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A guide to Edward Sang's tables and to their reconstructions

Denis Roegel

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Edward Sang (1805-1890) was probably the greatest calculator of logarithms of the 19th century [1, 2, 3, 4, 8, 9]. Sang spent 40 years computing tables of logarithms and trigonometric functions, with the assistance from his daughters Flora (1838-1925) and Jane (1834-1878). The result fills about 50 manuscript volumes, plus a number of transfer duplicates.

I have analyzed and reconstructed most of these tables, and in order to facilitate the journey through all these volumes, I provide here a guide to this collection. First, table 1 gives an overview of the main table manuscripts on logarithms and trigonometrical functions. These manuscripts are located in two different places, 42 volumes are found at the National Library of Scotland, Edinburgh (NLS), and 20 volumes at the Edinburgh University Library. In both places, there are also other volumes, but with which we are not concerned here.¹

1 The numbering of the manuscripts

Many of the manuscript volumes exist in two copies, the original manuscript, and a “transfer duplicate”. These duplicates are marked “t.d.” in my table.²

Those volumes not marked “t.d.” are original manuscripts. For instance, volume GEN 345 of the Edinburgh University Library is a transfer duplicate of volume Acc 10780/49 of the National Library of Scotland. Some of the volumes do not have transfer duplicates, for instance Acc 10780/54. All of the transfer duplicates are at Edinburgh University Library, and all but five volumes of the original manuscripts are at the National Library of Scotland. The original volumes GEN 319 to 323, kept at Edinburgh University Library, should in principle have been stored at the National Library of Scotland and may

¹Most of Sang's manuscripts have been consulted at Edinburgh University Library (July 31 and August 2, 2012) and at the National Library of Scotland (August 1–3 and 6–7, 2012). I am grateful to the libraries for facilitating the access to these archives, in particular to Derek Oliver (NLS, Manuscript and Map Collections) and Sally Pagan (Edinburgh University Library, Centre for Research Centre).

²These duplicates were produced using special thin paper and an adequate ink [10]. The thin paper was put *over* the original page, after it was completed, and the two were pressed together. The thin paper then carries a mirror image of the original page, but when looked through the other side, we have a copy of the page. This explains why the copies are made on thin translucent paper. This process is different from that of carbon paper, which would produce a positive image and not require thin paper, but would instead not work with a soft ink pen.

Sang's original denomination	Sang 1890 (Knott)	Actual volumes	
		NLS (Acc 10780)	Edinburgh Univ. Lib. (GEN)
Logarithms I	1	16	/
II	2	17	/
III	3	18	/
Logarithms. Primes.	4	19	/
Logarithms 0	5	20	/
Logarithms 1	6	21	/
Logarithms 10	7	/	319
11	8	/	320
12	9	/	321
13	10	/	322
14	11	/	323
Logarithms 10 (2nd c.)	12	22	} 310 (t.d.)
11 (2nd c.)	13	23	
12 (2nd c.)	14	24	
13 (2nd c.)	15	25	
14 (2nd c.)	16	26	} 311 (t.d.)
Logarithms 15	17	27	} 312 (t.d.)
16	18	28	
17	19	29	
18	20	30	
19	21	31	} 313 (t.d.)
20	22	32	
21	23	33	} 314 (t.d.)
22	24	34	
23	25	35	} 315 (t.d.)
24	26	36	
25	27	37	} 316 (t.d.)
26	28	38	
27	29	39	} 317 (t.d.)
28	30	40	
29	31	41	} 318 (t.d.)
30	32	42	
31	33	43	} 319 (t.d.)
32	34	44	
33	35	45	} 320 (t.d.)
34	36	46	
35	37	47	} 321 (t.d.)
36	38	48	
Log. Auxiliary table	39	49	345 (t.d.)
Sines	40	50	347 (t.d.)
Canon of sines 1'	41	51	} 343 (t.d.)
	42	52	
Log. sines and tangents	43	53	348 (t.d.)
Sines in degrees	44	54	/
Circular segments	45	55	344 (t.d.)
Mean anomalies (A)	46	56	/
Mean anomalies (B)	47	57	346 (t.d.)

Table 1: Correspondence table for Sang's manuscripts.

have been moved by mistake to Edinburgh University Library. It should also be noted that the transfer duplicates GEN 310 to 318 of Edinburgh University Library are bound differently from those of the National Library of Scotland, three volumes being bound in one. In addition to transfer duplicates, there are also volumes corresponding to two different computations: volumes Acc 10780/22 to 26 contain a second computation of the logarithms found in GEN 319 to 323, and volumes GEN 310 and 311 contain transfer duplicates of the second computation.³

Finally, there are up to four denominations of a certain table. There are first the shelf numbers of the actual volumes at the National Library of Scotland and at Edinburgh University Library. Then, there is Sang's original denomination, and finally there is Sang's 1890 numbering, taken over by Knott. For instance, the last volume of logarithms to 15 places is volume 36 in Sang's original numbering, because it contains the logarithms to 15 places of the integers 360000 to 370000. The original volume is Acc 10780/48, whereas the transfer duplicate is bound into volume GEN 318. And these volumes were given the number 38 by Sang in 1890. Since the latter numbers are the only ones covering the entire set of volumes, I will usually use them, writing K1 for the first volume, K2 for the second, up to K47 for the last one.

Knott's numbers have been added to some of the volumes. For instance, Acc 10780/16 has the mark "2.1", Acc 10780/17 has the mark "2.2", etc., Acc 10780/54 has the mark "2.44", meaning volumes 1, 2 and 44 of the second set of volumes catalogued by Knott. These are our volumes K1, K2, and K44. Knott has also organized the non tabular volumes and given them other numbers.

2 The reconstructions of the tables

During my work on Sang's tables, I have reconstructed a number of tables, and written a number of introductory notices and analyses. These are the following:

- General introduction to the computation of the logarithms of integers by Sang ([roegel2020sang-computation.pdf](#)) [15]
- Edward Sang's computation of sines ([roegel2020sang-sines.pdf](#)) [14]
- Table of logarithms to 7 places (1871) ([sang1871doc.pdf](#)) [12]
- Projected table of logarithms to 9 places (project 1872; planned, but not published; reconstructed) ([sang1872doc.pdf](#)) [11]
- Projected table of logarithms to 15 places (reconstructed, 90 volumes, corresponds to the original Sang volumes Log 10 to 99, of which only Log 10 to 36 exist) ([sang1873log15doc.pdf](#)) [20]
- All volumes K1 to K47, with approximations of K1, K2 and K3.

³Note that the page numbering of K7-K11 shows that these volumes predate K12-K16.

- K1, K2, and K3 record the steps for the construction of the logarithms of primes. These steps have approximately been reconstructed, but additional work is still required, as explained in the introduction to this reconstruction ([sang-K1-K2-K3-doc.pdf](#)) [16]
 - K4 ([sang-K4-doc.pdf](#)): table of the logarithms of the first 10000 primes [17]
 - K5 ([sang-K5-doc.pdf](#)): table of the logarithms of the first 10000 integers [18]
 - K6 ([sang-K6-doc.pdf](#)): table of the logarithms of the second 10000 integers [19]
 - K7 to K38: tables of logarithms to 15 places (files [sang1873log15vol10.pdf](#) to [sang1873log15vol199.pdf](#), as well as [sang1873log15all.pdf](#))
 - K39: A reconstruction of Edward Sang’s auxiliary table for logarithms of almost unitary values (1884) ([sang1884auxiliary-K39-doc.pdf](#)) [21]
 - K40: This volume was split in two reconstructions:
 - * A reconstruction of Edward Sang’s canon of sines (1876) ([sang1876canon-sines-K40-1-doc.pdf](#)): sines for every 25′ [22]
 - * A reconstruction of Edward Sang’s canon of sines (1877) ([sang1877canon-sines-K40-2-doc.pdf](#)): sines for every 5′ [23]
 - K41 and K42: A reconstruction of Edward Sang’s canon of sines (1881) ([sang1881canon-sines-K41-K42-doc.pdf](#)) (sines for every 1′, in two parts) [24]
 - K43: A reconstruction of Edward Sang’s table of logarithmic sines and tangents (1888) ([sang1888logsin-logtan-K43-doc.pdf](#)) [25]
 - K44: A reconstruction of Edward Sang’s table of sines in degrees (1879) ([sang1879sines-degrees-K44-doc.pdf](#)) (table giving the values of $\frac{200}{\pi} \sin \alpha$) [26]
 - K45: A reconstruction of Edward Sang’s table of circular segments (1879) ([sang1879segments-K45-doc.pdf](#)) [27]
 - K46: A reconstruction of Edward Sang’s table of mean anomalies, volume A (1880) ([sang1880anomalies-K46-doc.pdf](#)) [28]
 - K47: A reconstruction of Edward Sang’s table of mean anomalies, volume B (1880) ([sang1880anomalies-K47-doc.pdf](#)) [29]
- A reconstruction of Sang’s conversion tables between solar and sidereal time (1868) ([sang1868conversion-doc.pdf](#)) [13]

Not all is done, but all these reconstructions can now be used for a finer analysis of the original tables, and the gaps in K1, K2 and K3 should be filled. This should now be much easier than when I first worked through the tables.

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