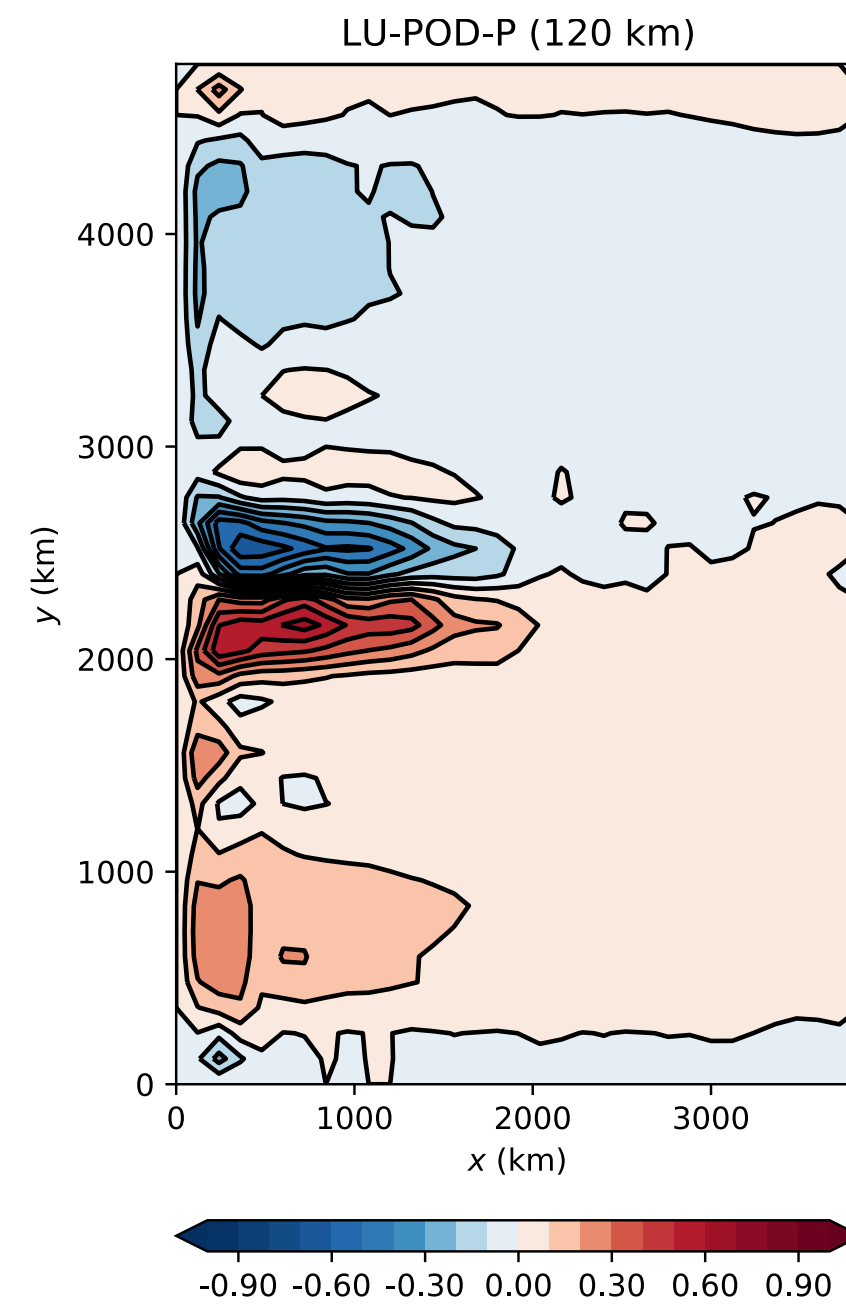
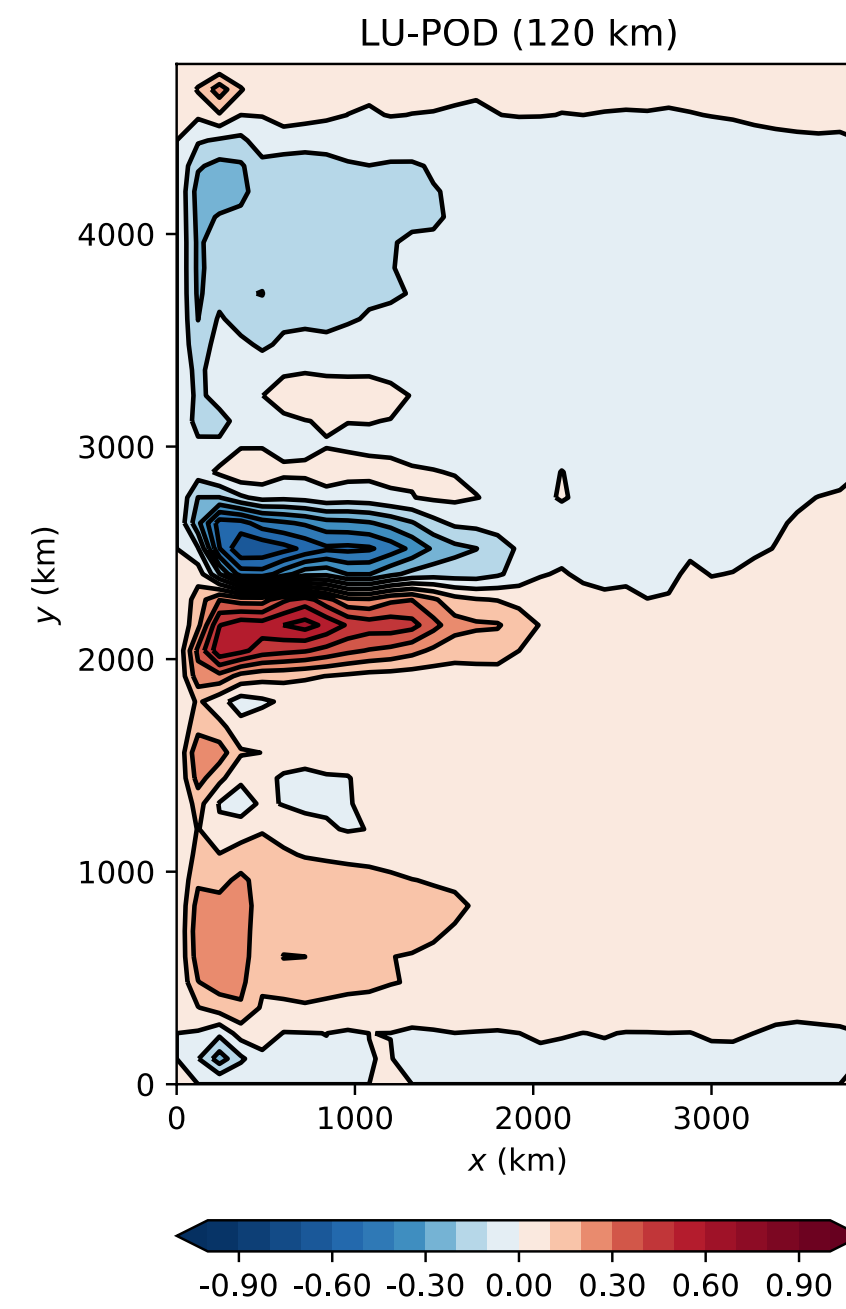
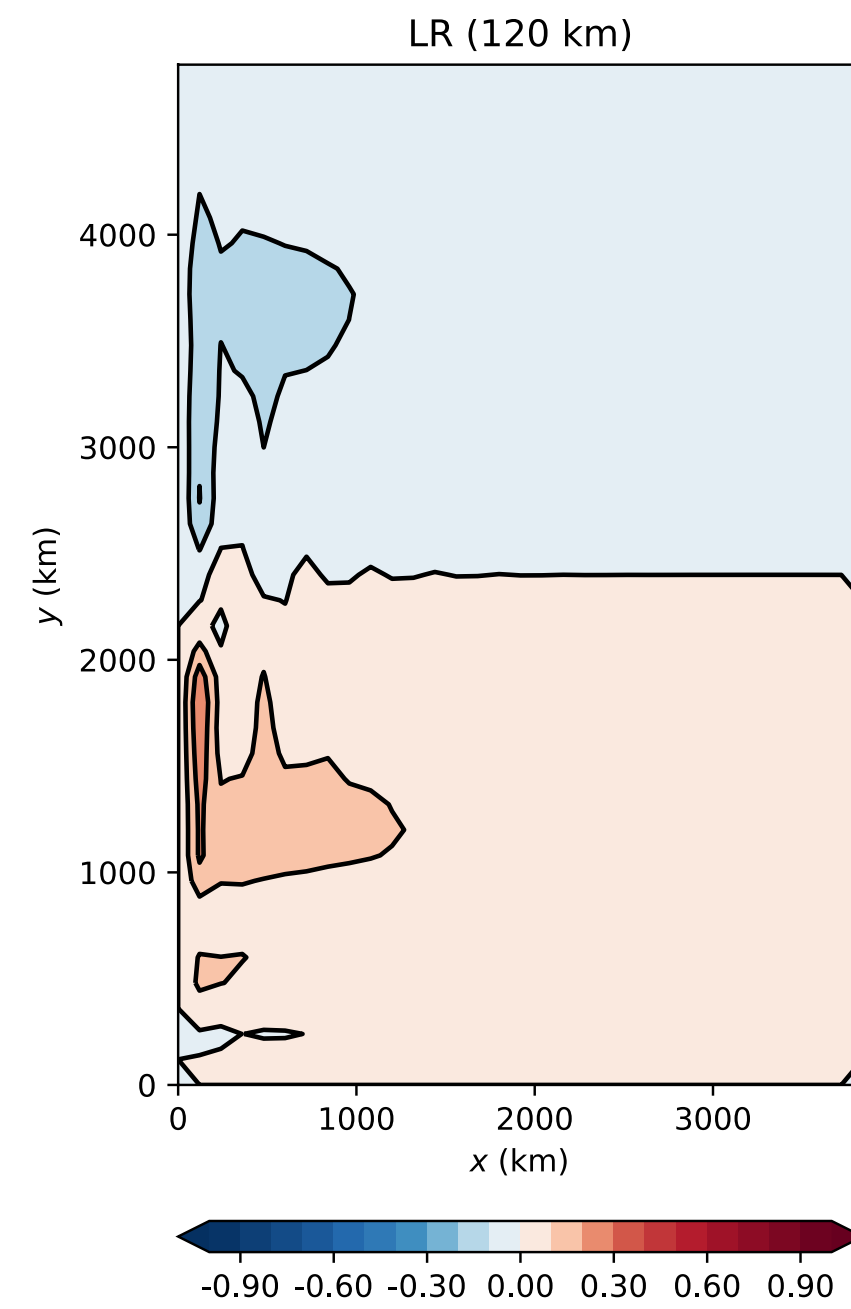


Comparison of mean (60-180 yrs) contour of pressure for coarse-model (120 km) simulations (Exp.1)

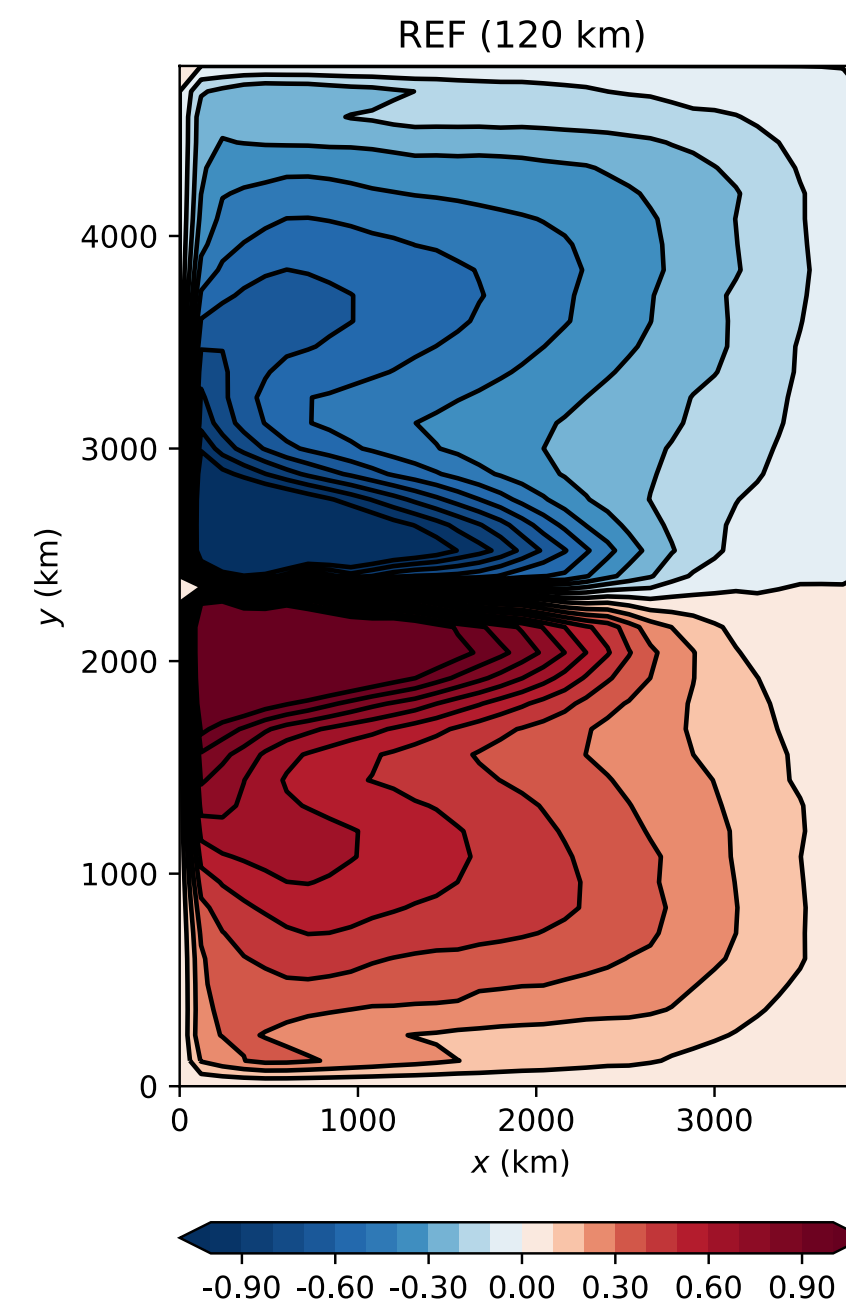
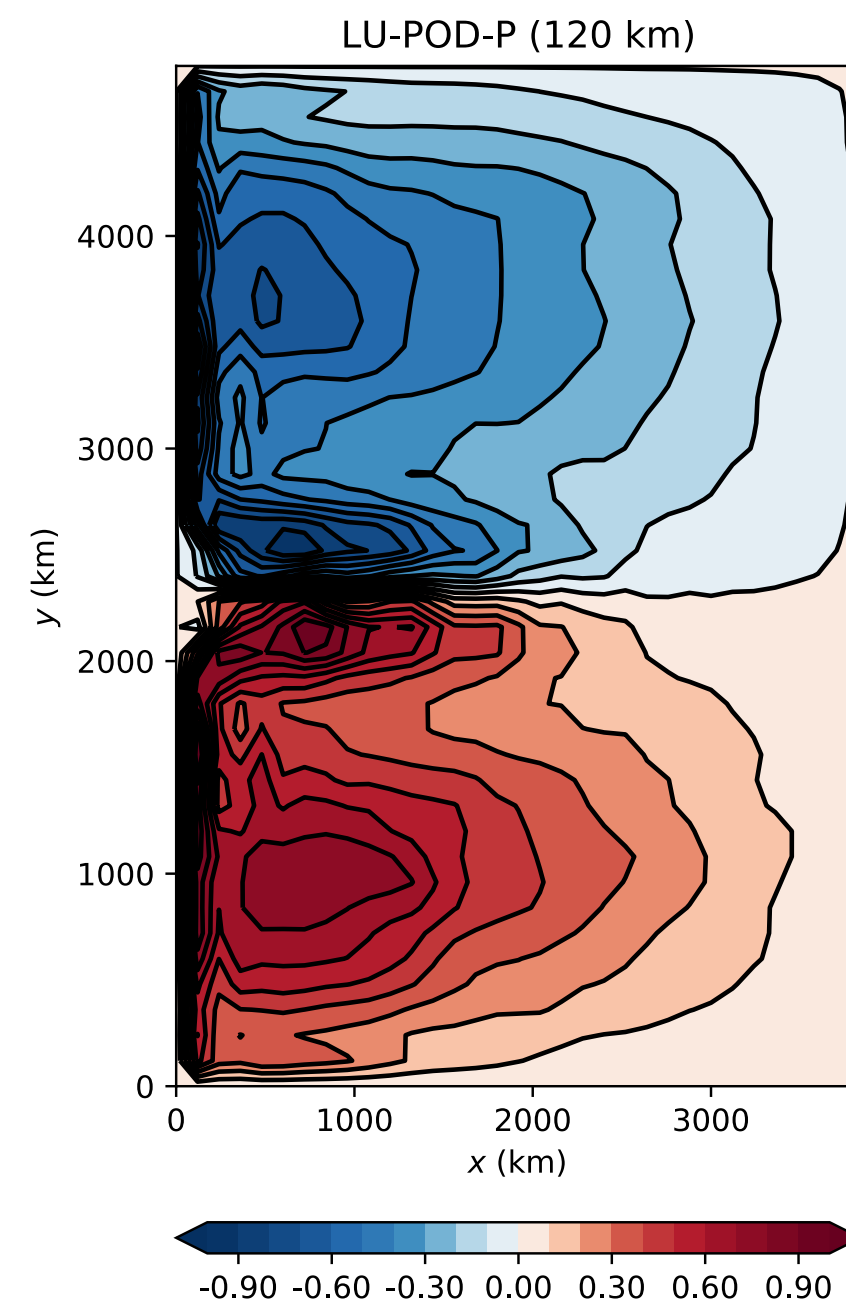
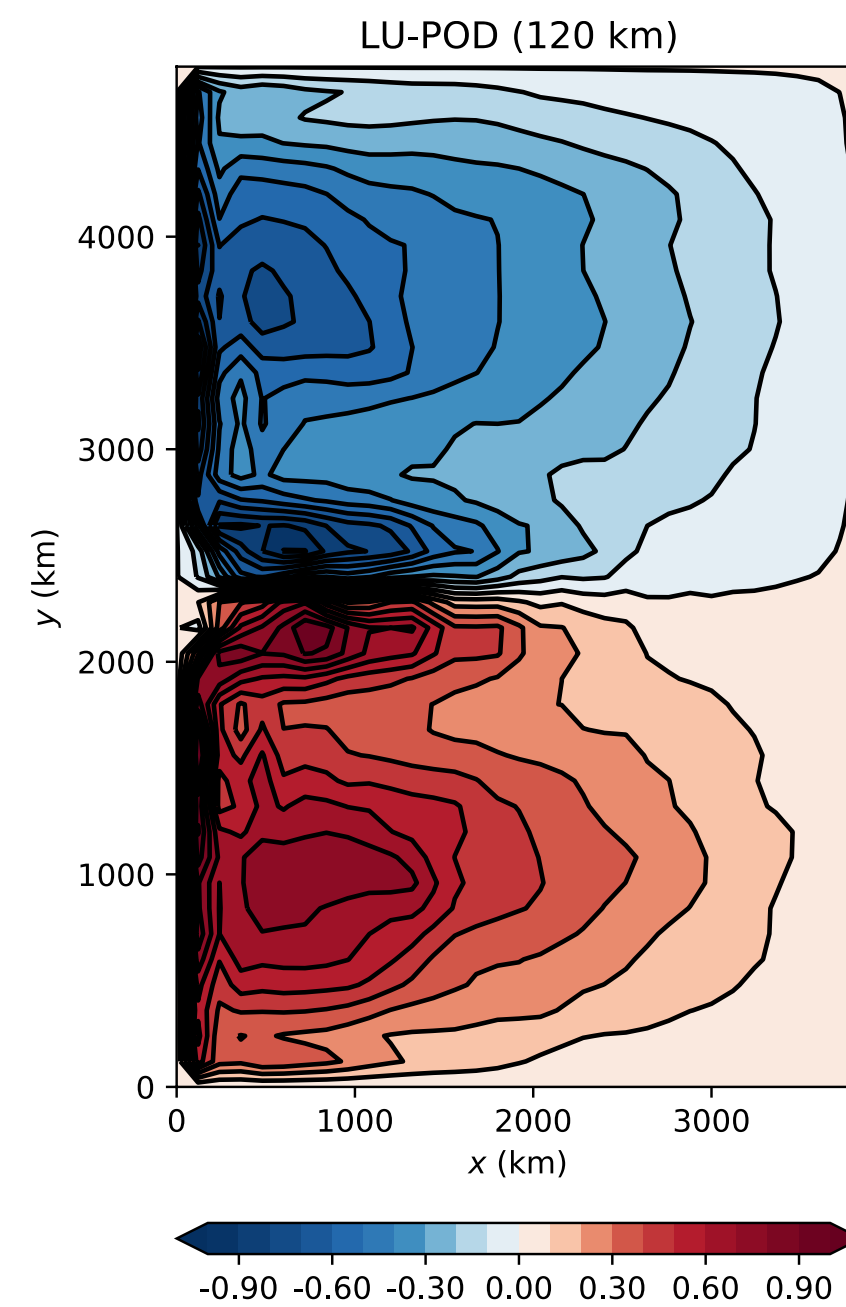
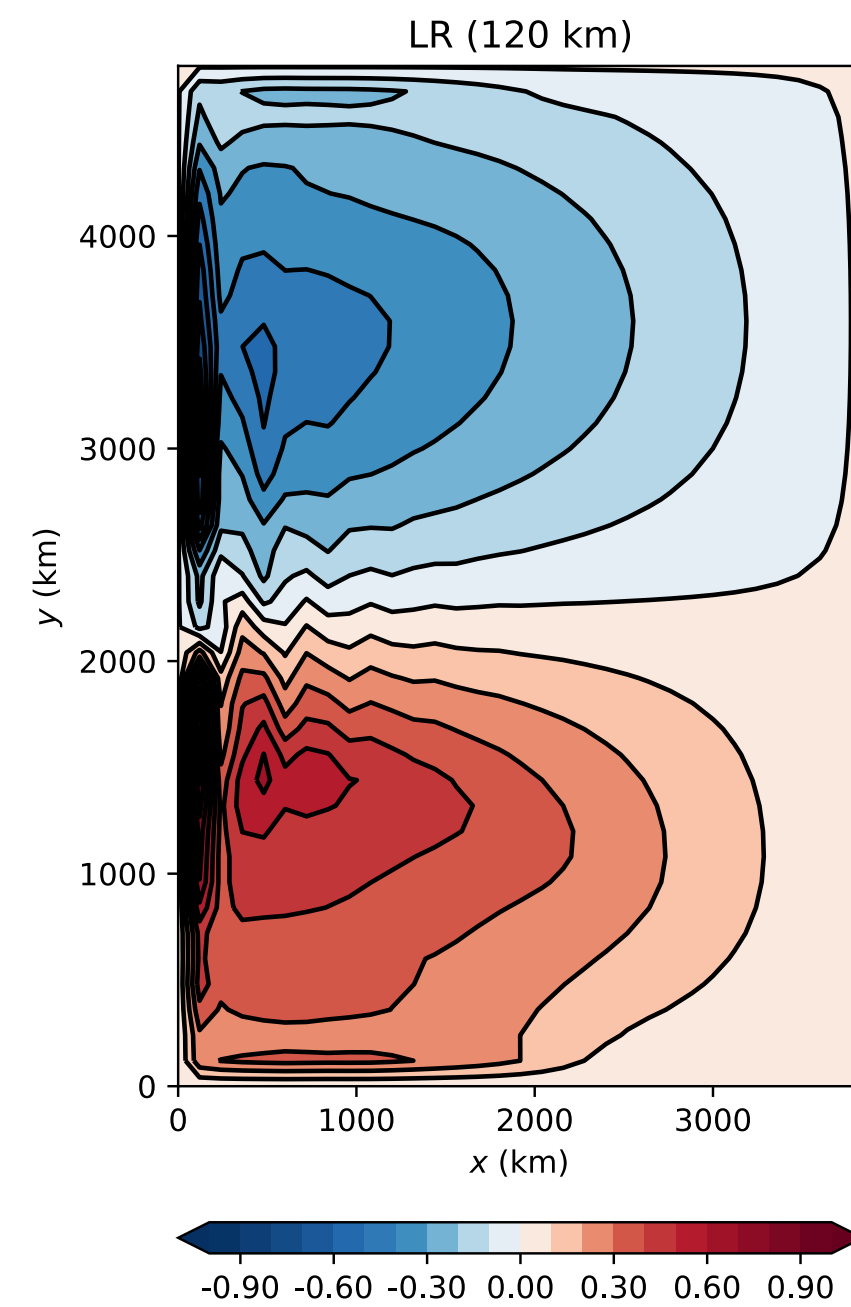


# Statistical prediction

**Barotropic mode**



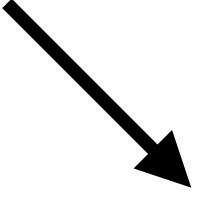
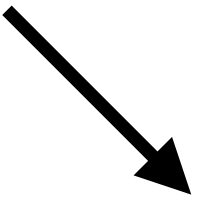
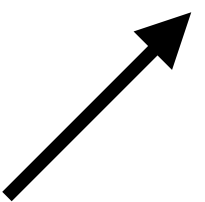
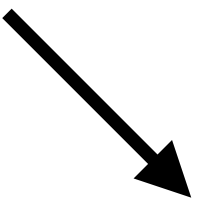
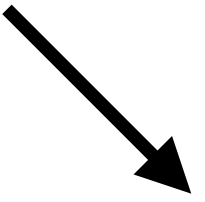
**1st Baroclinic mode**



Comparison of mean (60-180 yrs) contour of pressure for coarse-model (120 km) simulations (Exp.2)



# Measures of statistics

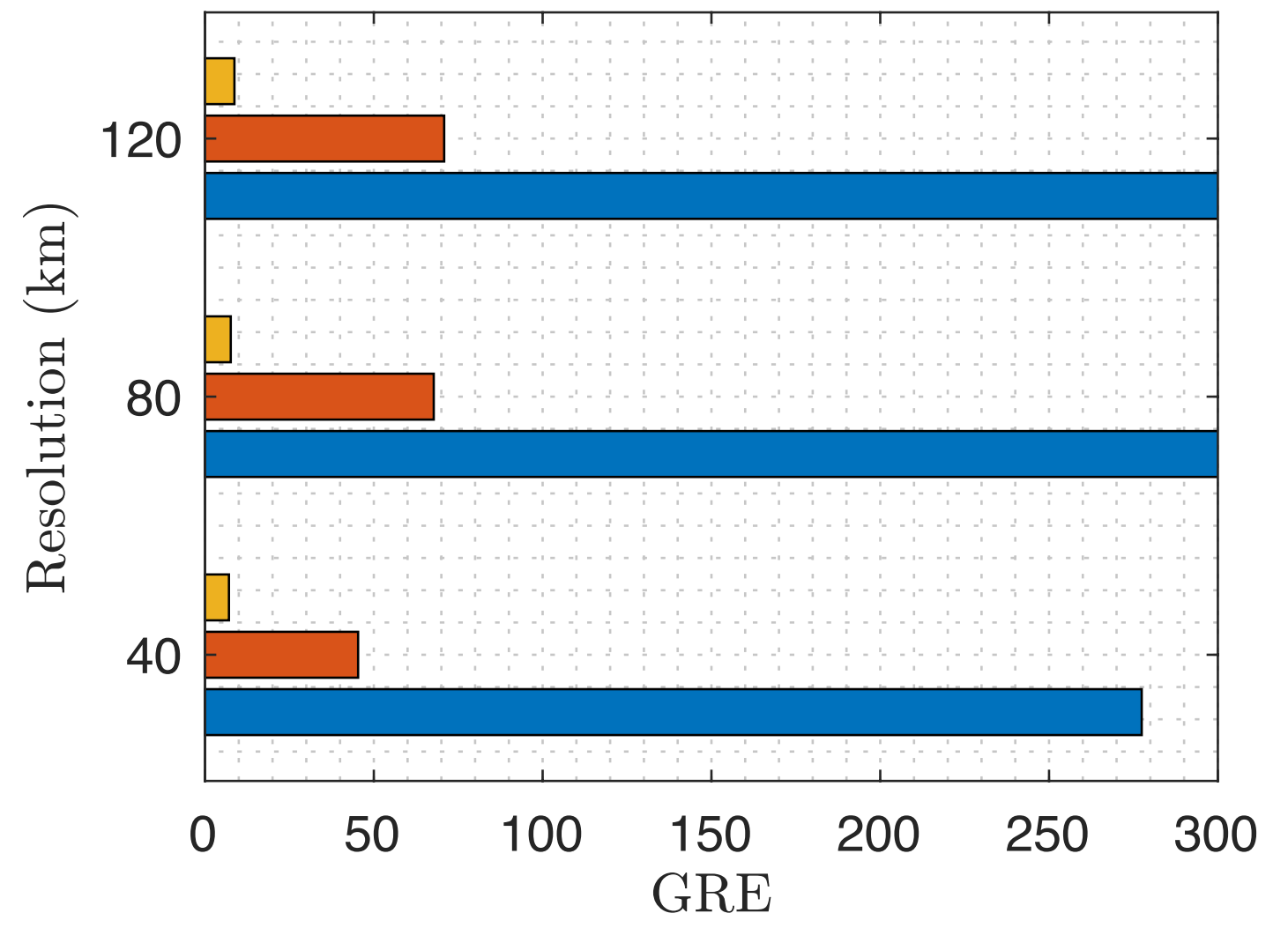
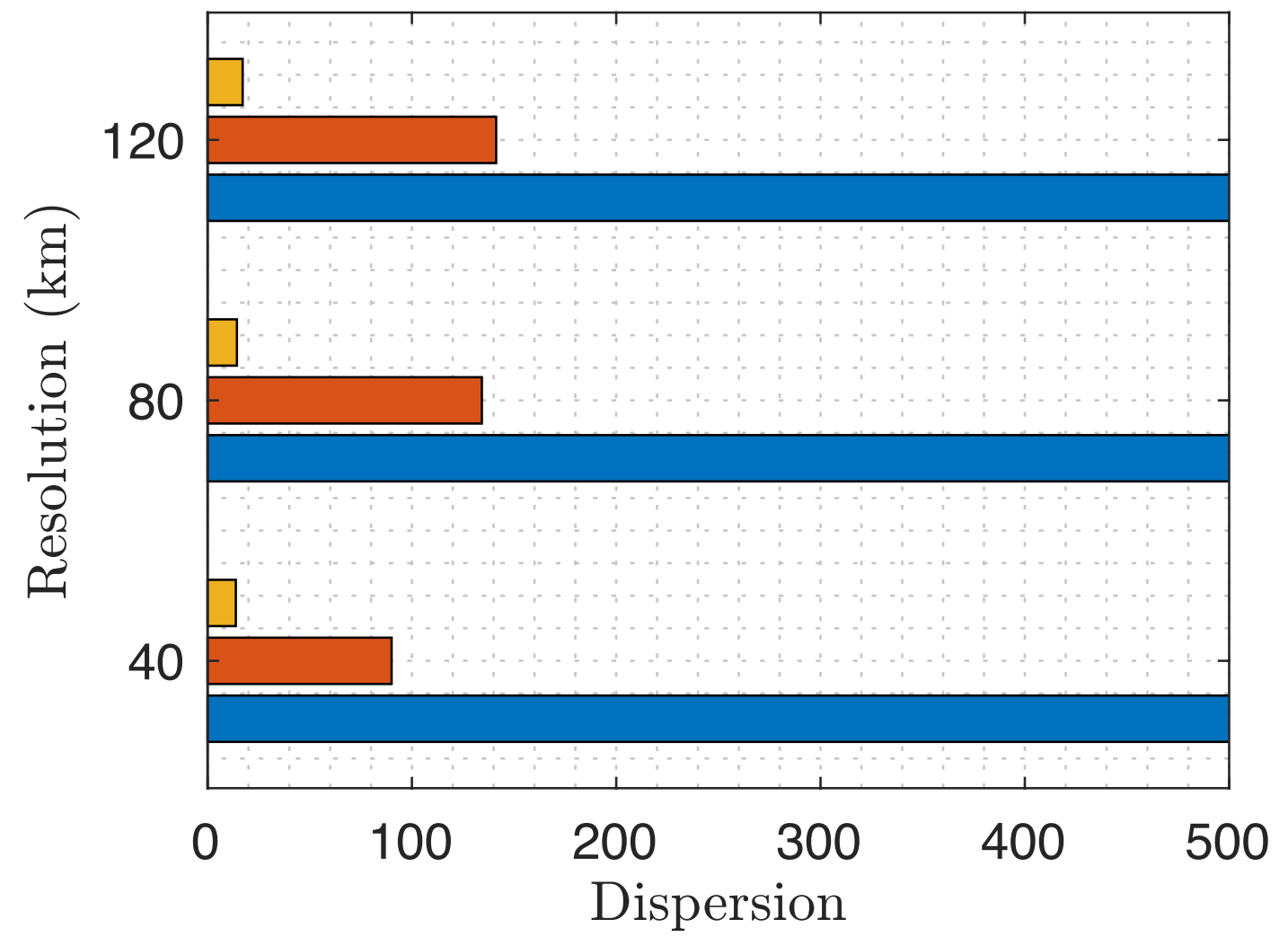
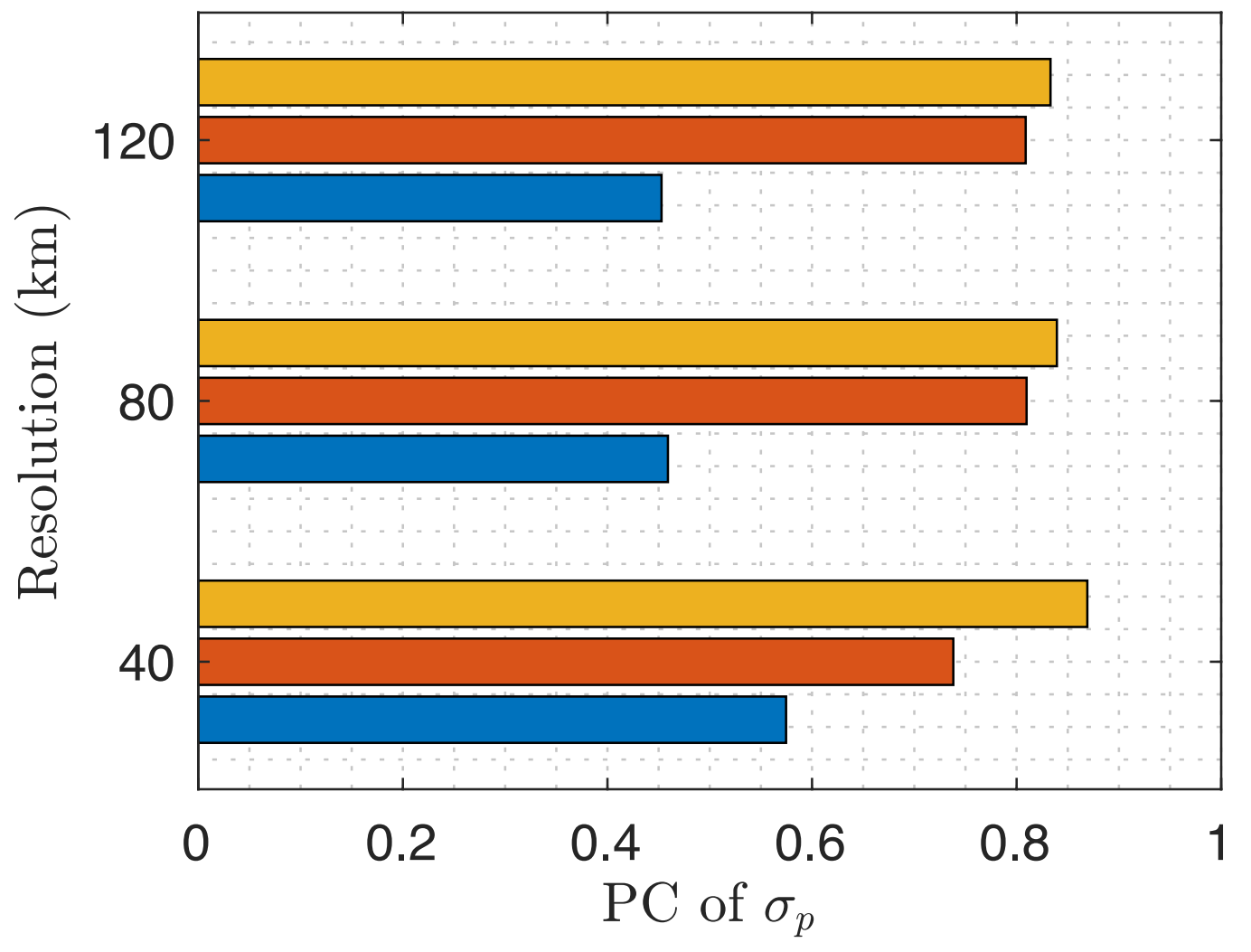
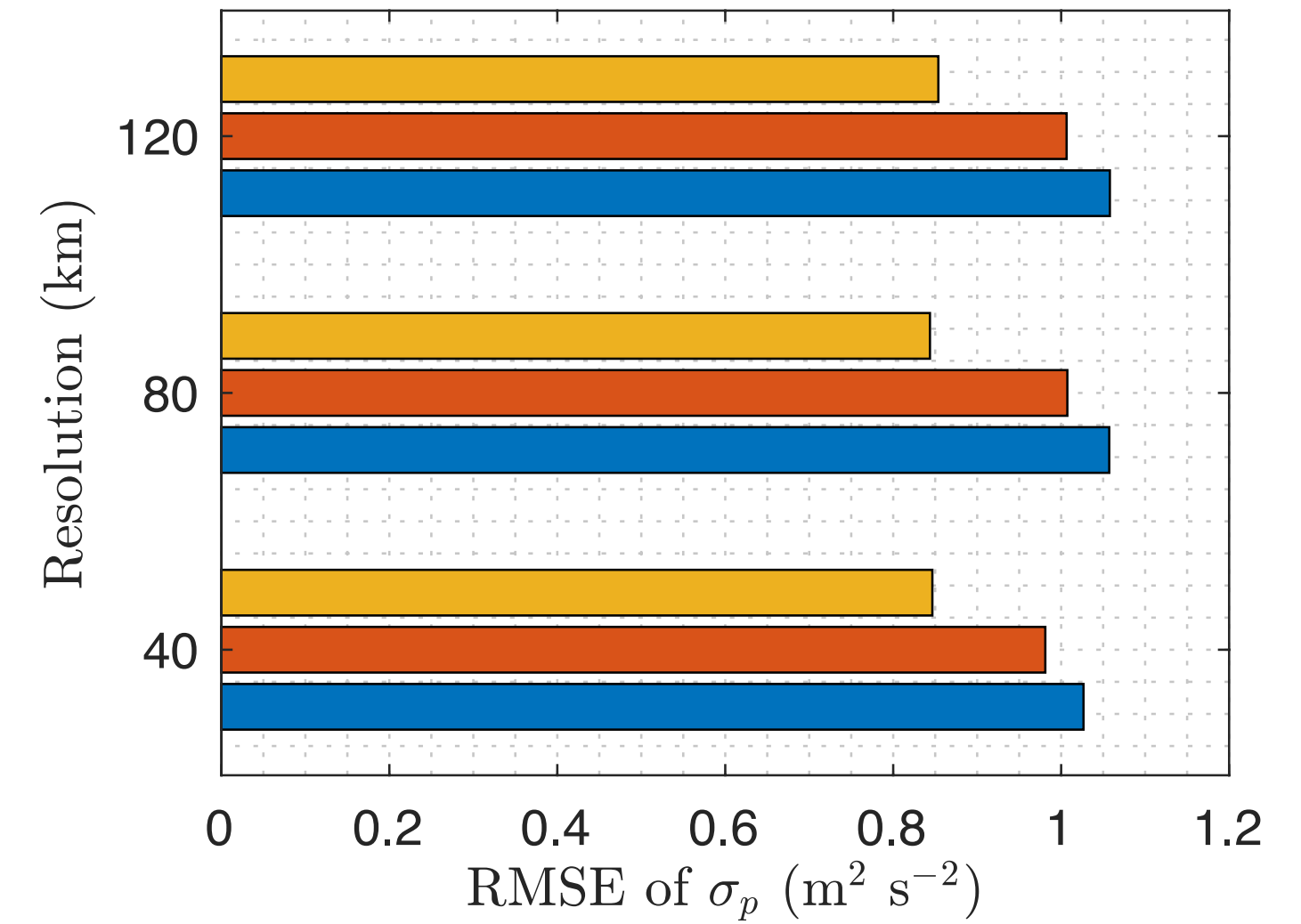
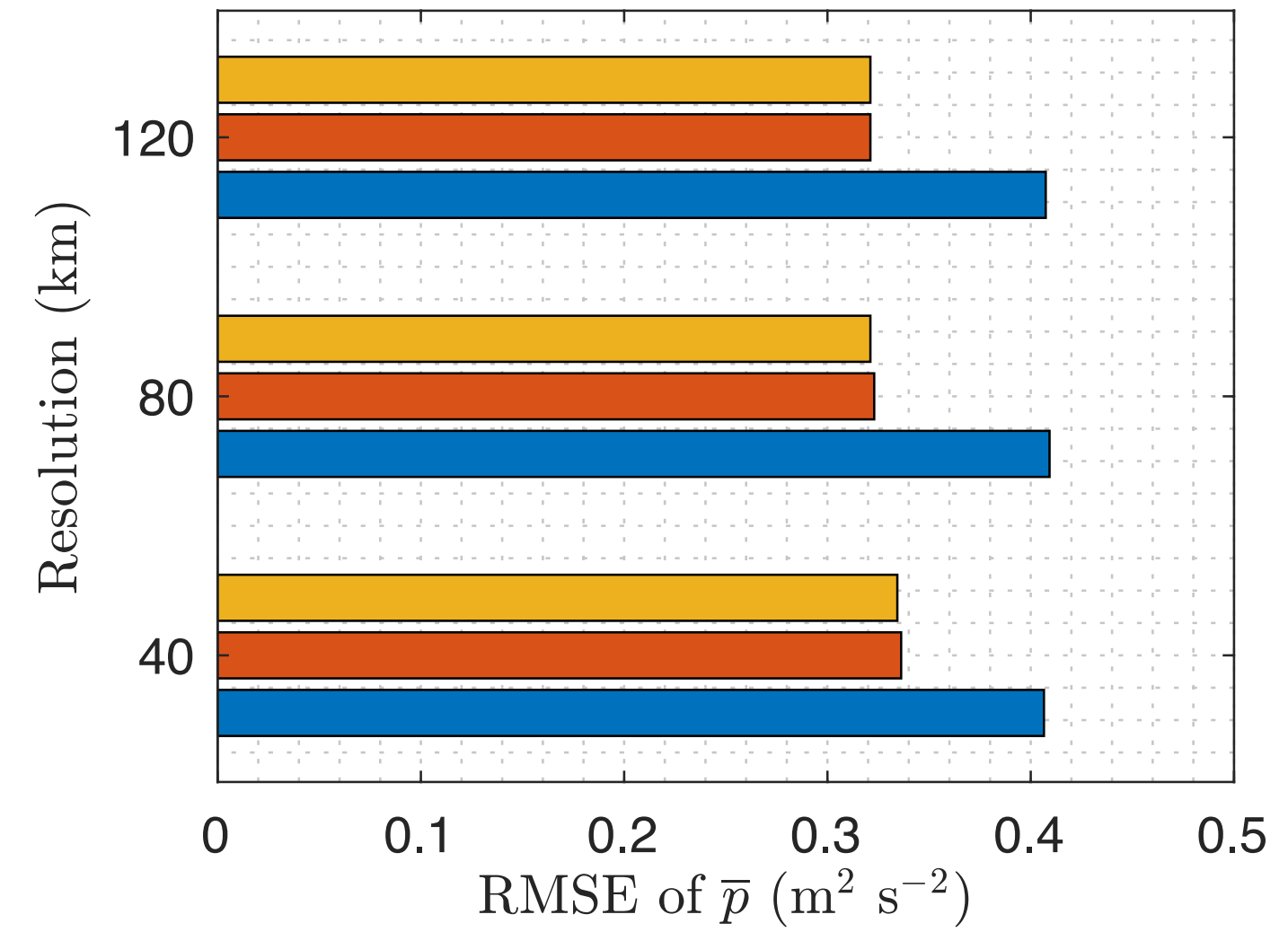
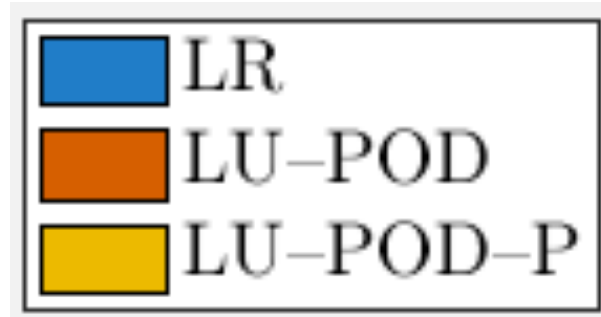
Measures	Values	Accuracy
RMSE of $\bar{p}_{\text{MOD}} = \ \bar{p}_{\text{MOD}} - \bar{p}_{\text{REF}}\ _A$		+
RMSE of $\sigma_{\text{MOD}} = \ \sigma_{\text{MOD}} - \sigma_{\text{REF}}\ _A$ , $\sigma = \overline{(p - \bar{p})^2}$		+
PC = $\frac{\ \sigma_{\text{REF}}\sigma_{\text{MOD}}\ _A^2}{\ \sigma_{\text{REF}}^2\ _A\ \sigma_{\text{MOD}}^2\ _A}$		+
Dispersion = $\frac{1}{ A } \iint \left( \frac{\sigma_{\text{REF}}^2}{\sigma_{\text{MOD}}^2} - 1 - \log \left( \frac{\sigma_{\text{REF}}^2}{\sigma_{\text{MOD}}^2} \right) \right) dA$		+
GRE = $\frac{1}{2} \left\  \frac{\bar{p}_{\text{REF}} - \bar{p}_{\text{MOD}}}{\sigma_{\text{REF}}} \right\ _A^2 + \frac{1}{2} \text{Dispersion}$		+

$$\|f\|_A = \left( \frac{1}{|A|} \iint f^2 dA \right)^{1/2}$$



# Statistical prediction

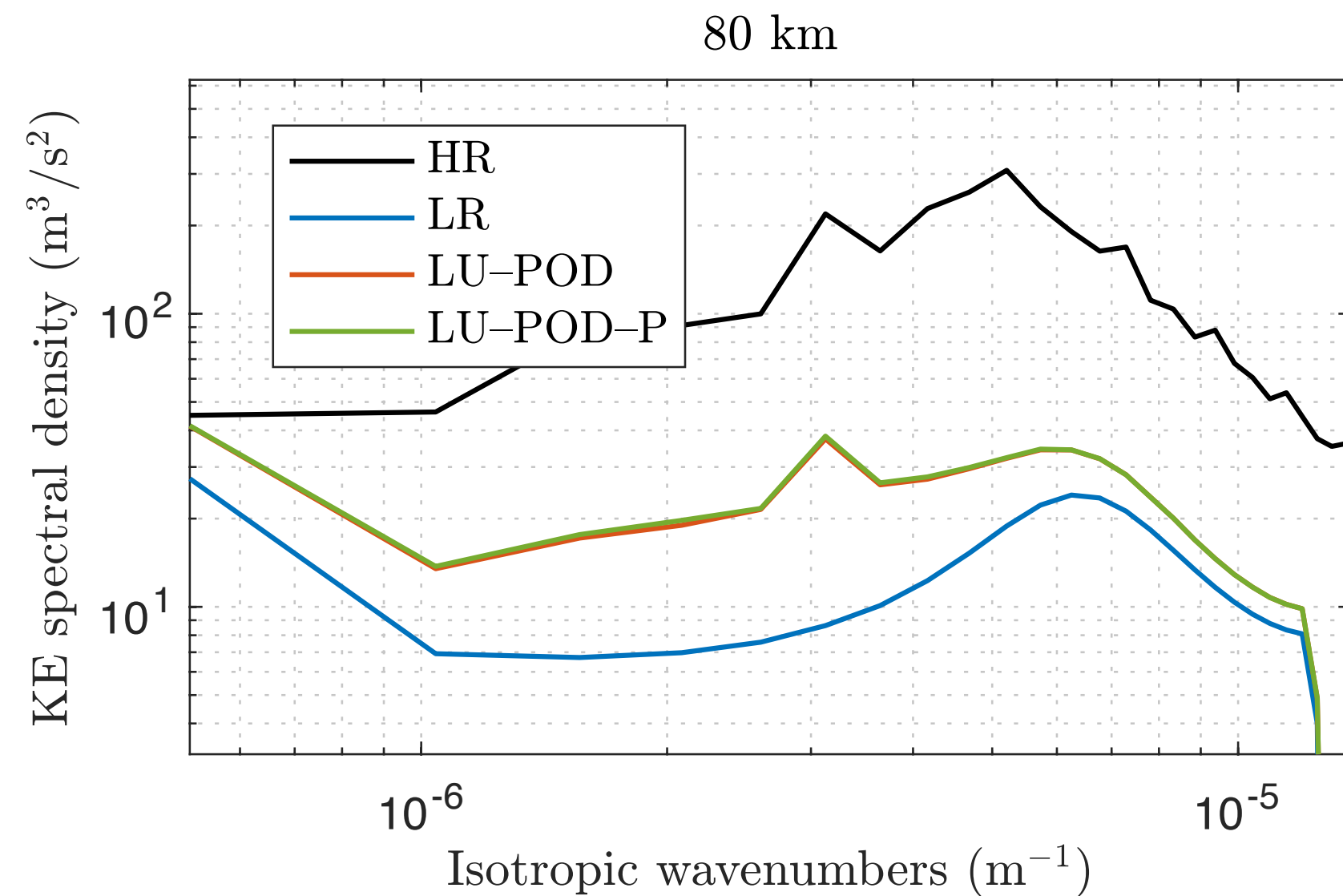
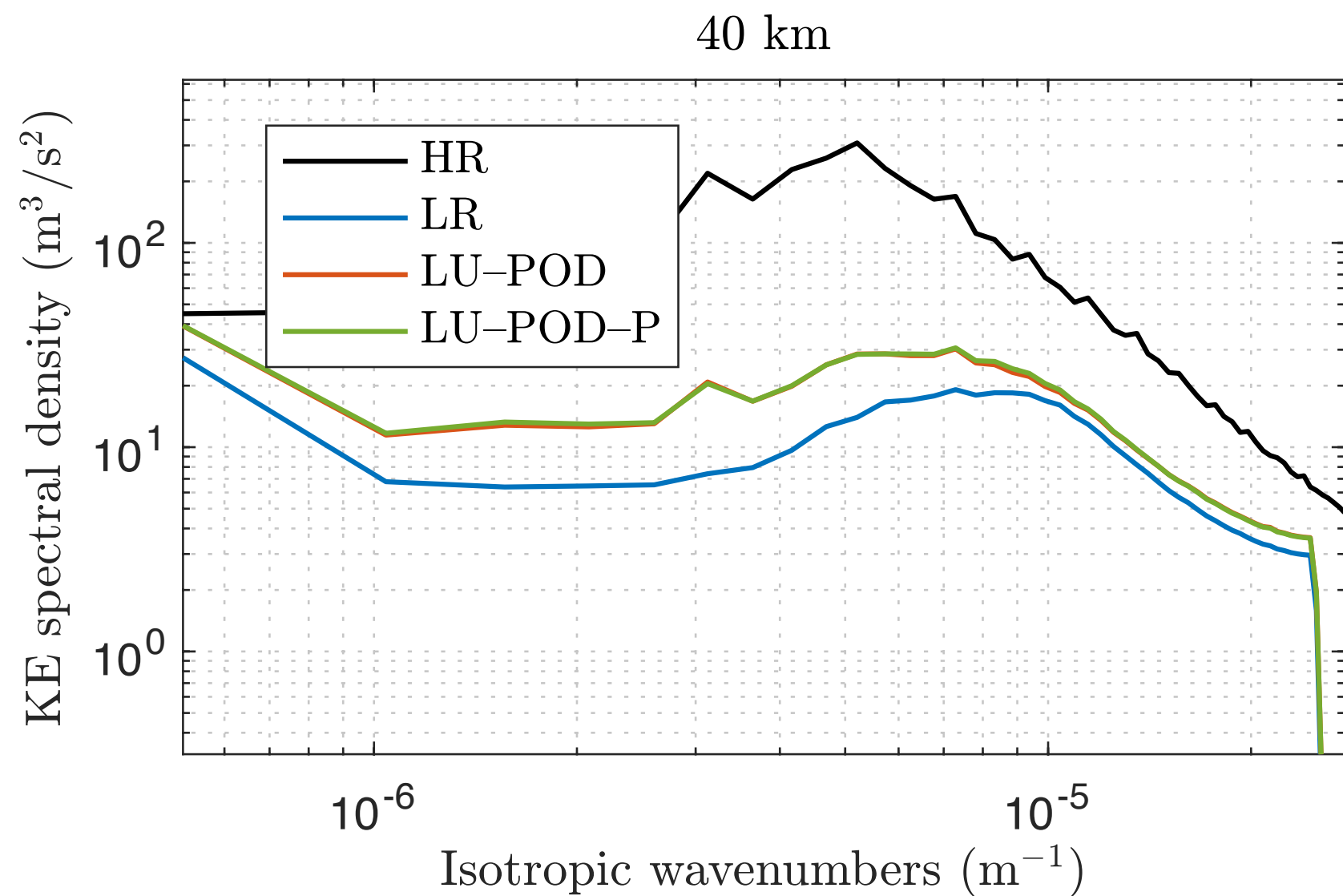
Comparison of statistical measures (integrated vertically) for pressures provided by different coarse models (Exp.2)





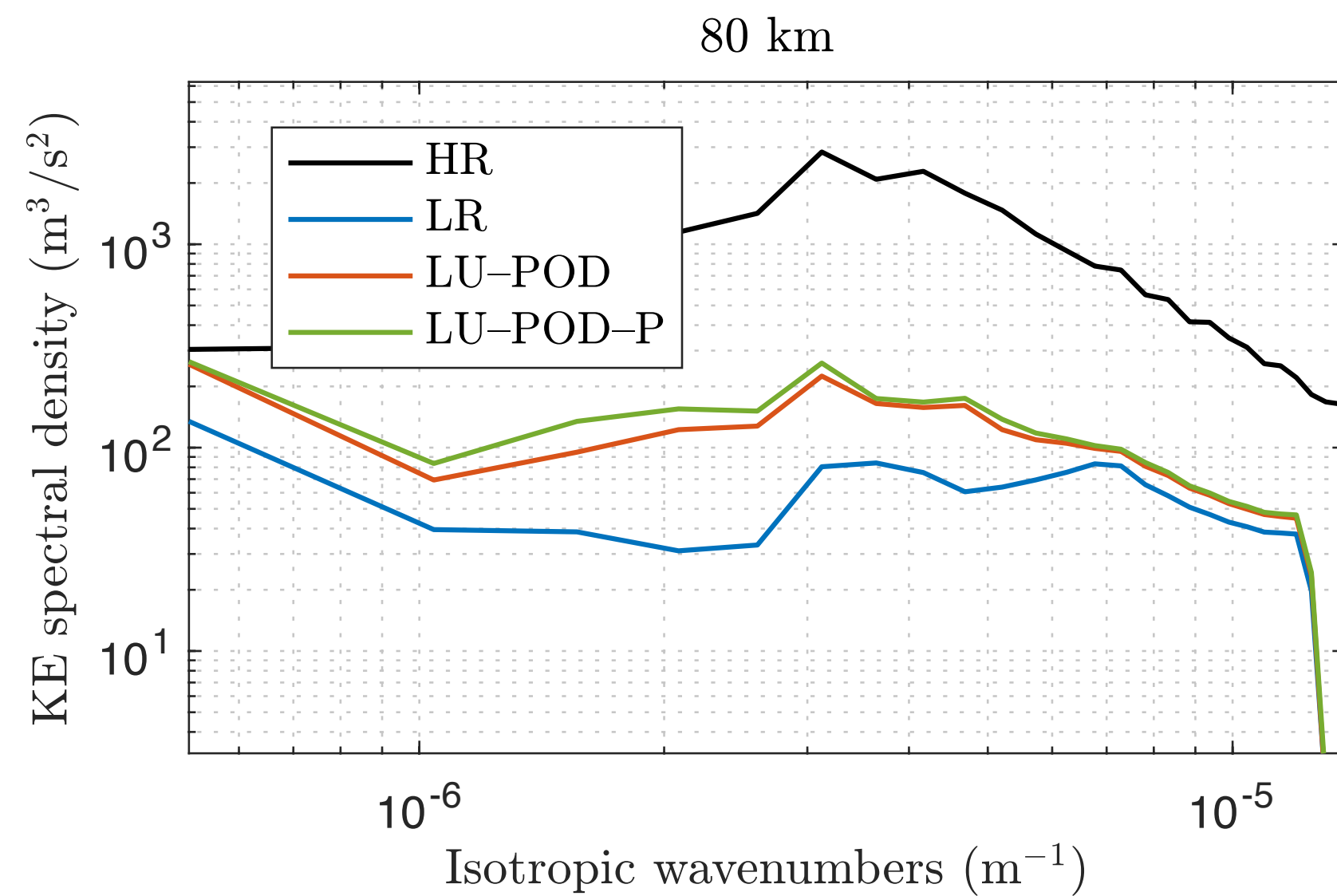
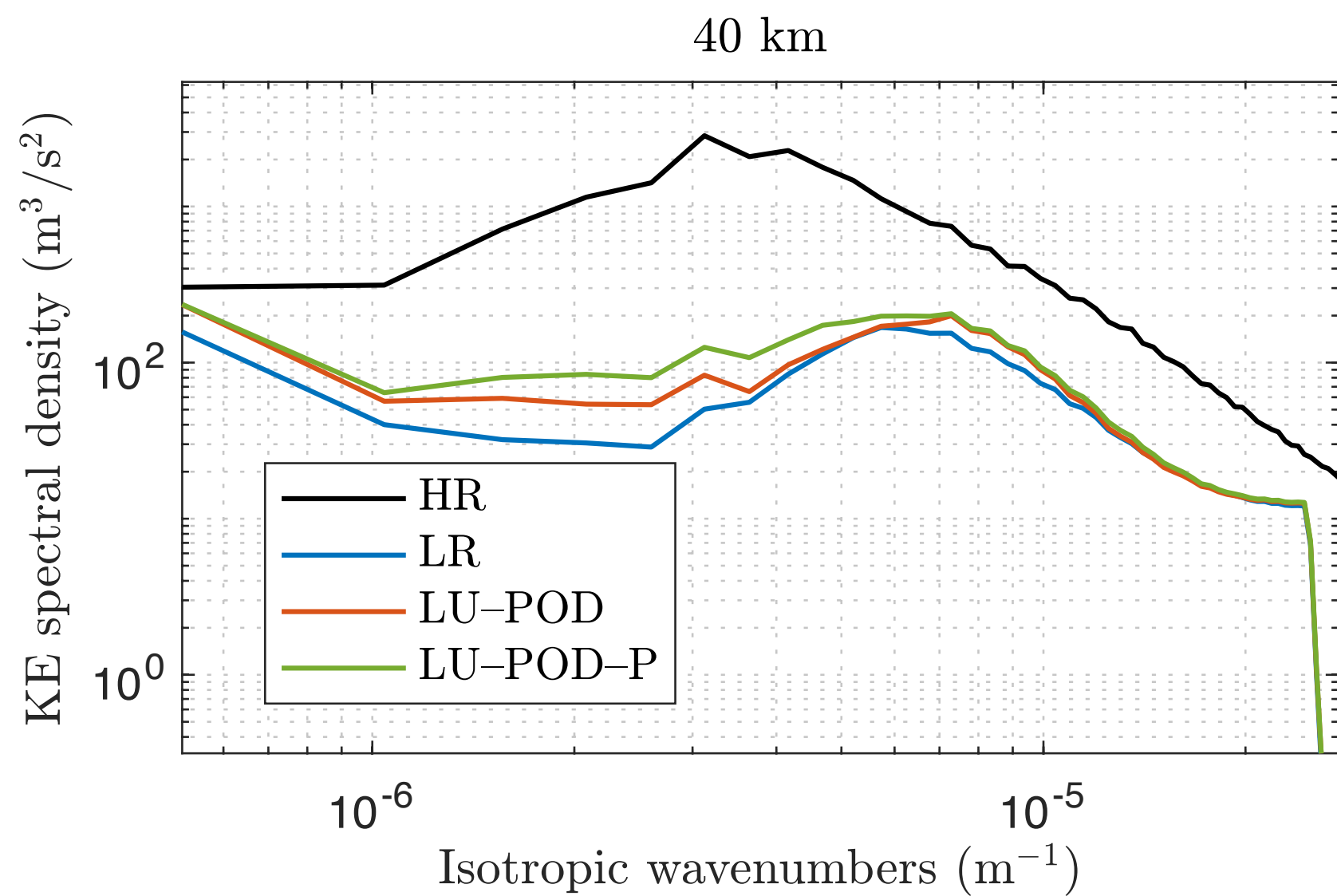
# Energy analysis

**Exp.1  
(without SST)**



**Comparison of  
KE spectral  
density  
averaged in  
time (60-75 yrs)  
and in layers  
for different  
coarse models**

**Exp.2  
(with SST)**



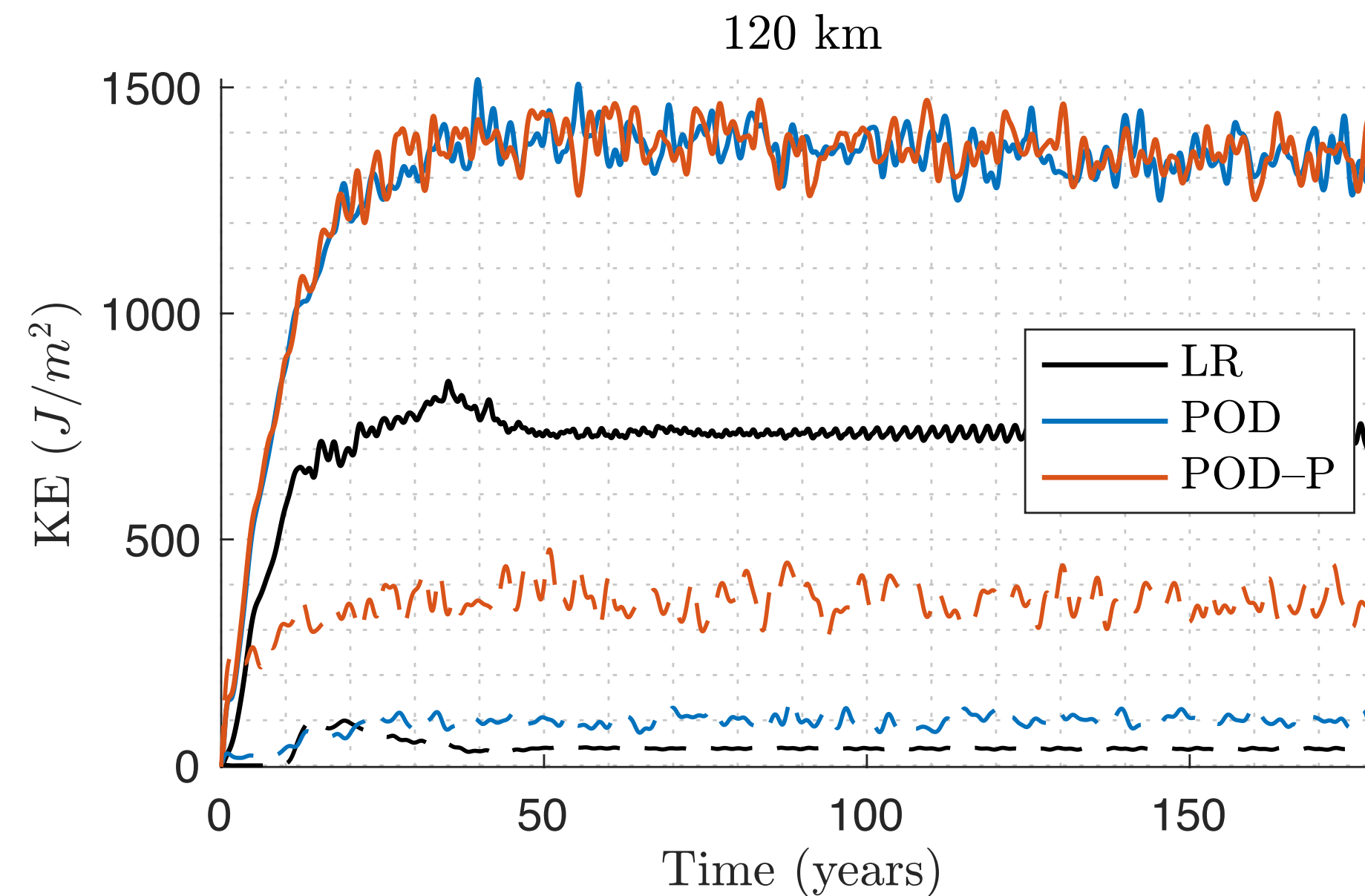
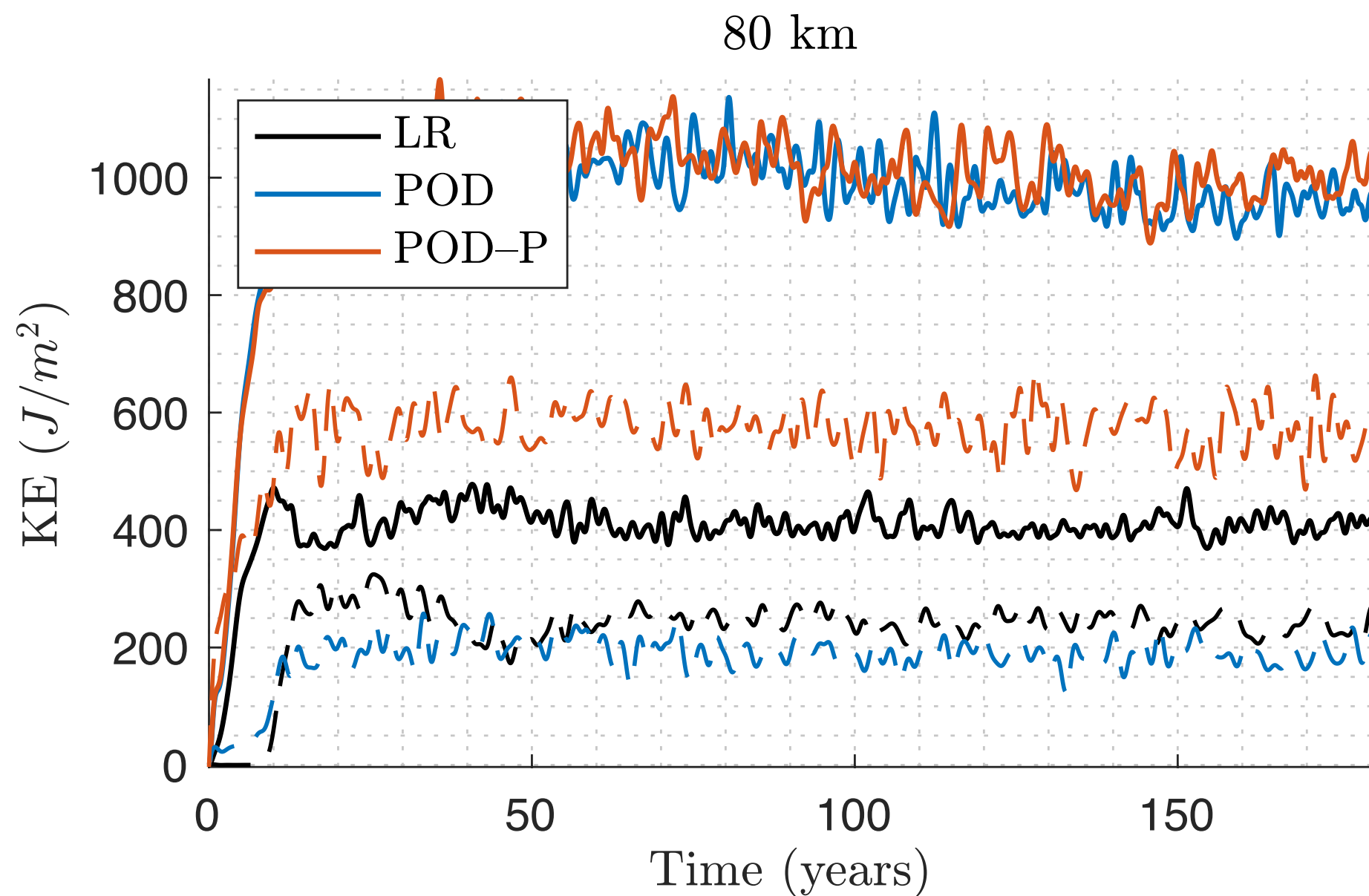


# Energy analysis

- KE decomposition

$$\mathbf{u}_k = \overline{\mathbf{u}}_k + \mathbf{u}'_k \quad \text{Low-pass filtering}$$

$$\text{EKE}_k = \underbrace{\frac{\rho_0 H_k}{2} \|\overline{\mathbf{u}}_k\|_A^2}_{\text{Standing EKE}} + \underbrace{\frac{\rho_0 H_k}{2} \|\mathbf{u}'_k\|_A^2}_{\text{Transient EKE}}$$



**Comparison of EKE (integrated over layers) for different coarse models (Solid lines — standing EKE, Dashed lines — transient EKE)**



# Energy analysis

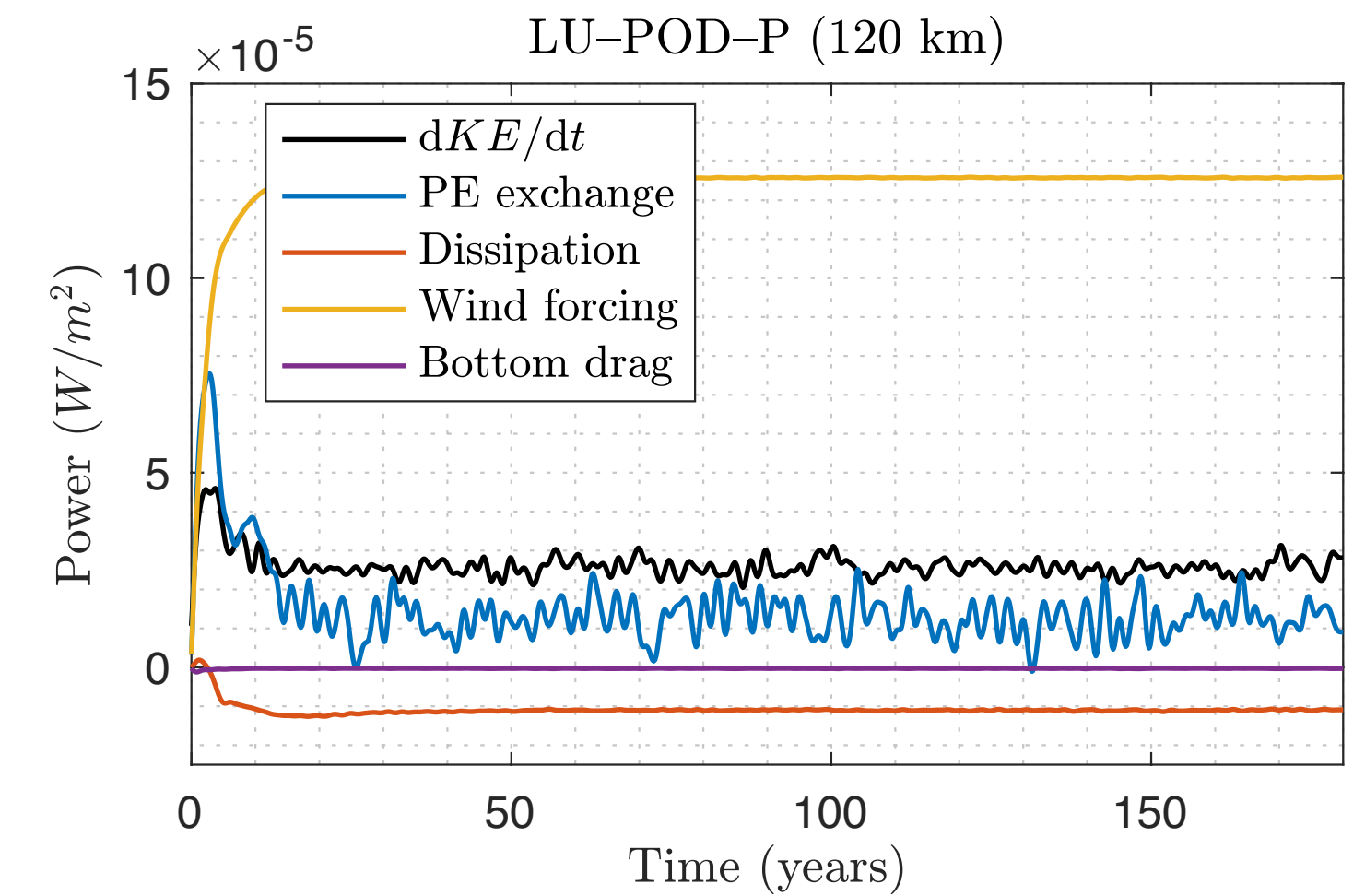
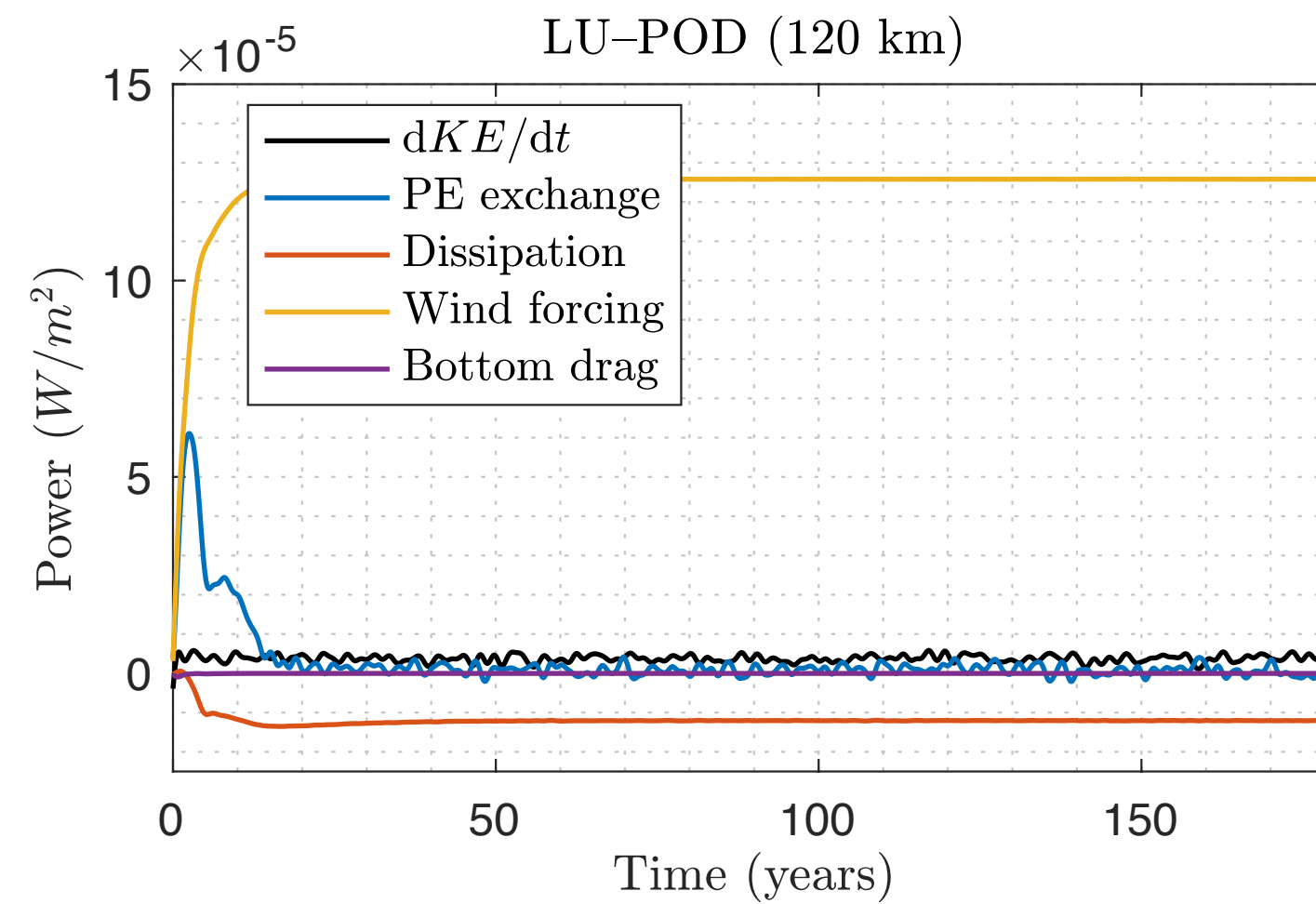
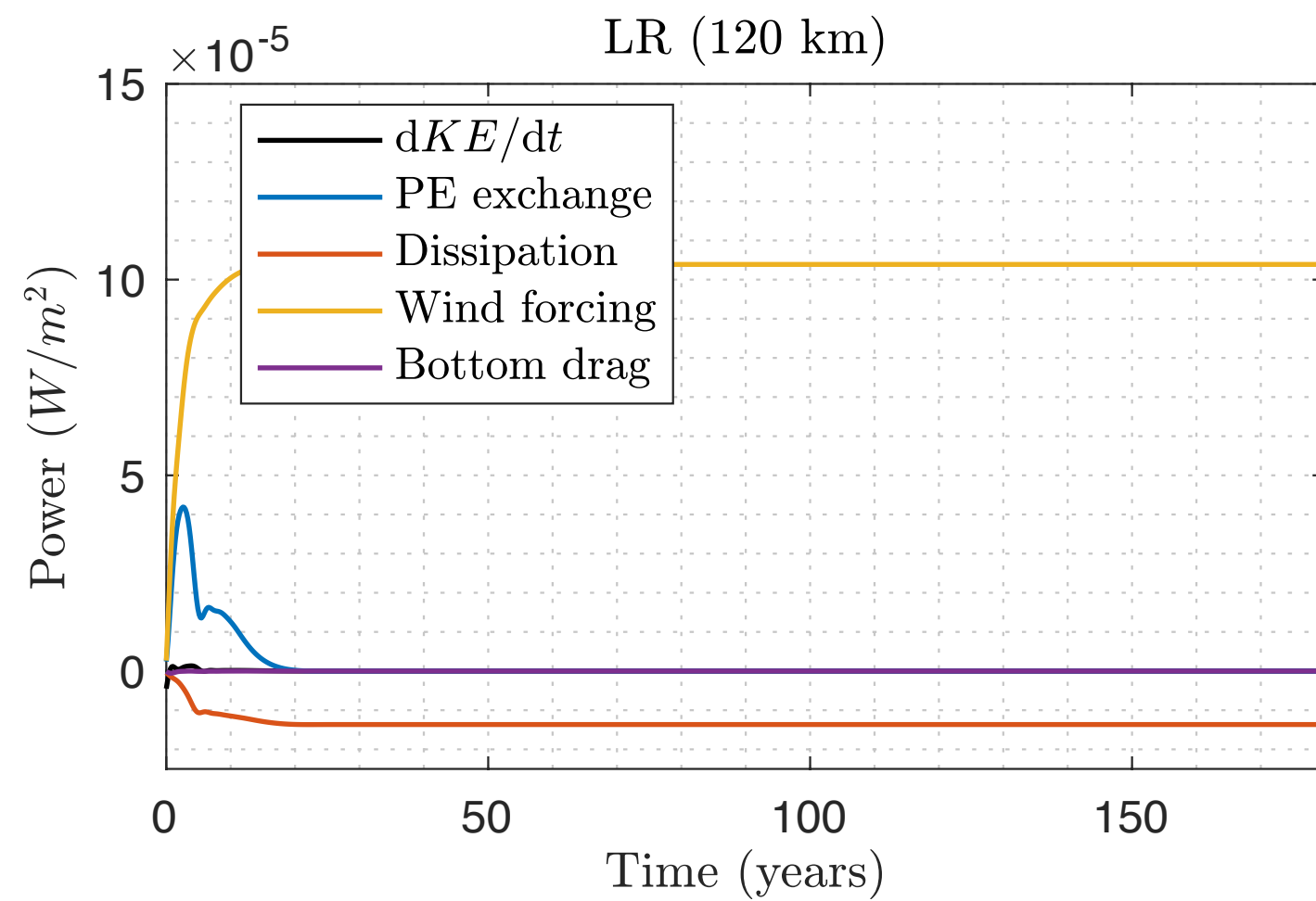
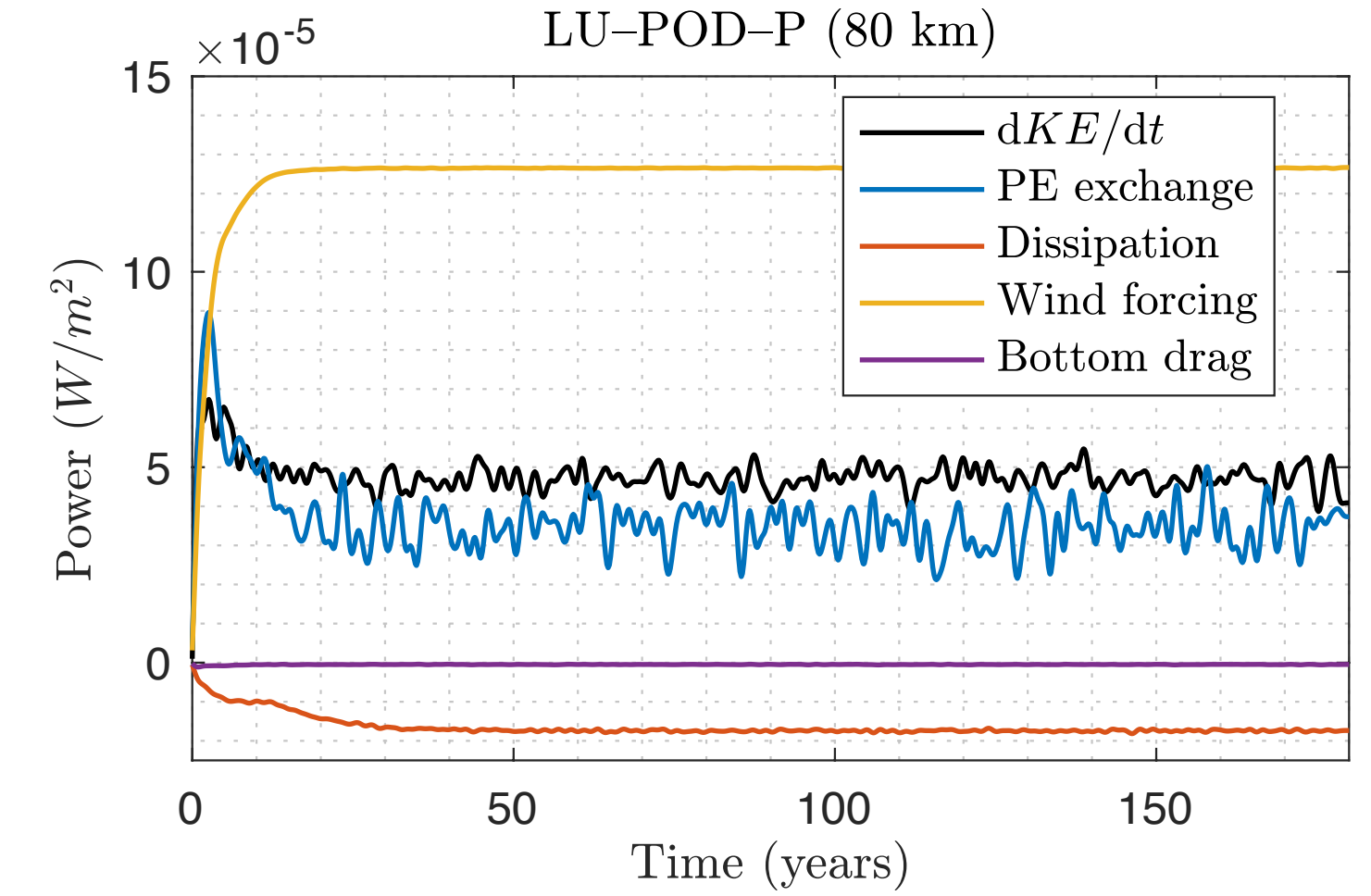
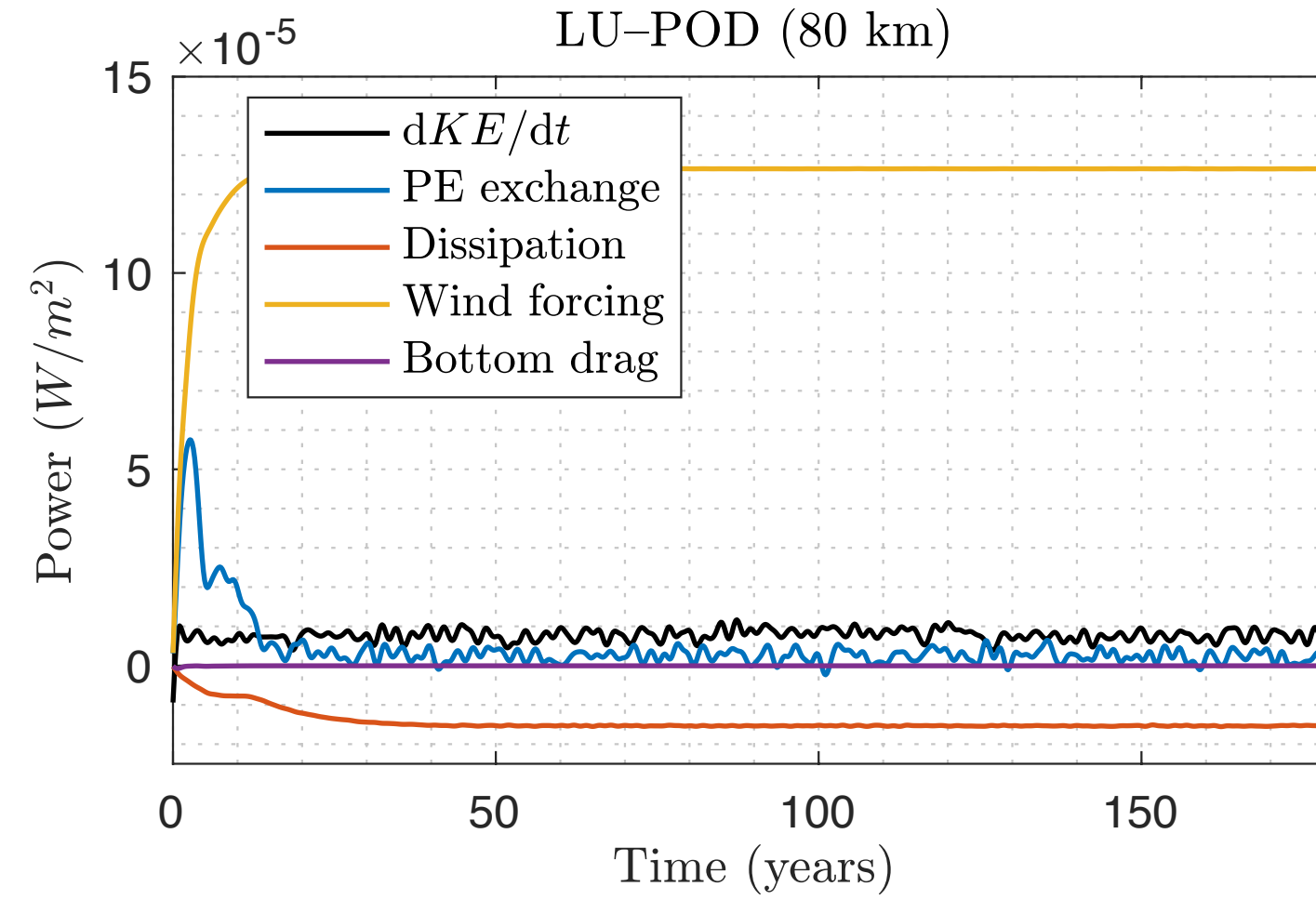
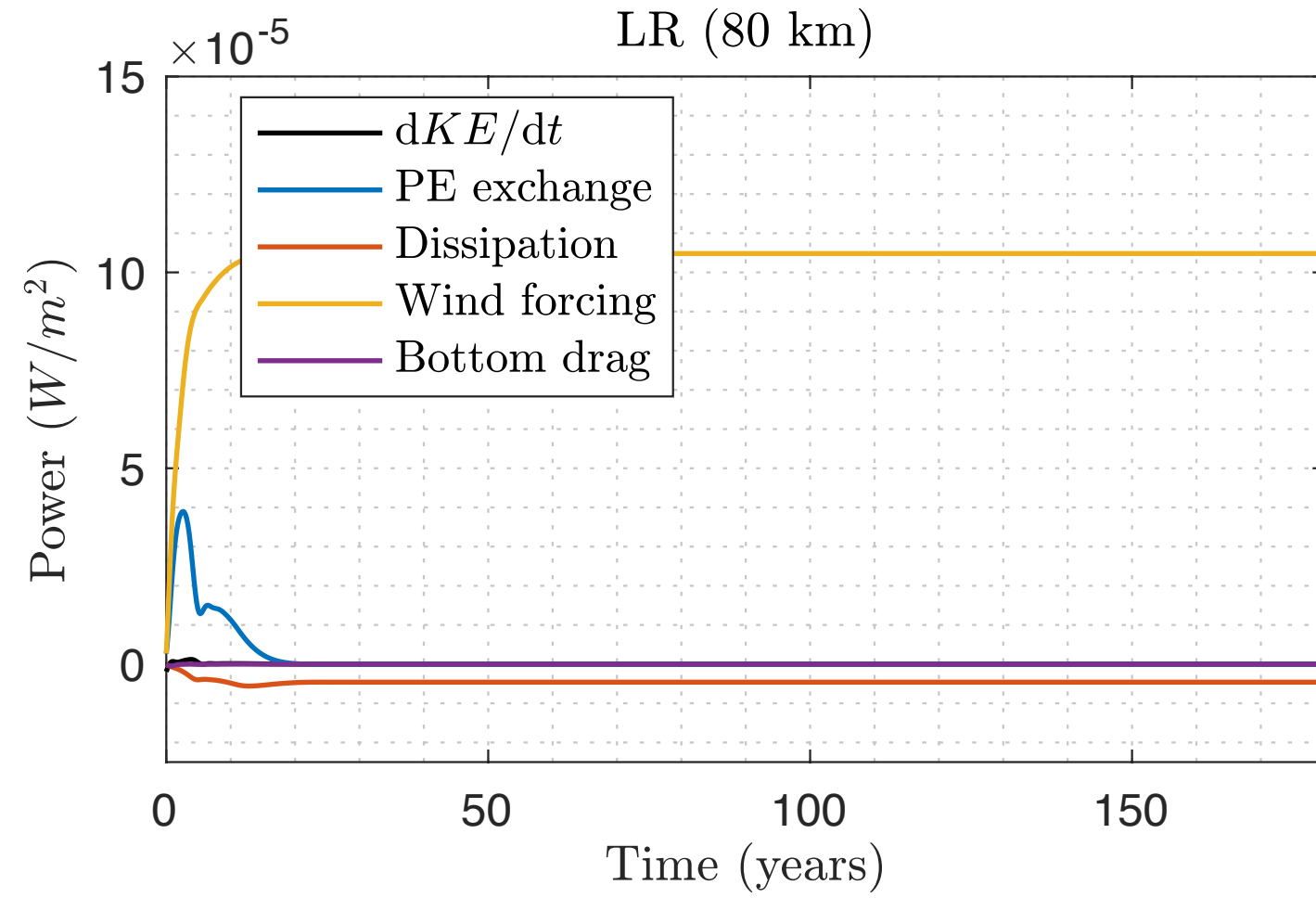
- KE contributions

$$\begin{aligned}
 & \text{KE} & \text{PE} & \text{Buoyancy forcing} & & (= 0 \text{ in Exp.1}) \\
 & \partial_t \sum_{k=1}^N \frac{\rho_0 H_k}{2} \|\mathbf{u}_k\|_A^2 = -\partial_t \sum_{k=1}^{N-1} \frac{\rho_0 g'_k}{2} \|\eta_k\|_A^2 - \sum_{k=1}^{N-1} \rho_0 g'_k \langle \eta_k, w_k \rangle_A \\
 & \text{Wind forcing} & + \rho_0 \langle \mathbf{u}_1, \boldsymbol{\tau} \rangle_A & - \frac{\rho_0 f_0 \delta_{ek}}{2} \|\mathbf{u}_N\|_A^2 & \text{Bottom drag} \\
 & + \sum_{k=1}^N \rho_0 H_k \langle \mathbf{u}_k, A_2 \nabla^2 \mathbf{u}_k - A_4 \nabla^4 \mathbf{u}_k \rangle_A & & & \text{Dissipation}
 \end{aligned}$$

$$\langle f, g \rangle_A = \frac{1}{|A|} \iint fg \, dA$$



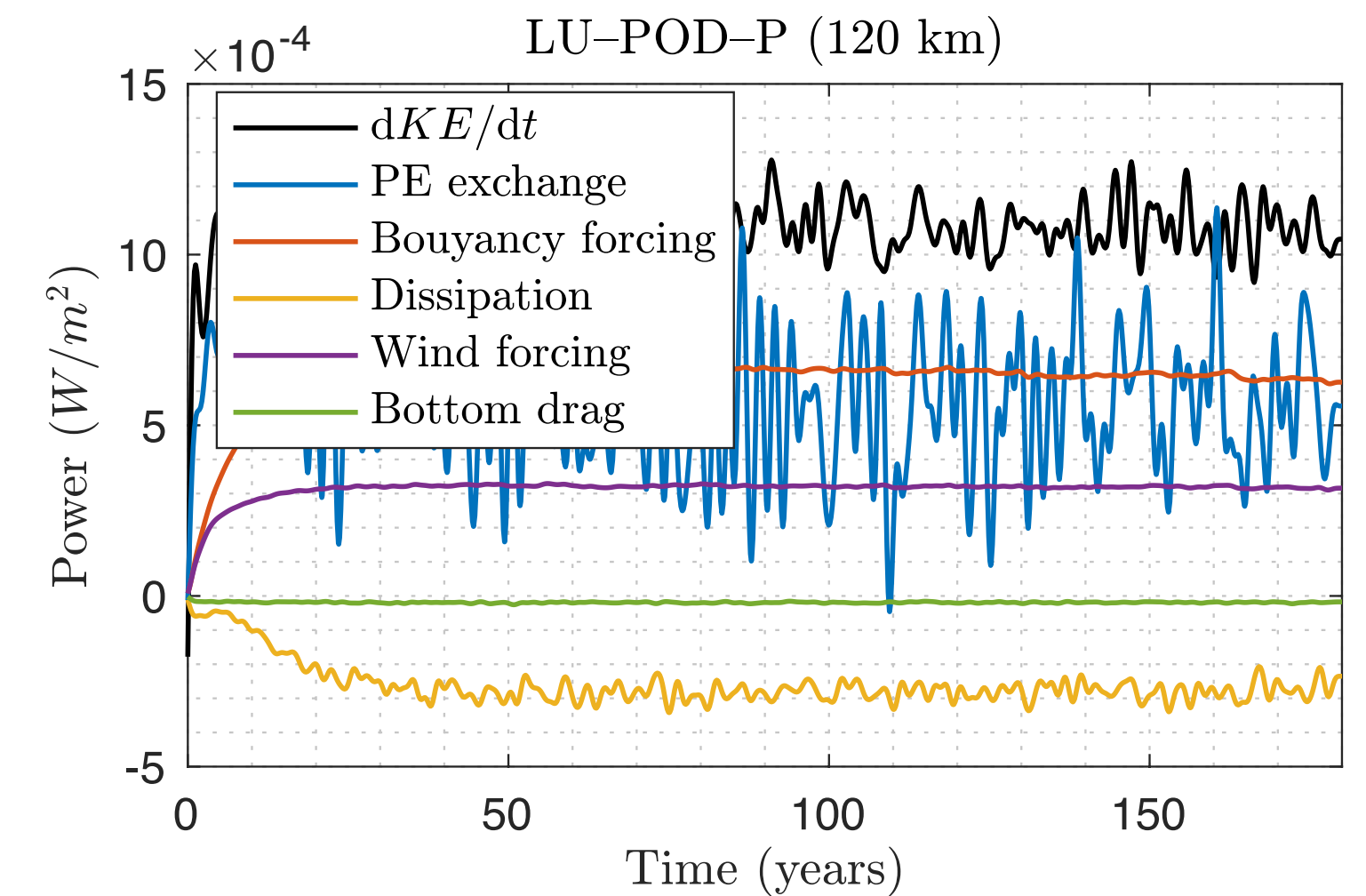
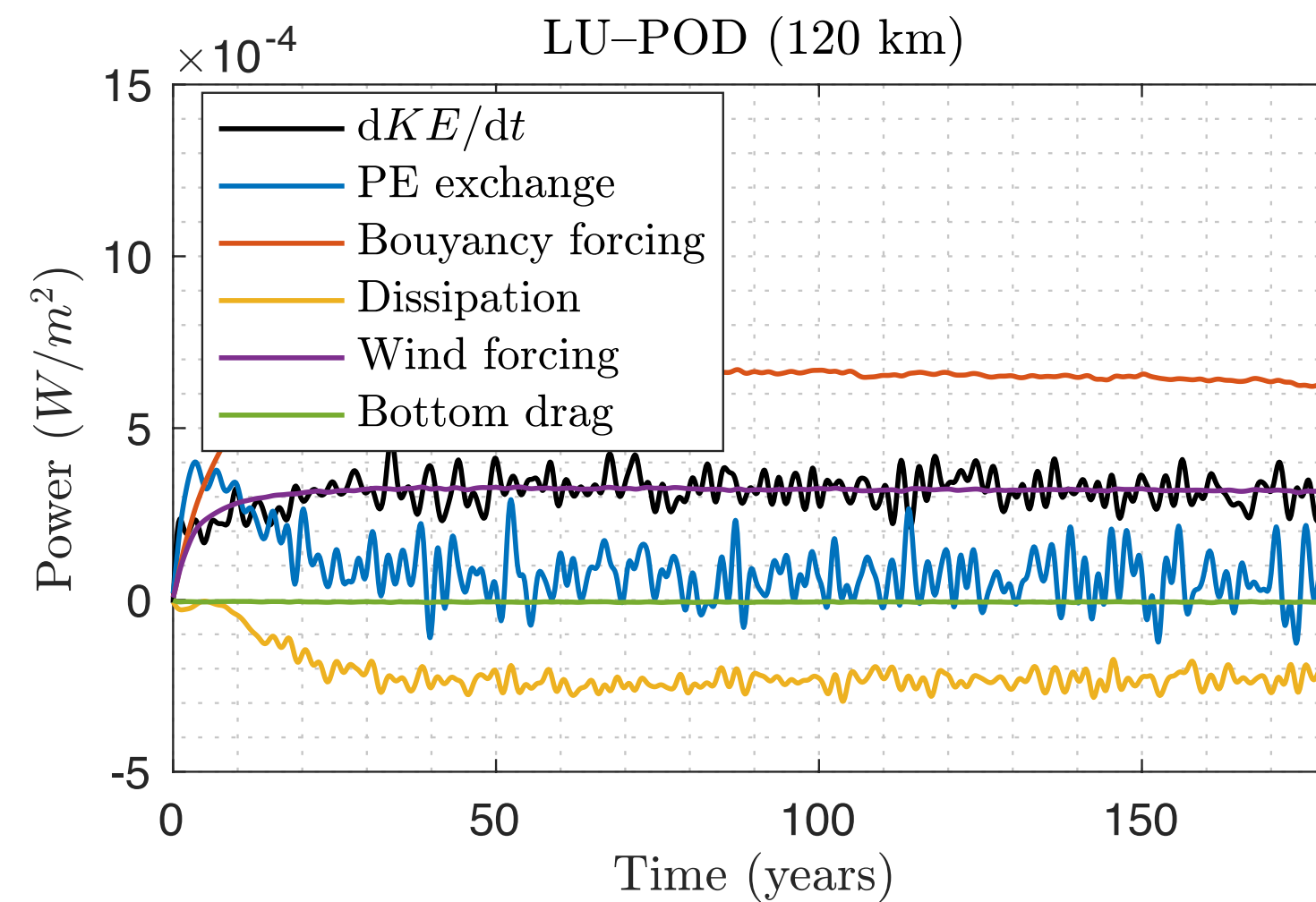
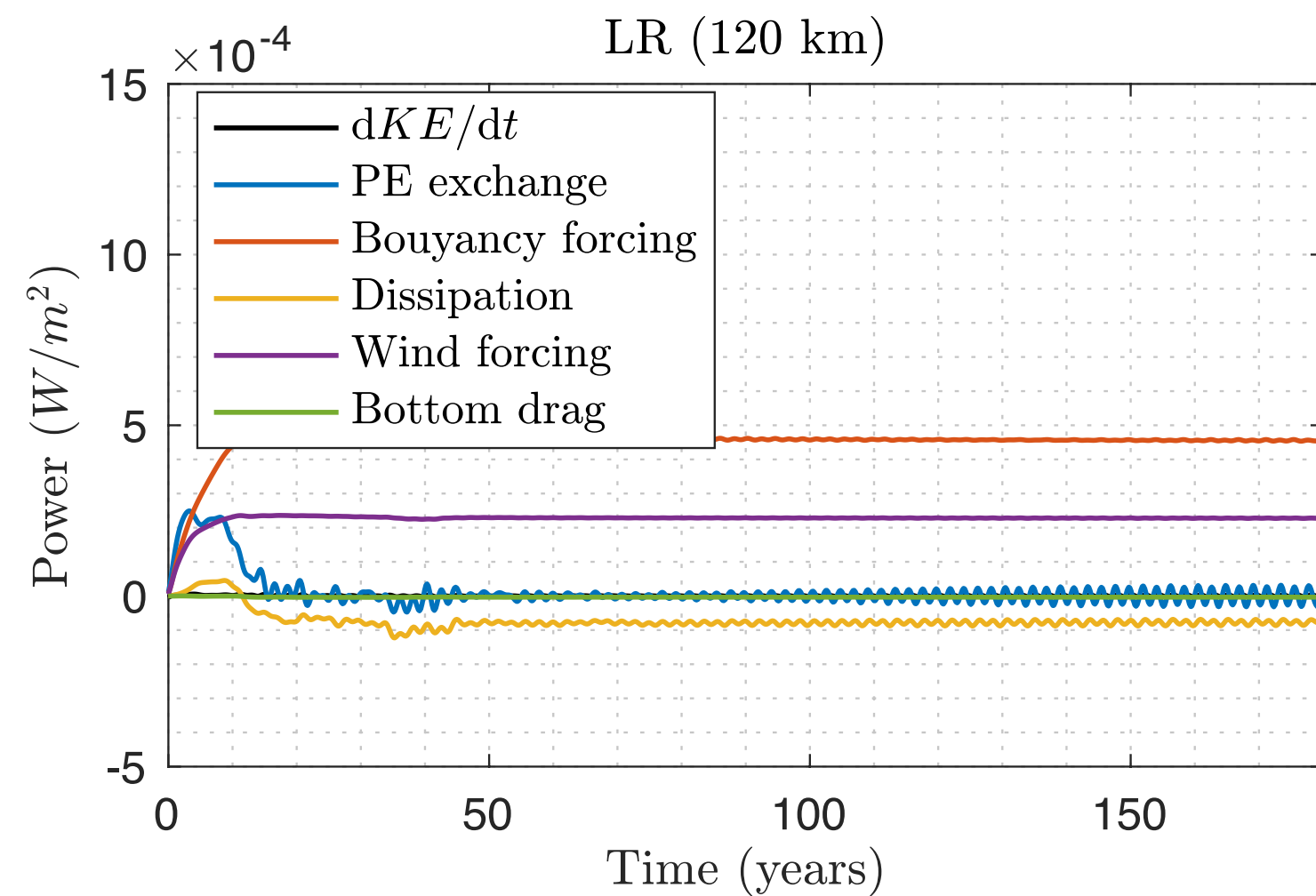
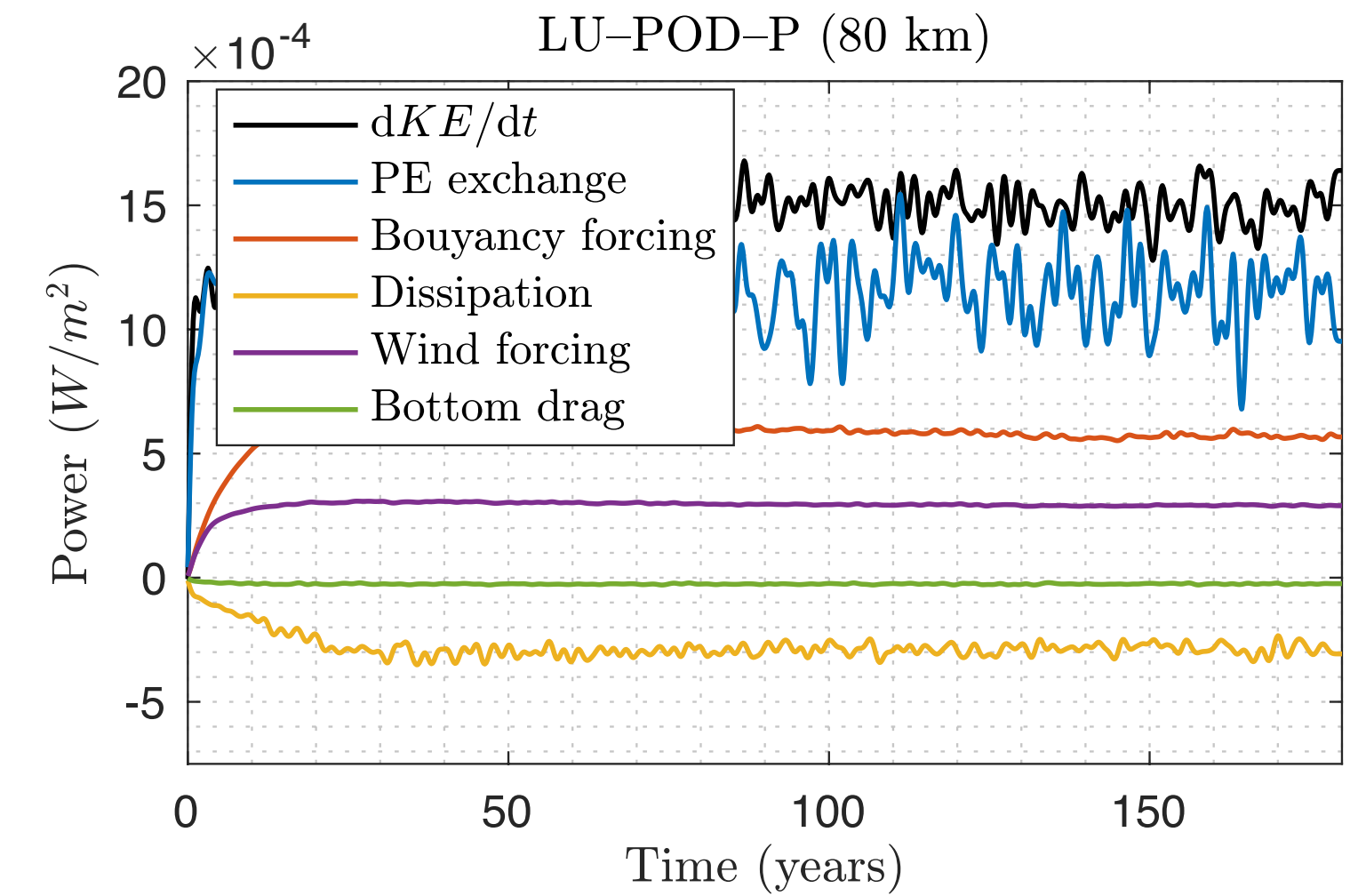
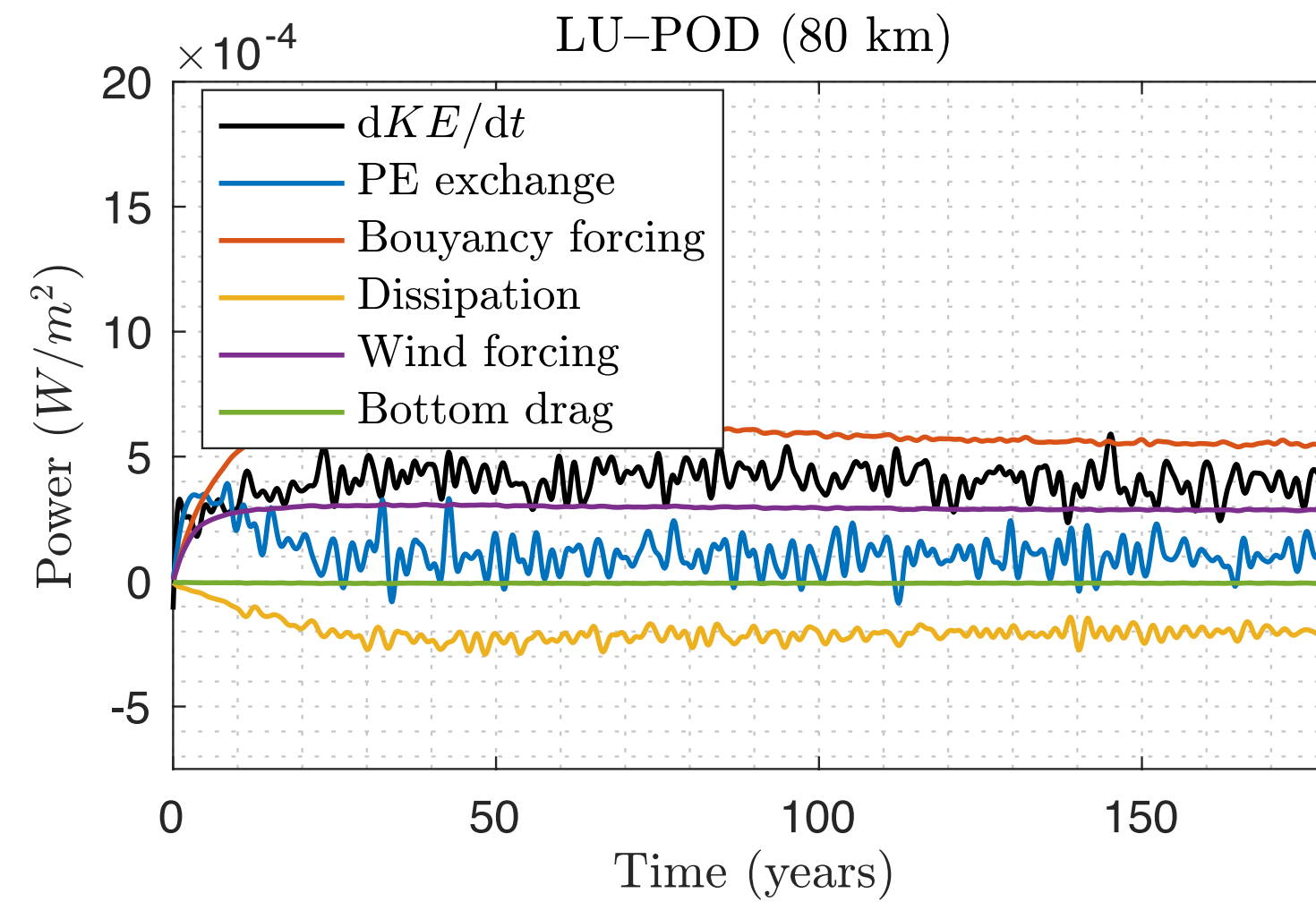
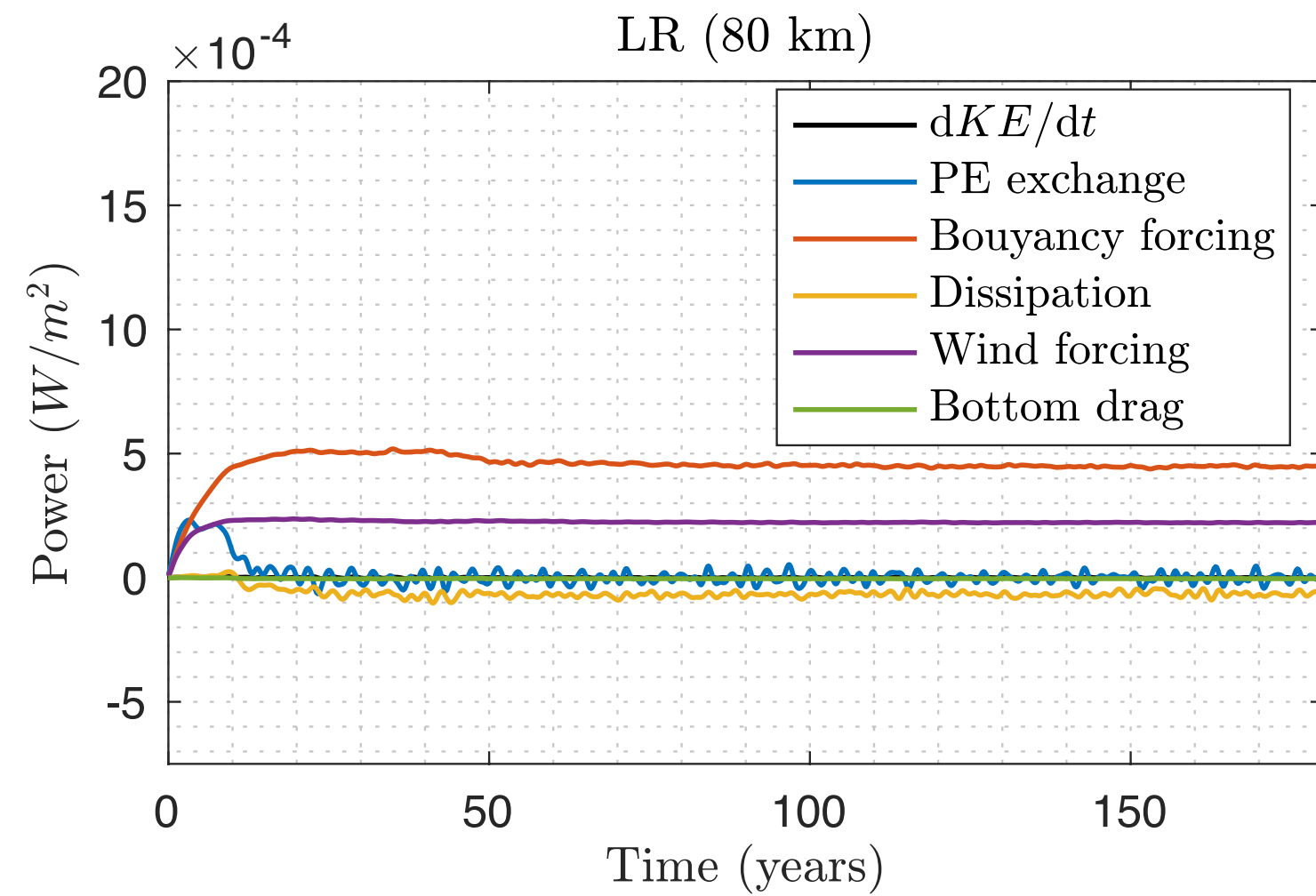
# Energy analysis



Comparison of contributions to the rate of KE for different coarse models (Exp.1)



# Energy analysis



Comparison of contributions to the rate of KE for different coarse models (Exp.2)

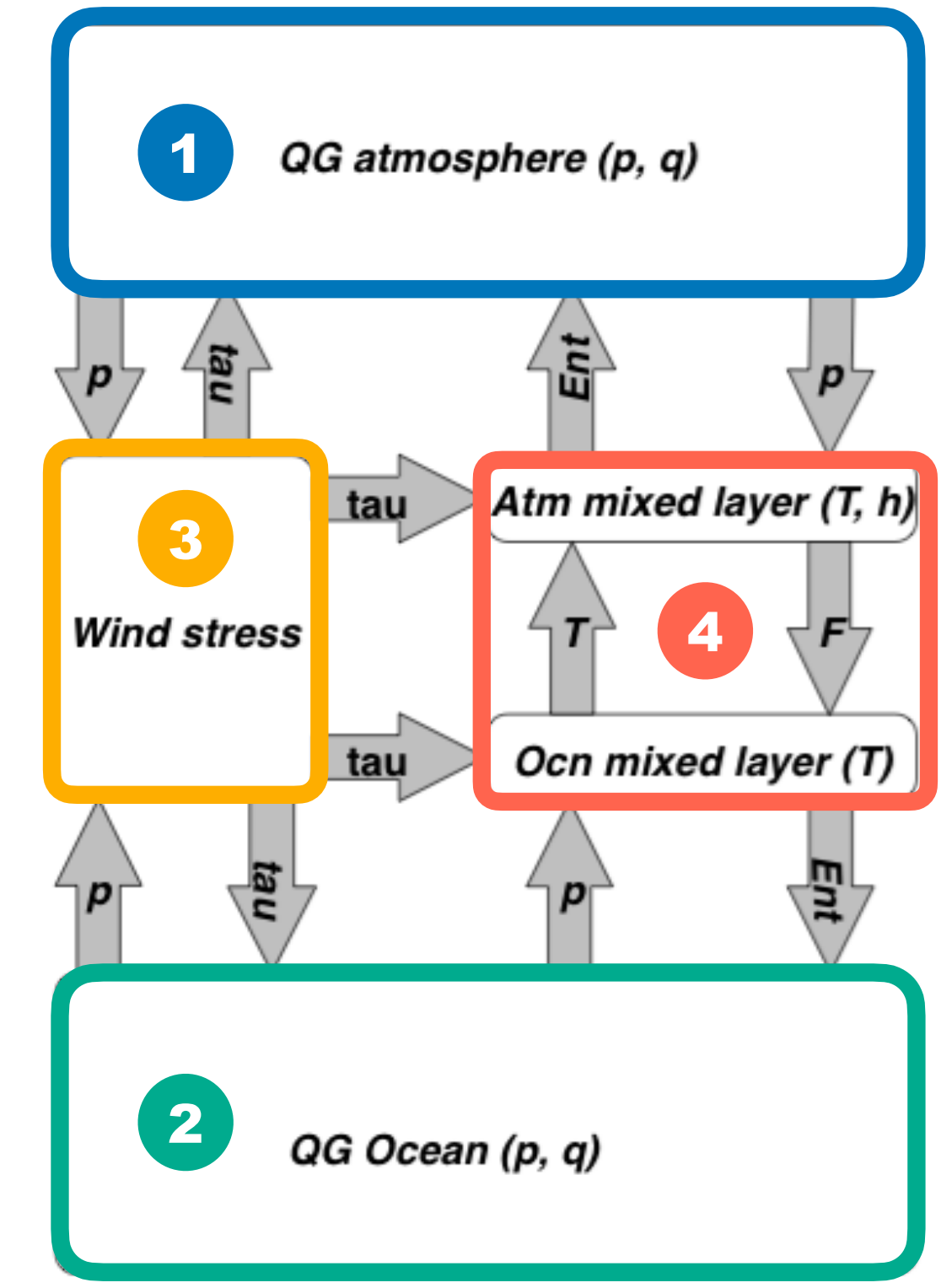
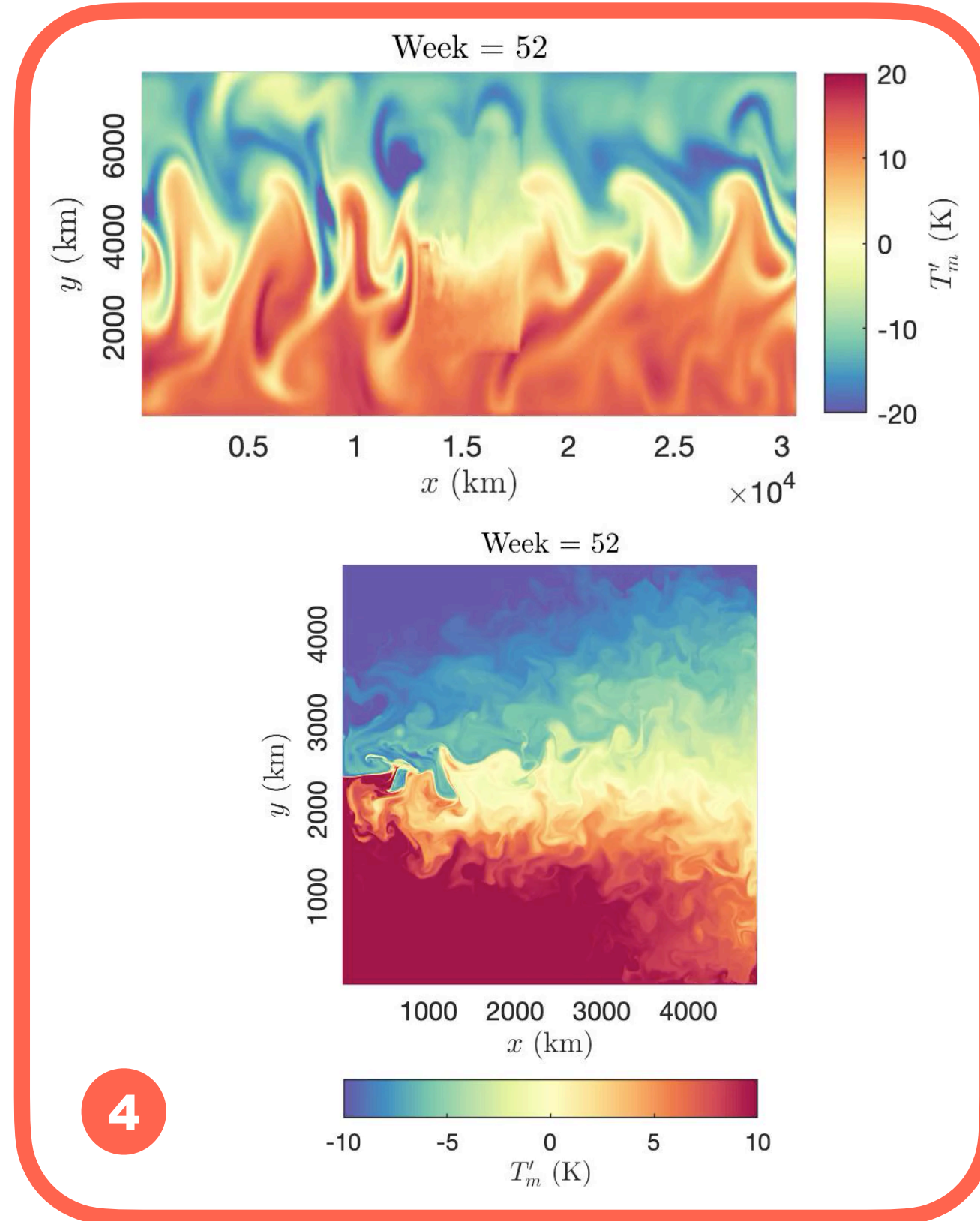
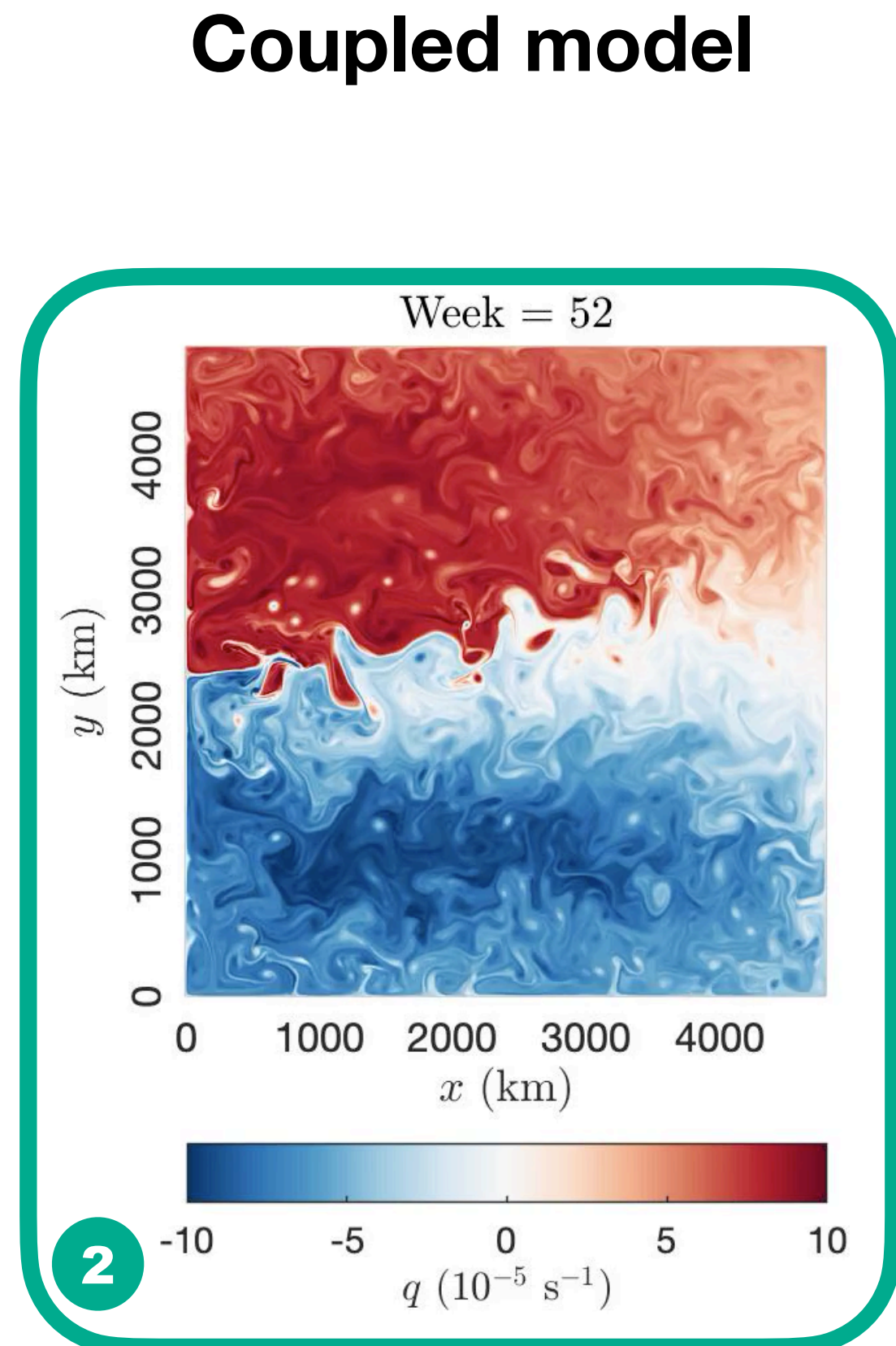
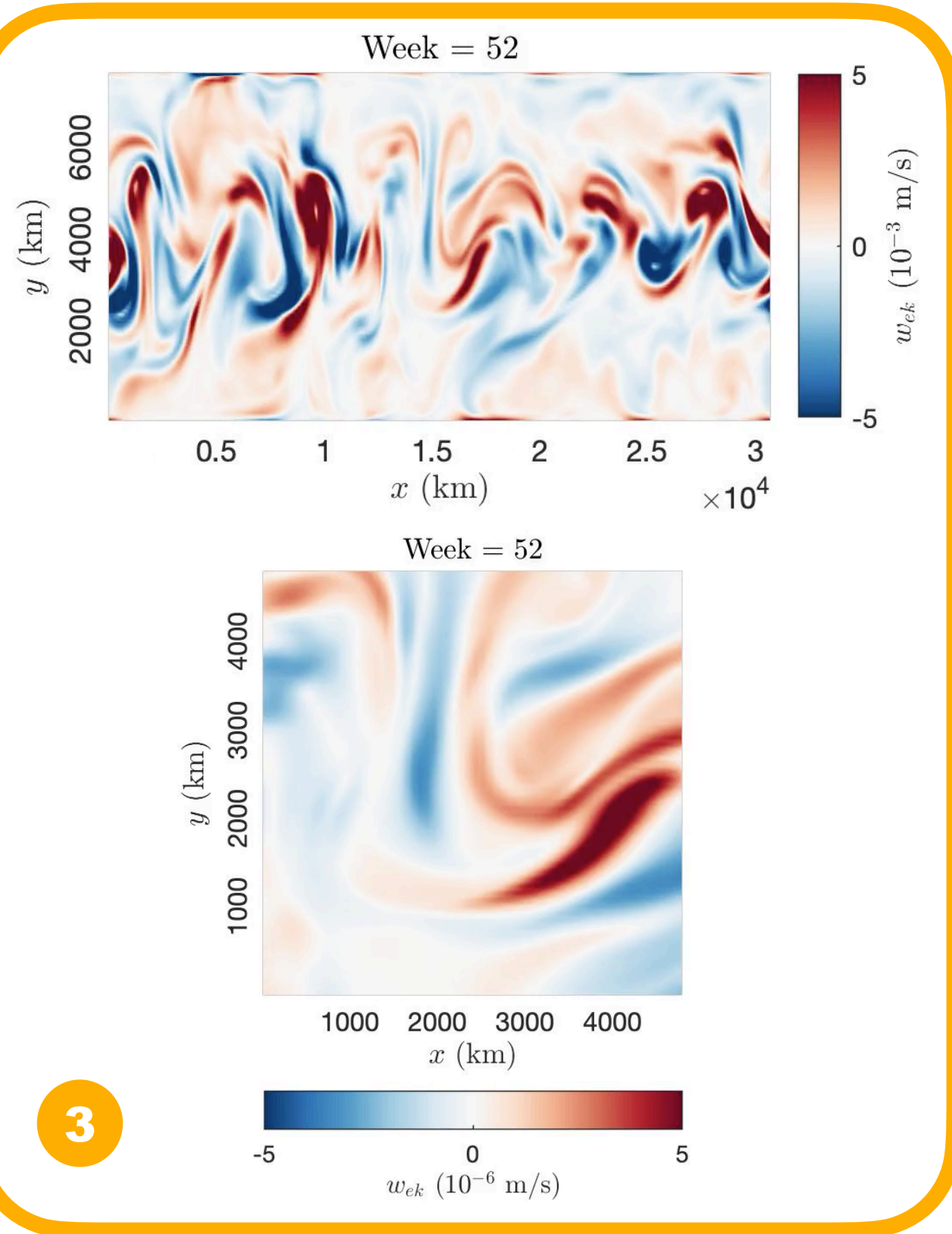
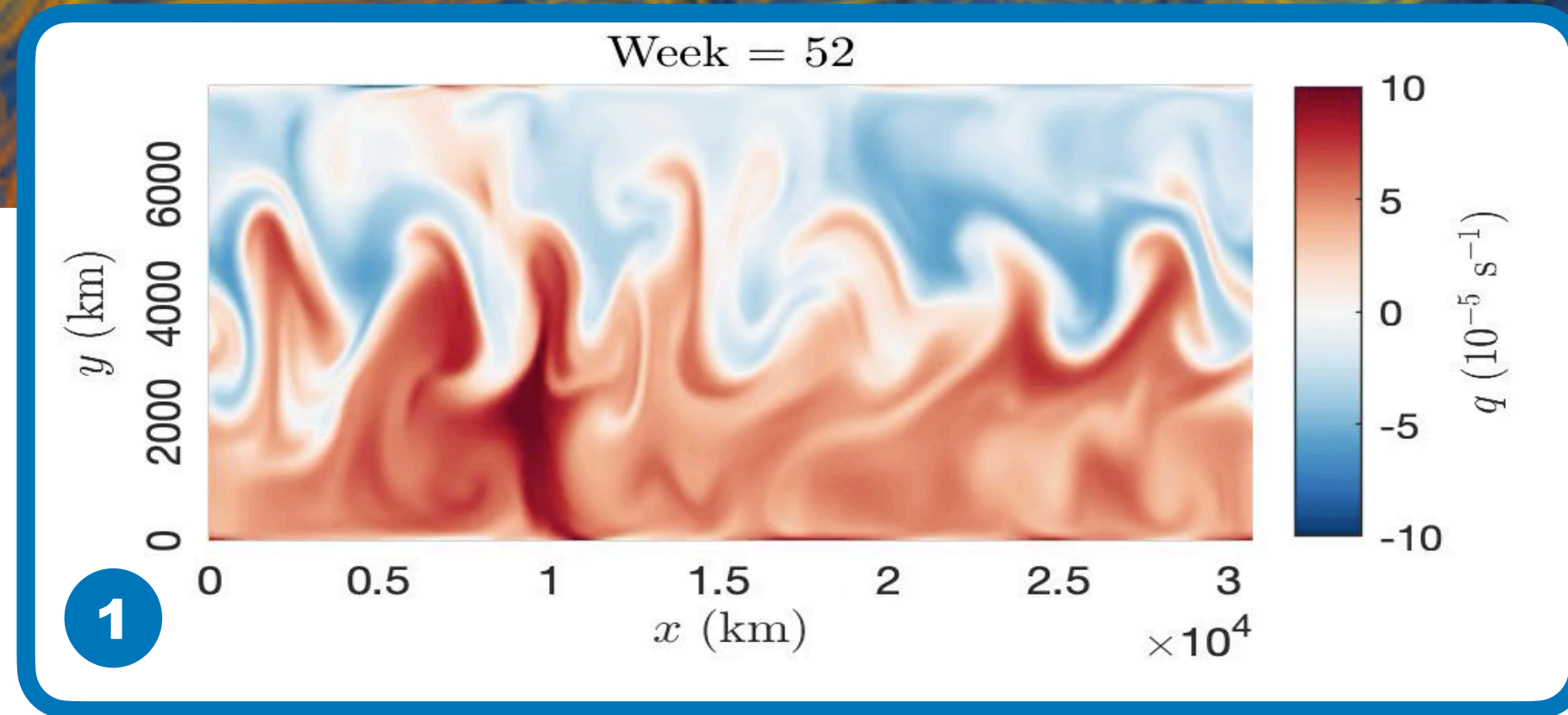


# Conclusion

- The correction drift is found to be important in reproducing the meandering jet on coarse mesh.
- The non-stationary noise based on the projection method enables us to improve the low-frequency variability of the large-scale circulation.



# Working on ...







**Thanks for your attention !**