



HAL
open science

State and parameter estimation in linear systems with delays

Qinghua Zhang

► **To cite this version:**

| Qinghua Zhang. State and parameter estimation in linear systems with delays. 2020. hal-02558797

HAL Id: hal-02558797

<https://inria.hal.science/hal-02558797>

Preprint submitted on 29 Apr 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

State and parameter estimation in linear systems with delays

Qinghua Zhang (Email: Qinghua.Zhang@inria.fr)

August 2, 2017

Consider systems in the form of

$$\dot{x}(t) = A(t)x(t) + B(t)x(t - \tau) + \Phi(t)\theta \quad (1a)$$

$$y(t) = C(t)x(t) + D(t)x(t - \tau) \quad (1b)$$

where θ is an *unknown* constant vector. The matrices $A(t), B(t), C(t), D(t), \Phi(t)$ are known, bounded and piecewise continuous. The delay τ is known.

Remark: $\Phi(t)\theta$ can be generalized to $\Phi_1(t)\theta_1 + \Phi_2(t - \tau)\theta_2$: let $\Phi(t) = [\Phi_1(t) \ \Phi_2(t - \tau)]$ and $\theta^T = [\theta_1^T \ \theta_2^T]$ so that $\Phi(t)\theta = \Phi_1(t)\theta_1 + \Phi_2(t - \tau)\theta_2$. More generally, it is possible to consider

$$\Phi(t)\theta = \Phi_1(t - \tau_1)\theta_1 + \dots + \Phi_m(t - \tau_m)\theta_m,$$

provided the delays are all known.

Assumption 1. In the particular case $\Phi(t)\theta \equiv 0$, a state observer is available in the form of

$$\dot{\hat{x}}_0(t) = A(t)\hat{x}_0(t) + B(t)\hat{x}_0(t - \tau) + L(t)[y(t) - C(t)\hat{x}_0(t) - D(t)\hat{x}_0(t - \tau)], \quad (2)$$

or in other words, the error dynamics

$$\dot{\eta}(t) = [A(t) - L(t)C(t)]\eta(t) + [B(t) - L(t)D(t)]\eta(t - \tau), \quad (3)$$

is such that

$$\lim_{t \rightarrow +\infty} \eta(t) = 0 \text{ exponentially.} \quad (4)$$

□

The adaptive observer for system (1):

$$\dot{\Upsilon}(t) = [A(t) - L(t)C(t)]\Upsilon(t) + [B(t) - L(t)D(t)]\Upsilon(t - \tau) + \Phi(t) \quad (5a)$$

$$\begin{aligned} \dot{\hat{x}}(t) = & A(t)\hat{x}(t) + B(t)\hat{x}(t - \tau) + \Phi(t)\hat{\theta}(t) + L(t)[y(t) - C(t)\hat{x}(t) + D(t)\hat{x}(t - \tau)] \\ & + \Upsilon(t)\hat{\theta}(t) + [B(t) - L(t)D(t)]\Upsilon(t - \tau)[\hat{\theta}(t) - \hat{\theta}(t - \tau)] \end{aligned} \quad (5b)$$

$$\dot{\hat{\theta}}(t) = \Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t - \tau)]^T \left\{ y(t) - C(t)\hat{x}(t) + D(t)\hat{x}(t - \tau) - D(t)\Upsilon(t - \tau)[\hat{\theta}(t) - \hat{\theta}(t - \tau)] \right\} \quad (5c)$$

where Γ is either a constant positive definite matrix or a recursively computed time varying matrix.

The **red terms** are unusual. They will be helpful for error dynamics analysis.

Define the estimation errors:

$$\tilde{x}(t) \triangleq x(t) - \hat{x}(t) \quad (6)$$

$$\tilde{\theta}(t) \triangleq \theta - \hat{\theta}(t). \quad (7)$$

Then (recall that θ is a constant vector)

$$\begin{aligned}\dot{\tilde{x}}(t) &= A(t)\tilde{x}(t) + B(t)\tilde{x}(t - \tau) + \Phi(t)\tilde{\theta} - L(t)C(t)\tilde{x}(t) - L(t)D(t)\tilde{x}(t - \tau) \\ &\quad + \Upsilon(t)\dot{\tilde{\theta}}(t) + [B(t) - L(t)D(t)]\Upsilon(t - \tau)[\tilde{\theta}(t) - \tilde{\theta}(t - \tau)].\end{aligned}\quad (8)$$

Now define

$$\eta(t) \triangleq \tilde{x}(t) - \Upsilon(t)\tilde{\theta}(t), \quad (9)$$

then

$$\begin{aligned}\dot{\eta}(t) &= [A(t) - L(t)C(t)]\eta(t) + [B(t) - L(t)D(t)]\eta(t - \tau) \\ &\quad + [A(t) - L(t)C(t)]\Upsilon(t)\tilde{\theta}(t) + [B(t) - L(t)D(t)]\Upsilon(t - \tau)\tilde{\theta}(t - \tau) \\ &\quad + \Phi(t)\tilde{\theta}(t) + \Upsilon(t)\dot{\tilde{\theta}}(t) \\ &\quad + [B(t) - L(t)D(t)]\Upsilon(t - \tau)[\tilde{\theta}(t) - \tilde{\theta}(t - \tau)] \\ &\quad - \dot{\Upsilon}(t)\tilde{\theta}(t) - \Upsilon(t)\dot{\tilde{\theta}}(t).\end{aligned}\quad (10)$$

In this last equation, the first occurrence of $[B(t) - L(t)D(t)]\Upsilon(t - \tau)\tilde{\theta}(t - \tau)$ cancels out with a later term, so does $\Upsilon(t)\dot{\tilde{\theta}}(t)$. Hence

$$\begin{aligned}\dot{\eta}(t) &= [A(t) - L(t)C(t)]\eta(t) + [B(t) - L(t)D(t)]\eta(t - \tau) \\ &\quad + \{[A(t) - L(t)C(t)]\Upsilon(t) + [B(t) - L(t)D(t)]\Upsilon(t - \tau) + \Phi(t) - \dot{\Upsilon}(t)\}\tilde{\theta}(t),\end{aligned}\quad (11)$$

which is then simplified, by taking into account (5a), to

$$\dot{\eta}(t) = [A(t) - L(t)C(t)]\eta(t) + [B(t) - L(t)D(t)]\eta(t - \tau). \quad (12)$$

According to Assumption 1, $\eta(t) \rightarrow 0$.

Now consider the error dynamics $\tilde{\theta}(t)$. It follows from (5c) that

$$\begin{aligned}\dot{\tilde{\theta}}(t) &= -\Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t - \tau)]^T \left\{ C(t)\tilde{x}(t) + D(t)\tilde{x}(t - \tau) - D(t)\Upsilon(t - \tau)[\hat{\theta}(t) - \hat{\theta}(t - \tau)] \right\} \\ &= -\Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t - \tau)]^T [C(t)\eta(t) + D(t)\eta(t - \tau)] \\ &\quad - \Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t - \tau)]^T \left\{ C(t)\Upsilon(t)\tilde{\theta}(t) + D(t)\Upsilon(t - \tau)\tilde{\theta}(t - \tau) + D(t)\Upsilon(t - \tau)[\tilde{\theta}(t) - \tilde{\theta}(t - \tau)] \right\} \\ &= -\Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t - \tau)]^T [C(t)\eta(t) + D(t)\eta(t - \tau)] \\ &\quad - \Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t - \tau)]^T [C(t)\Upsilon(t) + D(t)\Upsilon(t - \tau)]\tilde{\theta}(t).\end{aligned}\quad (13)$$

$$\begin{aligned}&= -\Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t - \tau)]^T [C(t)\eta(t) + D(t)\eta(t - \tau)] \\ &\quad - \Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t - \tau)]^T [C(t)\Upsilon(t) + D(t)\Upsilon(t - \tau)]\tilde{\theta}(t).\end{aligned}\quad (14)$$

The homogeneous part of this error dynamics (corresponding to $\eta(t) \equiv 0$) is stable under the following assumption.

Assumption 2. There exist $T > 0$ and $\alpha > 0$ such that, for all $t \geq t_0$,

$$\int_t^{t+T} [C(s)\Upsilon(s) + D(s)\Upsilon(s - \tau)][C(s)\Upsilon(s) + D(s)\Upsilon(s - \tau)]^T ds \geq \alpha I. \quad (15)$$

□

The remaining analysis is then similar to already published cases.