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Oceanic Dynamics under Location Uncertainty

Towards a consistent stochastic modelling

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- **Why stochastic modeling ?**

- **Take into account small-scale / unresolved processes**

- **Uncertainty Quantification, Ensemble Forecasts**

- **Why consistent model ?**

- **Both scales must respect appreciate physical laws**

- **Losing consistency may provide wrong statistics**

1 Governing Equations

- Location Uncertainty Principles
- Barotropic Quasi–Geostrophic Model under LU

2 Experimental Evaluations

- Structure—preserving of Rossby wave
- Ensemble forecasting verification of SQG_{MU}
- Time—statistics of wind—driven circulation

3 Summary

- **Stochastic flow :**

$$d\mathbf{X}_t = \underbrace{\mathbf{u}(\mathbf{X}_t, t)dt}_{\text{large-scale resolved}} + \underbrace{\boldsymbol{\sigma}(\mathbf{X}_t, t)d\mathbf{B}_t}_{\text{small-scale unresolved}}$$

- **Functional process :**

$$\boldsymbol{\sigma}(\mathbf{x}, t)d\mathbf{B}_t = \int_D \underbrace{\check{\boldsymbol{\sigma}}(\mathbf{x}, \mathbf{y}, t)}_{\text{symmetric kernel}} d\mathbf{B}_t(\mathbf{y})d\mathbf{y}$$

correlated in space
uncorrelated in time
space-time white noise

- **Variance tensor :**

$$\mathbf{a} \triangleq \boldsymbol{\sigma}\boldsymbol{\sigma}^T = \mathbb{E}[\boldsymbol{\sigma}d\mathbf{B}_t(\boldsymbol{\sigma}d\mathbf{B}_t)^T] / dt$$

homogeneous / heterogeneous

- **Transport of a random tracer :**

$$(\nabla \cdot \boldsymbol{\sigma} = 0)$$

Balanced Energy

➔

$\frac{d}{dt} \int_D \frac{1}{2} \theta^2 = 0$

($\nabla \cdot \mathbf{u}^* = 0$)

$$D_t \theta \triangleq d_t \theta + \underbrace{\mathbf{u}^* \cdot \nabla \theta dt}_{\text{corrected drift}} + \underbrace{\boldsymbol{\sigma} d\mathbf{B}_t \cdot \nabla \theta}_{\text{multiplicative noise}} - \underbrace{\frac{1}{2} \nabla \cdot (\mathbf{a} \nabla \theta) dt}_{\text{subgrid diffusion}} = 0$$

$\mathbf{u} - \frac{1}{2} \nabla \cdot \mathbf{a}$

- **Evolution of the potential vorticity with source processes :**

$$D_t q = S_1(\nabla \mathbf{u}) dt + S_2(\nabla \mathbf{u}) d\mathbf{B}_t$$

- **Evolution of the stream function :**

$$q = \Delta \psi - \psi / L_R^2 + f$$

- **Strong incompressible constraints :**

$$\mathbf{u} = \nabla^\perp \psi, \quad \sigma d\mathbf{B}_t = \nabla^\perp \varphi d\mathbf{B}_t, \quad \nabla \cdot \mathbf{u}^* = 0$$

- ➔ **Conservation of total energy :**

$$\frac{d}{dt} \int_D \frac{1}{2} \left[\|\nabla \psi\|^2 + (\psi / L_R^2)^2 \right] = 0$$

$$\left[\text{Sketch : } \int_D \psi \sigma \circ d\mathbf{B}_t \cdot \nabla (q - f) = \int_D \psi S_2(\nabla \mathbf{u}) \circ d\mathbf{B}_t \right]$$

- **Transport of a prognostic passive tracer :**

$$D_t q' = 0$$

- ➔ **Conservation of tracer energy :**

$$\frac{d}{dt} \int_D \frac{1}{2} (q')^2 = 0$$

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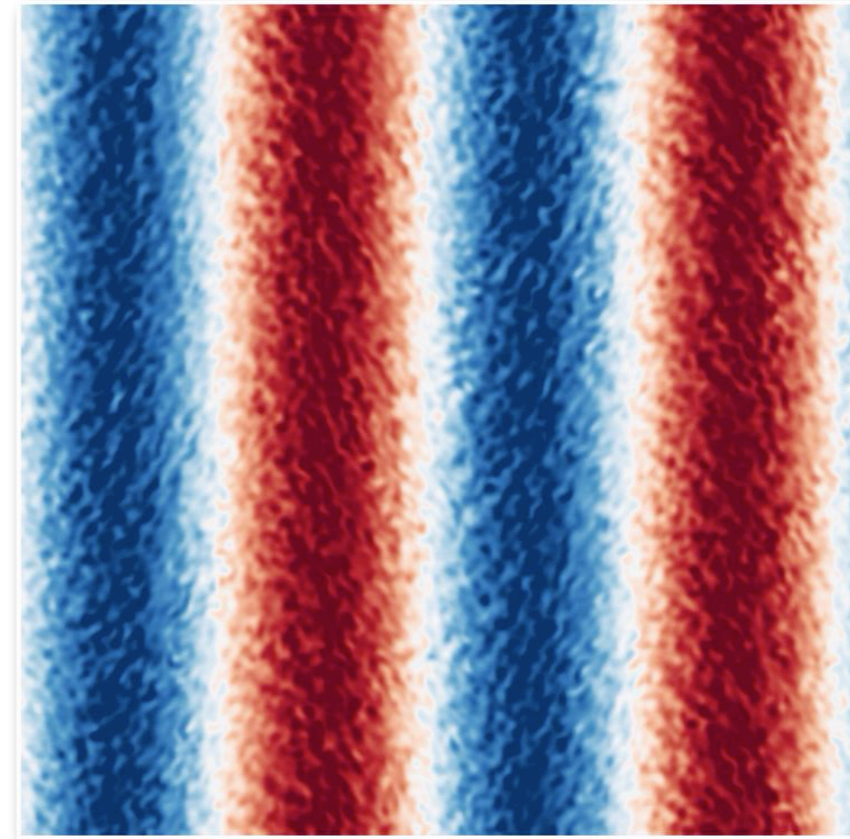
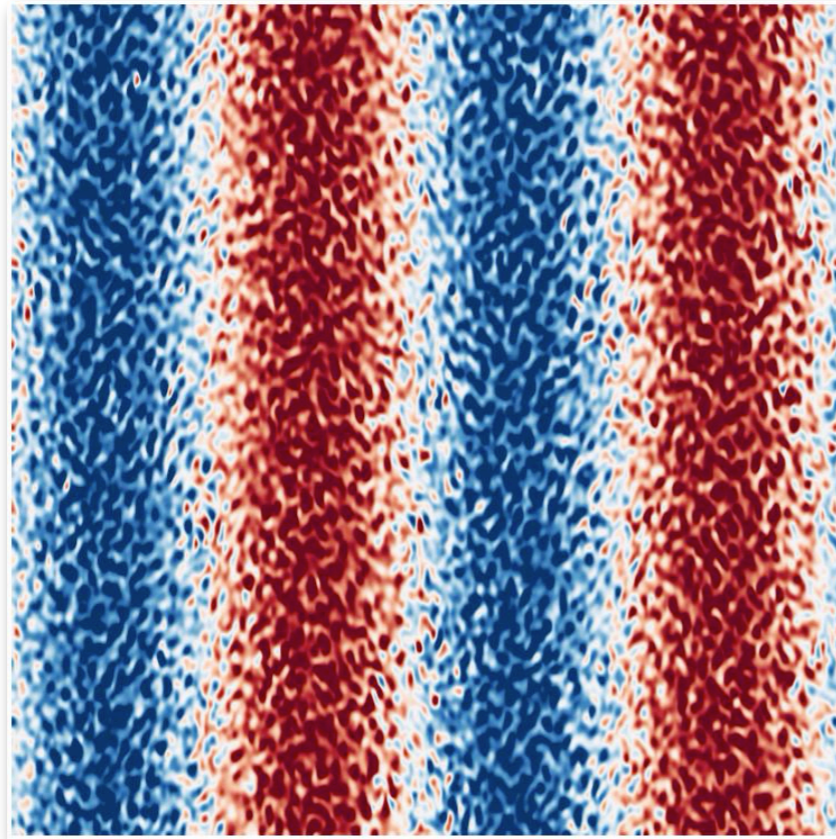
2 Experimental Evaluations

- Structure—preserving of Rossby wave
- Ensemble forecasting verification of SQG_{MU}
- Time—statistics of wind—driven circulation

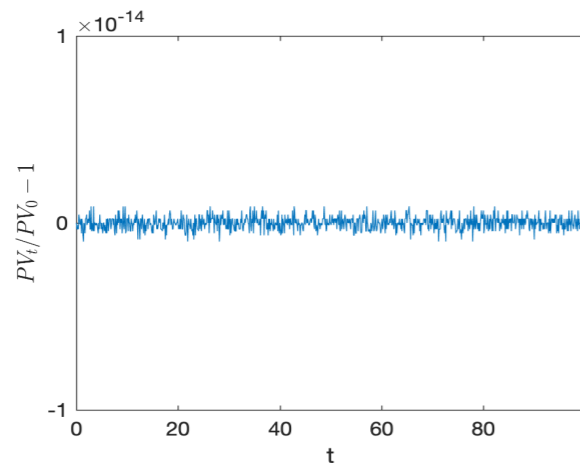
3 Summary

Structure-preserving of Rossby wave

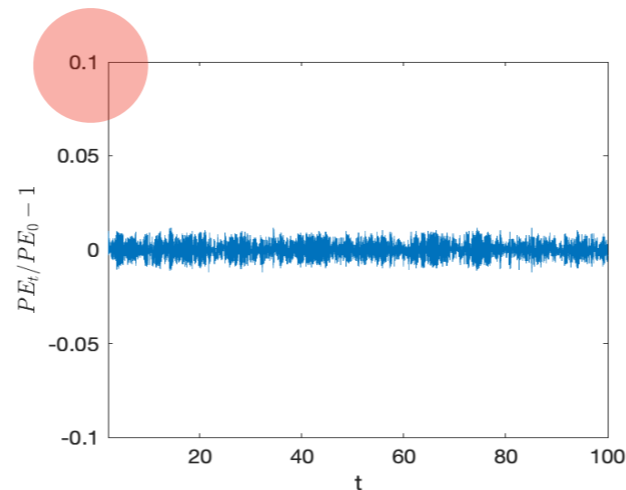
- Flowchart :



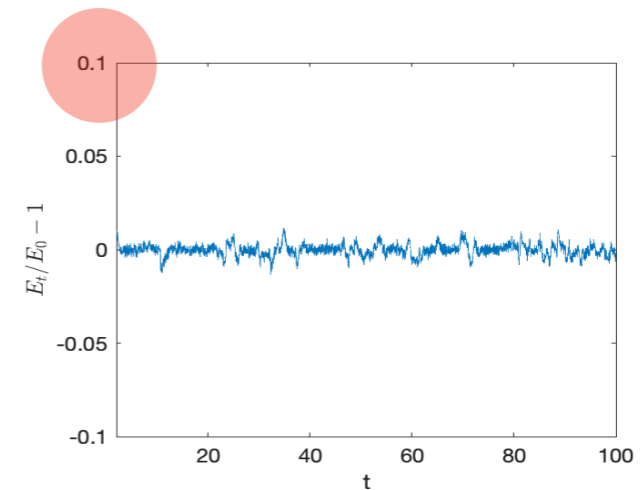
- Illustration of conservations :



Passive Tracer



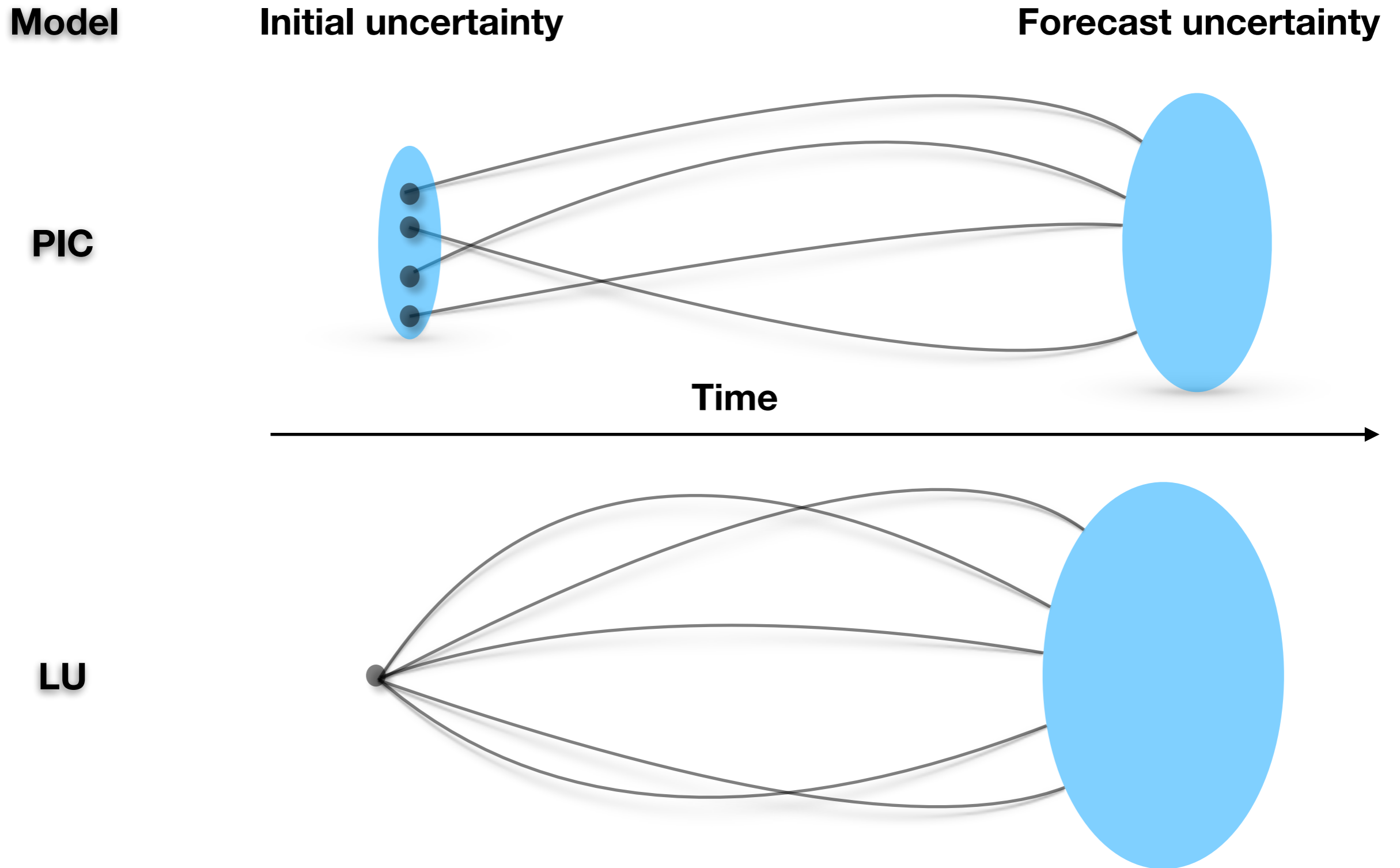
Tracer Energy



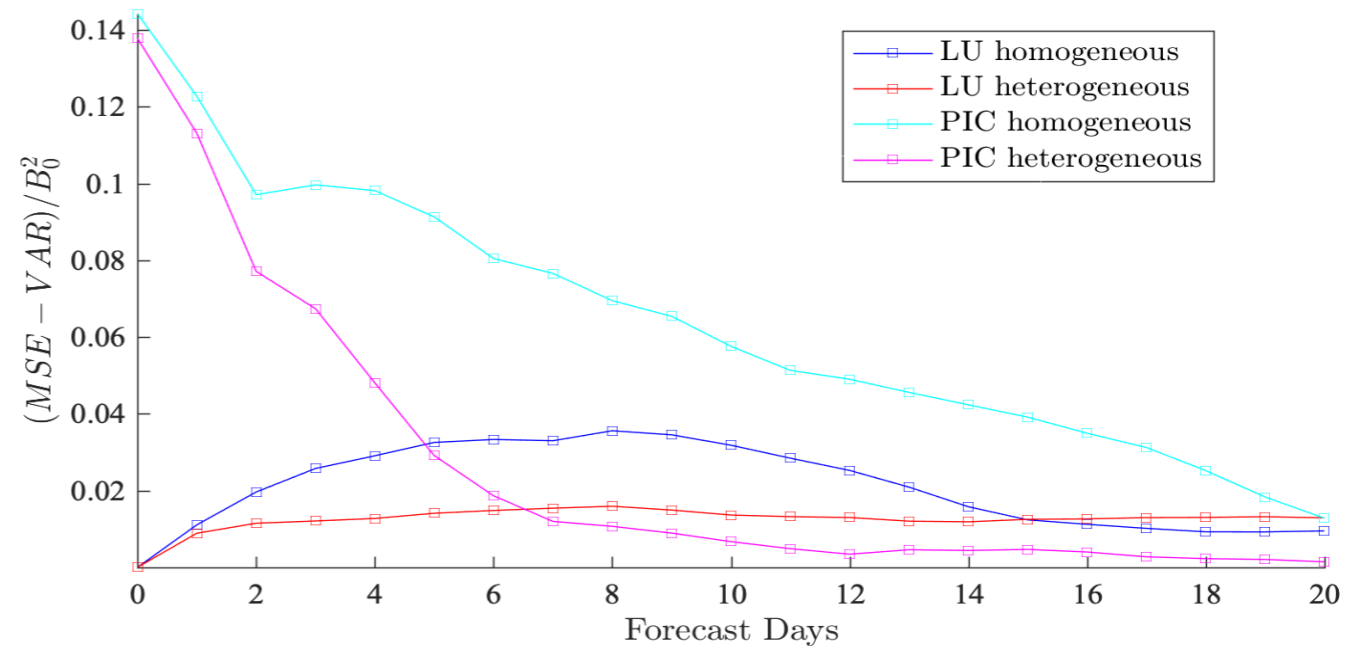
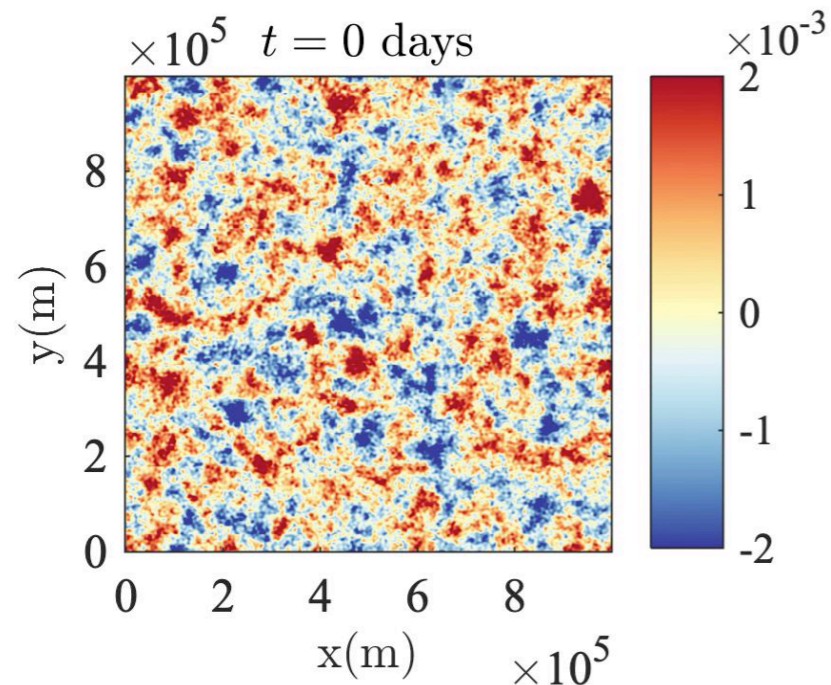
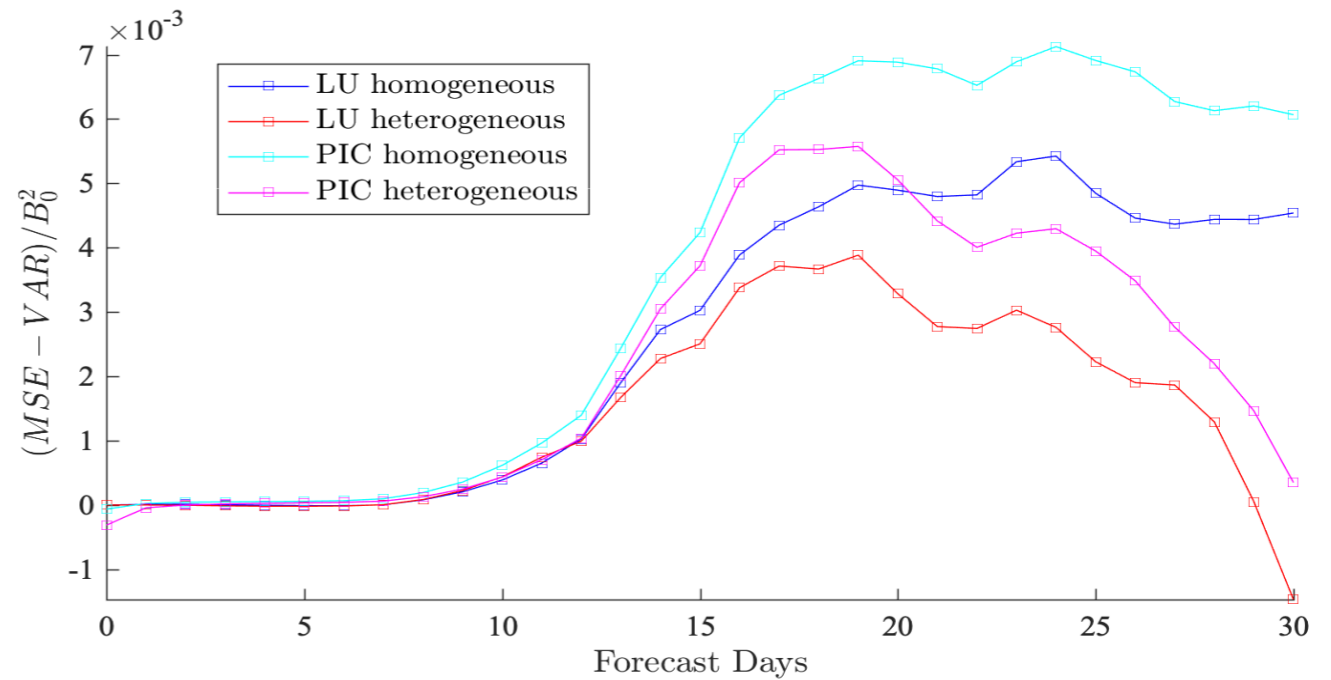
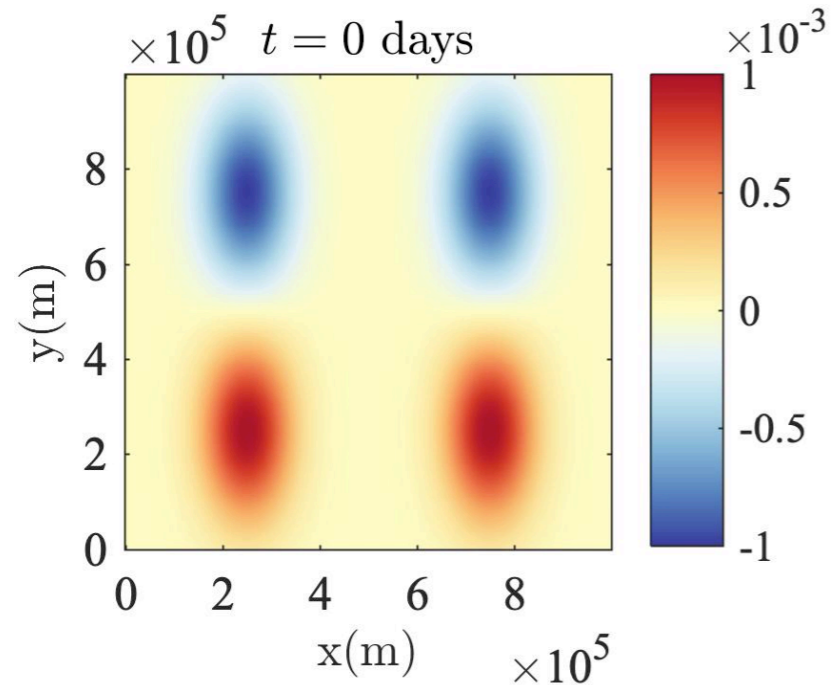
Total Energy

Ensemble forecasting verification of SQG_{MU}

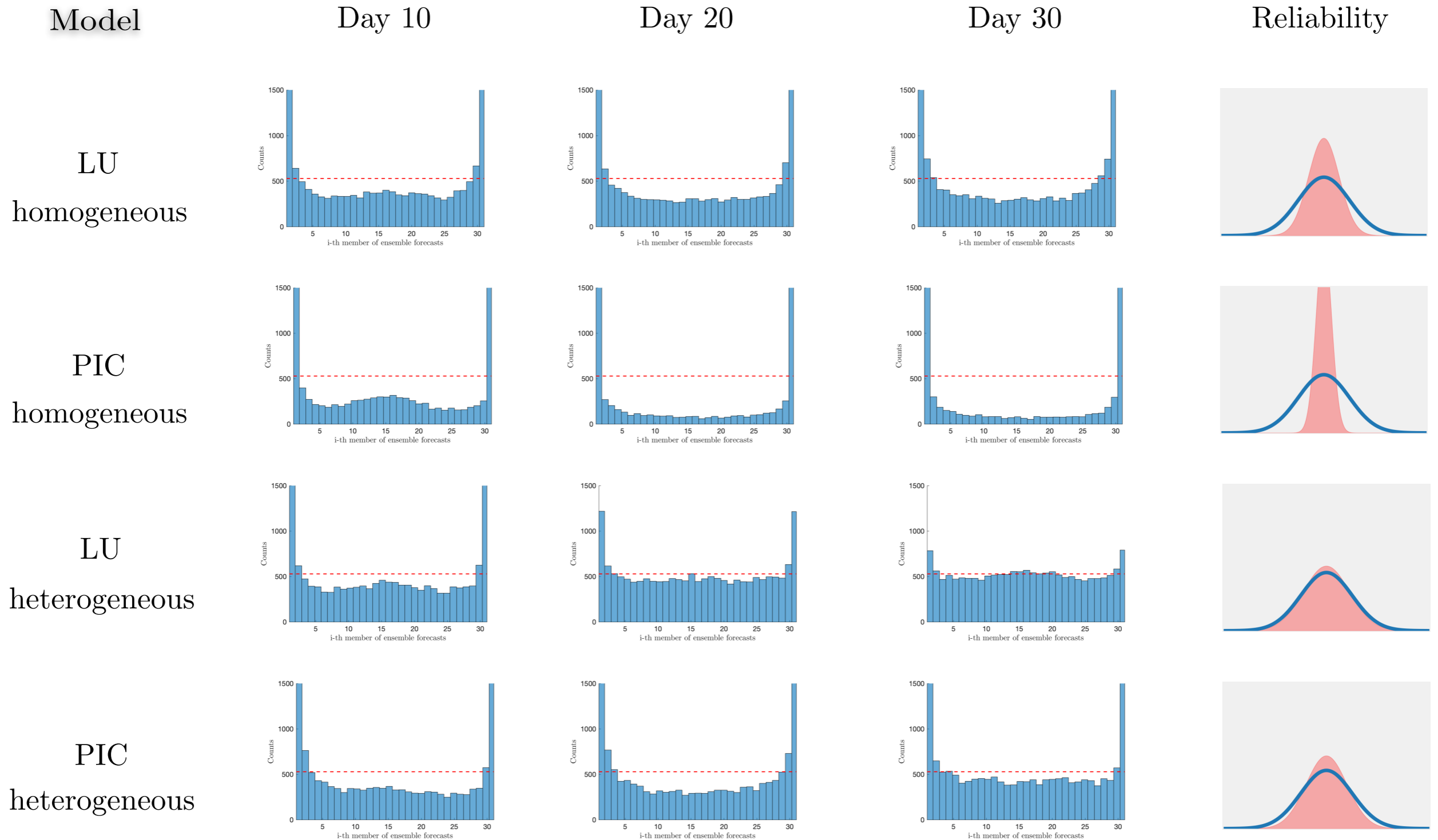
- Illustration of ensemble prediction systems :



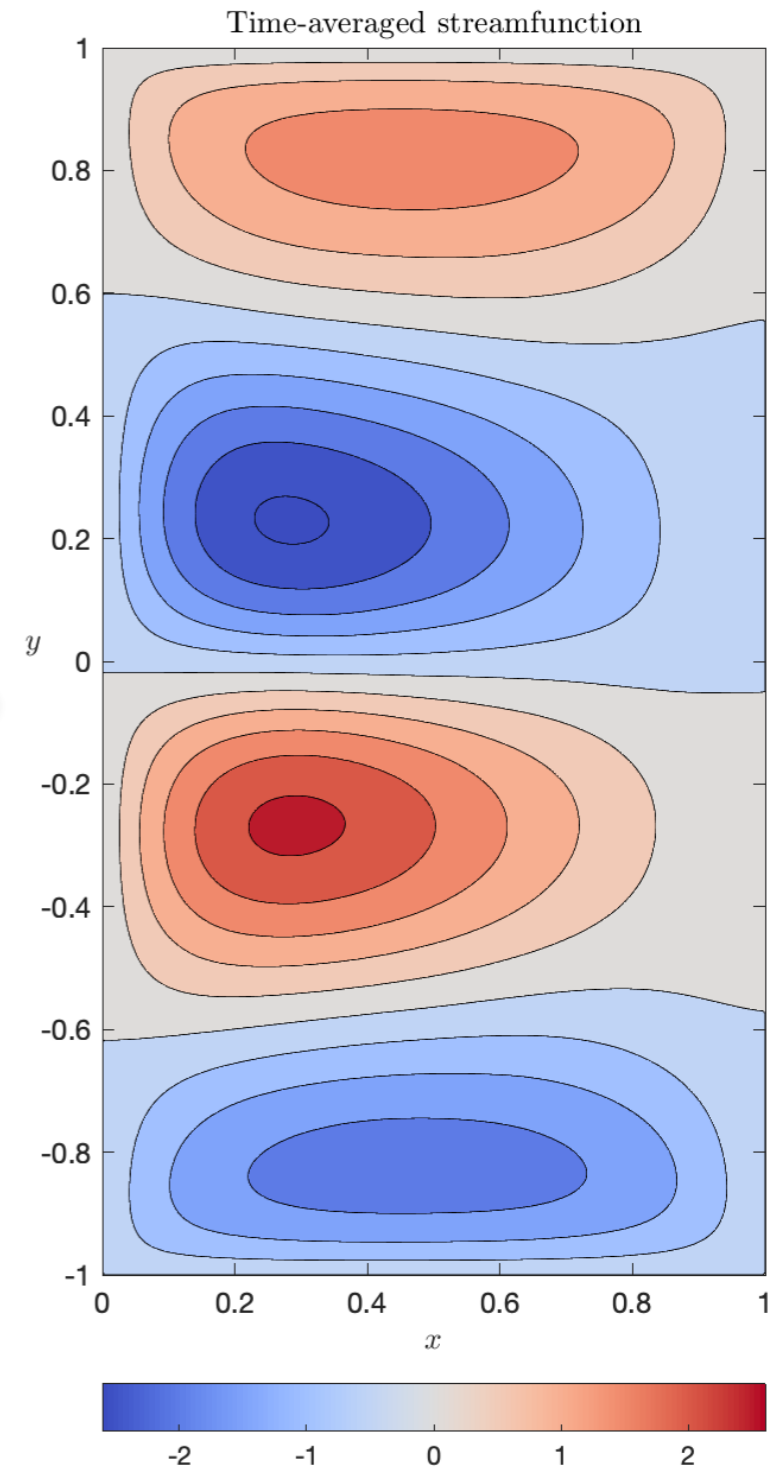
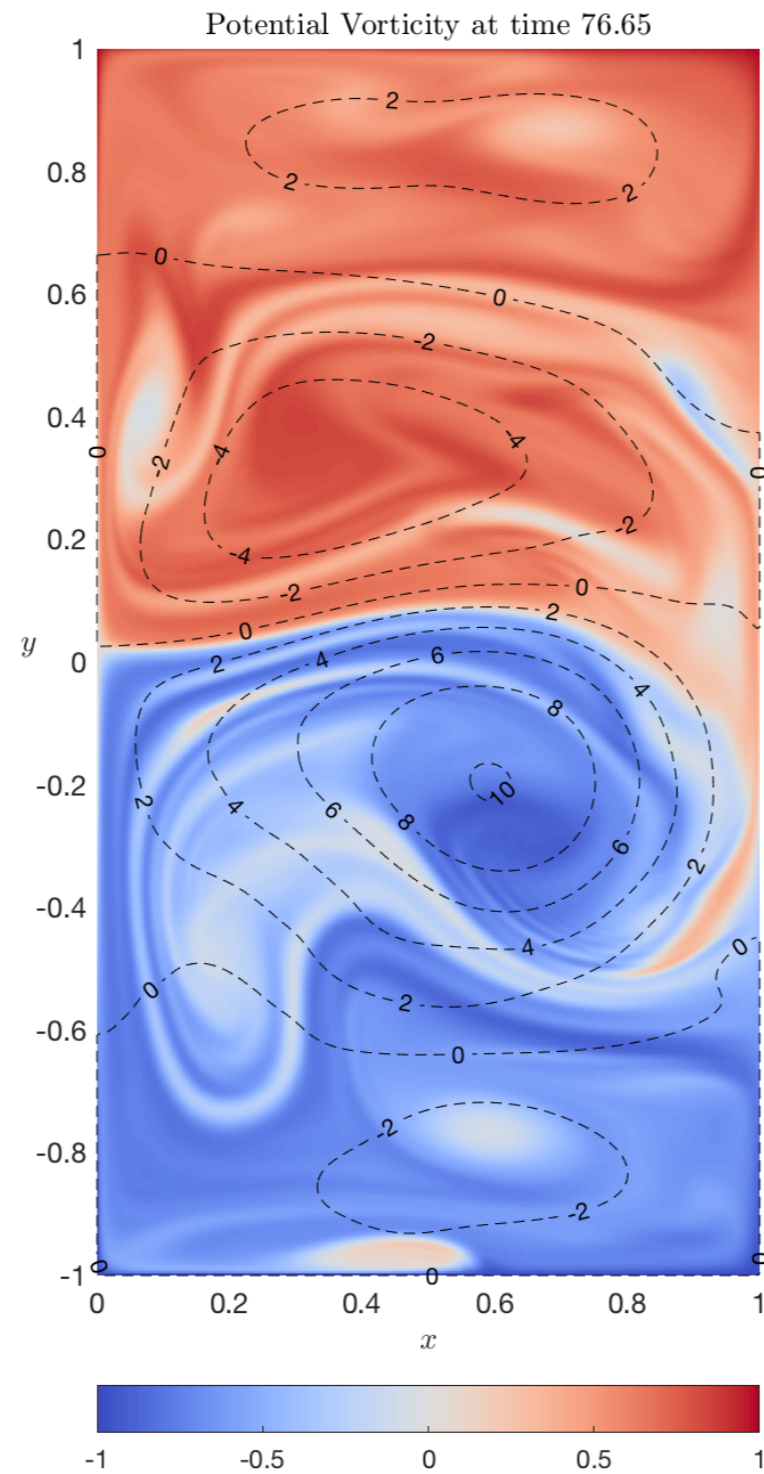
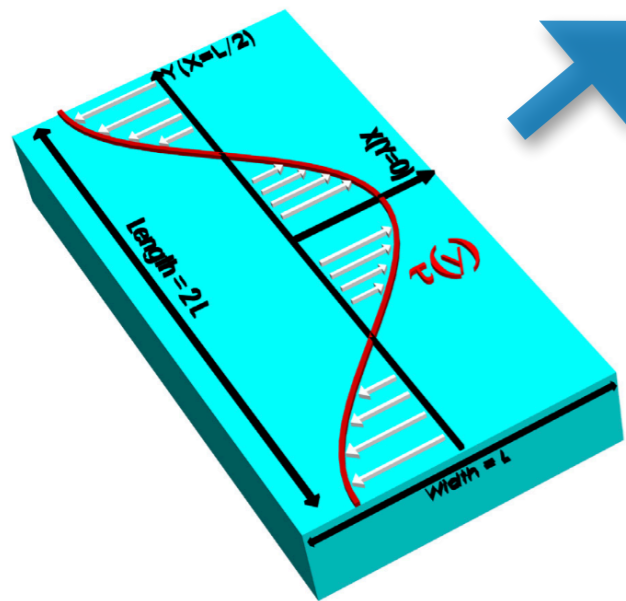
Reliability – Ensemble Spread match MSE of the Ensemble Mean :



Reliability — Ranked Histogram of Ensemble Members :

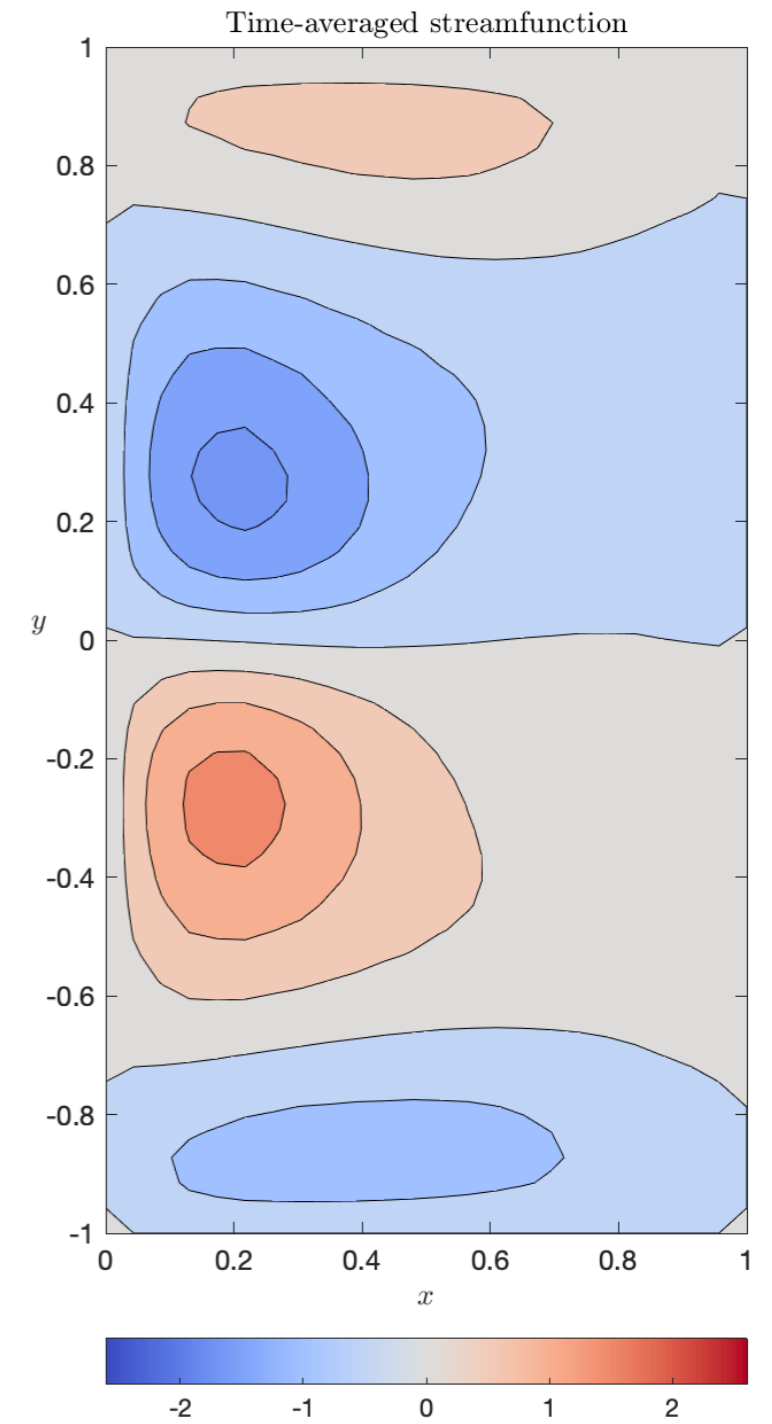
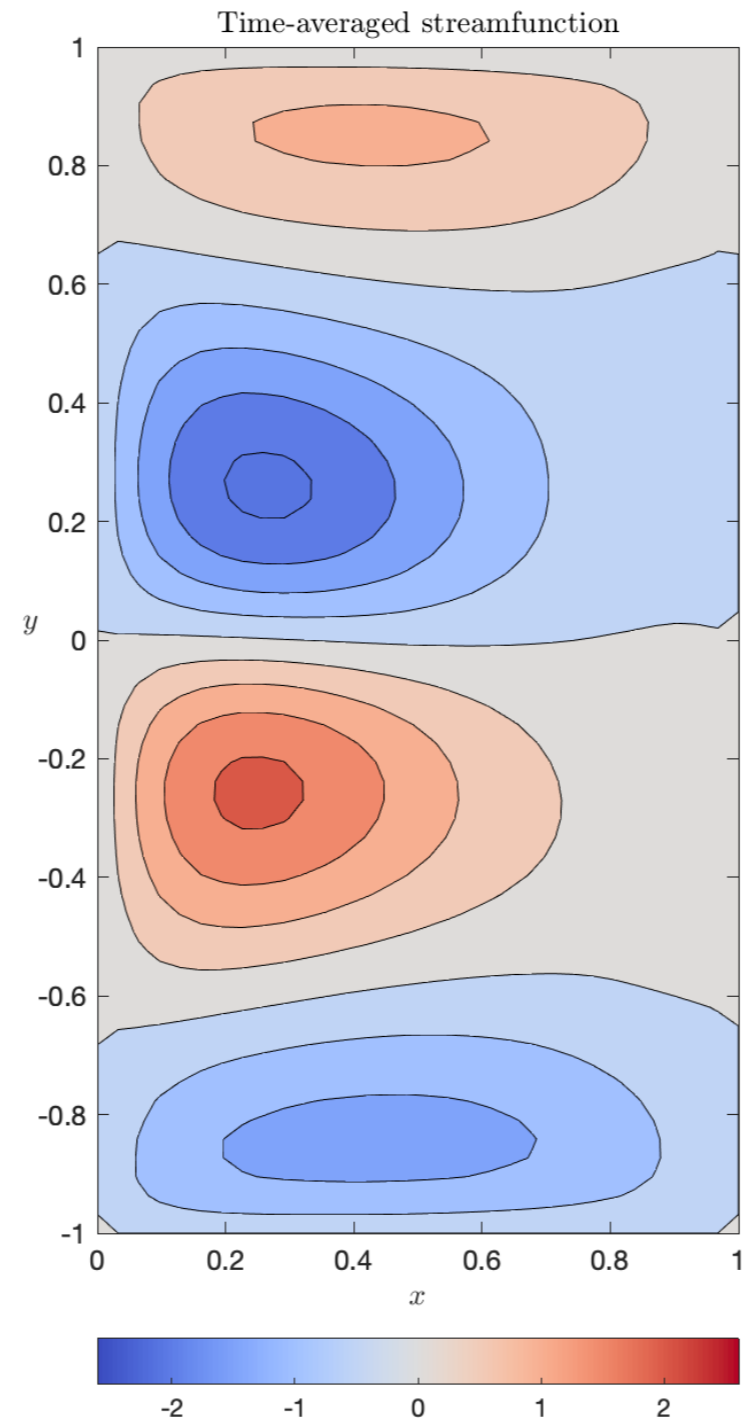
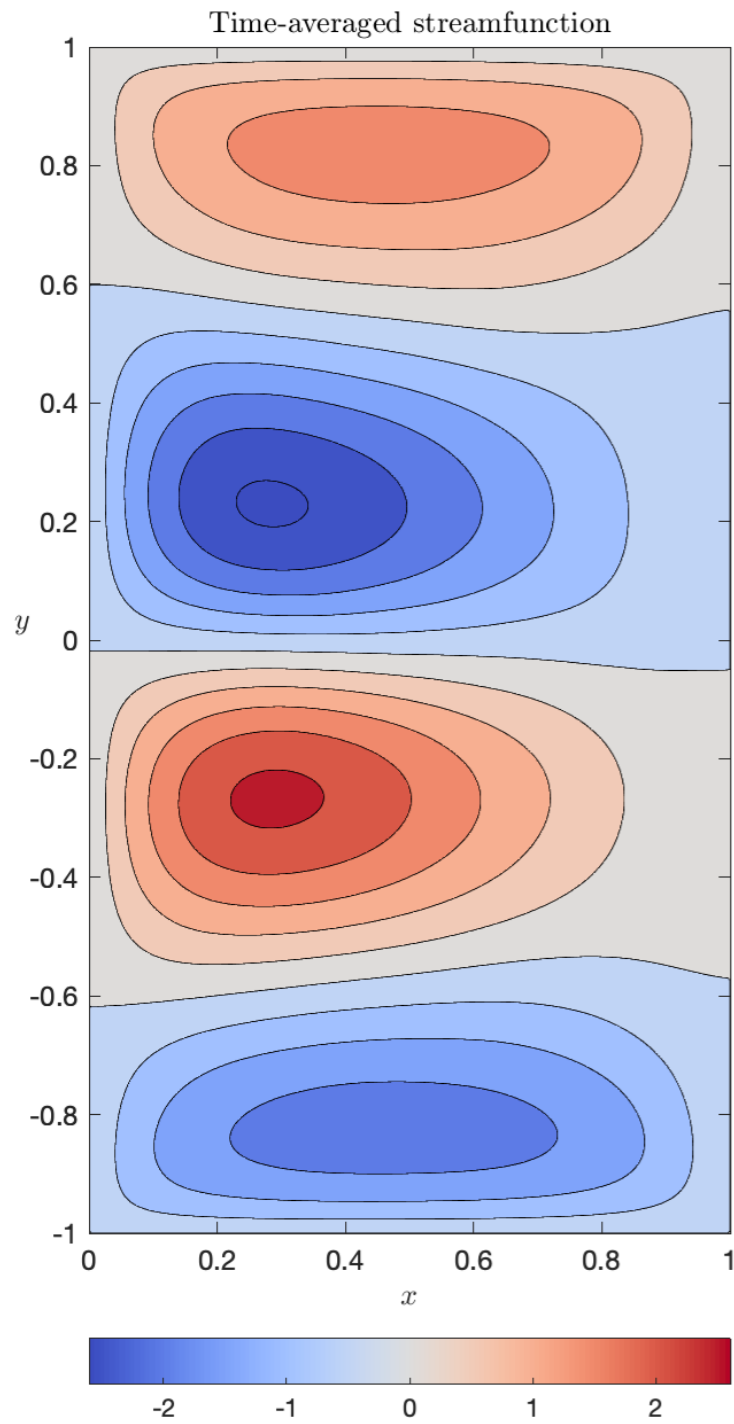


Time-statistics of wind-driven circulation



DNS 256×512 with $Re = 450$ and $Ro = 0.0016$

Time-statistics of wind-driven circulation



Truth



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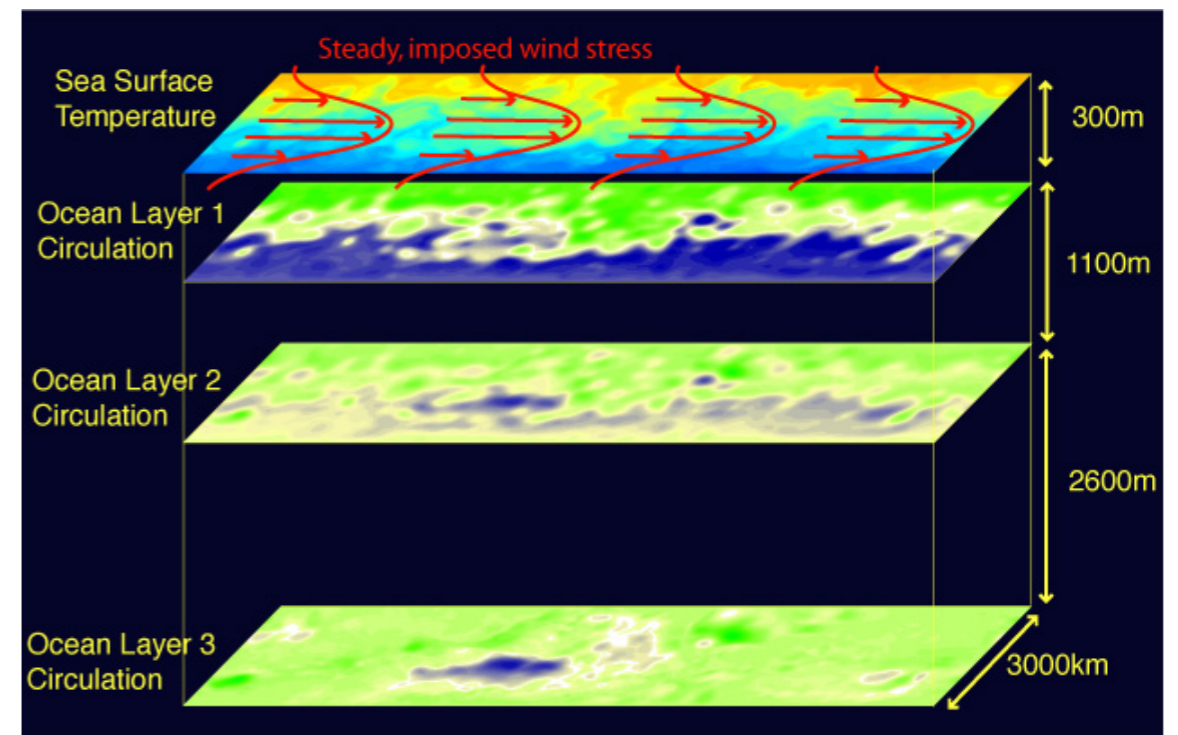
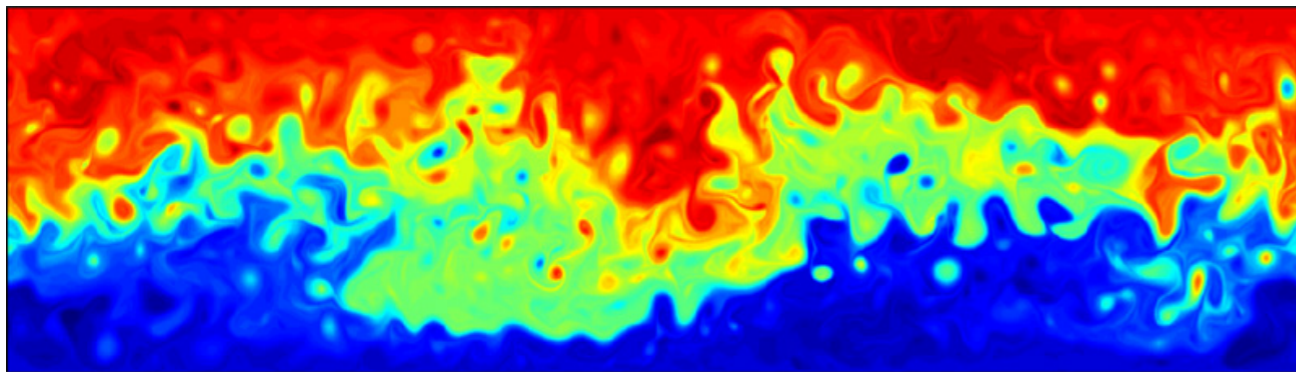
3 Summary

- **A consistent stochastic model :**
 - ➔ **Physical conservation laws satisfied**
 - ➔ **Better ensemble spread represented**
 - ➔ **Time-averaged profil well described on coarse mesh**

- **Future works :**

- ➔ **Multi-layers QG model under LU**

(Q-GCM Projects : Hogg et al. 2003)



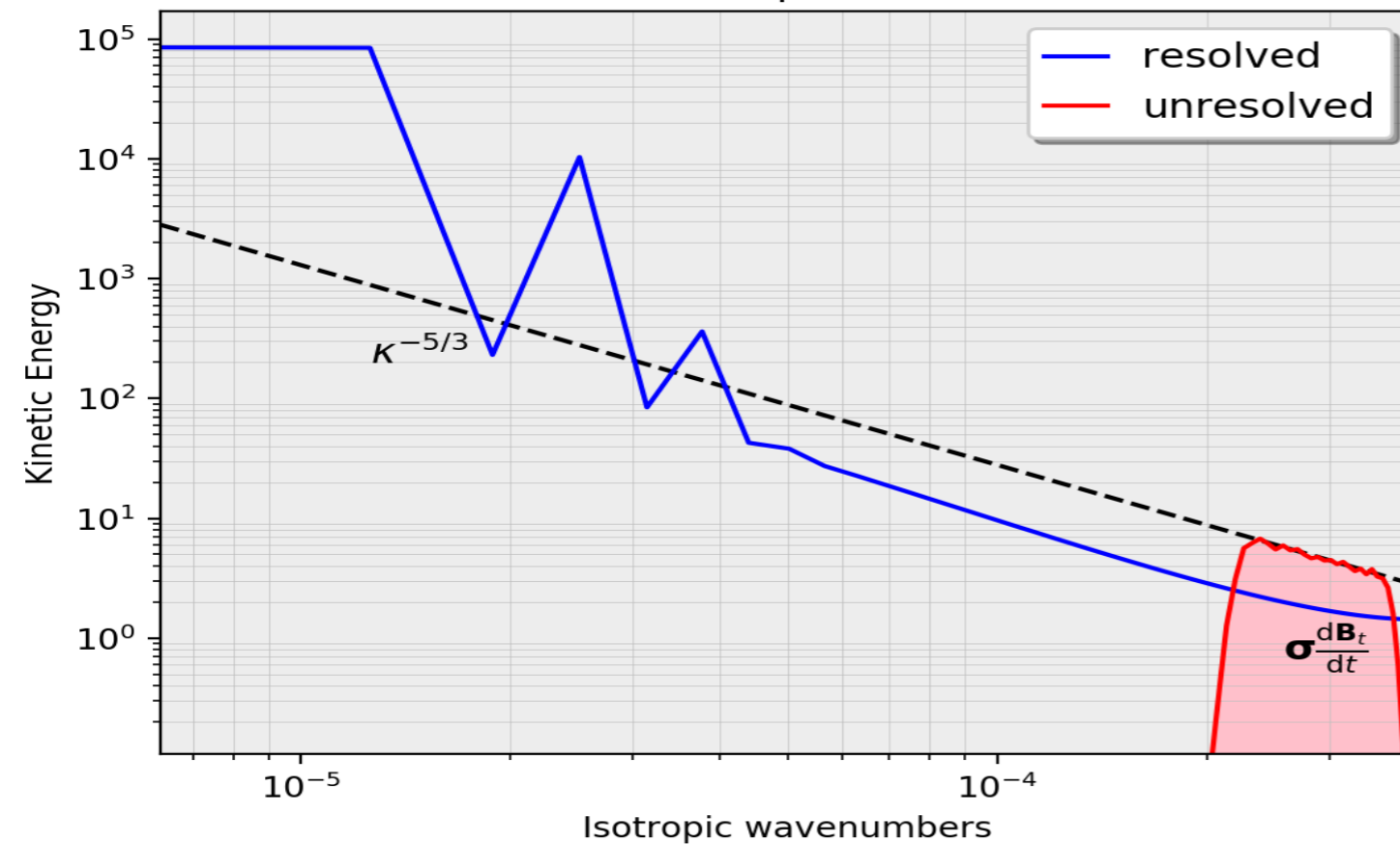
- ➔ **Data Assimilation with Particle Filters**

(Cotter et al. 2018)

Thanks for Your Attention !

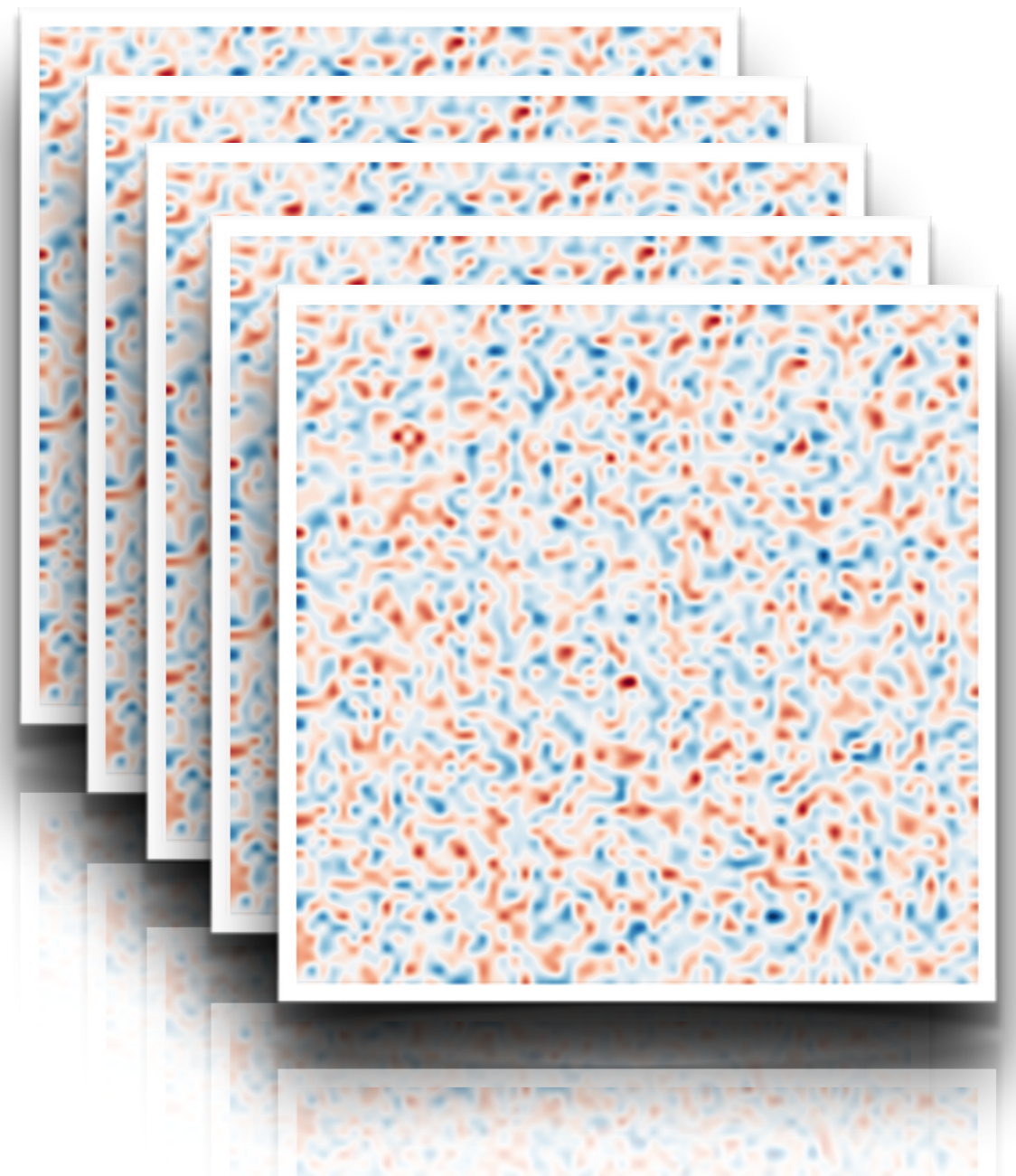
Homogeneous parameterization

Initial Spectrum

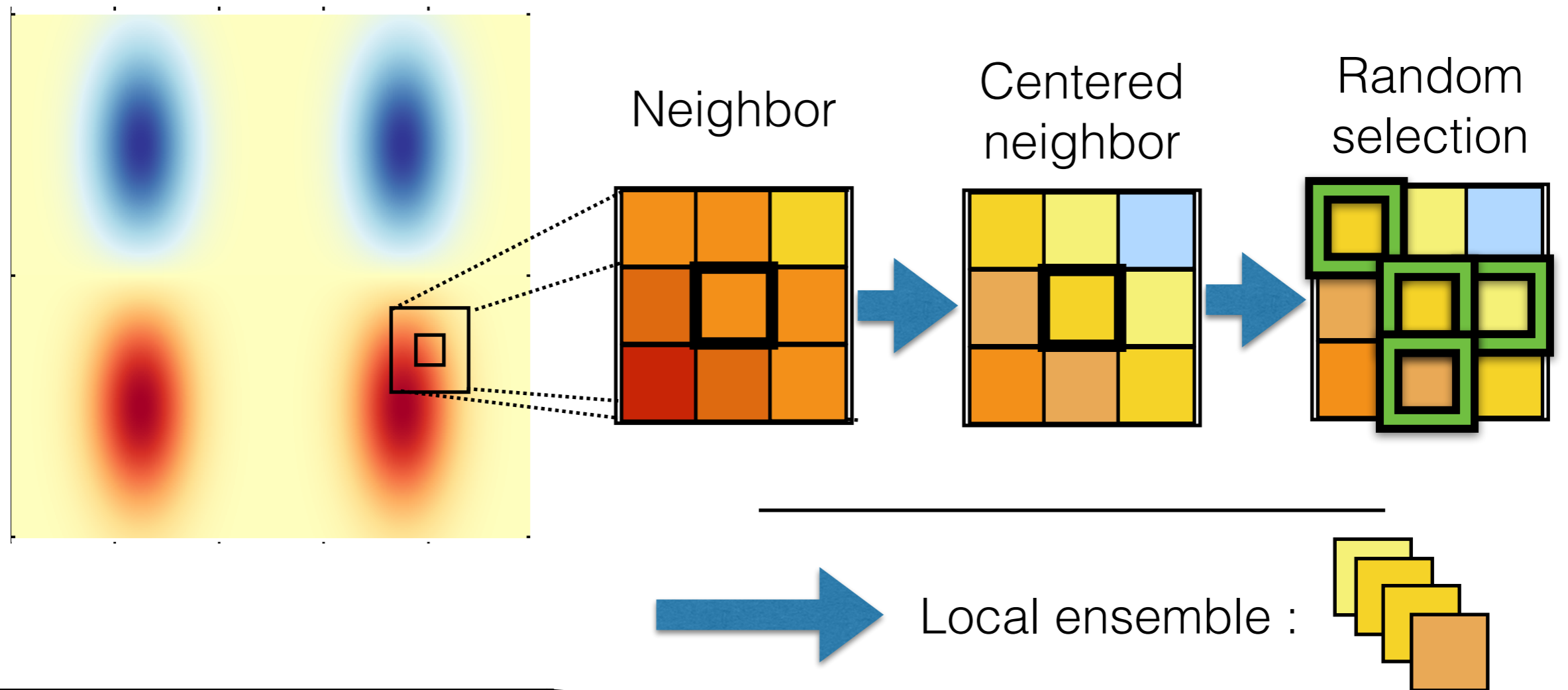


low-pass band filter \star **white noise**

Ensemble of random stream functions



Heterogeneous parameterization



Global ensemble

