



HAL
open science

Linear response for spiking neuronal networks with unbounded memory

Bruno Cessac, Rodrigo Cofré, Ignacio Ampuero

► **To cite this version:**

Bruno Cessac, Rodrigo Cofré, Ignacio Ampuero. Linear response for spiking neuronal networks with unbounded memory. Dynamics Days Europe, Aug 2020, Nice / Online, France. hal-02921842

HAL Id: hal-02921842

<https://inria.hal.science/hal-02921842>

Submitted on 25 Aug 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Linear response for spiking neuronal networks with unbounded memory

Bruno Cessac

Biovision Team, INRIA Sophia Antipolis, France.

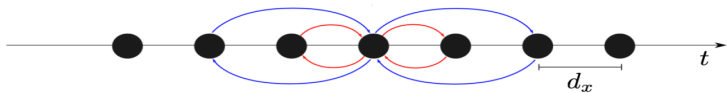
25-08-2020

with

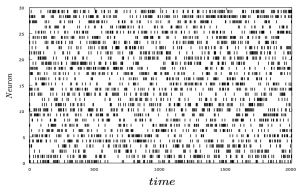
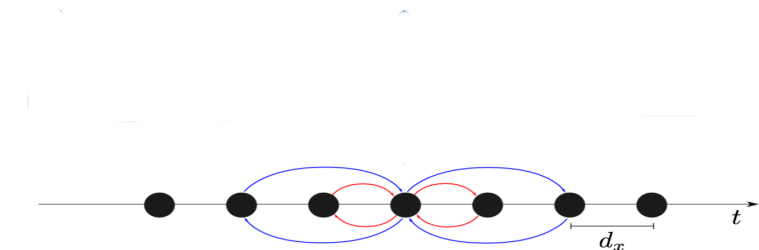
Rodrigo Cofré, CIMFAV, Facultad de Ingeniería, Universidad de Valparaíso,
Valparaíso, Chile. and

Ignacio Ampuero, Departamento de Informática, Universidad Técnica Federico Santa
María, Valparaíso, Chile

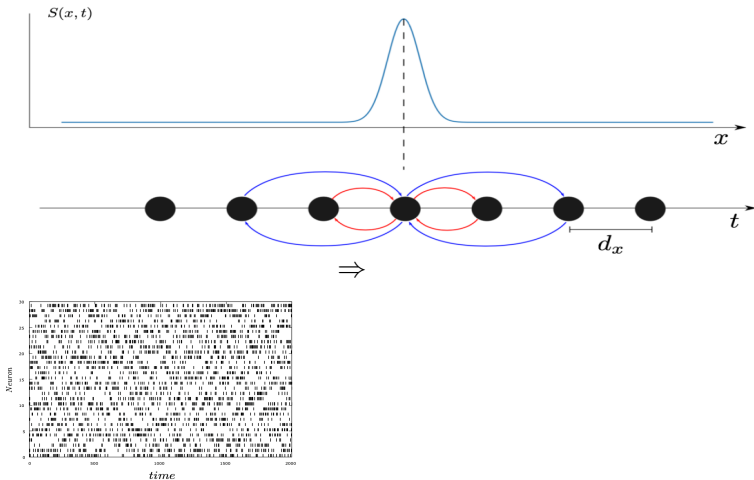
Spontaneous spiking activity



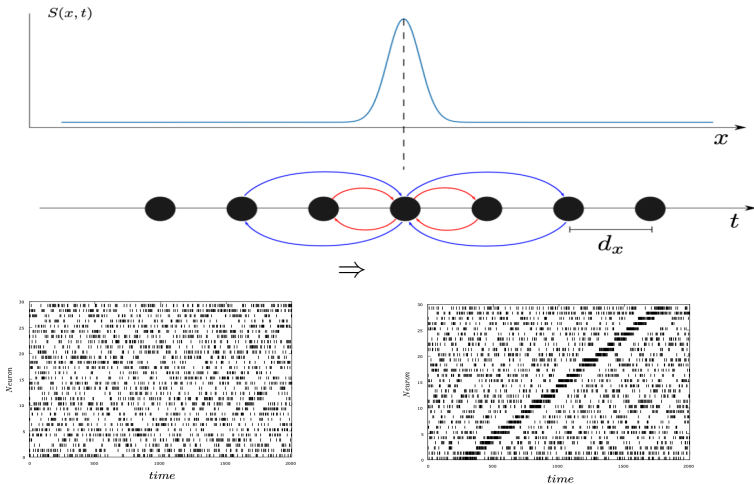
Spontaneous spiking activity



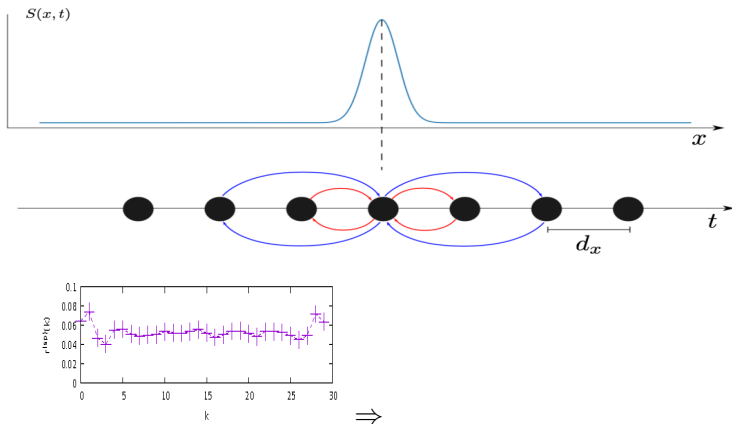
Spontaneous spiking activity



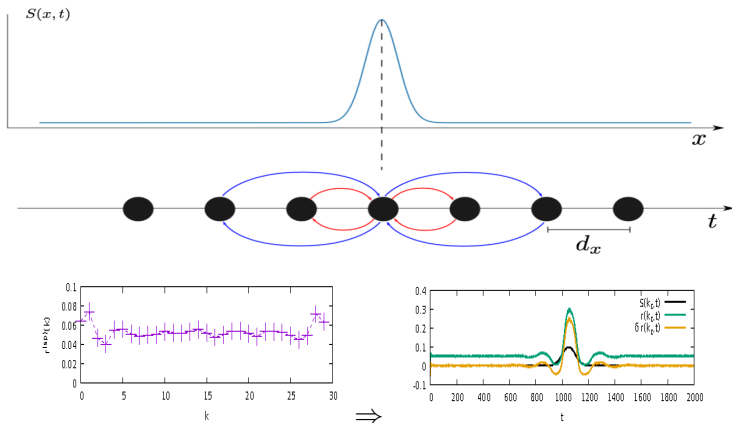
Spontaneous spiking activity



Spontaneous spiking activity

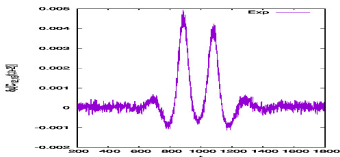
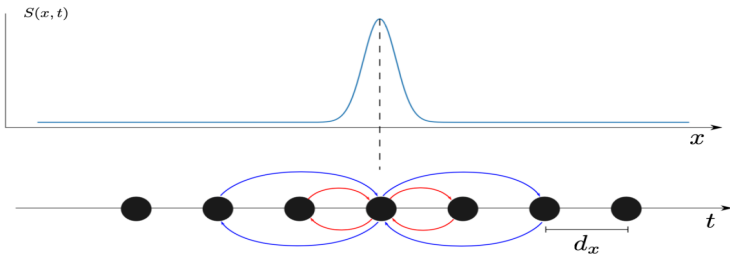


Spontaneous spiking activity



Sample averaging (non stationary)

Spontaneous spiking activity



Variation in the correlation $C_{12,15}(t, t + 2)$

Correlation variations are triggered by the stimulus, but they are constrained by the network dynamics.

Question

Question

How to compute the spatio-temporal response of a neuronal network to a time dependent stimulus ?

Question

How to compute the spatio-temporal response of a neuronal network to a time dependent stimulus ?

Can we compute the time variation of spike-observables as a function of the stimulus and spontaneous dynamics ?

Question

How to compute the spatio-temporal response of a neuronal network to a time dependent stimulus ?

Can we compute the time variation of spike-observables as a function of the stimulus and spontaneous dynamics ?

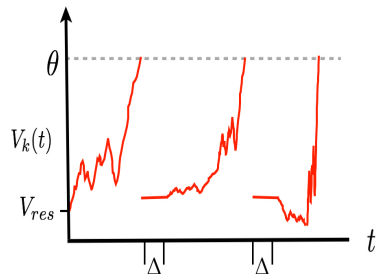
Linear response for spiking neuronal networks with unbounded memory,
Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré,

submitted to J. Math. Neuro, arXiv:1704.05344

From spiking neurons dynamics to linear response

An Integrate and Fire neural network model with unbounded memory

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013

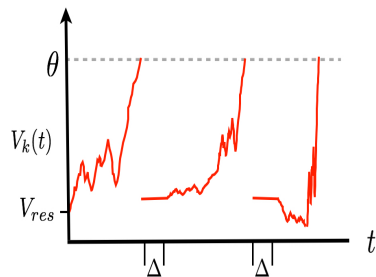


$\omega_k(n)$

0	0	1	0	0	0	0	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---	---	---

An Integrate and Fire neural network model with unbounded memory

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013



$\omega_k(n)$

0	0	1	0	0	0	0	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---	---	---

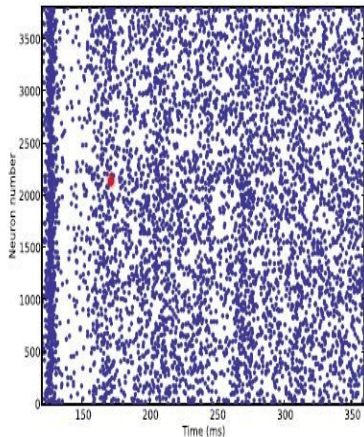
Spikes

- Voltage dynamics is time-continuous.
- Spikes are time-discrete events (time resolution $\delta > 0$).

$$t_k^{(l)} \in [n\delta, (n+1)\delta[\\ \Rightarrow \\ \omega_k(n) = 1$$

An Integrate and Fire neural network model with unbounded memory

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013

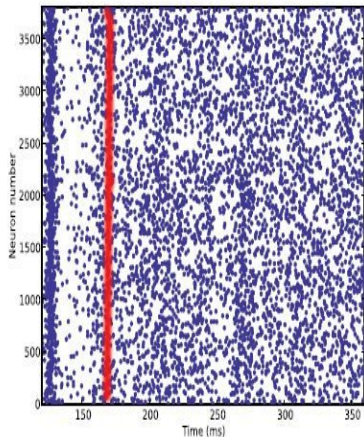


Spikes

- Voltage dynamics is time-continuous.
- Spikes are time-discrete events (time resolution $\delta > 0$).
- Spike state $\omega_k(n) \in \{0, 1\}$.

An Integrate and Fire neural network model with unbounded memory

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013

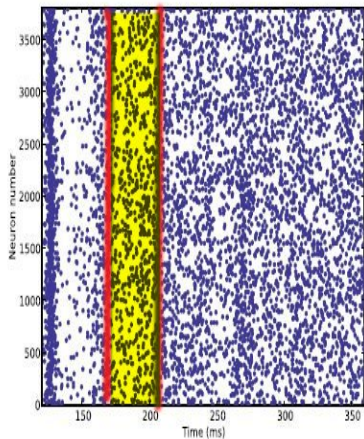


Spikes

- Voltage dynamics is time-continuous.
- Spikes are time-discrete events (time resolution $\delta > 0$).
- Spike state $\omega_k(n) \in \{0, 1\}$.
- Spike pattern $\omega(n)$.

An Integrate and Fire neural network model with unbounded memory

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013



Spikes

- Voltage dynamics is time-continuous.
- Spikes are time-discrete events (time resolution $\delta > 0$).
- Spike state $\omega_k(n) \in \{0, 1\}$.
- Spike pattern $\omega(n)$.
- Spike block ω_m^n .

A conductance-based Integrate and Fire model

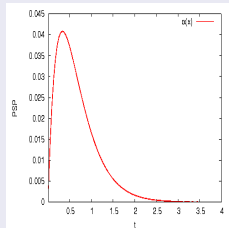
M. Rudolph, A. Destexhe, Neural Comput. 2006, (GIF model)

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} = -g_{L,k}(V_k - E_L) - \sum_j g_{kj}(t, \omega)(V_k - E_j)$$

Synapses



$$\alpha_{kj}(t) = \frac{t}{\tau} e^{-\frac{t}{\tau_{kj}}} H(t),$$

A conductance-based Integrate and Fire model

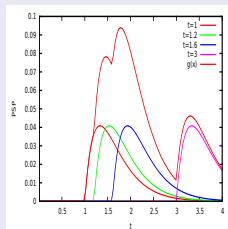
M. Rudolph, A. Destexhe, Neural Comput. 2006, (GIF model)

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} = -g_{L,k}(V_k - E_L) - \sum_j g_{kj}(t, \omega)(V_k - E_j)$$

Synapses



$$g_{kj}(t) = g_{kj}(t_j) + G_{kj}\alpha_{kj}(t - t_j)$$

$$t > t_j$$

A conductance-based Integrate and Fire model

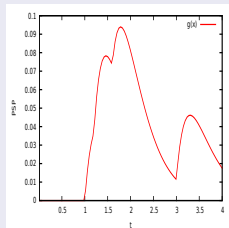
M. Rudolph, A. Destexhe, Neural Comput. 2006, (GIF model)

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} = -g_{L,k}(V_k - E_L) - \sum_j g_{kj}(t, \omega)(V_k - E_j)$$

Synapses



$$g_{kj}(t) = G_{kj} \sum_{n \geq 0} \alpha_{kj}(t - t_j^{(n)})$$

A conductance-based Integrate and Fire model

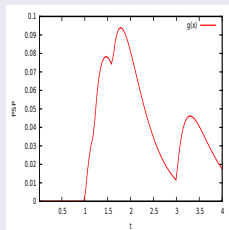
M. Rudolph, A. Destexhe, Neural Comput. 2006, (GIF model)

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} = -g_{L,k}(V_k - E_L) - \sum_j g_{kj}(t, \omega)(V_k - E_j)$$

Synapses



$$g_{kj}(t, \omega) = G_{kj} \sum_{n \geq 0} \alpha_{kj}(t - n\delta) \omega_j(n)$$

A conductance-based Integrate and Fire model

M. Rudolph, A. Destexhe, Neural Comput. 2006, (GIF model)

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013

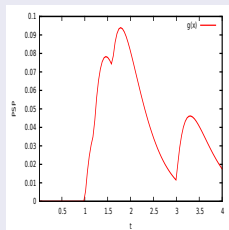
Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} = -g_{L,k}(V_k - E_L)$$

$$- \sum_j g_{kj}(t, \omega)(V_k - E_j)$$

$$+ S_k(t) + \sigma_B \xi_k(t)$$

Synapses



$$g_{kj}(t, \omega) = G_{kj} \sum_{n \geq 0} \alpha_{kj}(t - n\delta) \omega_j(n)$$

A conductance-based Integrate and Fire model

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t, \omega) V_k = i_k(t, \omega).$$

$$W_{kj} \stackrel{\text{def}}{=} G_{kj} E_j$$

$$\alpha_{kj}(t, \omega) = \sum_{n \geq 0} \alpha_{kj}(t - n\delta) \omega_j(n)$$

$$i_k(t, \omega) = g_{L,k} E_L + \sum_j W_{kj} \alpha_{kj}(t, \omega) + S_k(t) + \sigma_B \xi_k(t)$$

A conductance-based Integrate and Fire model

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t, \omega) V_k = i_k(t, \omega).$$

- Linear in V .
- Spike history-dependent.

Dynamics integration

$$\Gamma_k(t_1, t, \omega) = e^{-\frac{1}{C_k} \int_{t_1}^t V \tau_k(t, \omega) g_k(u, \omega) du}$$

A conductance-based Integrate and Fire model

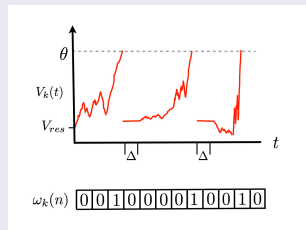
Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t, \omega) V_k = i_k(t, \omega).$$

- Linear in V .
- Spike history-dependent.

Dynamics integration

$$\Gamma_k(t_1, t, \omega) = e^{-\frac{1}{C_k} \int_{t_1}^t V \tau_k(t, \omega)} g_k(u, \omega) du$$



A conductance-based Integrate and Fire model

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t, \omega) V_k = i_k(t, \omega).$$

- Linear in V .
- Spike history-dependent.

Dynamics integration

$$\Gamma_k(t_1, t, \omega) = e^{-\frac{1}{C_k} \int_{t_1}^t V \tau_k(t, \omega) g_k(u, \omega) du}$$

$$V_k(t, \omega) = V_k^{(sp)}(t, \omega) + V_k^{(S)}(t, \omega) + V_k^{(noise)}(t, \omega)$$

A conductance-based Integrate and Fire model

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t, \omega) V_k = i_k(t, \omega).$$

- Linear in V .
- Spike history-dependent.

Dynamics integration

$$\Gamma_k(t_1, t, \omega) = e^{-\frac{1}{C_k} \int_{t_1}^t \nu \tau_k(t, \omega) g_k(u, \omega) du}$$

$$V_k(t, \omega) = \underbrace{V_k^{(sp)}(t, \omega) + V_k^{(S)}(t, \omega)}_{V_k^{(det)}(t, \omega)} + V_k^{(noise)}(t, \omega)$$

A conductance-based Integrate and Fire model

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t, \omega) V_k = i_k(t, \omega).$$

- Linear in V .
- Spike history-dependent.

Dynamics integration

$$\Gamma_k(t_1, t, \omega) = e^{-\frac{1}{C_k} \int_{t_1}^t \nu \tau_k(t, \omega) g_k(u, \omega) du}$$

$$V_k(t, \omega) = \underbrace{V_k^{(sp)}(t, \omega) + V_k^{(S)}(t, \omega)}_{V_k^{(det)}(t, \omega)} + V_k^{(noise)}(t, \omega)$$

$$V_k^{(sp)}(t, \omega) = V_k^{(syn)}(t, \omega) + V_k^{(L)}(t, \omega),$$

A conductance-based Integrate and Fire model

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t, \omega) V_k = i_k(t, \omega).$$

- Linear in V .
- Spike history-dependent.

Dynamics integration

$$\Gamma_k(t_1, t, \omega) = e^{-\frac{1}{C_k} \int_{t_1}^t V \tau_k(t, \omega)} g_k(u, \omega) du$$

$$V_k(t, \omega) = \underbrace{V_k^{(sp)}(t, \omega) + V_k^{(S)}(t, \omega)}_{V_k^{(det)}(t, \omega)} + V_k^{(noise)}(t, \omega)$$

$$V_k^{(sp)}(t, \omega) = V_k^{(syn)}(t, \omega) + V_k^{(L)}(t, \omega),$$

$$V_k^{(syn)}(t, \omega) = \frac{1}{C_k} \sum_{j=1}^N W_{kj} \int_{\tau_k(t, \omega)}^t \Gamma_k(t_1, t, \omega) \alpha_{kj}(t_1, \omega) dt_1$$

A conductance-based Integrate and Fire model

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t, \omega) V_k = i_k(t, \omega).$$

- Linear in V .
- Spike history-dependent.

Dynamics integration

$$\Gamma_k(t_1, t, \omega) = e^{-\frac{1}{C_k} \int_{t_1}^t V \tau_k(t, \omega)} g_k(u, \omega) du$$

$$V_k(t, \omega) = \underbrace{V_k^{(sp)}(t, \omega) + V_k^{(S)}(t, \omega)}_{V_k^{(det)}(t, \omega)} + V_k^{(noise)}(t, \omega)$$

$$V_k^{(sp)}(t, \omega) = V_k^{(syn)}(t, \omega) + V_k^{(L)}(t, \omega),$$

$$V_k^{(syn)}(t, \omega) = \frac{1}{C_k} \sum_{j=1}^N W_{kj} \int_{\tau_k(t, \omega)}^t \Gamma_k(t_1, t, \omega) \alpha_{kj}(t_1, \omega) dt_1$$

A conductance-based Integrate and Fire model

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t, \omega) V_k = i_k(t, \omega).$$

- Linear in V .
- Spike history-dependent.

Dynamics integration

$$\Gamma_k(t_1, t, \omega) = e^{-\frac{1}{C_k} \int_{t_1}^t \nu \tau_k(t, \omega)} g_k(u, \omega) du$$

$$V_k(t, \omega) = \underbrace{V_k^{(sp)}(t, \omega) + V_k^{(S)}(t, \omega)}_{V_k^{(det)}(t, \omega)} + V_k^{(noise)}(t, \omega)$$

$$V_k^{(sp)}(t, \omega) = V_k^{(syn)}(t, \omega) + V_k^{(L)}(t, \omega),$$

$$V_k^{(syn)}(t, \omega) = \frac{1}{C_k} \sum_{j=1}^N W_{kj} \int_{\tau_k(t, \omega)}^t \Gamma_k(t_1, t, \omega) \alpha_{kj}(t_1, \omega) dt_1$$

A conductance-based Integrate and Fire model

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k(t, \omega) V_k = i_k(t, \omega).$$

- Linear in V .
- Spike history-dependent.

Dynamics integration

$$\Gamma_k(t_1, t, \omega) = e^{-\frac{1}{C_k} \int_{t_1}^t V \tau_k(t, \omega)} g_k(u, \omega) du$$

$$V_k(t, \omega) = \underbrace{V_k^{(sp)}(t, \omega) + V_k^{(S)}(t, \omega)}_{V_k^{(det)}(t, \omega)} + V_k^{(noise)}(t, \omega)$$

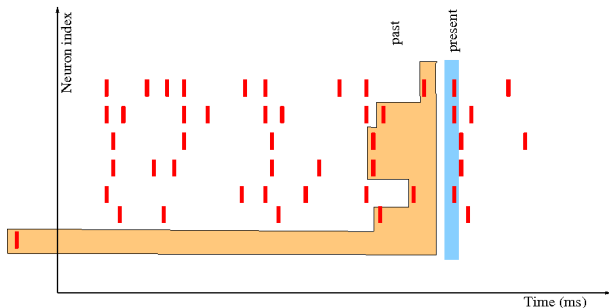
$$V_k^{(sp)}(t, \omega) = V_k^{(syn)}(t, \omega) + V_k^{(L)}(t, \omega),$$

$$V_k^{(syn)}(t, \omega) = \frac{1}{C_k} \sum_{j=1}^N W_{kj} \int_{\tau_k(t, \omega)}^t \Gamma_k(t_1, t, \omega) \alpha_{kj}(t_1, \omega) dt_1$$

A conductance-based Integrate and Fire model

Variable length Markov chain with unbounded memory

$$P_n [\omega(n) | \omega_{-\infty}^{n-1}] \equiv \Pi \left(\omega(n), \frac{\theta - V_k^{(det)}(n-1, \omega)}{\sigma_k(n-1, \omega)} \right)$$



Markov chains and Gibbs distributions

Markov chain: $P[\omega(n) \mid \omega_{n-D}^{n-1}] > 0 \Rightarrow \exists$ a unique, invariant, ergodic distribution μ .

Markov chains and Gibbs distributions

Markov chain: $P[\omega(n) \mid \omega_{n-D}^{n-1}] > 0 \Rightarrow \exists$ a unique, invariant, ergodic distribution μ .

$$\mu[\omega_m^n] = \prod_{l=m+D}^n P[\omega(l) \mid \omega_{l-D}^{l-1}] \mu[\omega_m^{m+D-1}], \quad \forall m < n \in \mathbb{Z}$$

Markov chains and Gibbs distributions

Markov chain: $P[\omega(n) \mid \omega_{n-D}^{n-1}] > 0 \Rightarrow \exists$ a unique, invariant, ergodic distribution μ .

$$\mu[\omega_m^n] = \prod_{l=m+D}^n P[\omega(l) \mid \omega_{l-D}^{l-1}] \mu[\omega_m^{m+D-1}], \quad \forall m < n \in \mathbb{Z}$$

$$\phi(\omega_{l-D}^l) = \log P[\omega(l) \mid \omega_{l-D}^{l-1}]$$

Markov chains and Gibbs distributions

Markov chain: $P \left[\omega(n) \mid \omega_{n-D}^{n-1} \right] > 0 \Rightarrow \exists$ a unique, invariant, ergodic distribution μ .

$$\mu \left[\omega_m^n \right] = \exp \sum_{l=m+D}^n \phi_l \left(\omega_{l-D}^l \right) \mu \left[\omega_m^{m+D-1} \right], \quad \forall m < n \in \mathbb{Z}$$

$$\phi \left(\omega_{l-D}^l \right) = \log P \left[\omega(l) \mid \omega_{l-D}^{l-1} \right]$$

Markov chains and Gibbs distributions

Markov chain: $P[\omega(n) \mid \omega_{n-D}^{n-1}] > 0 \Rightarrow \exists$ a unique, invariant, ergodic distribution μ .

$$\mu \left[\omega_m^n \mid \omega_m^{m+D-1} \right] = \exp \sum_{l=m+D}^n \phi_l \left(\omega_{l-D}^l \right), \quad \forall m < n \in \mathbb{Z}$$

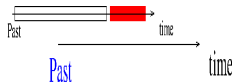
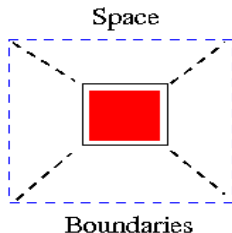
$$\phi \left(\omega_{l-D}^l \right) = \log P \left[\omega(l) \mid \omega_{l-D}^{l-1} \right]$$

Markov chains and Gibbs distributions

Markov chain: $P[\omega(n) | \omega_{n-D}^{n-1}] > 0 \Rightarrow \exists$ a unique, invariant, ergodic distribution μ .

$$\mu[\omega_m^n | \omega_m^{m+D-1}] = \exp \sum_{l=m+D}^n \phi_l(\omega_{l-D}^l), \quad \forall m < n \in \mathbb{Z}$$

$$\phi(\omega_{l-D}^l) = \log P[\omega(l) | \omega_{l-D}^{l-1}]$$



- Equilibrium stat. phys. (Max. Entropy Principle).
- Non equ. stat. phys. Onsager theory.

$$P[\omega] = \frac{1}{Z} e^{-\beta H\{\omega\}}$$

$$H\{\omega\} = \sum_{\alpha} \lambda_{\alpha} X_{\alpha}\{\omega\}$$

- Equilibrium stat. phys. (Max. Entropy Principle).
- Non equ. stat. phys. Onsager theory.

$$P[\omega] = \frac{1}{Z} e^{-\beta H\{\omega\}}$$

$$H\{\omega\} = \sum_{\alpha} \lambda_{\alpha} X_{\alpha}\{\omega\}$$

$X_{\alpha}(\omega)$ = Product of spike events

Hammersley, Clifford, 1971

- Equilibrium stat. phys. (Max. Entropy Principle).
- Non equ. stat. phys. Onsager theory.
- Ergodic theory.

$$P[\omega] = \frac{1}{Z} e^{-\beta H\{\omega\}}$$

$$H\{\omega\} = \sum_{\alpha} \lambda_{\alpha} X_{\alpha}\{\omega\}$$

$X_{\alpha}(\omega)$ = Product of spike events

Hammersley, Clifford, 1971

- Equilibrium stat. phys. (Max. Entropy Principle).
- Non equ. stat. phys. Onsager theory.
- Ergodic theory.
- Markov chains - finite memory.

$$P[\omega] = \frac{1}{Z} e^{-\beta H\{\omega\}}$$

$$H\{\omega\} = \sum_{\alpha} \lambda_{\alpha} X_{\alpha}\{\omega\}$$

$X_{\alpha}(\omega)$ = Product of spike events

Hammersley, Clifford, 1971

- Equilibrium stat. phys. (Max. Entropy Principle).
- Non equ. stat. phys. Onsager theory.
- Ergodic theory.
- Markov chains - finite memory.

- Chains with complete connections - infinite memory (Left Interval Specification).

$$P[\omega] = \frac{1}{Z} e^{-\beta H\{\omega\}}$$

$$H\{\omega\} = \sum_{\alpha} \lambda_{\alpha} X_{\alpha}\{\omega\}$$

$X_{\alpha}(\omega)$ = Product of spike events

Hammersley, Clifford, 1971

O. Onicescu and G. Mihoc. CRAS
Paris, 1935

R. Fernandez, G. Maillard, A. Le Ny,
J.R. Chazottes, ...

Response to stimuli

Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré, submitted to J. Math. Neuro, arXiv:1704.05344

Response to stimuli

Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré, submitted to J. Math. Neuro, arXiv:1704.05344

$$\phi(n, \omega) = \log P_n [\omega(n) \mid \omega_{-\infty}^{n-1}] \equiv \log \Pi \left(\omega(n), \frac{\theta - V_k^{(sp)}(t, \omega) - V_k^{(S)}(t, \omega)}{\sigma_k(n-1, \omega)} \right)$$

Response to stimuli

Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré, submitted to J. Math. Neuro, arXiv:1704.05344

$$\phi(n, \omega) = \log P_n [\omega(n) \mid \omega_{-\infty}^{n-1}] \equiv \log \Pi \left(\omega(n), \frac{\theta - V_k^{(sp)}(t, \omega) - V_k^{(S)}(t, \omega)}{\sigma_k(n-1, \omega)} \right)$$

Response to stimuli

Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré, submitted to J. Math. Neuro, arXiv:1704.05344

GIF model without stimulus \Rightarrow *stationary Gibbs distribution*.

time-dependent stimulus $S(t)$ \Rightarrow *non stationary Gibbs distribution*.

Response to stimuli

Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré, submitted to J. Math. Neuro, arXiv:1704.05344

GIF model without stimulus \Rightarrow *stationary Gibbs distribution*.

time-dependent stimulus $S(t) \Rightarrow$ *non stationary Gibbs distribution*.

How is the average of an observable $f(\omega, t)$ affected by the stimulus ?

Response to stimuli

Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré, submitted to J. Math. Neuro, arXiv:1704.05344

GIF model without stimulus \Rightarrow *stationary Gibbs distribution*.

time-dependent stimulus $S(t) \Rightarrow$ *non stationary Gibbs distribution*.

How is the average of an observable $f(\omega, t)$ affected by the stimulus ?

If S is weak enough: $\delta\mu[f(t)] = [\kappa_f * S](t)$, (linear response).

Response to stimuli

Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré, submitted to J. Math. Neuro, arXiv:1704.05344

GIF model without stimulus \Rightarrow *stationary Gibbs distribution*.

time-dependent stimulus $S(t) \Rightarrow$ *non stationary Gibbs distribution*.

How is the average of an observable $f(\omega, t)$ affected by the stimulus ?

If S is weak enough: $\delta\mu[f(t)] = [\kappa_f * S](t)$, (linear response).

$$\kappa_{k,f}(t - t_1) = \frac{1}{C_k} \sum_{r=-\infty}^{n=[t]} \mathcal{C}^{(sp)} \left[f(t, \cdot), \frac{\mathcal{H}_k^{(1)}(r, \cdot)}{\sigma_k(r-1, \cdot)} \int_{\tau_k(r-1, \cdot)}^{r-1} S_k(t_1) \Gamma_k(t_1, r-1, \cdot) dt_1 \right]$$

Response to stimuli

Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré, submitted to J. Math. Neuro, arXiv:1704.05344

GIF model without stimulus \Rightarrow *stationary Gibbs distribution*.

time-dependent stimulus $S(t) \Rightarrow$ *non stationary Gibbs distribution*.

How is the average of an observable $f(\omega, t)$ affected by the stimulus ?

If S is weak enough: $\delta\mu[f(t)] = [\kappa_f * S](t)$, (linear response).

$$\kappa_{k,f}(t - t_1) = \frac{1}{C_k} \sum_{r=-\infty}^{n=[t]} C^{(sp)} \left[f(t, \cdot), \frac{\mathcal{H}_k^{(1)}(r, \cdot)}{\sigma_k(r-1, \cdot)} \int_{\mathcal{T}_k(r-1, \cdot)}^{r-1} S_k(t_1) \Gamma_k(t_1, r-1, \cdot) dt_1 \right]$$

History dependence.

Response to stimuli

Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré, submitted to J. Math. Neuro, arXiv:1704.05344

GIF model without stimulus \Rightarrow *stationary Gibbs distribution*.

time-dependent stimulus $S(t) \Rightarrow$ *non stationary Gibbs distribution*.

How is the average of an observable $f(\omega, t)$ affected by the stimulus ?

If S is weak enough: $\delta\mu[f(t)] = [\kappa_f * S](t)$, (linear response).

$$\kappa_{k,f}(t - t_1) = \frac{1}{C_k} \sum_{r=-\infty}^{n=[t]} \mathcal{C}^{(sp)} \left[f(t, \cdot), \frac{\mathcal{H}_k^{(1)}(r, \cdot)}{\sigma_k(r-1, \cdot)} \int_{\tau_k(r-1, \cdot)}^{r-1} S_k(t_1) \Gamma_k(t_1, r-1, \cdot) dt_1 \right]$$

History dependence, observable

Response to stimuli

Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré, submitted to J. Math. Neuro, arXiv:1704.05344

GIF model without stimulus \Rightarrow *stationary Gibbs distribution*.

time-dependent stimulus $S(t) \Rightarrow$ *non stationary Gibbs distribution*.

How is the average of an observable $f(\omega, t)$ affected by the stimulus ?

If S is weak enough: $\delta\mu[f(t)] = [\kappa_f * S](t)$, (linear response).

$$\kappa_{k,f}(t - t_1) = \frac{1}{C_k} \sum_{r=-\infty}^{n=[t]} C^{(sp)} \left[f(t, \cdot), \frac{\mathcal{H}_k^{(1)}(r, \cdot)}{\sigma_k(r-1, \cdot)} \right] \int_{\tau_k(r-1, \cdot)}^{r-1} S_k(t_1) \Gamma_k(t_1, r-1, \cdot) dt_1$$

History dependence, observable, network dynamics

Response to stimuli

Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré, submitted to J. Math. Neuro, arXiv:1704.05344

GIF model without stimulus \Rightarrow *stationary Gibbs distribution*.

time-dependent stimulus $S(t) \Rightarrow$ *non stationary Gibbs distribution*.

How is the average of an observable $f(\omega, t)$ affected by the stimulus ?

If S is weak enough: $\delta\mu[f(t)] = [\kappa_f * S](t)$, (linear response).

$$\kappa_{k,f}(t - t_1) = \frac{1}{C_k} \sum_{r=-\infty}^{n=[t]} \mathcal{C}^{(sp)} \left[f(t, \cdot), \frac{\mathcal{H}_k^{(1)}(r, \cdot)}{\sigma_k(r-1, \cdot)} \int_{\tau_k(r-1, \cdot)}^{r-1} S_k(t_1) \Gamma_k(t_1, r-1, \cdot) dt_1 \right]$$

Spontaneous correlation between observable and network dynamics

An illustrative example

The simplest example of the gIF model is the ... discrete time
leaky-integrate and fire model

$$V_k(n+1) = \gamma V_k(n) + \sum_j W_{kj} \omega_j(n) + I_0 + S_k(t) + \sigma_B \xi_k(n), \quad \text{if } V_k(n) < \theta$$

Linear response

$$\delta\mu^{(1)} [f(n)] \sim - \sum_{k=1}^N \sum_{m=1}^{D+1} \sum_{l=0}^{\frac{1}{\nu_k}} \gamma^l \mathcal{K}_{km}^{(1)} S_k(n-m-l)$$

$$\mathcal{K}_{k,m}^{(1)} = \mathcal{C}^{(sp)} [f(m, \cdot), \zeta_k(0, \cdot)]$$

$$\zeta_k(r-1, \omega) = \frac{\mathcal{H}_k^{(1)}(r, \omega)}{\sigma_k(r-1, \omega)}$$

Firing rates

Computations have been done using the INRIA Pranas software, B. Cessac et al, *Frontiers in Neuroinformatics*, Vol 11, page 49, (2017).

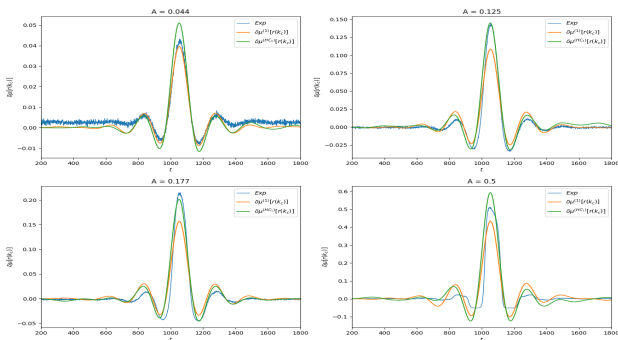


Figure: Linear response of $f(\omega, n) = \omega_{k_c}(n)$ for different values of stimulus amplitude A . From <https://arxiv.org/abs/1704.05344>

Pairwise correlations

Computations have been done using the INRIA Pranas software, B. Cessac et al, *Frontiers in Neuroinformatics*, Vol 11, page 49, (2017).

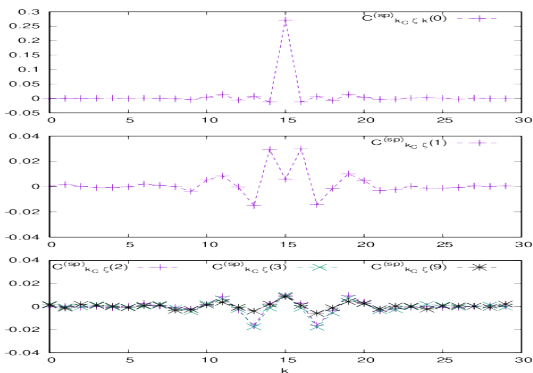


Figure: Correlation functions corresponding to the firing rate of the neuron $k_C = \frac{N}{2}$ as a function of the neuron index k (abscissa), for different values of the time delay m . From <https://arxiv.org/abs/1704.05344>

Higher order observable

Computations have been done using the INRIA Pranas software, B. Cessac et al, *Frontiers in Neuroinformatics*, Vol 11, page 49, (2017).

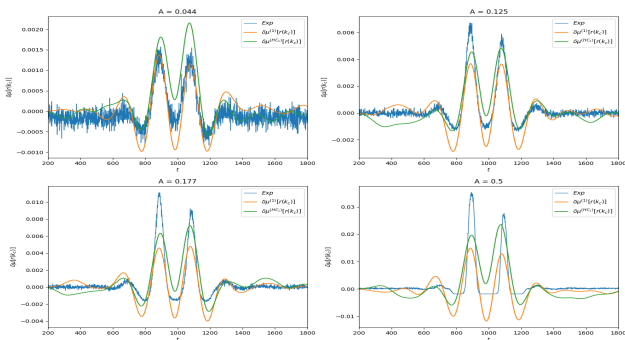


Figure: Linear response of the observable $f(n, \omega) = \omega_{k_C-2}(n-3)\omega_{k_C}(n)$.

Range of validity

Computations have been done using the INRIA Pranas software, B. Cessac et al, *Frontiers in Neuroinformatics*, Vol 11, page 49, (2017).

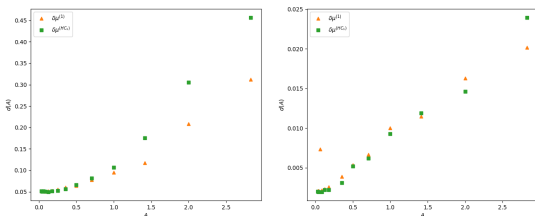


Figure: L^2 distance between the curves $\delta\mu^{(1)} [f(n)]$, $\delta\mu^{(HC_1)} [f(n)]$ and the empirical curve, as a function of the stimulus amplitude A . Left panel show distance between rate curves and right panel distance between pairwise observable with delay.

Main conclusions

Main conclusions

- The response to a time dependent stimulus is expressed in terms of correlations of the unperturbed dynamics.

Main conclusions

- The response to a time dependent stimulus is expressed in terms of correlations of the unperturbed dynamics.
- Linear response is characterized by a convolution kernel depending on the observable, on the network dynamics and the afferent Gibbs distribution.

Main conclusions

- The response to a time dependent stimulus is expressed in terms of correlations of the unperturbed dynamics.
- Linear response is characterized by a convolution kernel depending on the observable, on the network dynamics and the afferent Gibbs distribution.
 - How to determine the potential (from data) ?

Main conclusions

- The response to a time dependent stimulus is expressed in terms of correlations of the unperturbed dynamics.
- Linear response is characterized by a convolution kernel depending on the observable, on the network dynamics and the afferent Gibbs distribution.
 - How to determine the potential (from data) ?
The potential is shaped from dynamics.

Main conclusions

- The response to a time dependent stimulus is expressed in terms of correlations of the unperturbed dynamics.
- Linear response is characterized by a convolution kernel depending on the observable, on the network dynamics and the afferent Gibbs distribution.
 - How to determine the potential (from data) ?

The potential is shaped from dynamics. There is no "thermodynamic principle" to constrain its shape.

Main conclusions

- The response to a time dependent stimulus is expressed in terms of correlations of the unperturbed dynamics.
- Linear response is characterized by a convolution kernel depending on the observable, on the network dynamics and the afferent Gibbs distribution.
 - How to determine the potential (from data) ?

The potential is shaped from dynamics. There is no "thermodynamic principle" to constrain its shape.

Hammersley-Clifford expansion, whose first non trivial term is the "Ising" term, contains too much terms. (Cofre, Cessac, PRE, 2014; arXiv:1405.3621)

Main conclusions

- The response to a time dependent stimulus is expressed in terms of correlations of the unperturbed dynamics.
- Linear response is characterized by a convolution kernel depending on the observable, on the network dynamics and the afferent Gibbs distribution.

- How to determine the potential (from data) ?

The potential is shaped from dynamics. There is no "thermodynamic principle" to constrain its shape.

Hammersley-Clifford expansion, whose first non trivial term is the "Ising" term, contains too much terms. (Cofre, Cessac, PRE, 2014; arXiv:1405.3621)

Which ones are relevant ? \Rightarrow Requires a method to reduce the "dimensionality of the potential"

Main conclusions

- The response to a time dependent stimulus is expressed in terms of correlations of the unperturbed dynamics.
- Linear response is characterized by a convolution kernel depending on the observable, on the network dynamics and the afferent Gibbs distribution.
 - How to determine the potential (from data) ?
 - Higher order terms in the response ?

Main conclusions

- The response to a time dependent stimulus is expressed in terms of correlations of the unperturbed dynamics.
- Linear response is characterized by a convolution kernel depending on the observable, on the network dynamics and the afferent Gibbs distribution.
 - How to determine the potential (from data) ?
 - Higher order terms in the response ?
 - Similar to Volterra-expansion.
 - Higher order terms computed by D. Ruelle (Nonlinearity, 11(1):518, 1998.) for hyperbolic dynamical systems \Rightarrow Complex expansion hard to handle from data.
 - Sigmoid are "hard" to approximate with Taylor expansions.

Main conclusions

- The response to a time dependent stimulus is expressed in terms of correlations of the unperturbed dynamics.
- Linear response is characterized by a convolution kernel depending on the observable, on the network dynamics and the afferent Gibbs distribution.
 - How to determine the potential (from data) ?
 - Higher order terms in the response ?
- Toward an extension of the concept of receptive-field ?